"AN INVESTIGATION INTO THE ROLE THAT KNOWLEDGE OF THE NATURE OF MATHEMATICS AND OTHER FACTORS PLAY IN DETERMINING MOTIVATION FOR FURTHER STUDY OF MATHEMATICS AFTER STD 7 IN SELECTED PROVINCIAL SECONDARY SCHOOLS."

by

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CONTENTS

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ABSTRACT

This study involves an investigation into reasons why pupils continue with mathematics after Std 7. The sample, consisting of Std 8 and Std 10 mathematics pupils at six academic, English-medium high schools in Port Elizabeth, had to complete a short questionnaire during the third term of 1985. It was found that most pupils continue with mathematics because of requirements for certain post-matric courses, to improve their career options and for other reasons classified in this investigation as "extrinsic", i.e. reasons that are not directly related to the nature of mathematics.

An attempt is made to determine the extent of the pupils' knowledge of the nature of mathematics. Questions relating to the nature of mathematics are poorly answered by almost all of the pupils, thereby suggesting that this aspect of mathematics is sadly neglected in mathematics courses. The suggestion is made that this lack of knowledge of the nature of mathematics is linked to pupils' decisions to continue with the subject. The conclusion is drawn that unless more attention is given to conveying to pupils something of the nature of mathematics, there will be an increasing proportion of pupils who choose to continue with mathematics for the "wrong" reasons. This, in turn, has possible negative implications for mathematics education in general.

Attention is also paid to differences re the above in respect of groupings based on standard, mathematical grade, sex, mathematical achievement and type of schooling. Although no conclusive findings are made other than with regard to specific questions, enough evidence of differences in certain of the groupings is produced to warrant further investigation.

INTRODUCTION

"I hate maths" is a statement many mathematics teachers have often heard pupils utter. As mathematics is commonly accepted as being a difficult subject for many people, this statement is perhaps not all that surprising. It is, however, disturbing to have heard it uttered by so many pupils who could previously have dropped the subject if they so wished.

Mathematics is a compulsory part of the normal curriculum up until the end of Std 7 in schools under the control of the Cape Education Department. Pupils are then free to stop mathematics although there are a few schools at which mathematics is compulsory right up until Std 10.

If pupils "choose" to continue with mathematics, why do they continue with a subject that so many of them apparently do not enjoy or even hate? One possibility is that no viable alternative exists. The child therefore continues with the lesser of two or more "evils", depending on the options available in terms of time-table exigincies. (Schools divide subjects into different groups or levels, with pupils having to choose a subject from each level.)

Notwithstanding the above, it is my suspicion that most pupils choose to continue with mathematics for extrinsic rather than intrinsic reasons, i.e. because of social pressures and mercenary reasons rather than a love for mathematics itself. I believe that there is an overemphasis on continuing with mathematics because of its value rather than because of its nature. As a result pupils have little or no knowledge of the nature of mathematics. In deciding whether or not to continue with mathematics they thus have little idea of <u>what</u> they are continuing.

Furthermore, I suspect that there may be correlations between the above-mentioned reasons and particular groups of pupils depending upon their sex, the standard in which they are, the grade at which they are tackling the subject, the type of school that they attend and their achievement in mathematics.

I previously used the word "choose" in inverted commas. It is debatable just how much say a Std 7 pupil actually has in his or her choice of subjects for Std 8. The pupil might state that parents had little or no say in the matter but this would be difficult to prove. One would indeed hope that in the light of the tender age of most Std 7's and the magnitude of the decision, parents are in fact involved in the choice. I am not so much concerned with the choice process as with the reason why the pupil is continuing with mathematics. If the pupil continues because his or her parents, or anyone else, said that the pupil must, this reason will be classified as being an extrinsic one.

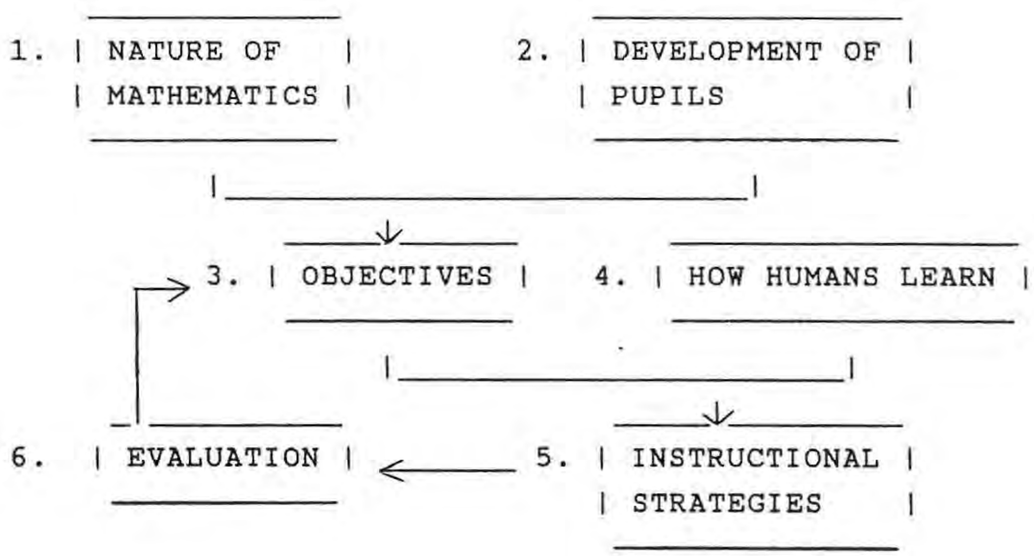
The purpose of this study, which took the form of a questionnaire administered to Std 8 and Std 10 pupils at six English-medium high schools in Port Elizabeth, is thus to investigate the factors that play a role in determining motivation for further study of mathematics after Std 7. By analysing the responses to the questionnaires, I hope to be able to show that extrinsic factors are dominant and that, linked to this, pupils have little or no concept of the nature of mathematics. I also wish to examine the possibility of factors having varying importance within the five groups of pupils previously mentioned.

The main implication of this study is that if pupils continue with mathematics for extrinsic reasons, this could be affecting the teaching of mathematics in a very negative manner. Mathematics teachers have very little motive, other than self-pride, to improve the quality of their teaching or make their subject as interesting as possible if pupils continue with mathematics for the "wrong" reasons. There is surely the danger that pupils are going to continue with mathematics irrespective of the quality of the teaching, thereby indirectly promoting its deterioration. If teachers were to place greater emphasis on conveying to pupils the nature of mathematics, greater appreciation for the subject may ensue. This appreciation may make their decisions to continue with mathematics better founded.

CHAPTER 1

THE NATURE OF MATHEMATICS

The learning of mathematics is a complicated process as is well illustrated in the following flow-chart of Farrell and Farmer:



(Farrell and Farmer 1980, p1)

The interdependence of these six blocks should be clear. Most student teachers cover blocks 2, 4, 5 and 6 in fair detail during teacher training courses but I believe that little or no attention is given to blocks 1 and 3. Students are taught to set objectives for specific lessons and know the objectives for various sections of the syllabus, yet how many of them know the general aims and objectives of mathematics as stated in the beginning of a syllabus? Furthermore, how many prospective as well as experienced teachers have a concept of what the nature of mathematics is? Failure to find answers to these questions will, in terms of the previous diagram, have negative implications for the whole teaching process. As is stated by M.Baron:

> "Unless we give some careful thought to such questions our teaching is likely to remain at a mediocre level and our day-to-day procedures will be based on tradition, convention and imitation."
>
> (Baron in Chapman 1972, p21-22)

Having an idea about the nature of mathematics is necessary before objectives can be set, hence my concern with this aspect of the teaching process.

When considering the nature of mathematics, I am basically trying to determine what mathematics _is_. A large and varied body of thought has grown from earliest times purporting to answer this question. A danger exists of confusing the nature of mathematics with content. The list of the various topics included in the field of mathematics adds light to the nature of the subject but mathematics is more than just mere topics.

1.1. Mathematics: Product or Process?

To the ancient Greeks, mathematics was as much a tool as a field of knowledge. Through the ages we have corrupted this idea and thus the balance between process and product has often been lost. Mathematics is thought by some to be a study of general abstract systems. Those holding this view consider useful mathematics such as the theory of relativity to be inferior and debased, i.e. emphasis is placed on mathematics as a product. Others regard mathematics as an activity of the mind involving actions as well as objects, rather than a body of knowledge to be passed on from teacher to pupil, i.e. the emphasis is on mathematics as a process.

It would probably be true to say that the modern view has tended much more to seeing mathematics as an activity to be experienced rather than a body of knowledge to be acquired. According to this view the processes of abstraction, representation, generalisation and proof are more important than the particular ideas and skills contained in mathematics. Baron (in Chapman 1972) makes the point that it is always easier to say what we can do with a piece of mathematics than to say what it is, while G. Williams states emphatically that:

> "mathematics is what mathematicians do".
>
> (Williams 1983, p170)

Many writers have considered this debate in varying detail, e.g. Courant (1947) and van den Berg (1977). The modern emphasis on the process aspect of mathematics unfortunately seems to cast more light upon the nature of mathematisation than upon the nature of mathematics.

1.2. Mathematics: Invention or Discovery?

Another important debate involved in discussions about the nature of mathematics is that of whether mathematics is an invention or a discovery. Due to the apparent close relationship that mathematics has with the external world, some say that it has an existence of its own and has merely been discovered by man. This view is known as Platonism. Formalists such as Einstein and Klein, on the other hand, believe that mathematics is manmade. Einstein reckoned mathematics to be:

> "a free invention of the human intellect".
> (in Lamon 1972, p40)

In an attempt to bring these two views together, Davis and Hersh hold that discovery and invention are merely opposite sides of the same coin, saying that:

> "we invent ideal objects and then try to discover the facts about them".
> (in de Villiers 1984, p42)

1.3. Deductive vs. Inductive Nature of Mathematics

A third conflict worth examining is that of whether mathematics has a deductive or inductive nature. Once again a balance between the two views is probably desirable. While many agree that mathematics in the making is an experimental, inductive science, they also agree that it is presented as a systematic, deductive science. The overemphasis by many on the deductive nature of mathematics is because most people, although they think and discover facts inductively, tend to shroud this in the language of deductive thought (Mercer in Chapman 1972, p94).

A danger of this overemphasis is that, as Freudental pointed out, having logical structures dished up to young people contradicts their natural inclinations, while according to Wheeler, mathematical thinking needs freedom for thought and freedom to make conjectures (in van den Berg 1977). Another danger, to quote Morris Kline, is that:

> "the deductive presentation of mathematics...... leads students to believe that mathematics is created by geniuses who start with axioms and reason directly and flawlessly to theorems".
>
> (in de Villiers 1984)

This could lead to feelings of inferiority on the part of the student.

It thus appears that while mathematics is presented to us as a rigorous proof structure having a systematic, deductive and abstract nature, mathematics in the making is experimental, inductive and knows no laws (Baron in Chapman 1972, p35). It is therefore important that we acknowledge both the deductive and inductive nature of mathematics.

1.4. Axiomatic Nature of Mathematics

Most discussions about the nature of mathematics involve the use of the word "axiom", as used by Euclid in his book "The Elements". What is an axiom? The classical belief was that:

> "the axioms of Euclidian geometry were meant to be self-evident truths derived from physical experience".
>
> (Attiyah in de Villiers 1984, p39)

The modern belief is summed up by Hilton when he says that:

> "axioms are postulates about undefined entities and not self-evident truths about actual objects".
> (in de Villiers 1984, p40)

J.Mercer expresses the same thought differently by saying:

> "in mathematics we define things to mean what we want them to mean".
> (in Chapman 1972, p104)

The link between these contrasting views and the creation/discovery dichotomy should be obvious.

1.5. <u>Definitions of Mathematics</u>

The aforegoing are some of the issues involved in discussions about the nature of mathematics, issues which are not independent of each other. Various mathematicians have gone a step further and tried to define mathematics. Examples of these definitions are as follows:

> "Mathematics really is mostly deductions from axiom systems."
> (C. Kilmister in Chapman 1972, p10)

> "Mathematics is the study of .... numbers and their relationships."
> (de Villiers 1984, p10)

> "Mathematics is the classification and study of all possible patterns."
> (Sawyer 1955, p12)

Bertrand Russell is perhaps hedging a little when he says the following:

> "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we say is true."
>
> (Baron in Chapman 1972, p31)

Literature reveals that it is very difficult to define mathematics. Every attempted definition, such as the preceding, can and has been criticised from various angles. G. Hann (1972), in considering this difficulty, argues that there is no reason to suppose that mathematics can have only one identifying description. He uses the analogy of trying to define a steam locomotive, pointing out that various people will have different definitions of the locomotive, depending on their field of interest. For example, an engineer would probably define the locomotive in terms of its mechanical functioning. A sociologist may base his definition upon communication and economic factors. The point is that, despite the differences in definition, the steam locomotive has not changed in form or function.

1.6. The Importance of the Nature of Mathematics

One might ask what the point of trying to determine the nature of mathematics is when the previous sections point to so much uncertainty and so many conflicting views. Opinions about the nature of mathematics, ranging from those of Pythagoras to those of modern day mathematical philosophers, reveal that in this regard, according to Kasner and Newman:

> "it is easier to be clever than clear".
>
> (Baron in Chapman 1972, p21)

There does not seem to be one correct answer to the question: "What is mathematics?". Are we thus to ignore something that is so integral a part of the whole teaching process? (cf. the flowchart at the beginning of this chapter.)

Although questions regarding the nature of mathematics are not normally explicitly discussed in class, they must surely influence the teaching of the subject. It is not important whether a pupil believes that mathematics is invented or discovered, deductive or inductive, a product or a process. It is also not important what specific definition a pupil has for mathematics. It is, however, important that teachers encourage pupils to consider these and other issues and thus come to some understanding of the nature of mathematics, albeit only a small understanding. This will help them to place the various aspects of their mathematics syllabus in perspective, in accordance with this understanding. Perhaps pupils will then be in a better position to <u>know</u> mathematics as well as be able to <u>do</u> mathematics. They may also be in a better position to decide whether or not to continue with mathematics after Std 7 because they should have a better idea of what they are letting themselves in for.

