ANLYSIS OF GRADE 10 MATHEMATICAL LITERACY STUDENTS' ERRORS IN FINANCIAL MATHEMATICS

By

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DECLARATION

I hereby declare that this thesis is my own unaided work and that any assistance received has been fully acknowledged in the text. No part of this thesis has been previously submitted to any Higher Education institution inside the country and abroad.

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January 2014

ABSTRACT

The main aim of the study was (1) to identify errors committed by learners in financial mathematics and (2) to understand why learners continue to make such errors so that mechanisms to avoid such errors could be devised. The following has been hypothesised; (1) errors committed by learners are not impact upon by language difficulties, (2) errors committed by learners in financial mathematics are not due to prerequisite skills, facts and concepts, (3) errors committed by learners in financial mathematics are not due to the application of irrelevant rules and strategies.

Having used Polya's problem-solving techniques, Threshold Concept and Newman's Error Analysis as the theoretical frameworks for the study, a four-point Likert scale and three content-based structured-interview questionnaires were developed to address the research questions. The study was conducted by means of a case study guided by the positivists' paradigm where the research sample comprised of 105 Grade-10 Mathematics Literacy learners as respondents. Four sets of structured-interview questionnaires were used for collecting data, aimed at addressing the main objective of the study. In order to test the reliability and consistency of the questionnaires for this study, Cronbach's Alpha was calculated for standardised items ($\alpha = 0.705$).

Content analysis and correlation analysis were employed to analyse the data. The three hypotheses of this study were tested using the ANOVA test and hence revealed that, (1) errors committed by learners in financial mathematics are not due to language difficulties, as all the variables illustrated a statistical non-significance (2) errors committed by learners in financial mathematics are not due to prerequisite skills, facts and concepts, as the majority of the variables showed non-significance and (3) errors committed by learners in financial mathematics were due to the application of irrelevant rules and strategies, as 66.7% of the variables illustrated a statistical significance to the related research question.

Keywords: Errors analysis, financial mathematics, mathematical literacy, irrelevant rules, language difficulty,

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DEDICATIONS:

To my late sister Unathi and my late grandmother Lathiwe Coderlia Khalo

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CHAPTER 1

BACKGROUND OF THE STUDY

1.0 INTRODUCTION

The main objective of the study was (1) to explore errors that learners commit when confronted by financial mathematics questions in different forms of assessment and (2) to develop an understanding of the reasons why learners continue to make such errors so that mechanisms to avoid such errors could be devised. In light of the objectives, the current research sought to explore (1) the underlying factors that relate to the identified types of errors that learners commit in financial mathematics, (2) bring about the findings and the recommendation to educators as well as learners as to the methods of eliminating those errors. This section of the study gives a background by means of an elaborated definition of the concept error, mathematics, mathematical literacy, financial mathematics, hypothetical errors, the statement of the problem, research questions and the hypothesis, the significance, scope and limitations of the study.

In 2006, South Africa implemented a new curriculum; the National Curriculum Statements (NCS) with compulsory mathematics learning in the Further Education and Training (FET) band. All learners are required to take either Mathematics or Mathematical Literacy (ML) as one of the fundamental subjects.

> Mathematical Literacy provides learners with awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications in mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems (Department of Education, 2003a: 9).

To elaborate on the purpose of its introduction, North (2005: 34) states "Mathematical Literacy is an attempt to provide students in South Africa, the majority of who will never go to university, with skills to be able to function properly in the workplace and their lives." It is an attempt to educate our population about things like debts, loans, tax, interest rate, graphs and other mathematical issues we encounter on a daily basis. Mathematical Literacy is about helping our students to become mathematical literate people who are able to function in society.

Mathematical Literacy is needed in order to make sense of the mathematical content we come across in our everyday lives, for instance graphs and tables which always form part of data representation in newspapers and magazines. It is aimed at transforming the society and equip learners with mathematical skills to be used in real-life.

> "Learners who are mathematically literate should have the capacity and confidence to interpret any real-life context that they encounter and be able to identify and perform the techniques, calculations and/or other considerations needed to make sense of the context" (DoE, 2010:11).

It therefore equips' learners with problem-solving skills for mathematical content and the context of the problem. Most importantly learners should have the ability to apply both mathematical and non-mathematical techniques in any context when solving a problem and making informed decisions.