There is little evidence of research having been done into pupils' views about the nature of mathematics. Some of the aforementioned issues have been touched upon by M. de Villiers (1984), but he dealt with the views of prospective mathematics teachers at universities and not those of pupils.

The point of this investigation is thus not to decide whether or not pupils have correct views about various aspects of the nature of mathematics. In the light of the conflicting arguments this would be unjustified. I am merely looking for evidence that some thought has been given to the nature of mathematics during the pupil's study of the subject.

CHAPTER 2

WHY MATHEMATICS?

The importance of teaching/learning mathematics should be related to the nature of the subject. Plunket agrees that:

> "It must be very important that we think as carefully as we can about the nature of mathematics, and attempt to relate our conclusions on that point to our ideas about why we should teach it."
>
> (in de Villiers 1984, p32)

Our ideas about why it is important to <u>teach</u> mathematics may be very different from the reasons why pupils want to <u>take</u> mathematics. It is thus necessary to examine both perspectives, as well as examine mathematics from a sociological point of view.

2.1. <u>Why Teach Mathematics?</u>

The ancient Egyptians saw mathematics as being important for practical reasons related to commerce, farming and building, as well as for aesthetic reasons. These aesthetic reasons included interests in relationships, forms and patterns. The early Greeks, however, emphasised the aesthetic importance, with particular attention being paid to style of argument and to rational discussion. Plato, for example, said that:

> "The study of mathematics develops and sets into operation a mental organism more valuable than a thousand eyes, because through it alone can truth be apprehended."
>
> (Baron in Chapman 1972, p24)

Until approximately 50 years ago it was agreed that mathematics was taught mainly because it "trained the mind". While people such as Mercer believe that the role of logic in mathematics is so great that some people see mathematics as simply a branch of logic (in Chapman 1972, p66), there is little evidence that mathematics does in fact train the mind.

In 1940 the Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics (also in America) decided that mathematics is important because the average citizen needs considerable mathematical knowledge in the activities and experiences of everyday life. It also decided that mathematics supplies a means of understanding important aspects of the world (in Breslich 1966, p465). This statement is representative of the modern shift towards the practical importance of mathematics.

Other reasons for teaching mathematics include the following beliefs: it is a language because it is a means of communicating quantifiable ideas; it is involved in every subject that a pupil does, even if this involvement is only trivial; it is a tool for use in various occupations; it is the foundation for scientific study; as a subject itself its new techniques and concepts may have economic consequences; and it is worthwhile simply because it is an enjoyable and satisfying activity. (Hann 1972; Dainton Committee Report in Griffiths and Howson 1974; Bishop and Nickson 1983)

2.2. Why Take Mathematics?

I have chosen to use the word "take", rather than "do" or "learn", because I do not wish to choose sides in the product vs process debate mentioned in 1.1. It should be clear how this debate and other aspects of the nature of mathematics are linked to views about the importance of the subject. For example, somebody in favour of the "product" argument would be more inclined to stress the aesthetic importance of mathematics rather than its practical importance.

The reasons for teaching mathematics are not necessarily the reasons why pupils take mathematics. Some writers suggest that children see mathematics as being important for reasons that are not always as "pure" or "noble" as those of their teachers. These reasons are often connected to careers and to ways of pleasing and impressing others (Beldon 1981). Both sets of reasons can be distorted by examinations. Teachers and pupils might become so engrossed in the examination syllabus that noble ideas about why mathematics is taught or taken soon become forgotten. There is the danger that, as M. Tuck puts it:

> "Children are made to fit in with the needs of the syllabus, not the syllabus with the needs of the children".
>
> (Tuck 1981, p3)

The children, sucked into this "examination syllabus syndrome", cannot be blamed for seeing mathematics as a means towards an end, with examinations merely obstacles en route towards that end.

There is a large field of literature available dealing with reasons why pupils choose to <u>stop</u> doing mathematics. Little research, however, seems to have been done into why pupils choose to <u>continue</u> with mathematics. Reasons for continuing with mathematics should theoretically be the same as the reasons for teaching the subject. I have offered some reasons to suggest that they are probably not the same. This investigation is an attempt to determine what they in fact are.

2.3. Mathematics as a Social Agent

The importance of mathematics, as discussed above, has made the subject an integral part of most curriculi. This inclusion in the curriculum has led to mathematics being manipulated by society to the point that it has gained importance in ways other than originally intended.

It has already been mentioned that mathematics is a tool for use in various careers. Probably no discipline affords more career versatility than training in mathematics, with the general career outlook for people with mathematical training getting better each year. In fact, many employers are now finding that it is easier and more effective to hire a mathematician and teach him the engineering he needs, than it is to hire an engineer and give him the necessary special mathematical training, says D. Lick (Lick 1971, p86). There is, however, the danger of mathematics being linked to careers, not because the subject will better equip an applicant to do a job, but rather because it can be a useful criterion in the selection process. Thus, because mathematics is agreed to be important, it leads to more people doing the subject, which in turn leads to outsiders using mathematics as a measuring stick of general ability. This use of mathematics as some sort of "career filter" results in the subject becoming even more important in a rather false way.

Some people contend that mathematics has also gained importance of a political nature. Jenny Maxwell says that:

> "it seems impossible that such a central part, mathematics, of such a political institution, education, should really be neutral".
> . (Maxwell 1985, p18)

This political dimension of mathematics is usually covert and is inter-related with the aims and values of the society within which it exists (Dunn 1977). While many believe mathematics to be a pure, objective discipline above and beyond such matters, others say that it is naive to think that there are no social and political consequences linked to it. Others go further and say that these consequences have contributed to the alleged crisis in Western society, in that mathematics, as part of a technocratic ideology, has led to the dehumanising of society (in de Villiers 1984).

Another social connotation of mathematics is the question of stratification of subjects within schools. There are those that contend that mathematics is the most important subject at school which, they argue, leads to mathematics teachers being important.

> "As a specialist, the mathematics teacher is probably given more unequivocal recognition of his specialisation than most teachers in schools."
> (R. Williams in Wain 1978, p49)

A. Caldwell (in Chapman 1972) believes that large social pressures operate through the mathematics teacher in the classroom, with mathematics itself being incidental. Mathematics, according to him, is often a discriminating factor in the classroom with those who are non-mathematicians constantly being reminded that they are intelligent, and yet constantly being spoken to as if they were not. Pupils and teachers would probably support this notion of academic snobbery being closely linked to mathematics.

The existence of many of these "non-educational" factors is debatable and, if they do exist, they are probably morally indefensible. Nevertheless, because at the very least they are perceived to exist, they add to the importance of the subject and thus probably influence the reasons why people teach or take mathematics.

CHAPTER 3

ATTITUDES TOWARDS MATHEMATICS

Mathematics is in many ways an emotional subject. Despite its importance, its nature causes people to vary considerably in their attitudes towards it. These attitudes have a great influence on decisions re the continuation of mathematics.

According to Aiken (1970), an attitude is:

> "a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or other person".
>
> (Moodley in Rajah 1981, p41)

In this case we are considering a learned predisposition or tendency on the part of pupils to respond positively or negatively to mathematics.

In most schools where mathematics is not a compulsory subject up until Std 10, achievement in mathematics is an important consideration when deciding whether or not to continue with the subject after Std 7. A pupil whose mathematics results have been poor up until Std 7 will probably be discouraged from continuing with the subject. Yet one cannot categorically say that such a pupil does not have mathematical ability. Perhaps his or her achievement has been negatively affected by some particular attitude towards mathematics.

There is, however, little research basis for believing that attitudes towards mathematics and achievement in mathematics are causally related (Knaupp 1973), with only low to moderate correlations between attitude and achievement having been shown by various studies (Stephens 1960; Spickerman 1970; Wilson 1973; Johnson 1977). An interesting finding of Saburoh Minato and Shyoichi Yanase is that if learning of mathematics and attitudes are related, the relationship is different for different levels of pupil intelligence, with the attitudes of low intelligence pupils being more critical (Minato and Yanase 1984, p319).

Many negative attitudes are related to what is known as "maths anxiety". Maths anxiety is not necessarily related to general anxiety and nor is it related to general intelligence (Lazarus 1974; Kogelman & Warren 1978; Tobias 1978). Maths anxiety can often be traced to negative experiences during early school years but fortunately various writers have suggested constructive techniques and strategies that teachers can use to help prevent the development of maths anxiety (Morris 1981; Greenwood 1984).

Positive attitudes towards mathematics as a school subject peak in early adolescence and decline through high school (Carpenter et al 1980). An investigation by W. Callahan (1971) in America, found that grades 6 and 7 were the most important for developing attitudes towards mathematics. 50% of his sample of 366 pupils recognised a change of attitude in the 8th grade. Relating this to the South African context, it would seem that pupils' attitudes are well established before the end of Std 7. Callahan also found that positive attitudes seem to relate to the practical nature of mathematics and the fact that mathematics is needed in everyday life. Negative attitudes were found to be related to feelings of inadequacy linked to learning and memorising mathematics, as well as to the alleged boring and repetitive nature of the subject (Callahan 1971, p754-755).

Just as it has been difficult to confirm the connection between pupil attitude and achievement, there is also no clear evidence of the effect of teachers' attitudes on pupils' attitudes. The matter is complicated by pupils having different reactions to different facets of mathematics. Brown and Abel (1965) were, however, able to determine that the correlation between attitude and achievement is higher for arithmetic, a facet of mathematics, than for spelling, reading or language acquisition (Moodley in Rajah 1981, p44). It has also been established that positive attitudes towards mathematics and the perceived usefulness of the subject are highly correlated with participation in mathematics courses (Carpenter et al 1980).

Attitudes towards mathematics, as discussed above, will thus have to be taken into account when considering why pupils choose to continue with mathematics after Std 7.

CHAPTER 4

GROUP DIFFERENCES

I have discussed a number of different reasons why pupils may choose to continue with mathematics, as well as some factors influencing this choice. It is now necessary to consider whether these reasons and factors are uniform for all pupils or differ according to certain groupings. In my study I will be looking at five groupings: boys vs girls; Higher Grade vs Standard Grade; Std 8 vs Std 10; groups based on achievement in mathematics examinations; pupils attending boys' schools vs girls' schools vs co-educational schools.

4.1. Boys vs Girls

Much of the research related to my particular investigation has been done in the field of sex differences in mathematics.

There is considerable evidence available showing that the female proportion of all pupils taking mathematics decreases as the level of the course increases (Noble 1974; Leder 1980; Visser 1985; Visser 1986). Despite the large amount of research attention directed at sex differences in participation, agreement has not been reached on the extent of these differences or on the relative importance of factors contributing to them (Leder 1985, p304).

There is also lack of clarity about sex-related differences in achievement in mathematics but there is no doubt that they do exist. These differences begin to appear in junior high school, a period usually coinciding with the onset of puberty, and evidence points to this difference being in favour of the boys (Maccoby & Jacklin 1974; Callahan and

Glennon 1975; Mullis 1975; Fox 1976; Visser 1986). Recent investigation suggests that these differences may in fact be diminishing (Oberholster 1985).

Noble (1982) divides the reasons for the differences into four categories: ability, socialisation influences, attitude towards mathematics and impact of schooling. All of these may influence decisions to continue with mathematics and I will therefore consider them briefly.

4.1.1. Ability

Is there something in the nature of mathematics that precludes females? Some famous figures from history believed this to be the case. Rousseau reckoned that feeble brains made females unfit for research into abstract and speculative truth, or the principles and axioms of science. Martin Luther, who believed that women were created with big hips so that they could stay home and sit on them, declared that:

> "no dress or garment is less becoming to a female than a show of intelligence".
>
> (in Burton 1979, p262)

Aristotle and Descartes had similar chauvinistic views.

While smaller heads and delicate nervous systems of females have been suggested as being the reasons for the differences (Leland 1904; Smith-Rosenberg & Rosenberg 1974), there is really little evidence that innate or genetic differences account for boy/girl differences in mathematical achievement. The case for spatial-visualisation differences between the sexes is, for example, not conclusive (Oberholster 1985).

What seems to have a far greater effect on achievement are socialisation influences.

4.1.2. Socialisation Influences

Traditionally, society has seen woman as having closer links with her home and family than with a career. While this view has changed considerably, particularly during the past few decades, it still seems to influence females in many ways. Isaacson and Freeman (1980) found that girls may drop mathematics and change career choices if mathematics is an entry qualification to a particular career, primarily because their careers are not that important to them.

Indications are that much of society still sees mathematics as being more important for boys than for girls. Delene Visser, in a South African study carried out in 1984, found that there is a greater correlation between mathematics achievement and intended career among girls than boys. She also found that the overwhelming majority of parents involved in her research regarded mathematics as being more important and useful for their sons than for their daughters. She concludes that:

> "although research has shown that parents' own achievements in and their attitudes towards mathematics do not affect the outlook of their daughters as such, parental expectations and the values that parents attach to their daughters' ongoing involvement with mathematics do have a very strong influence".
>
> (Visser 1986)

These findings agree with those of Fennema & Sherman (1977) and Luchins & Luchins (in Fox 1980).

This notion that mathematics is more a male domain than a female one has led to some females developing a syndrome known as "fear of success", fearing that success in mathematics may bring their femininity into question (Horner 1968). Visser did not find that fear of success is greater for girls than for boys but Marjorie Carrs, an Australian, maintains that there is considerable evidence to suggest that girls are concerned about possible negative effects of success in mathematics. If there is a conflict between images of academic success and being viewed as feminine, they often choose the latter (in Zweng 1983). Lyn Osen goes as far as saying that:

> "Many women in our present culture value mathematical ignorance as if it were a social grace".
> (in Burton 1979, p263)

It thus seems that social influences contribute greatly to sex differences re enrolment and achievement in mathematics.