It is so unfortunate that learners seem to struggle to demonstrate competence in the subject, the reason being learners continue to commit errors in financial mathematics problems. This study sought to identify the type of errors learners commit and the underlying reason for committing those errors. Mathematical skills are needed to demonstrate competency in Mathematical Literacy and learners need to eliminate errors in their work.

1.1 BACKGROUND

A Mathematical Literacy (ML) teacher always administers different assessment tasks throughout the course of the year as per requirements of the National Curriculum Statements (NCS). These tasks are administered in order to determine learners' understanding of the concepts taught inside and outside the classroom. According to the Department of Education (2010: 101), "assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and thereby assist the learners' development in order to improve the process of learning and teaching".

Surprisingly, learners who seem to follow the trend of the lessons, commit errors when working out the tasks assigned. That stimulated the researcher to critique, understand and conduct research study to try and find answers to what type of errors are common among grade 10 ML learners in financial mathematics. Financial mathematics accounts for 35% weighting of the topics in the examination which indicates that it is very valuable in ML studies. It encompasses a number of basic mathematical skills such as: interpreting, communicating answers and calculating, number and calculations with numbers. This is where learners lose marks in their assessment tasks.

Financial mathematics is categorised as an Application Topic which according to DoE (2011: 13) "contain[s] the contexts related to scenarios involving daily life, workplace and business environment, and wider social, national and global issues that learners are expected to make sense of content and context". It includes: financial documents, tariff systems, income, expenditure, profit/loss, income-and-expenditure statements, budget, interest, banking, loans and investments.

This study focused on income, expenditure, profit/loss and interest as identified sections where the errors are most common and these types of errors are discussed extensively (cf Chapter 2, section 2.7).

There are different definitions of errors based on context, and the researcher sought to relate errors, mathematics, mathematical literacy and financial mathematics as alluded above.

1.2 ERRORS, MATHEMATICS, MATHEMATICAL LITERACY AND FINANCIAL MATHEMATICS

Error is (1) a mistake, (2) the condition of being wrong in opinion or conduct, (3) the amount of inaccuracy in a calculation or measuring device (Pollard, 1994: 270).

Cambourne (1988) as cited in Killen (2007: 3) defined learning as “a process that involves making connections, identifying patterns, and organizing previously unrelated bits of knowledge, behaviour and action into new patterned whole”. Learners do make mistakes in the aforementioned process of learning. In Mathematics and Mathematical Literacy they are expected to employ different algorithms, adhere to inculcated rules and strategies.

Mathematics is the abstract science of numbers, quantity and space either as abstract concepts (pure mathematics) or as applied to other disciplines such as physics and engineering (applied mathematics). The dissimilarity between Mathematics and Mathematical Literacy is the content; the complexity and the fact that ML is more contextual whereas Mathematics is more abstract (Wiens, 2007).

Financial mathematics is one of the topics common to the school subject as it is a collection of mathematical techniques that find application in finances. Thus the research sought to interrogate the concerns (cf chapter 1 section 1.6): If learners are made aware of the kind of errors they commit and taught skills to avoid them, would that bring about change in their performance in the subject? If these errors keep appearing in all assessments, how much effect do they have on learner performance in formal assessment?
Wiens (2007:1) states that “assessment is such an important piece of educating my students and the careless errors made on these assessments need to be addressed.” Financial mathematics which is an Application Topic constitutes 35% in the weighting per topic in the examination. Learners need to put more focus on and concentration on this topic in order to avoid unnecessary loss of marks. “…Teachers’ understanding of learner errors and misconceptions are

key to reform the visions in many countries" (Brodie, 2005:2-178). These errors according to Brodie (2005:2-179) are systematic and consistent across time and place, remarkably resistant to instruction, and extremely reasonable when viewed from the perspective of the learner.

Case in point: The following questions were included in Grade 10 ML learners' assessment task in Simple and Compound interest:

1. How long will it take R5100, invested at 9% simple interest per year to amount to R7 854?

In working out the first question, most learners only calculated 9% of R5100 and did not know what to do with that value. Some knew that they have to find the difference between R7854 and R5100, and could not proceed. Brodie (2005: 177) categorised that type of error as missing information.