4.1.3. Attitude

Sex-related differences with regards to attitudes towards mathematics have also been found to exist. Noble, for example, states that girls display more anxiety and less confidence in their mathematical ability than do boys and that this occurs even before signs of diminishing mathematical competence appear (Noble 1982, p3). Males, on the other hand, have been found to view mathematics as being more useful than do girls. This is important in the light of the findings that there is a positive correlation between perceived usefulness of mathematics and achievement (Fennema & Sherman 1977). No sex-related differences have been found to exist with regards to the actual enjoyment of mathematics.

Research has shown that the majority of people are uncomfortable when handling mathematical material (Lazarus 1974; Tobias 1982; McDaniel 1982) and there seems to be an unusual willingness to discuss these feelings of discomfort. Within this phenomenon it has been found that more females than males openly acknowledge that mathematics causes them anxiety (Lazarus 1974; Tobias 1982; McDaniel 1982), but whether this is because more females experience maths anxiety (Maccoby & Jacklin 1974) or because they are simply more open about it (Sarrason & Winkel 1966), seems unclear. This maths anxiety, an irrational fear of mathematics, has been shown to be a better predictor of choice among girls than boys (Brush 1979). Furthermore, the nature of mathematics is believed to be one of the causes of maths anxiety (Visser 1985).

4.1.4. Impact of Schooling

The school system itself seems to account to some extent for sex-related differences in achievement as well as participation in mathematics, with prejudice in the system seeming to be in favour of the boys. Schonborn (1975) found that teachers pay more attention to males than to females, while a study by Gregory (1977) revealed that teachers are less inclined to provide remedial help to females. Underachieving females are less likely to be behaviour problems in class and are thus less visible to their teachers than are underachieving males (Gregory 1977).

Early this century it was found that problems used in mathematics textbooks were biased in favour of males (E. Sidgwick and L. Story in Clements 1979). There also seems to be a general shortage of female role-models in the mathematics world. Despite this shortage of famous female mathematicians, their contribution to mathematics has not been insignificant. Yet how many pupils (or teachers) have heard of names such as Hypatia, Maria Agnesi, Sophie Germain, Emmy Noether and Sonja Kovalevsky? This shortcoming is related to our efforts, or lack of them, to make pupils aware of the nature of mathematics, as well as to the absence of any study of the history of mathematics in our school syllabusses.

Another example of how schooling affects sex-related differences in mathematics is the question of whether girls should attend co-educational or girls' schools. Those attending girls' schools are reported to have greater confidence and higher self-esteem regarding mathematics. The findings of J. Harding (in Kelly 1981) and H. Shuard (in Cockroft 1982) support the view that girls are disadvantaged in a mixed school setting. Dale (1974), on the other hand, found that girls perform better in mixed schools. This divided opinion is perhaps due to insufficient attention having been paid to socio-economic differences in schools (Leder 1985, p307).

It is my belief that all of the previously discussed interdependent factors either directly or indirectly affect the choice of whether or not to continue with mathematics after Std 7. These factors thus throw light on possible sex-related differences in this regard. Research on Std 7's by Visser found that this choice is guided more by emotional, social and attitudinal factors among girls, and more by intellectual capabilities in the case of boys (Visser 1986).

4.2. Higher Grade vs Standard Grade

It seems that there has been no research into the reasons why H.G. as opposed to S.G. pupils decide to continue with mathematics after Std 7. The closest comparison to our H.G./ S.G. differentiation is the A- and O-Levels used particularly in the United Kingdom. In an enquiry by Jack Selkirk (1974) into the attitudes of A-Level students in Northumberland, it was found that attitudes towards A-Level mathematics were more unfavourable than those that they, in retrospect, had towards O-Level mathematics. These unfavourable attitudes were unrelated to mathematical achievement. Mathematics was recorded less often than other subjects as the pupils' most enjoyable subject. 56% of the sample regarded mathematics as their most difficult subject while 80% of them would have retained mathematics if allowed to choose again (Selkirk 1974).

The relevance of these findings to my investigation is small because the attitudes compared by Selkirk were those held by the same pupils, in that all the A-Level pupils did O-Levels originally. H.G. and S.G. pupils, on the other hand, are normally distinct groupings. Many of the S.G. pupils in my study, particularly those in Std 10, may however have started to take mathematics on the H.G. in Std 8 and dropped to S.G. later on.

4.3. Std 8 vs Std 10

Once again little literature exists that is related to the purpose of this investigation. An assessment done by the National Assessment of Educational Progress in the U.S.A. (1970) compared attitudes of 13 and 17 year old pupils towards mathematics as opposed to science, social studies, English and physical education. Mathematics was the most disliked subject of the 17 year olds but there were two subjects more disliked by the 13 year olds. The 17 year olds saw mathematics as the most difficult of the five subjects but also the most important. Science and social studies were more difficult from the 13 year olds' point of view, but they also viewed mathematics as the most important of the five. 63% of the 17 year olds disagreed that they were taking mathematics because they had to, with 80% of them agreeing that the subject had practical use (Carpenter et al 1980).

Most Std 8's are 15 or 16 years old, while most Std 10's turn 18 in that year. Despite the discrepancy in the particular ages of the two pairs of groups of pupils, the above study is at least an example of age being used as a criterion to compare attitudes towards mathematics.

4.4. Achievement in Mathematics Examinations

The relationship between attitude towards mathematics and achievement in mathematics has been touched on previously in this chapter. I am particularly interested in the possibility of there being a correlation between the marks that pupils obtain in examinations and the reasons why they are doing mathematics. There seems to be little available literature in this regard.

4.5. Boys' Schools vs Girls' Schools vs Co-ed. Schools

Although the impact of schooling on possible sex differences in mathematics education has already been mentioned (see 4.1.4.), I shall be looking specifically at these groupings to see if differences exist re reasons for doing mathematics and knowledge of the nature of mathematics.

CHAPTER 5

RESEARCH DESIGN

5.1. Aims

The general aims of this investigation may be summarised as follows:

(a) To determine reasons why pupils choose to continue with mathematics after Std 7.

(b) To determine the extent of pupils' understanding of the nature of mathematics.

(c) To determine whether significant differences with regards to (a) and/or (b) exist between:

- (i) Std 8 and Std 10 pupils
- (ii) Higher Grade and Standard Grade pupils
- (iii) boys and girls
- (iv) groups based on achievement in mathematics examinations
- (v) pupils attending boys' schools, girls' schools and co-educational schools.

5.2. The Sample

In order to obtain the views of pupils who had recently gone through the whole process of deciding whether or not to continue with mathematics, as well as the views of pupils who were coming to the end of their school mathematics career, I decided to use Std 8 and Std 10 mathematics pupils in my investigation. These pupils came from all six Cape Education Department English-medium, academic (as opposed to technical, commercial, etc.) high schools in Port Elizabeth. Four of the schools are co-educational, another a boys' school and the last a girls' school. The sample covered a wide spectrum of socio-economic groupings.

Not every mathematics pupil in Std 8 and Std 10 at the schools in question did in fact complete a questionnaire. This was due to circumstances at the individual schools and not to any particular sampling technique of mine.

The following are the more important characteristics of the sample:

STANDARD

Std 8	Std 10	Total
550	250	800

SEX

Male	Female	Total
448	349	797*

SCHOOL

Boys	Girls	Co-ed.	Total
142	86	571	799*

Boys	Girls	?
306	263	2

GRADE

H.G.	S.G.	Total
370	427	797*

* The discrepancies in the totals are due to errors by pupils when completing the questionnaires. Rather than completely scrap responses that may otherwise be relevant to the investigation, it was decided to take note of the errors but prevent them from influencing any subsequent statistics.

5.3. The Questionnaire

The questionnaire was divided into two sections, Part A and Part B. Part A was subdivided into a section dealing with the personal particulars of the pupils and a section that aimed at obtaining some opinions of the pupils with regards to mathematics. It was largely by means of this section of Part A that I wished to determine whether pupils continue with mathematics for extrinsic or intrinsic reasons. Part B consisted mainly of open-ended questions relating to the nature of mathematics.

5.4. Administration of the Questionnaire

The questionnaire was administered by teachers at the schools during the third term of 1985. The majority of the pupils filled in the questionnaire during part of a mathematics lesson, with a few completing it at home. There was no evidence of reference books having been used to answer questions such as Question 3 of Part B where the temptation to do so may have been great.

5.5. Scoring of the Questionnaire

PART A: The optional answers to each question were coded with the actual responses being noted accordingly. Errors in responses, such as no answer or more than one answer, were recorded but not used in subsequent Chi-squared tests. Many errors occurred in the answering of question 2.3, with a number of respondents failing to indicate their most important reason for choosing to continue with mathematics. In questions 2.4 to 2.6 I was merely interested in the position of mathematics in relation to the pupils' other five subjects.

PART B: Questions in Part B were considered by me to be related to the nature of mathematics as follows:

Definition of mathematics	-	Question 1
History of mathematics	-	Questions 2 & 3
Mathematics: Invention or discovery?	-	Question 4
Mathematics: Product or process?	-	Questions 4 & 5
Mathematics as a tool/ career filter	-	Questions 6, 7 & 8
Mathematics: A multi-faceted subject	-	Question 9
Axiomatic structure of mathematics	-	Questions 10, 11, 12 & 13
Mathematics: Deductive or inductive?	-	Questions 14 & 15

Subsequent to the administering of the questionnaire, yet prior to the analysis of the ensuing data, it was decided to ignore questions 4, 5, 9, 11, 12 and 13. It was somewhat belatedly felt that the links of these questions with the nature of mathematics were not as clear as originally thought. The responses to these questions were also ignored in order to make the investigation less unwieldy.

It has previously been pointed out that there is no one correct answer to questions about the definition of mathematics. I thus coded answers to Question 1, and others in Part B, as follows:

1 - an answer that was complete, having evidence that thought had been given to the question; an answer containing no obviously incorrect statement
2 - an incomplete answer; evidence of an attempted answer
3 - no attempt at answering the question; an absurd answer, e.g. names of staff members at the particular schools being given as names of famous mathematicians

The coding of much of Part B of the questionnaire was thus, to a large degree, a subjective act on my part. In question 8 ("involvement of mathematics in other subjects"), I allocated a code of "1" if at least three acceptable examples were given. The required answers to the two parts of Question 15 were "no" and "false" respectively. A "1" was allocated if both correct answers were given and a "2" if only one answer was correct. My answers were based on the view that pattern is not sufficient reason for mathematical proof. It may be argued that, based on the principles of mathematical induction, the answers "yes" and "true" are the correct ones. 391 pupils gave this latter pair of answers, although their motivation for doing so is unclear.

Using a computer program known as BMDP, I was able to obtain the following information:

(a) overall frequencies of responses to the following twenty items in the questionnaire:

Part A - Questions 1.1.; 1.3.; 1.4.; 1.5.; 1.6.; 2.1.; 2.2.; 2.3.; 2.4.; 2.5.; 2.6.

Part B - Questions 1; 2; 3; 6; 7; 8; 10; 14; 15.

(b) frequencies of responses of

(i) Std 8 vs Std 10

(ii) boys vs girls

(iii) Higher Grade vs Standard Grade

(iv) boys' school vs girls' school vs co-educational schools

(v) pupils obtaining 60%-100% vs 40%-59% vs 33,3%-39% vs 30%-33,3% vs 0%-29%

(vi) combinations of (ii) and (iv), i.e. sex and school

to Questions 2.1, 2.2, 2.3, 2.4, 2.5 and 2.6 in Part

A and Questions 1, 2, 3, 6, 7, 8, 10, 14 and 15 of Part B

(c) the results of Chi-squared tests for all the comparisons done in (b)

When employing Chi-squared tests, I used 5% and 1% levels of confidence to decide whether or not significant differences existed between the groupings being compared. In tests where there was only one degree of freedom, Yates' corrected Chi-squares were calculated and subsequently used.

CHAPTER 6

RESULTS

6.1. Reasons Why Children Choose To Continue With Mathematics After Std 7

The most important question relating to this aspect of the investigation was Question 2.3 of Part A. In this question pupils were asked to specifically say why they are doing mathematics. Most of the responses to Question 2.39 can be labeled as "extrinsic" or "intrinsic" reasons without much debate. It is, however, necessary for me to explain why I classified three of the reasons as I did.

Option 1: "Mathematics is compulsory at my school"
Despite the fact that 143 of the respondents attended a school where mathematics is a compulsory subject, thus having no option but to carry on with the subject, only 40 of them said that they were doing mathematics because it was compulsory. The majority of these respondents therefore seemed to consider all the options and did not feel restricted to Option 1. Thus, whenever this reason was given for continuing with mathematics, I classified it as being an "extrinsic" one.

Option 8: Mathematics is "a useful subject"
I classified this as an "extrinsic" reason believing that pupils who offered this reason did not view mathematics as being a valuable subject in its own right, but rather saw it as a valuable tool for use in other subjects, in careers and in everyday life. This belief is perhaps supported by the fact that only 18,5% of the pupils who wrote down this option

did not consider mathematics to be useful outside the mathematics classroom (cf Question 6 of Part B).

Option 9: Mathematics "trains me to think logically"
It has already been pointed out that mathematics does not necessarily train people to think logically (cf 2.1.). It is nevertheless the belief of many that it does "train the mind". I classified this as an "intrinsic" reason for doing mathematics, a reason closely bound to the nature of mathematics and not a means towards any specific end.

A summary of the responses to Question 2.3, as well as the classification of each response, is to be found in Table 1.

Table 1.

Response	Frequency	Classification
1. mathematics is compulsory at my school	40	extrinsic
2. my family persuaded me to carry on with the subject	21	extrinsic
3. my teacher persuaded me to carry on with the subject	1	extrinsic
4. I enjoy the subject	135	intrinsic
5. it is an easy subject	6	intrinsic
6. it is necessary for job application/university entrance purposes	287	extrinsic
7. I like my mathematics teacher	3	extrinsic
8. it is a useful subject	138	extrinsic
9. it trains me to think logically	46	intrinsic
Errors	123	
TOTAL	800	

According to these classifications, the relationship between the extrinsic and intrinsic reasons is such that the extrinsic reasons are well in the majority (see Table 2).

Table 2

	Frequency	%
Extrinsic reasons	490	72,4
Intrinsic reasons	187	27,6

The answers to Questions 2.1, 2.2, 2.4, 2.5 and 2.6 give an indication of the pupils' general attitudes towards mathematics, which are indirectly linked to reasons why pupils are doing mathematics. It would, for example, be a contradiction if the majority of pupils were found to be doing mathematics for intrinsic reasons (according to answers to Question 2.3), yet said that mathematics was uninteresting, a waste of time and their least enjoyable subject (according to Questions 2.1, 2.2 and 2.5 respectively).

An examination of the responses to the above questions (see Appendix 2) shows that the majority of pupils consider mathematics to be an interesting and worthwhile subject, despite it being their most difficult subject. Mathematics was rated as their most useful subject by most pupils. Ratings of mathematics re enjoyment were not as favourable, with the majority of pupils considering it to be their third most enjoyable subject.

I will now deal with the results in respect of the different groups already mentioned (cf. Chapter 4). Detailed tables relating to the following can be found in Appendix 2.

6.1.1. Std 8 vs Std 10

76% of the Std 8's as opposed to 65% of the Std 10's gave an extrinsic reason for doing mathematics. Chi-squared tests showed this difference to be significant at better than the 1% level of significance.

Table 3

Standard	Extrinsic	Intrinsic	Total
Std 8	351 / 76%	113 / 24%	464
Std 10	139 / 65%	74 / 35%	213
Total	490 / 72%	187 / 28%	677

Yates' corrected Chisq. = 7,41
giving $p < 0{,}01$

Significant differences were also found to exist in the responses to the following questions:

Question 2.1. (Interest) - More Std 8's (86%) than Std 10's (78%) considered mathematics to be interesting. ($p < 0{,}05$)

Question 2.2. (Worth) - More Std 8's (93%) than Std 10's (88%) considered mathematics to be worthwhile as opposed to a waste of time. ($p < 0{,}05$)

Question 2.5. (Enjoyment) - Pupils in the two standards had different views about the enjoyment of mathematics, with a greater percentage of Std 10's ranking mathematics as either their most or least enjoyable subject. ($p < 0,01$) .

No significant differences re the difficulty and usefulness of mathematics were found to exist.

6.1.2. Higher Grade (H.G.) vs Standard Grade (S.G.)

More S.G. pupils (76%) than H.G. pupils (68%) offered extrinsic reasons ($p < 0,05$). This difference was found to be significant at better than the 5% level.

Table 4

Grade	Extrinsic	Intrinsic	Total
H.G.	215 / 68%	101 / 32%	316
S.G.	272 / 76%	86 / 24%	358
Total	487 / 72%	187 / 28%	674

Yates' corrected Chisq. = 4,87
giving $p < 0,05$

Further significant differences:

Question 2.1. (Interest) - More H.G. pupils rated mathematics as being interesting (87% vs 80%). ($p < 0,05$)

Question 2.2. (Worth)	- More H.G. pupils considered mathematics to be worthwhile (94% vs 89%). ($p < 0,05$)
Question 2.4. (Enjoyment)	- H.G. pupils generally ranked mathematics higher re enjoyment than did S.G. pupils. ($p < 0,05$) This ranking seems to be linked to achievement in mathematics in the case of both grades. Some pupils rated mathematics as their most enjoyable subject despite having obtained poor marks in examinations, but the frequency of low ratings is greater as marks decrease in both the H.G. and S.G. courses. (See Appendix 3)

No significant differences were found in connection with difficulty and usefulness of mathematics.

6.1.3. Boys vs Girls

76% of the boys as opposed to 68% of the girls gave an extrinsic reason for doing mathematics. This difference was found to be significant at better than the 1% level.

Table 5

Sex	Extrinsic	Intrinsic	Total
Boys	291 / 76%	93 / 24%	384
Girls	197 / 68%	94 / 32%	291
Total	488 / 72%	187 / 28%	675

Yates' corrected Chisq. = 5,01
giving $p < 0,01$

With regards to schooling, it is interesting to note that boys attending the boys' school were more inclined to offer extrinsic reasons for doing mathematics than were those boys at co-educational schools. In contrast to this, a smaller percentage of girls at the girls' school, as opposed to the co-educational schools, gave an extrinsic reason for doing mathematics.

Table 6

Boys

School	Extrinsic	Intrinsic	Total
Boys'	109 / 87%	16 / 13%	125
Co-ed.	182 / 70%	77 / 30%	259
Total	291 / 76%	93 / 24%	384

Girls

	Extrinsic	Intrinsic	Total
Girls'	47 / 64%	27 / 36%	74
Co-ed.	150 / 69%	67 / 31%	217
Total	197 / 68%	94 / 32%	291

The only other significant difference was in the way that the pupils ranked their subjects in terms of perceived usefulness (Question 2.6). 44,2% of the boys regarded mathematics as their most useful subject. Only 28,9% of the girls ranked mathematics in this position, although this was also their most popular ranking. A greater percentage of girls attending the girls' school as opposed to co-ed. schools (41% vs 25%), rated mathematics as their most useful subject. The opposite was true of the boys, with 39% of those at the unisex school ranking mathematics as "most useful" and 46% of those at co-ed. schools doing so. (See Appendix 3)

6.1.4. Differences according to examination results

A pupil doing mathematics on the Higher Grade has to obtain a minimum of 40% to pass. If the pupil gets 30% or more (but under 40%), he or she fails on the Higher Grade but passes on the Standard Grade. 33,3% is the pass mark for a pupil doing mathematics on the Standard Grade. I bore these "critical points" in mind when deciding on my groupings of the examination marks. The 60% - 100% grouping was intended to group together pupils who were achieving good to excellent results for mathematics. Table 7 should show that as the examination results weaken, the percentage of pupils giving extrinsic reasons for doing mathematics increases.

Table 7

Mark	Extrinsic	Intrinsic	Total
60%-100%	155 / 64%	86 / 36%	241
40%-59%	186 / 73%	68 / 27%	254
33,3%-39%	55 / 76%	17 / 24%	72
30%-33%	29 / 85%	5 / 15%	34
0%-29%	61 / 88%	8 / 12%	69
Total	486 / 73%	184 / 27%	670

Chisq. = 20,18
giving $p < 0,01$

Further analysis of these results with respect to the grade on which the results were obtained, shows that both H.G. and S.G. display similar patterns (see Table 8).

Table 8

Higher Grade

	Extrinsic	Intrinsic	Total
Pass H.G.	186 / 68%	86 / 32%	272
Fail H.G./ Pass S.G.	39 / 69%	13 / 31%	52
Fail	13 / 87%	2 / 13%	15
Total	238 / 70%	101 / 30%	339

Table 8 (cont.)

Standard Grade

Pass S.G.	194 / 72%	74 / 28%	268
Fail	75 / 87%	11 / 13%	86
Total	269 / 76%	85 / 24%	354

When scoring the responses of the various groups, it was found that n < 5 in many of the cells. Use of Chi-squared tests would thus have been invalid. It could, however, be used in Question 2.1 ("Interest") and Question 2.2 ("Worth") - $p < 0,01$ in both cases. The higher achievers generally saw mathematics as being more interesting and more worthwhile than did those obtaining lower marks. This was true for both H.G. and S.G. pupils.

6.1.5. Boys' school vs girls' school vs co-ed. schools

87% of the respondents at the boys' school, 64% of those at the girls' school and 70% of the respondents at co-ed. schools gave an extrinsic reason for doing mathematics.

Table 9

School	Extrinsic	Intrinsic	Total
Boys'	109 / 87%	16 / 13%	125
Girls'	47 / 64%	27 / 36%	74
Co-ed.	334 / 70%	144 / 30%	478
Total	490 / 72%	187 / 28%	677

Chisq. = 18,16

giving $p < 0,01$

A greater percentage of pupils at the girls' school than at the other two types of school, considered mathematics to be interesting and worthwhile ($p < 0,01$ in both cases). A greater proportion of the girls attending the unisex school found mathematics to be interesting (94% vs 83%) and worthwhile (99% vs 89%). This pattern was not evident in the case of the boys. A slightly greater percentage of the boys at co-ed. schools considered mathematics to be interesting (82% vs 79%), whereas a greater percentage of boys at the boys' school described mathematics as worthwhile (95% vs 89%). These findings may be linked to the particular schools involved in the investigation, not necessarily due to sex-related issues, and must thus be treated with caution. (See "Sex and Schooling" in Chapter 7.)

No significant differences were found in the responses to Questions 2.4, 2.5 and 2.6.

6.1.6. A Summary of Significant Differences in Responses to Part A of the Questionnaire

The significant differences identified in 6.1.1 - 6.1.5 above are summarised in Table 10.

Table 10

Summary of Significant Differences - Part A

Questions	Std	Grade	Sex	Mark	School
2.1 Interest	p<0,05	p<0,05	-	p<0,01	p<0,01
2.2 Worth	p<0,05	p<0,05	-	p<0,01	p<0,01
2.3 Reason (Extr. vs Intr.)	p<0,01	p<0,05	p<0,01	p<0,01	p<0,01
2.4 Difficulty	-	-	-	- n<5	-
2.5 Enjoyment	p<0,01	p<0,05	-	- n<5	-
2.6 Usefulness of all subjects	-	-	p<0,01	- n<5	- n<5

6.2. Pupils' Understanding of the Nature of Mathematics

As was explained in Section 5.5 of Chapter 5, certain items in Part B of the questionnaire were examined with a view to gauging the extent of pupils' understanding of the nature of mathematics. Specific frequencies of responses to the various questions can be found in Appendix 2. I will proceed to discuss some of the more interesting characteristics of the ways in which the questions were answered, as well as look at some group differences that were found to exist.

* Question 1: Definition of mathematics.

40% of the pupils could not offer even a partial definition of mathematics, despite having done the subject for at least 9 years. Most of the definitions offered by the pupils included phrases such as "the study of numbers", and the inclusion of key words such as "logic", "numerals", "symbols", "figures" and "shapes".

* Question 2: Male mathematicians.

The names of many prominent mathematicians are commonly known today. Many of them also gained fame in the field of science. This, together with the fact that names such as Pythagoras and Euclid are specifically mentioned in the syllabus, contributed to 75% of the pupils being able to name at least one famous male mathematician. In the light of the above, the percentage who were not able to name anyone (25%) can be considered to be large. The more common, acceptable answers that were given were the names of Pythagoras, Einstein, Newton, Archimedes and Galileo.

* Question 3: Female mathematicians.

Not one pupil was able to name a famous female mathematician.

Marie Curie's name was often given, but not accepted by me, with many pupils being tempted to write down names of female mathematics teachers at their schools.

* Question 6: Is mathematics useful outside the classroom? The overwhelming majority of pupils (79%) who answered the question believed mathematics to be useful outside the classroom.

* Question 7: Careers for which mathematics is essential. Almost all the pupils (97%) were able to name at least one career, with the majority naming three. The more commonly mentioned careers were medicine, architecture, engineering, quantity surveying and mathematics teaching.

* Question 8: Involvement of mathematics in other subjects. The positive response to this item was good, although not as good as in Question 7. Science (physics), geography (mapwork) and biology (genetics) were common answers.

* Question 10: Definition of "axiom".
Despite the fact that all mathematics pupils deal with axioms in geometry, which is a substantial part of their syllabus, 73% of the pupils could not define the concept. Some of the pupils confused "axioms" with "axes", thus giving definitions in terms of graphs.

* Question 14: Definition of "deductive reasoning".
The essence of what I was looking for was reasoning "from the general to the specific". I was thus wanting a reasonably strict mathematical definition of "deductive reasoning". Less than 1% of the pupils could give such a definition. 70% of the pupils that attempted a definition vaguely related the adjective "deductive" to the verb "deduce", saying that

"deductive reasoning" means to work out (deduce) an answer from given information. Because I was linking this question to the nature of mathematics, I rejected this more simplistic interpretation.

* Question 15: Mathematical proof.

In this question I was testing whether or not the pupils realised that mathematical proof does not depend on specific examples. 76% of the pupils gave an answer of "no" for the first part of the question or "false" for the second part. Only 12% answered both parts correctly. An answer to the second part of "not necessarily" was accepted as being correct.

6.2.1. Std 8 vs Std 10

Significant differences were found in responses to the following questions:

* Question 1 - Proportionately more Std 8's than Std 10's were able to give a partial or complete definition of mathematics (65% vs 48%). ($p < 0,01$)

* Question 2 - Proportionately fewer Std 8's (72%) than Std 10's (83%) were able to name at least one famous male mathematician, with twice as many Std 10's able to name three (36% vs 18%). ($p < 0,01$)

* Question 6 - Proportionately more of the Std 8's viewed mathematics as being useful outside the mathematics classroom (81% vs 73%). ($p < 0,05$)

* Question 7 - Proportionately slightly more of the Std 10's were able to name three careers (78% vs 73%), but 25% of the Std 8's named one or two careers as opposed to only 18% of the Std 10's. $(p < 0,05)$

* Question 10 - 35% of the Std 10's were able to at least partially define the term "axiom". Only 24% of the Std 8's were able to do so. $(p < 0,01)$

* Question 15 - More Std 8's (14% vs 7%) gave correct answers to both parts of the question, but this was balanced out to a certain extent by more Std 10's (83% vs 73%) being able to answer only one part correctly. $(p < 0,01)$

6.2.2. <u>Higher Grade (H.G.) vs Standard Grade (S.G.)</u>

Significant differences:

* Question 1 - A greater percentage of H.G. pupils were able to give a partial or complete definition of mathematics. $(p < 0,01)$
  - A breakdown of the responses according to examination results showed a similar pattern in both H.G. and S.G. pupils of a generally decreasing ability to define mathematics as the marks decreased.