2. If R12000 is invested at 9,5% simple interest per year, calculate the value of investment after 9 years and three months.

Most learners did not have a problem in calculating the interest but they could not calculate it over a period. They committed an error in writing 9 years 3 months as either a common fraction or a decimal. Most of them just wrote 9.3 as the value of period (n). That is an error due to deficient mastery of prerequisite skills, facts and concepts (Radatz, 1979: 164).

The response to the percentage reduction question, where learners had to calculate the amount excluding VAT from an amount where VAT was already included is always poorly answered. All of these suggest some variations or hypothetical error types discussed below (cf Chapter 1 section 1.4).

1.3 HYPOTHETICAL ERRORS

In classifying errors according to individual difficulties of learners, one should, of course, acknowledge that errors are also a function of other variables in the education process (Radatz, 1979: 164) where he classifies errors according to information processing such as:

Errors Due to Language Difficulties

In solving word problems learners should be familiar with the language used, mathematical concepts, symbols and the vocabulary. A misunderstanding of semantics of the mathematical text could be a source of learner errors. The research interrogated this form of error and its significant impact (cf Chapter 2).

Errors Due to Deficient Mastery of Prerequisite Skills, Facts, and Concepts

Deficit in basic prerequisites includes ignorance of algorithms, inadequate mastery of basic facts, incorrect procedures in applying mathematical techniques, and insufficient knowledge of necessary concepts and symbols. The research explored this form of error and its significant importance (cf Chapter 2).

Errors Due to Incorrect Association or Rigidity of Thinking

In elaborating on the Errors due to incorrect association or rigidity of thinking Radatz quoted Pippig (1975) in classifying this type of error: error of perseveration, error of interference, error of assimilation and error of negative transfer. The research explored this form of error and its significant importance (cf Chapter 2).

Error Due to the Application of Irrelevant Rules or Strategies

Use of irrelevant rules, incorrect algorithms and application of inadequate strategies is a cause of this particular type of error. The research explored this form of error and its significant importance (cf Chapter 2).

Additionally, Brodie (2005: 2-179) brought into the debate of learner errors “Situative” perspectives: Situative perspectives argue that what a learner says and does in the classroom make sense from the perspective of his/her current ways of knowing and being, his/her developing identity in relation to mathematics and to his/her previous experiences of learning mathematics, both in and out of school.

Brodie's coding scheme categorizes learner's contribution (cf Chapter 2, section 2.7) which he developed when engaged with learners in a discussion of a particular topic in class.

Furthermore, Melis (2003: 4) discovered the following type of errors for derivative problems and for computation with fractions:

- Missing or erroneous condition
- Misconception of quantifiers
- Missing case splits or missing sub-proofs (taken for granted)
- Erroneous variable handling
- Misconception of proof by refutation (strategic error)

For the domain 'derivation of (composite) functions' the type of errors are different. They include

- Misconception of the interpretation of the notion 'derivative'
- Misconception of composite functions
- Erroneous assumptions about composite functions
- Application of wrong derivation rules or wrong application of such rules
- Misconception about variables
- Misconception about variables missing domain conditions
- Slips in computations
- Arithmetic or algebraic errors.

From the discussion, it can be asserted that learners are confronted by a challenge of subjects, new approaches, which is more contextual based with more word problems. Sometimes teachers neglect to stress the importance of going through the work before submitting it in order to minimise error made in tasks submitted.

1.4 STATEMENT OF THE RESEARCH PROBLEM

The Grade 12 learners were introduced to Mathematical Literacy in 2010 when they were in Grade 10. They are supposed to have eliminated the errors they commit during their problem-solving in the subject. The above-mentioned synopsis suggests that learners continue to commit similar errors in their work when they are assessed even with the best teaching and learning strategies. Learners are also sometimes not even aware of the errors they commit. There is also a repetitive error in most of their assessment tasks throughout the year. However there is a negligible number of South African studies on the technical know-how of error analysis in particular with regard to G10 ML.

To be able to reduce and/or eliminate these errors, both learners and educators need to be able to (1) identify them and (2) understand why learners continue to make them and then be able to avoid them. The research focused on the mechanisms involved in errors as applied in financial mathematics.

Following the discussion thus far, the list below seeks to pose the research questions.