* Question 2 - 20% of the H.G. pupils were unable to name even one famous male mathematician compared to 30% of the S.G. pupils. $(p < 0,01)$

* Question 6 - More of the H.G. pupils considered mathematics to be useful outside the mathematics classroom (86% vs 72%). ($p < 0{,}01$)

* Question 8 - More of the H.G. pupils (89%) than S.G. pupils (80%) were able to give at least one example of how mathematics is used in other subjects. ($p < 0{,}01$)

* Question 10 - H.G. pupils were also better at defining the term "axiom" (40% vs 17%). ($p < 0{,}01$)

* Question 15 - Approximately the same percentage of H.G. (11,6%) and S.G.(12,2%) pupils were able to answer both questions, but 80% of the H.G. pupils could give only one correct answer as opposed to 73% of the S.G. pupils. ($p < 0{,}05$)

6.2.3. Boys vs Girls

Significant differences:

* Question 2 - A considerably greater percentage of boys were able to name at least one famous male mathematician (81% vs 67%). ($p < 0{,}01$)

* Question 7 - 8% more boys than girls (78% vs 70%) named three careers, but this was partly balanced out by 5% more of the girls being able to name one or two careers (20,3% of the boys vs 25,5% of the girls). ($p < 0{,}05$)

- Type of schooling had no influence on the boys in this question, but there was a significant difference in the way that the girls responded. A smaller percentage of girls at the girls' school as opposed to the co-ed. schools named three careers, but a greater percentage were able to name one or two careers. ($p < 0,05$)

* Question 8 - A similar pattern was found in responses to this question, with a greater percentage of the boys giving three examples but a greater percentage of the girls giving one or two examples. Approximately the same percentage of the boys (15,4%) and of the girls (16%) could not give an example. ($p < 0,01$)

* Question 10 - A greater percentage of boys were able to give at least a partially acceptable definition of "axiom" (33% vs 21%). ($p < 0,01$)
  - Boys attending the boys' school answered the question better than did those attending co-ed. schools. ($p < 0,01$)

6.2.4. <u>Differences based on achievement in mathematics examinations</u>

Significant differences:

* Question 2 - A greater percentage of pupils obtaining high marks for mathematics were able to

name one or more famous male mathematicians. As the examination result decreased so too did the percentage of those who were able to respond positively to the question.
($p < 0,05$)

* Question 6 - A similar pattern existed in the responses to this question. 84,5% of the pupils who obtained a mark of 60% to 100% for mathematics considered the subject to be useful outside the classroom. This percentage decreased as the marks decreased with only 62,5% of the pupils that obtained from 0% to 29% answering the question positively.
($p < 0,01$)
82% of H.G. pupils that obtained a mark of 0% to 29% viewed mathematics as being useful as opposed to only 59% of the S.G. pupils that obtained similar marks. It must be remembered, however, that the number of H.G. pupils in this grouping was only 11 compared with 68 S.G. pupils. ($p < 0,05$)

* Question 15 - A greater percentage of those obtaining marks in the 60%-100% range were able to answer one or two of the questions correctly. There was no pattern to the responses of the other groupings.
($p < 0,05$)

6.2.5. Boys' school vs girls' school vs co-ed. schools

Significant differences:

* Question 1 - A smaller percentage of the pupils at co-ed. schools (56%) were able to give at least a partial definition of mathematics. This was in contrast to 69% of pupils at the boys' school and 71% of those at the girls' school. $(p < 0{,}01)$
  - There was little difference in the way that boys and girls at the co-ed. schools answered the question. (See Appendix 3)

* Question 2 - A far greater percentage of the pupils at the boys' school (95%) named one or more famous male mathematicians than did pupils at the girls' school (79%) and at the co-ed. schools (69%). $(p < 0{,}01)$
  - Within the co-ed. schools, 74% of the boys as opposed to 63% of the girls were able to provide at least one name. $(p < 0{,}01)$

* Question 6 - Pupils at the unisex schools were more inclined to believe that mathematics is useful outside the mathematics classroom, with 86% of pupils at both the boys' and girls' schools answering "yes". 76% of the pupils at the co-ed. schools answered likewise. $(p < 0{,}01)$
  - 79% of the boys at these co-ed. schools answered "yes" in contrast with 72% of the girls. There was a significant difference

in the way that the girls at the girls' school as opposed to the co-ed. schools answered the question. ($p < 0{,}05$)

- No significant difference was found between boys in the boys' school and boys in the co-ed. schools.

* Question 7 - A greater percentage of pupils at the boys' school (84%) named three careers than did pupils at co-ed. schools (73%) and at the girls' school (66%). ($p < 0{,}01$)
  - Boys and girls at the co-ed. schools responded in similar ways - 75% and 71% respectively naming three careers and 23% of each naming one or two careers.

  (See Appendix 3)

* Question 8 - Pupils at the boys' school and at the co-ed. schools responded slightly better to this question than did pupils at the girls' school. ($p < 0{,}01$)
  - There was no significant difference in the way that girls from the two types of schools responded to the question.

  (See Appendix 3)

6.2.6. A Summary of Significant Differences in Responses to Part B of the Questionnaire

The significant differences identified in 6.2.1 - 6.2.5 above are summarised in Table 11.

Table 11

Summary of Significant Differences - Part B.

Question	Std	Grade	Sex	Mark	School
1 Def. of mathematics	p<0,01	p<0,01	-	- n<5	p<0,01
2 Male mathematicians	p<0,01	p<0,01	p<0,01	p<0,05	p<0,01
3 Female mathematicians	-	-	-	-	-
6 Usefulness of mathematics	p<0,05	p<0,01	-	p<0,01	p<0,01
7 Careers	p<0,05	-	p<0,05	-	p<0,01
8 Use in other subjects	-	p<0,01	p<0.01	-	p<0,01
10 Definition of axiom	p<0,01	p<0,01	p<0,05	- n<5	- n<5

Table 11 (cont.)

Question	Std	Grade	Sex	Mark	School
14 Deductive reasoning	- n<5	- n<5	- n<5	- n<5	- n<5
15 Mathematical proof	$p<0,01$	$p<0,05$	-	$p<0,05$	- n<5

CHAPTER 7

CONCLUSION AND RECOMMENDATIONS

According to my classification of reasons for continuing with mathematics, far more pupils were found to be studying mathematics for extrinsic reasons (72,4%), as opposed to intrinsic reasons (27,6%). (This finding will be discussed in more detail further on in this chapter.)

This investigation found that most pupils considered mathematics to be interesting and worthwhile. It was also found that pupils generally ranked mathematics as their most difficult but also most useful subject. These last two characteristics of mathematics have to work in tandem because I believe that most pupils would avoid choosing a difficult subject if it were not very useful as well. Not many pupils would be keen to continue with a subject simply because it presents a difficult challenge. Unfortunately this belief was not really tested in this investigation.

Another omission in the questionnaire was to not specifically ask pupils whether they considered mathematics to be enjoyable. The ranking by the majority of pupils of mathematics as their third most enjoyable subject, does not really tell us enough about whether pupils enjoy mathematics as such.

Answers to questions in Part B of the questionnaire that relate to key aspects of the nature of mathematics, suggest that pupils have little concept of this aspect of the subject. Furthermore, responses to Questions 6, 7 and 8, which deal with the utility aspect of mathematics, were the questions in Part B that were best and most positively answered. It thus seems that pupils know what to <u>do</u> with mathematics (how the subject is used), but know very little about what mathematics is.

The following are conclusions that can be drawn about the different groupings that were examined:

Std 8 vs Std 10

Responses of pupils from the various standards to questions relating to the interest and enjoyment value of mathematics, as well as to whether or not the subject is worthwhile, suggest that attitudes towards mathematics still seem to change after Std 8. If this is the case it makes it that much more difficult to expect a sound decision from a Std 7 pupil re continuation of mathematics as a subject up to matric.

The difference between the two groups with regards to reasons for continuing with mathematics, is interesting. The results show that a greater percentage of the Std 8's gave an extrinsic reason for continuation than did Std 10's. Perhaps the almost three years that had elapsed between the actual decision to continue with mathematics and the completion of this questionnaire, had served to dull the memories of the Std 10's. On the other hand, perhaps a change had taken place with the extrinsic value of mathematics becoming less important to the Std 10's than it was when they were in Std 8.

The Std 10's were generally better able to answer questions relating to the nature of mathematics. Perhaps this is because of the extra two years of mathematics that they have experienced, which has possibly given them greater insight into the subject.

I can offer no explanation as to why the Std 8's were better able to define mathematics (see Question 1 of Part B).

Grade and Mathematical Achievement

A greater percentage of S.G. pupils as opposed to H.G. pupils gave an extrinsic reason for continuing with mathematics. The attitudes of S.G. pupils re interest, worth and enjoyment of mathematics, were also more negative than those of H.G. pupils. It might be argued that it is to be expected that H.G. pupils have more positive attitudes towards mathematics because they are generally better mathematicians than are S.G. pupils. On the other hand, the strain of doing mathematics on the H.G. could encourage negative attitudes. The opposite may also be true. The pressures of H.G. mathematics may in fact promote positive attitudes, at least superficially. In an attempt to combat the thought of dropping to S.G. mathematics, H.G. pupils might overemphasise the positive aspects of the subject.

A more plausible reason for a greater percentage of the S.G. pupils admitting extrinsic reasons for continuing with mathematics and for their more negative attitudes towards the subject, is perhaps the following. Many of the pupils who struggle with mathematics seem to be studying mathematics for extrinsic reasons. This is verified by an analysis of reasons for doing mathematics of groups based on achievement in mathematical examinations. In general, the lower the mark obtained, the greater the percentage of pupils who gave an extrinsic reason for continuing with mathematics. Most pupils obtaining poor marks end up doing mathematics on the S.G., and thus it is on this grade that we expect to find most of the pupils who would not have continued with mathematics if they were truly free to make their own choice. Attitudes towards mathematics also seem to deteriorate as marks decrease, irrespective of the grade on which mathematics is being studied. No conclusive decision about this can be reached because my sample was such that the use of Chi-squared tests could not always be warranted. (N was too small in certain of the cells.)

Those pupils, both on the H.G. and S.G., achieving good marks for mathematics, had a greater knowledge of the nature of mathematics, while H.G. pupils generally showed up better in this regard than did S.G. pupils. I believe that knowledge of the nature of mathematics should not depend on mathematical ability. It would be more desirable, in my opinion, if knowledge of what mathematics <u>is</u> leads to better achievement in the subject rather than vice versa, i.e. mathematical ability should be more dependent on knowledge of the nature of mathematics.

<u>Sex and Schooling</u>

76% of the boys gave an extrinsic reason for continuing with mathematics, as opposed to 68% of the girls. Boys were also more inclined to rate mathematics as a useful subject. Perhaps society still views mathematics as being more important for boys and thus more boys are pressurised into taking the subject. This could mean that there is a greater percentage of girls who are studying mathematics because they <u>want</u> to rather than because they feel that they <u>have</u> to.

Results showed that the percentage of boys from the boys' school that gave an extrinsic reason for continuing with mathematics was greater than boys at co-ed. schools, while a smaller percentage of the girls at the girls' school as opposed to the co-ed. schools gave an extrinsic reason. This possibly supports the belief that boys are put under greater pressure to continue with mathematics. At co-ed. schools, where girls are also present, this pressure might be less than in the case of a boys-only setting. This is, however, mere speculation on my part and could be the subject of future research.

In my sample I ended up with only one boys' school and one girls' school. All of my findings with regards to type of schooling may thus have little or nothing to do with the sex aspect of the school and more to do with the standard of education and educational philosophy of the respective schools. Significant differences re reasons for continuing with mathematics and knowledge of the nature of mathematics, that involve type of schooling, may thus not be all that significant.

Recommendations for Future Research

This investigation had two main purposes, namely to look at reasons why pupils continue with mathematics and to examine the extent of pupils' knowledge of the nature of mathematics. Specialisation in one of these two areas, particularly the first area, should produce more conclusive results. Pupils could be offered more options in a questionnaire as to why they chose to continue with mathematics and/or they could be given more open-ended questions in this regard. The options could be worded in such a way that it is very clear whether an extrinsic or intrinsic reason is being dealt with.

Std 9's could be included in an investigation and, if the investigation is done over a few years, the same pupils could be asked to complete further questionnaires. This might produce evidence of changing attitudes to mathematics and a change in reasons why pupils study mathematics. Another possibility is that the opinions of Std 7's be obtained shortly before or after they make their subject choice for Std 8.

My sample was adequate for many of the purposes of my investigation but had shortcomings when it came to certain of the groups that I was comparing. The problems of the sample with regards to type of schooling have already been mentioned. My sample also had, I felt, too many Std 8's in relation to the number of Std 10's. Future researchers examining matters related to this investigation would be wise to specialise in one or just a few of the groupings that I dealt with, and ensure that the sample used is truly representative of these groupings. I do, however, believe that it was necessary to deal with all the groupings that I did in order to lay the foundation for any further research.

My assessment of achievement in mathematics was based on one examination result. Although I asked pupils to comment if the result used was not in keeping with their normal results, the use of just one result can be criticised. Researchers should examine ways of solving this problem.

This investigation was limited to Cape Education Department <u>English</u>-medium, <u>academic</u> schools situated in <u>Port Elizabeth</u>. A broadening or change of sample should thus be considered.

Another aspect worthy of attention in any related investigation is the actual extent of parental involvement in subject choice. In my study I had the feeling that pupils were not being honest about how much say their family had had in their decision to continue with mathematics. I feel that pupils might think it embarrassing to admit that their subject choice was influenced by their parents. The problems of adolescent/parent relationships and the adolescent's striving towards independence need not be discussed here.

Recommendations for Mathematics Education

> "If there were true freedom who would join us to do mathematics?"
>
> (J.Dunn 1977, p25)

This true freedom will obviously never exist. Pupils find themselves in a society that places various pressures on them. It is thus not surprising that many pupils continue with mathematics because of social pressures rather than because of personal preference. Despite shortcomings in the investigation, I believe that I have given enough evidence to show that an excessive number of pupils continue with mathematics because of extrinsic rather than intrinsic reasons. To expect no pupils to continue for extrinsic reasons would be idealistic, but to have pupils who give extrinsic reasons outnumber those that give intrinsic reasons to the extent that they evidently do, should be cause for concern.

This investigation has also highlighted the lack of knowledge of the nature of mathematics that seems to exist in our schools. I have suggested that there is a link between this lack of knowledge of the nature of mathematics and extrinsic reasons for studying mathematics. If this is the case, it is necessary that teachers pay attention to this aspect of mathematics. This, in turn, implies more attention being paid to the nature of mathematics during teacher training courses and provision for this topic being made in the mathematics syllabus. There is a need for a section on, for example, the history of mathematics to be placed somewhere in the syllabus. This, and similar topics, might be particularly helpful to slower pupils who "can't do maths". Consideration of the nature of mathematics may in fact lead to an improved ability to do mathematics.

As educators it is important that we do not fall into a "numbers trap". Where there is an adequate supply of mathematics teachers and where mathematics classrooms are full, it is dangerous to presume that mathematics education is healthy. We must avoid reaching the stage (if we are not there already), of having student teachers obtain credits in mathematics simply because it improves their chances of obtaining a teaching bursary; of teachers being promoted largely because they are mathematics teachers; of pupils doing mathematics because of the "reward" awaiting them at the end of Std 10 or because of a supposed superior status of mathematics over other subjects in the school curriculum. Pupils must learn to appreciate and enjoy mathematics <u>itself</u>. This can only happen if teachers convey to pupils something of this appreciation and enjoyment.

It is important that teachers develop their own, clear ideas about the nature of mathematics and also know why they are teaching mathematics, i.e. know what the objectives of teaching mathematics are. E. Begle said the following:

> "We have learnt a great deal about how to teach better mathematics, but very little about how to teach mathematics better."
>
> (in Hill 1976, p.444)

Unless we become better teachers, partly by paying attention to some of the shortcomings exposed by this investigation, I believe the future of mathematics education to be a bleak one.

APPENDIX 1

MATHEMATICS QUESTIONNAIRE

PART A
Please place a tick in the appropriate block.

1. PERSONAL PARTICULARS

1.1. Standard

8	550	68,75%
10	250	31,25%

1.2. Language of Mathematics Class

Afr.	-	-
Eng.	800	100%

1.3. Sex

Male	448	56,2%
Female	349	43,8%

(Errors - 3)

1.4. Type of School (a)

Boys	142	17,8%
Girls	86	10,8%
Co-Ed.	571	71,4%

(Errors - 1)

(b)

Academic	800	100%
Commercial	-	-
Technical	-	-

1.5. Mathematical Grade

H.G.	370	46,4%
S.G.	427	53,6%

(Errors - 3)

1.6. "In my last mathematics examination I obtained ...."
(If this mark was significantly higher or lower than your usual mark, please write "higher" or "lower" behind your tick.)

80-100% (A) 70-79% (B) 60-69% (C)	281 35,5%
50-59% (D) 40-49% (E)	296 37,4%
33,3-39%(F)	86/10,9%
30-33% (FF)	43/ 5,4%
20-29% (G) 0-19% (H)	85 10,8%

1.7. "I am completing this questionnaire ...."

at home	
at school	

PERSONAL OPINIONS

2.1. "I find mathematics to be ...."

interesting	652 83,3%
uninteresting	131 16,7%

(Errors - 17)

2.2. "I find mathematics to be ...."

worthwhile	721 91,2%
pointless/a waste of time	70 8,8%

(Errors - 9)

2.3. "I am doing mathematics because ...."
N.B. (Place a "1" against the most important reason and "ticks" against less important reasons.)

mathematics is compulsory at my school.	40 5,9%
my family persuaded me to carry on with the subject.	21 3,2%
my teachers persuaded me to carry on with the subject.	1 0,1%
I enjoy the subject.	135/9,9%
it is an easy subject.	6/0,9%
it is necessary for job application/university entrance purposes.	287 42,4%
I like my mathematics teacher.	3 0,4%
it is a useful subject.	138 20,4%
it trains me to think logically.	46 6,8%

Other reasons..............
................0..........
(Errors - 123)

2.4. Rate your subjects in your order of difficulty.

Easiest → Most difficult

54 / 6,8%
79 / 10,0%
109 / 13,7%
135 / 17,0%
195 / 24,6%
221 / 27,9%

2.5. Rate your subjects in your order of enjoyment.

Least enjoyable → Most enjoyable

94 / 11,9%
115 / 14,6%
148 / 18,7%
169 / 21,4%
160 / 20,2%
104 / 13,2%

2.6. Rate your subjects in your order of usefulness.

Least useful

↓

Most useful

56 /	7,0%
56 /	7,0%
67 /	8,5%
117 /	14,8%
200 /	25,3%
296 /	37,4%

PART B

Please answer the following questions.

1. Define mathematics as briefly as possible.

....
.... Full ..... 179 / 22,4% ....
.... Part ..... 301 / 37,6% ....
.... X .... 320 / 40,0% ....

2. Name three famous male mathematicians who have made important contributions to mathematics.

....
..... 3 .... 189 / 23,6% ....
... 1 or 2 .... 411 / 51,4% ....
..... X .... 200 / 25,0% ....

3. Name three famous female mathematicians who have made important contributions to mathematics.

....
..... 3 .... 0 ....
... 1 or 2 .... 0 ....
..... X .... 800 / 100% ....

4. Is man's knowledge of mathematics complete or is mathematics being developed all the time?

....
....

5. "The important thing is to do mathematics and understand later what one has done." Is this statement true or false?

....
....

6. Do you believe that the mathematics that you learn at school can be useful outside the mathematics classroom? Yes or no?

....
....

Yes – 617 / 78,7%
No – 167 / 21,3%
Errors - 16

7. Name <u>three</u> careers for which mathematics is essential.

....
.... 3 ..... 595 / 74,4% ....
.... 1 or 2 ... 181 / 22,6% ....
.... X ..... 24 / 3,0% ....

8. Give <u>one</u> example of how mathematics is used/involved in each of your other subjects.

....
.... 3 .... 200 / 25,0% ....
.... 1 or 2 ...... 475 / 59,4% ....
.... X .... 125 / 15,6% ....
....
....
....

9. If a pupil uses algebra to solve a geometric problem or vice versa, should the pupil be penalised, i.e. marks deducted?

....
....

10. Give a brief definition of the concept "axiom".

....
.... Full .... 143 / 17,9% ....
.... Part .... 76 / 9,5% ....
.... X .... 581 / 72,6% ....

11. What is the difference between a "theorem" and a "theory"?

....
....
....

12. Is the following an example of an axiom, theorem or theory?

X Y Z $\qquad\qquad X\hat{Y}Z = 180^\circ$

....
....

13. What is the most important difference between a theorem and other geometric problems?

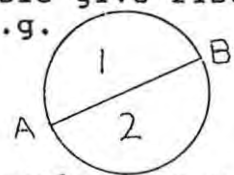
....
....
....

14. Give a brief definition of "deductive reasoning".

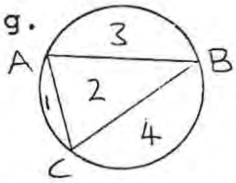
....
.... Full .... 5 / 0,6% ....
.... Part .... 189 / 23,6% ....
.... X .... 606 / 75,8% ....

15. The following is an example of inductive reasoning (generalising):

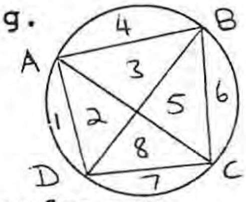
Lines joining two points on a circle give rise to two (2^1) regions in the circle, e.g.



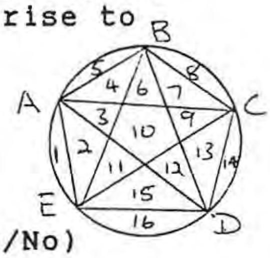
Three points ⟶ four (2^2) regions, e.g.



Four points ⟶ eight (2^3) regions, e.g.



We seem to be concerned with powers of two, therefore we would expect lines joining five points to give rise to 16 (2^4) regions. This is in fact correct, e.g.



Does this form of reasoning prove results? (Yes/No)
....
....

From the examples given above, is it true that lines joining 9 points would give rise to 256 (or 2^8) regions? (True/False)
....
....

Both correct	95 / 11,9%	
One correct	609 / 76,1%	
X	96 / 12,0%	

****

THANK YOU FOR YOUR CO-OPERATION

APPENDIX 2

RESPONSES TO PART A - FREQUENCIES AND PERCENTAGES

Question 2.1. - Interest

		Interesting	Uninteresting	Total
Std	8	458/85,6%	77/14,4%	535
	10	194/78,2%	54/21,8%	248
	Total	652/83,3%	131/16,7%	783

Yates' corr. Chisq. = 6,108 ; p = 0,0135

		Interesting	Uninteresting	Total
Grade	HG	316/87,1%	47/12,9%	363
	SG	335/80,3%	82/19,7%	417
	Total	651/83,5%	129/16,5%	780

Yates, corr. Chisq. = 5,865 ; p = 0,0154

		Interesting	Uninteresting	Total
Sex	Boys	353/81,3%	81/18,7%	434
	Girls	296/85,5%	50/14,5%	346
	Total	649/83,2%	131/16,8%	780

Yates, corr. Chisq. = 2,153 ; p = 0,1423

		Interesting	Uninteresting	Total
Marks	60-100	259/92,8%	20/ 7,2%	279
	40-59	240/83,9%	46/16,1%	286
	33,3-39	68/79,1%	18/20,9%	86
	30-33	25/61,0%	16/39,0%	41
	0-29	55/66,3%	28/33,7%	83
	Total	647/83,5%	128/16,5%	775

Chisq. = 51,847 ; p = 0,0000

		Interesting	Uninteresting	Total
School	Boys'	109/79,0%	29/21,0%	138
	Girls'	81/94,2%	5/ 5,8%	86
	Co-ed.	461/82,6%	97/17,4%	558
	Total	651/83,2%	131/16,8%	782

Chisq. = 9,335 ; p = 0,0094

Question 2.2. - Worth

		Worthwhile	Waste of time	Total
Std	8	504/92,6%	40/ 7,4%	544
	10	217/87,9%	30/12,1%	247
	Total	721/91,2%	70/ 8,8%	791

Yates' corr. Chisq. = 4,262 ; p = 0,039

Grade	HG	344/94,0%	22/ 6,0%	366
	SG	375/88,9%	47/11,1%	422
	Total	719/91,2%	69/ 8,8%	788

Yates' corr. Chisq. = 5,822 ; p = 0,0158

Sex	Boys	404/90,8%	41/ 9,2%	445
	Girls	314/91,5%	29/ 8,5%	343
	Total	718/91,1%	70/ 8,9%	788

Yates' corr. Chisq. = 0,06 ; p = 0,8066

Marks	60-100	266/95,3%	13/ 4,7%	279
	40-59	274/93,5%	19/ 6,5%	293
	33,3-39	74/88.1%	10/11,9%	84
	30-33	33/76,7%	10/23,3%	43
	0-29	65/78,3%	18/21,7%	83
	Total	712/91,0%	70/ 9,0%	782

Chisq. = 36,705 ; p = 0,0000

School	Boys'	135/95,1%	7/ 4,9%	142
	Girls'	85/98,8%	1/ 1,2%	86
	Co-ed.	500/89,0%	62/11,0%	562
	Total	720/91,1%	70/ 8,9%	790

Chisq. = 12,309 ; p = 0,0021

Question 2.4. - Difficulty

		Frequencies					
	Most difficult A	B	C	D	E	Least difficult F	Total
Std 8	152	136	101	73	49	34	545
10	69	59	34	36	30	20	248
Total	221	195	135	109	79	54	793

Chisq. = 5,069 ; p = 0,4082

	A	B	C	D	E	F	Total
Grade HG	102	95	66	50	32	21	366
SG	119	99	68	59	47	32	424
Total	221	194	134	109	79	53	790

Chisq. = 3,052 ; p = 0,6919

	A	B	C	D	E	F	Total
Sex Boys	115	111	74	65	47	33	445
Girls	105	84	59	44	32	21	345
Total	220	195	133	109	79	54	790

Chisq. = 2,833 ; p = 0,7258

	A	B	C	D	E	F	Total
Mark 60-100	33	54	53	50	48	41	279
40-59	77	82	57	42	27	10	295
33,3-39	32	28	13	11	1	1	86
30-33	24	10	3	2	1	1	41
0-29	52	19	7	3	2	1	84
Total	218	193	133	108	79	54	785

No Chisq. - n < 5

	A	B	C	D	E	F	Total
School Boys'	23	36	25	28	17	12	141
Girls'	27	25	14	10	5	5	86
Co-ed.	171	133	96	71	57	37	565
Total	221	194	135	109	79	54	792

Chisq. = 16,943 ; p = 0,0756

Question 2.4. - Difficulty

Percentages

		Most difficult A	B	C	D	E	Least difficult F	Total
Std	8	27,9	25,0	18,5	13,4	9,0	6,2	545
	10	27,8	23,8	13,7	14,5	12,1	8,1	248
								793
	Total	27,9	24,6	17,0	13,7	10,0	6,8	

Chisq. = 5,069 ; p = 0,4082

		A	B	C	D	E	F	Total
Grade	HG	27,9	26,0	18,0	13,7	8,7	5,7	366
	SG	28,1	23,4	16,0	13,9	11,1	7,5	424
								790
	Total	28,0	24,6	16,9	13,8	10,0	6,7	

Chisq. = 3,052 ; P = 0,6919

		A	B	C	D	E	F	Total
Sex	Boys	25,9	24,9	16,6	14,6	10,6	7,4	445
	Girls	30,4	24,3	17,1	12,8	9,3	6,1	345
								790
	Total	27,9	24,7	16,8	13,8	10,0	6,8	

Chisq. = 2,833 ; P = 0,7258

		A	B	C	D	E	F	Total
Mark	60-100	11,8	19,4	19,0	17,9	17,2	14,7	279
	40-59	26,1	27,8	19,3	14,2	9,2	3,4	295
	33,3-39	37,2	32,5	15,1	12,8	1,2	1,2	86
	30-33	58,6	24,4	7,3	4,9	2,4	2,4	41
	0-29	61,9	22,6	8,3	3,6	2,4	1,2	84
								785
	Total	27,7	24,6	16,9	13,8	10,1	6,9	