1.5 RESEARCH QUESTIONS

The following are the two main research questions of the study:

1. What errors do Grade 10 Mathematical Literacy learners commit in Financial Mathematics?
2. Why do learners commit errors on given tasks in financial mathematics?

The following are the five sub questions to the second research question:

2.1 What is/are the underlying factor(s) related to the errors due to incorrect association or rigidity of thinking?

2.2 What is/are the underlying factor(s) related to the errors due to the application of irrelevant rules or strategies?

2.3 What is/are the underlying factor(s) related to the errors due to deficient mastery of prerequisite skills, facts and concepts?

2.4 What is/are the underlying factor(s) related to the errors due to language difficulties?

2.5 What degree of predictability and hence strategies underpin error analysis in questions 1-4?

1.6 RESEARCH HYPOTHESES

Based on the background, statement of the problem and research questions, the following have been hypothesised:

HYPOTHESI 1

H_0: Errors committed by learners in financial mathematics are due to language difficulties.

H_1: Errors committed by learners in financial mathematics are not impacted upon by language difficulties.

HYPOTHESIS 2

H_0: Errors committed by learners in financial mathematics are due to prerequisite skills, facts and concepts.

H_1: Errors committed by learners in financial mathematics are not due to prerequisite skills, facts and concepts.

HYPOTHESIS 3

H_0: Errors committed by learners in financial mathematics are due to the application of irrelevant rules and strategies.

H_1: Errors committed by learners in financial mathematics are not due to the application of irrelevant rules and strategies.

1.7 SIGNIFICANCE OF THE STUDY

The research findings and recommendations may in multifaceted ways be useful for teachers of Grade 10, 11 and 12. With a greater level of predictability, these will include among others, to empower learners to go back over the task and to look for errors made before they hand in their work. That will encourage learners to self reflect on their work and therefore reduce errors in their work. The research findings will be valuable to the curriculum developers and policy makers of South Africa. The findings could be included in the Curriculum of Higher Education Institutions that train educators, thus equipping educators in training with skills and the knowledge of dealing with learner errors.

1.8 SCOPE AND LIMITATIONS OF THE STUDY

The current study was focused on Grade 10 Mathematical Literacy learners in the East London district of the Eastern Cape Province in South Africa. Mathematical Literacy is a new subject which has only been introduced in South Africa six years ago; therefore not much has been researched in the subject. This study focused on financial mathematics which constitutes 35% weighting in the assessment programme but seems to pose a challenge to learners.

Previous research has been done mainly on Mathematics not Mathematical Literacy and most academic articles on error analysis are based on Mathematics. Hence, the research will predominately source primary research based on international studies.

1.9 RESEARCH METHODOLOGY

A positivist paradigm which included a quantitative approach was used for the measurement of data in order to discover and confirm causes and effects. The selection of the case purposively included one East London district school; however, the respondents were selected using a simple random sample technique (cf Chapter 3).

1.9.1 Case study

Research was conducted with learners in Grade 10 ML from a secondary school which the researcher conveniently chose in the East London district of the Eastern Cape Province in South Africa. The researcher considered the accessibility, travel costs and the time frame when choosing this particular school (cf Chapter 3, sub-section 3.3.1).

1.9.2 Sample size (n) and Justification

This school currently has 5 Grade 10 ML classes with 186 learners that is population (N) of Grade 10 ML. There are 104 girls and 82 boys with ages ranging from 14 to 18 years. The researcher adopted the simplified formula by Yamane (Yamane 1967: 886) for proportions to determine the sample size (n), where e is the level of precision.

$$n = \frac{N}{1 + N(e)^2}$$

$$n = \frac{186}{1 + 186(0.05)^2}$$

$$n = 126.96 \approx 127 \text{ participants}$$

Hence the sample size will be nearly 127 where, N is the population size and assuming that confidence level is 95% and the level of precision is .5 (Yamane 1967: 886).

The calculated sample size was increased by 30% to compensate for non-participation of randomly selected respondents. After the sample size has been determined, the participants were selected by a simple random selection method (cf Chapter 3).

1.9.3 Data-collection methods

Data was collected by means of structured-interview questionnaires and documentary studies (examiners reports and other document on the subject published by the Department of Education). The documentary analysis was based on (1) why do learners commit errors on given tasks in financial mathematics (2) errors due to incorrect association or rigidity of thinking (3) errors due to the application of irrelevant rules or strategies (4) errors due to deficient mastery of prerequisite skills, facts and concepts and lastly (5) errors due to language difficulties (Cf Chapter 3).