No Chisq. - n < 5

		A	B	C	D	E	F	Total
School	Boys'	16,3	25,5	17,7	19,9	12,1	8,5	141
	Girls'	31,4	29,1	16,3	11,6	5,8	5,8	86
	Co-ed.	30,3	23,5	17,0	12,6	10,1	6,5	792
								792
	Total	27,9	24,5	17,0	13,8	10,0	6,8	

Chisq. = 16,943 ; p = 0,0756

Question 2.5. - Enjoyment

		Frequencies					
	Most enjoyable A	B	C	D	E	Least enjoyable F	Total
Std 8	60	103	127	115	81	56	542
10	44	57	42	33	34	38	248
Total	104	160	169	148	115	94	790

Chisq. = 19,864 ; p = 0,0013

	A	B	C	D	E	F	Total
Grade HG	60	76	85	61	50	33	365
SG	44	83	84	87	63	61	422
Total	104	159	169	148	113	94	787

Chisq. = 13,12 ; p = 0,0223

	A	B	C	D	E	F	Total
Sex Boys	69	81	103	78	61	52	444
Girls	34	78	66	70	53	42	343
Total	103	159	169	148	114	94	787

Chisq. = 9,299 ; p = 0,0977

	A	B	C	D	E	F	Total
Mark 60-100	55	65	66	51	29	13	279
40-59	32	65	61	59	48	29	294
33,3-39	6	17	17	16	14	15	85
30-33	1	5	6	7	8	14	41
0-29	10	6	18	14	15	21	84
Total	104	158	168	147	114	92	783

No Chisq. - n < 5

	A	B	C	D	E	F	Total
School Boys'	15	24	29	29	26	17	140
Girls'	11	18	23	12	14	7	85
Co-ed.	78	118	116	107	75	70	564
Total	104	160	168	148	115	94	789

Chisq. = 7,885 ; p = 0,6401

Question 2.5. - Enjoyment

	Percentages Most enjoyable A	B	C	D	E	Least enjoyable F	Total
Std 8	11.1	19,0	23,5	21,2	14,9	10,3	542
10	17,8	23,0	16,9	13,3	13,7	15,3	248
							790
Total	13,1	20,3	21,4	18,7	14,6	11,9	

Chisq. = 19,864 ; p = 0,0013

	A	B	C	D	E	F	Total
Grade HG	16,5	20,8	23,3	16,7	13,7	9,0	365
SG	10,4	19,7	19,9	20,6	14,9	14,5	422
							787
Total	13,2	20,2	21,5	18,8	14,4	11,9	

Chisq. = 13,12 ; p = 0,0223

	A	B	C	D	E	F	Total
Sex Boys	15,6	18,2	23,2	17,6	13,7	11,7	444
Girls	9,9	22,8	19,2	20,4	15,5	12,2	343
							787
Total	13,1	20,2	21,5	18,8	14,5	11,9	

Chisq. = 9,299 ; p = 0,0977

	A	B	C	D	E	F	Total
Mark 60-100	19,7	23,2	23,7	18,3	10,4	4,7	279
40-59	10,9	22,1	20,7	20,1	16,3	9,9	294
33,3-39	7,1	20,0	20,0	18,8	16,5	17,6	85
30-33	2,5	12,2	14,6	17,1	19,5	34,1	41
0-29	11,9	7,1	21,4	16,7	17,9	25,0	84
							783
Total	13,2	20,2	21,5	18,8	14,6	11,7	

No Chisq. - n < 5

	A	B	C	D	E	F	Total
School Boys'	10,7	17,2	20,7	20,7	18,6	12,1	140
Girls'	12,9	21,2	27,1	14,1	16,5	8,2	85
Co-ed.	13,8	20,9	20,6	19,0	13,3	12,4	564
							789
Total	13,1	20,3	21,3	18,8	14,6	11,9	

Chisq. = 7,885 ; p = 0,6401

Question 2.6. - Usefulness

		Frequencies						
		Most useful A	B	C	D	E	Least useful F	Total
Std	8	217	136	73	48	34	35	543
	10	79	64	44	19	22	21	249
	Total	296	200	117	67	56	56	792

Chisq. = 8,041 ; p = 0,1540

		A	B	C	D	E	F	Total
Grade	HG	145	95	51	28	27	20	366
	SG	150	104	66	38	29	36	423
	Total	295	199	117	66	56	56	789

Chisq. = 4,478 ; p = 0,4828

		A	B	C	D	E	F	Total
Sex	Boys	197	105	56	24	29	35	446
	Girls	99	93	61	43	26	21	343
	Total	296	198	117	67	55	56	789

Chisq. = 29,495 ; p = 0,0000

		A	B	C	D	E	F	Total
Mark	60-100	120	65	45	21	12	17	280
	40-59	105	87	37	23	26	17	295
	33,3-39	25	22	19	8	8	4	86
	30-33	16	9	4	5	2	4	40
	0-29	27	17	10	9	6	14	83
	Total	293	200	115	66	54	56	784

No Chisq. - n < 5

		A	B	C	D	E	F	Total
School	Boys'	56	32	24	5	11	14	142
	Girls'	35	20	14	10	4	3	86
	Co-ed.	204	148	79	52	41	39	563
	Total	295	200	117	67	56	56	791

No Chisq. - n < 5

Question 2.6. - Usefulness

		Most useful A	B	C	D	E	Least useful F	Total
Std	8	40,0	25,0	13,5	8,8	6,3	6,4	543
	10	31,7	25,7	17,7	7,6	8,9	8,4	249
								792
	Total	37,4	25,3	14,8	8,5	7,0	7,0	

Chisq. = 8,041 ; p = 0,1540

		A	B	C	D	E	F	Total
Grade	HG	39,6	25,9	13,9	7,7	7,4	5,5	366
	SG	35,4	24,6	15,6	9,0	6,9	8,5	423
								789
	Total	37,4	25,2	14,8	8,4	7,1	7,1	

Chisq. = 4,478 ; p = 0,4828

		A	B	C	D	E	F	Total
Sex	Boys	44,2	23,5	12,6	5,4	6,5	7,8	446
	Girls	28,9	27,1	17,8	12,5	7,6	6,1	343
								789
	Total	37,5	25,1	14,8	8,5	7,0	7,1	

Chisq. = 29,495 ; p = 0,0000

		A	B	C	D	E	F	Total
Mark	60-100	42,8	23,2	16,1	7,5	4,3	6,1	280
	40-59	35,6	29,5	12,5	7,8	8,8	5,8	295
	33,3-39	29,1	25,6	22,1	9,3	9,3	4,6	86
	30-33	40,0	22,5	10,0	12,5	5,0	10,0	40
	0-29	32,5	20,5	12,1	10,8	7,2	16,9	83
								784
	Total	37,4	25,5	14,7	8,4	6,9	7,1	

No Chisq. - n < 5

		A	B	C	D	E	F	Total
School	Boys'	39,5	22,5	16,9	3,5	7,7	9,9	142
	Girls'	40,7	23,2	16,3	11,6	4,7	3,5	86
	Co-ed.	36,2	26,3	14,0	9,3	7,3	6,9	563
								791
	Total	37,3	25,3	14,8	8,4	7,1	7,1	

No Chisq. - n < 5

RESPONSES TO PART B - FREQUENCIES AND PERCENTAGES

Question 1 - Definition of mathematics

		Full	Part	X	Total
Std	8	117/21,3%	242/44,0%	191/34,7%	550
	10	62/24,8%	59/23,6%	129/51,6%	250
	Total	179/22,4%	301/37,6%	320/40,0%	800

Chisq. = 32,199 ; p = 0,0000

		Full	Part	X	Total
Grade	HG	96/25,9%	158/42,7%	116/31,4%	370
	SG	82/19,2%	142/33,3%	203/47,5%	427
	Total	178/22,3%	300/37,7%	319/40,0%	797

Chisq. = 21,716 ; p = 0,0000

		Full	Part	X	Total
Sex	Boys	98/21,8%	175/39,1%	175/39,1%	448
	Girls	81/23.2%	124/35,5%	144/41,3%	349
	Total	179/22,5%	299/37,5%	319/40,0%	797

Chisq. = 1,045 ; p = 0,5931

		Full	Part	X	Total
Mark	60-100	86/30,6%	110/39,2%	85/30,2%	281
	40-59	67/22,6%	110/37,2%	119/40,2%	296
	33,3-39	12/14,0%	29/33,7%	45/52,3%	86
	30-33	7/16,2%	18/41,9%	18/41,9%	43
	0-29	5/ 5,9%	31/36,5%	49/57,6%	85
	Total	117/22,4%	298/37,7%	316/39,9%	791

Chisq. = 39,683 ; p = 0,0000

		Full	Part	X	Total
School	Boys'	35/24,6%	63/44,4%	44/31,0%	142
	Girls'	28/32,5%	33/38,4%	25/29,1%	86
	Co-ed.	116/20,3%	204/35,7%	251/44,0%	571
	Total	179/22,4%	300/37,5%	320/40,1%	799

Chisq. = 15,347 ; p = 0,0040

Question 2 - Male mathematicians

		3	1 or 2	X	Total
Std	8	100/18,2%	296/53,8%	154/28,0%	550
	10	89/35,6%	115/46,0%	46/18,4%	250
	Total	189/23,6%	411/51,4%	200/25.0%	800

Chisq. = 30,453 ; p = 0,0000

		3	1 or 2	X	Total
Grade	HG	105/28,4%	193/52,2%	72/19,4%	370
	SG	83/19,4%	216/50,6%	128/30,0%	427
	Total	188/23,6%	409/51,3%	200/25,1%	797

Chisq. = 15,551 ; p = 0,0004

		3	1 or 2	X	Total
Sex	Boys	131/29,2%	232/51,8%	85/19,0%	448
	Girls	57/16,3%	177/50,7%	115/33,0%	349
	Total	188/23,6%	409/51,3%	200/25,1%	797

Chisq. = 29,177 ; p = 0,0000

		3	1 or 2	X	Total
Mark	60-100	77/27,4%	152/54,1%	52/18,5%	281
	40-59	71/24,0%	155/52,4%	70/23,6%	296
	33,3-39	17/19,8%	40/46,5%	29/33,7%	86
	30-33	8/18,6%	21/48,8%	14/32,6%	43
	0-29	15/17,7%	38/44,7%	32/37,6%	85
	Total	188/23,8%	406/51,3%	197/24,9%	791

Chisq. = 19,661 ; p = 0,0117

		3	1 or 2	X	Total
School	Boys'	47/33,1%	88/62,0%	7/ 4,9%	142
	Girls'	23/26,8%	45/52,3%	18/20,9%	86
	Co-ed.	119/20,8%	277/48,5%	175/30,7%	571
	Total	189/23,7%	410/51,3%	200/25,0%	799

Chisq. = 42,344 ; p = 0,0000

Question 6 - Usefulness outside the classroom

		Yes	No	Total
Std	8	437/81,1%	102/18,9%	539
	10	180/73,5%	65/26,5%	245
	Total	617/78,7%	167/21,3%	784

Yates' corr. Chisq. = 5,369 ; p = 0,0159

		Yes	No	Total
Grade	HG	316/86,1%	51/13,9%	367
	SG	298/72,0%	116/28,0%	414
	Total	614/78,6%	167/21,4%	781

Yates' corr. Chisq. = 22,25 ; p = 0,0000

		Yes	No	Total
Sex	Boys	355/81,2%	82/18,8%	437
	Girls	261/75,9%	83/24,1%	344
	Total	616/78,9%	165/21,1%	781

Yates' corr. Chisq. = 3,009 ; p = 0,0828

		Yes	No	Total
Mark	60-100	234/84,5%	43/15,5%	277
	40-59	233/79,8%	59/20,2%	292
	33,3-39	63/75,0%	21/25,0%	84
	30-33	30/71,4%	12/28,6%	42
	0-29	50/62,5%	30/37,5%	80
	Total	610/78,7%	165/21,3%	775

Chisq. = 20,265 ; p = 0,0004

		Yes	No	Total
School	Boys'	120/86,3%	19/13,7%	139
	Girls'	74/86,0%	12/14,0%	86
	Co-ed.	422/75,6%	136/24,4%	558
	Total	616/78,7%	167/21,3%	783

Chisq. = 10,73 ; p = 0,0047

Question 7 - Use in careers

		3	1 or 2	X	Total
Std	8	400/72,7%	137/24,9%	13/ 2,4%	550
	10	195/78,0%	44/17,6%	11/ 4,4%	250
	Total	595/74,4%	181/22,6%	24/ 3,0%	800

Chisq. = 7,077 ; p = 0,0291

		3	1 or 2	X	Total
Grade	HG	281/76,0%	80/21,6%	9/ 2,4%	370
	SG	312/73,1%	100/23,4%	15/ 3,5%	427
	Total	593/74,4%	180/22,6%	24/ 3,0%	797

Chisq. = 1,273 ; p = 0,5292

		3	1 or 2	X	Total
Sex	Boys	349/77,9%	91/20,3%	8/ 1,8%	448
	Girls	245/70,2%	89/25,5%	15/ 4,3%	349
	Total	594/74,5%	180/22,6%	23/ 2,9%	797

Chisq. = 8,190 ; p = 0,0167

		3	1 or 2	X	Total
Mark	60-100	224/79,7%	54/19,2%	3/ 1,1%	281
	40-59	216/73,0%	68/23,0%	12/ 4,0%	296
	33,3-39	61/70,9%	24/27,9%	1/ 1,2%	86
	30-33	27/62,8%	13/30,2%	3/ 7,0%	43
	0-29	63/74,1%	18/21,2%	4/ 4,7%	85
	Total	591/74,7%	177/22,4%	23/ 2,9%	791

No Chisq. - n < 5

		3	1 or 2	X	Total
School	Boys'	119/83,8%	22/15,5%	1/ 0,7%	142
	Girls'	57/66,3%	29/33,7%	0/ 0,0%	86
	Co-ed.	418/73,2%	130/22,8%	23/ 4,0%	571
	Total	594/74,3%	181/22,7%	24/ 3,0%	799

No Chisq. - n < 5

Question 8 - Use in other subjects

		3	1 or 2	X	Total
Std	8	136/24,7%	330/60,0%	84/15,3%	550
	10	64/25,6%	145/58,0%	41/16,4%	250
	Total	200/25,0%	475/59,4%	125/15,6%	800

Chisq. = 0,308 ; p = 0,8573

		3	1 or 2	X	Total
Grade	HG	101/27,3%	229/61,9%	40/10,8%	370
	SG	98/23,0%	244/57,2%	85/10,8%	427
	Total	199/25,0%	473/59,3%	125/15,7%	797

Chisq. = 12,709 ; p = 0,0017

		3	1 or 2	X	Total
Sex	Boys	135/30,1%	244/54,5%	69/15,4%	448
	Girls	65/18,6%	228/65,3%	56/16,1%	349
	Total	200/25,1%	472/59,2%	125/15,7%	797

Chisq. = 14,318 ; p = 0,0008

		3	1 or 2	X	Total
Mark	60-100	81/21,8%	165/58,7%	35/12,5%	281
	40-59	70/23,6%	179/60,5%	47/15,9%	296
	33,3-39	16/18,6%	58/67,4%	12/14,0%	86
	30-33	11/25,6%	24/55,8%	8/18,6%	43
	0-29	22/25,9%	43/50,6%	20/23,5%	85
	Total	200/25,3%	469/59,3%	122/15,4%	791

Chisq. = 11,129 ; p = 0,1945

		3	1 or 2	X	Total
School	Boys'	36/25,3%	92/64,8%	14/ 9,9%	142
	Girls'	10/11,6%	60/69,8%	16/18,6%	86
	Co-ed.	154/27,0%	322/56,4%	95/16,6%	571
	Total	200/25,0%	474/59,3%	125/15,7%	799

Chisq. = 14,038 ; p = 0,0072

Question 10 - Definition of axiom

		Full	Part	X	Total
Std	8	73/13,3%	58/10,5%	419/76,2%	550
	10	70/28,0%	18/ 7,2%	162/64,8%	250
	Total	143/17,9%	76/ 9,5%	581/72,6%	800

Chisq. = 25,946 ; p = 0,0000

		Full	Part	X	Total
Grade	HG	100/27,0%	48/13,0%	222/60,0%	370
	SG	43/10,1%	28/ 6,5%	356/83,4%	427
	Total	143/18,0%	76/ 9,5%	578/72,5%	797

Chisq. = 55,255 ; p = 0,0000

		Full	Part	X	Total
Sex	Boys	83/18,5%	63/14,1%	302/67,4%	448
	Girls	60/17,2%	13/ 3,7%	276/79,1%	349
	Total	143/18,0%	76/ 9,5%	578/72,5%	797

Chisq. = 25,865 ; p = 0,0000

		Full	Part	X	Total
Mark	60-100	78/27,8%	25/ 8,9%	178/63,3%	281
	40-59	48/16,2%	31/10,5%	217/73,3%	296
	33,3-39	10/11,6%	10/11,6%	66/76,8%	86
	30-33	1/ 2,3%	5/11,6%	37/86,1%	43
	0-29	6/ 7,0%	5/ 5,9%	74/87,1%	85
	Total	143/18,1%	76/ 9,6%	572/72,3%	791

No Chisq. - n < 5

		Full	Part	X	Total
School	Boys'	29/20,4%	46/32,4%	67/47,2%	142
	Girls'	14/16,3%	2/ 2,3%	70/81,4%	86
	Co-ed.	100/17,5%	28/ 4,9%	443/77,6%	571
	Total	143/17,9%	76/ 9,5%	580/72,6%	799

No Chisq. - n < 5

Question 14 - Definition of "deductive reasoning"

		Full	Part	X	Total
Std	8	4/ 0,7%	157/28,6%	389/70,7%	550
	10	1/ 0,4%	32/12,8%	217/86,8%	250
	Total	5/ 0,6%	189/23,6%	606/75,8%	800

No Chisq. - n < 5

		Full	Part	X	Total
Grade	HG	4/ 1,1%	132/35,7%	234/63,2%	370
	SG	1/ 0.2%	56/13,1%	370/86,7%	427
	Total	5/ 0,6%	188/23,6%	604/75,8%	797

No Chisq. - n < 5

		Full	Part	X	Total
Sex	Boys	4/ 0,9%	125/27,9%	319/71,2%	448
	Girls	1/ 0,3%	63/18,0%	285/81,7%	349
	Total	5/ 0,6%	188/23,6%	604/75,8%	797

No Chisq. - n < 5

		Full	Part	X	Total
Mark	60-100	2/ 0,7%	87/31,0%	192/68,3%	281
	40-59	3/ 1,0%	60/20,3%	233/78,7%	296
	33,3-39	0/ 0,0%	20/23,3%	66/76,7%	86
	30-33	0/ 0,0%	9/20,9%	34/79,1%	43
	0-29	0/ 0,0%	12/14,1%	73/85,9%	85
	Total	5/ 0,6%	188/23,8%	598/75,6%	791

No Chisq. - n < 5

		Full	Part	X	Total
School	Boys'	2/ 1,4%	68/47,9%	72/50,7%	142
	Girls'	0/ 0,0%	9/10,5%	77/89,5%	86
	Co-ed.	3/ 0,5%	112/19,6%	456/79,9%	571
	Total	5/ 0,6%	189/23,7%	605/75,7%	799

No Chisq. - n < 5

Question 15 - Mathematical proof

		2 correct	1 correct	X	Total
Std	8	77/14,0%	402/73,1%	71/12,9%	550
	10	18/ 7,2%	207/82,8%	25/10,0%	250
	Total	95/11,9%	609/76,1%	96/12,0%	800

Chisq. = 10,033 ; p = 0,0066

		2 correct	1 correct	X	Total
Grade	HG	43/11,6%	296/80,0%	31/ 8,4%	370
	SG	52/12,2%	310/72,6%	65/15,2%	427
	Total	95/11,9%	606/76,0%	96/12,1%	797

Chisq. = 9,188 ; p = 0,0101

		2 correct	1 correct	X	Total
Sex	Boys	52/11,6%	344/76,8%	52/11,6%	448
	Girls	43/12,3%	262/75,1%	44/12,6%	349
	Total	95/11,9%	606/76,0%	96/12,1%	797

Chisq. = 0,323 ; p = 0,8510

		2 correct	1 correct	X	Total
Mark	60-100	32/11,4%	226/80,4%	23/ 8,2%	281
	40-59	36/12,2%	221/74,6%	39/13,2%	296
	33,3-39	8/ 9,3%	68/79,1%	10/11,6%	86
	30-33	9/20,9%	24/55,8%	10/23,3%	43
	0-29	9/10,6%	64/75,3%	12/14,1%	85
	Total	94/11,9%	603/76,2%	94/11,9%	791

Chisq. = 15,529 ; p = 0,0496

		2 correct	1 correct	X	Total
School	Boys'	14/ 9,8%	114/80,3%	14/ 9,9%	142
	Girls'	16/18,6%	67/77,9%	3/ 3,5%	86
	Co-ed.	65/11,4%	427/74,8%	79/13,8%	571
	Total	95/11,9%	608/76,1%	96/12,0%	799

No Chisq. - n < 5

APPENDIX 3

RESPONSES (IN PERCENTAGES) TO CERTAIN QUESTIONS ACCORDING TO MARK/GRADE GROUPINGS

Part A: Question 2.1. - Interest

	H.G.			S.G.		
Marks	Int.	Unint.	N	Int.	Unint.	N
60-100	92	8	182	95	5	97
40-59	84	16	131	84	16	155
33,3-39	83	17	29	77	23	57
30-33	50	50	8	66	34	32
0-29	82	18	11	65	35	71
			[361]			[412]

No Chisq.'s - n < 5

Part A: Question 2.2. - Worth

	H.G.			S.G.		
Marks	Worthwhile	Waste	N	Worthwhile	Waste	N
60-100	96	4	181	95	5	98
40-59	96	4	134	91	9	159
33,3-39	86	14	29	89	11	55
30-33	67	33	9	79	21	33
0-29	82	18	11	79	21	71
			[364]			[416]

No Chisq.'s - n < 5

Part A: Question 2.5. - Enjoyment

H.G.

Marks	A (Most enjoyable)	B	C	D	E	F (Least enjoyable)	N
60-100	19	20	26	16	13	6	182
40-59	14	21	21	19	16	9	135
33,3-39	14	28	18	4	18	18	28
30-33	0	13	13	37	0	37	8
0-29	18	10	27	18	0	27	11
							[364]

No Chisq. - n < 5

S.G.

Marks	A (Most enjoyable)	B	C	D	E	F (Least enjoyable)	N
60-100	21	29	20	23	5	2	97
40-59	8	23	20	21	17	11	159
33,3-39	4	16	21	25	16	18	57
30-33	3	13	16	13	22	33	32
0-29	11	7	21	17	19	25	72
							[417]

No Chisq. - n < 5

Part B: Question 1 - Definition of mathematics

Marks	H.G. 3	H.G. 1or2	H.G. X	H.G. N	S.G. 3	S.G. 1or2	S.G. X	S.G. N
60-100	32	45	23	183	28	29	43	98
40-59	24	40	36	136	22	34	44	160
33,3-39	10	38	52	29	16	32	52	57
30-33	12	44	44	9	16	42	42	33
0-29	0	55	45	11	7	34	59	73
				[368]				[421]

No Chisq.'s - n < 5

Part B: Question 15 - Mathematical proof

Marks	H.G. 2 correct	H.G. 1 correct	H.G. X	H.G. N	S.G. 2 correct	S.G. 1 correct	S.G. X	S.G. N
60-100	14	82	4	183	6	78	16	98
40-59	11	80	9	136	13	70	17	160
33,3-39	3	80	17	29	12	79	9	57
30-33	12	44	44	9	24	58	18	33
0-29	0	82	18	11	12	74	14	73
				[368]				[421]

No Chisq.'s - n < 5

RESPONSES (IN PERCENTAGES) TO CERTAIN QUESTIONS
ACCORDING TO SEX/SCHOOL GROUPINGS

Part A: Question 2.1. - Interest

School	Boys Int.	Boys Unint.	Boys N	Girls Int.	Girls Unint.	Girls N
Boys'	79	21	138	--	--	---
Girls'	--	--	---	94	6	86
Co-ed.	82	18	296	83	17	259
			[434]			[345]

Boys: Chisq. = 0,527
p = 0,4678

Girls: No Chisq.
n < 5

Part A: Question 2.2. - Worth

School	Boys: Worthwhile	Boys: Waste	Boys: N	Girls: Worthwhile	Girls: Waste	Girls: N
Boys'	95	5	142	--	--	---
Girls'	--	--	---	99	1	86
Co-ed.	89	11	303	89	11	256
			[445]			[342]

No Chisq.'s - n < 10

Part A: Question 2.6. - Usefulness of various subjects

Boys

School	A (Most useful)	B	C	D	E	F (Least useful)	N
Boys'	38	23	17	4	8	10	142
Co-ed.	46	24	11	6	6	7	304
							[446]

Chisq. = 7,189
p = 0,2069

Girls

School	A (Most useful)	B	C	D	E	F (Least useful)	N
Girls'	41	23	16	12	5	3	86
Co-ed.	25	28	18	13	9	7	256
							[342]

No Chisq. - n < 5

Part B: Question 1 - Definition of mathematics

School	Boys: Full	Boys: Part	Boys: X	Boys: N	Girls: Full	Girls: Part	Girls: X	Girls: N
Boys'	25	44	31	142	--	--	--	---
Girls'	--	--	--	---	33	38	29	86
Co-ed.	21	36	43	306	20	34	46	262
				[448]				[348]

Boys: Chisq. = 5,699
p = 0,0579

Girls: Chisq. = 8,709
p = 0,0129

Part B: Question 2 - Male mathematicians

School	Boys: 3	Boys: 1or2	Boys: X	Boys: N	Girls: 3	Girls: 1or2	Girls: X	Girls: N
Boys'	33	62	5	142	--	--	--	---
Girls'	--	--	--	---	27	52	21	86
Co-ed.	27	48	25	306	13	50	37	262
				[448]				[348]

Boys: Chisq. = 26,834, p = 0,0000

Girls: Chisq. = 12,636, p = 0,0018

Part B: Question 6 - Usefulness outside the classroom.

School	Boys: Yes	Boys: No	Boys: N	Girls: Yes	Girls: No	Girls: N
Boys'	86	14	139	--	--	---
Girls'	--	--	---	86	14	86
Co-ed.	79	21	298	72	28	257
			[437]			[343]

Boys: Chisq. = 2,999, p = 0,0833

Girls: Chisq. = 5,843, p = 0,0156

Part B: Question 7 - Use in careers

School	Boys: 3	Boys: 1or2	Boys: X	Boys: N	Girls: 3	Girls: 1or2	Girls: X	Girls: N
Boys'	84	15	1	142	--	--	--	---
Girls'	--	--	-	---	66	34	0	86
Co-ed.	75	23	2	306	71	23	6	262
				[448]				[348]

No Chisq.'s - n < 5

Part B: Question 8 - Use in other subjects

School	Boys: 3	Boys: 1or2	Boys: X	Boys: N	Girls: 3	Girls: 1or2	Girls: X	Girls: N
Boys'	25	65	10	142	--	--	--	---
Girls'	--	--	--	---	12	70	18	86
Co-ed.	32	15	18	306	21	64	15	262
				[448]				[348]

Boys: Chisq. = 9,793, p = 0,0075

Girls: Chisq. = 3,849, p = 0,1460

Part B: Question 10 - Definition of axiom

School	Boys: Full	Boys: Part	Boys: X	Boys: N	Girls: Full	Girls: Part	Girls: X	Girls: N
Boys'	20	32	48	142	--	-	--	---
Girls'	--	--	--	---	16	2	82	86
Co-ed.	18	6	76	306	18	4	78	262
				[448]				[348]

Boys: Chisq. = 62,703, p = 0,0000

Girls: No Chisq. - n < 5

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