Data-collection instruments

Four sets of structured-interview questionnaires (see Appendices A, B, C and D) were used; three content-based questionnaires where respondent were expected to work out financial mathematics problems and one set which includes the possible underlying factors related to the different types of errors learners commit. The fourth questionnaire with rating scale questions using a Likert scale will be used to collect data from the respondents (Grade 10 ML learners of participating school in the East London district).

The fourth questionnaire contains Likert scale type of questions and which has been used as a follow-up questionnaire. This questionnaire was based on answering (1) Why do learners commit errors on given tasks in financial mathematics? (2) The underlying factors related to the errors due to incorrect association or rigidity of thinking (3) The underlying factors related to the errors due to the application of irrelevant rules or strategies (4) The underlying factors related to errors due to deficient mastery of prerequisite skills, facts and concepts and lastly (5) The underlying factors related to errors due to language difficulties. The main discussion of this section is found in Chapter 4 of this study.

1.10 DATA ANALYSIS

In the first, second and third questionnaires which are content based the researcher will be guided by the Newman's error analysis in content analysis and identification of errors committed. Quantitative analysis with descriptive statistics which describe the distribution, the relationship among variables and variability through the use of frequencies will be used to analyse the fourth questionnaire. Statistical Package of Social Sciences (SPSS) version 21 will be used for correlation coefficient analysis to measure the relationship between variables of each of the afore-stated research questions. Analysis of Variance (ANOVA) will be used for testing the hypotheses of the study. The main discussion of this section is found in Chapter 4 of this study.

Management of TYPE I and TYPE II Errors in data analysis

TYPE I error is a type of error that occurs in the data-analysis stage where a researcher rejects the null hypothesis when in fact it is true, whereas TYPE II error is committed where a researcher accepts the null hypothesis when it is in fact not true. These errors directly affect the validity of the study.

The researcher has to be mindful of the place and significance of test, not forgetting the problem of the Hawthorne effect operating negatively or positively on students who have to undertake the tests (Cohen et al, 2007: 116). The Hawthorne effect is the phenomenon in which participants alter their behaviour as a result of being part of the study. The researcher ensured standardized procedures in administering the test. In the data-analysis stage, the researcher will avoid TYPE I and / or TYPE II errors by presenting the data without misrepresenting its meaning. By a pilot study the researcher ensured that the invalidity is minimized as much as possible throughout the study. That shows that validity of the study cannot be achieved through tests only but when the results of different tools (i.e. tests and questionnaires) used should be analysed concurrently.

"For research to be reliable it must be carried out on a similar group of respondents in a similar context (however defined), then similar results would be found" (Cohen et al, 2007: 117). To test the reliability and validity of the instruments the questionnaires was developed and administered as a pilot study. The main discussion of this section is found in Chapter 4 of this study.

1.11 CHAPTER DEMARCATION

Chapter 1: This chapter give an outline of the study and consists of the following section: Introduction, Background of the Study, Statement of the Research Problem, Significance of the Study, Limitations of the Study, Research Questions and Definition of Terms.

Chapter 2: This chapter interrogate the previous literature and the theoretical framework that guides this study through the following sub section: Review of Literature and Theoretical Framework of the Study.

Chapter 3: In addressing the research questions, different stages such as Research Methodology, Data-collection Methods and the description of the Research Participants and their involvement in the study is discussed in this chapter.

Chapter 4: Data collected has been analysed and the analysis and the illustration of that data is presented in this section as the following main sections: Data Presentation and Analysis

Chapter 5: The findings are discussed in this section under the following sub-heading: Discussion of Research Findings

Chapter 6: Summary of the study, Conclusions and Recommendations form part of this section

CHAPTER 2

LITERATURE REVIEW AND HYPOTHESES OF THE STUDY

2.0 INTRODUCTION

A number of articles have been published on error analysis in Mathematics but little has been published specifically on Mathematical Literacy. This Chapter brings conceptualization of Mathematical Literacy and Mathematics in a South African perspective by means of different definitions. In conceptualising the two subjects, the writer has to delve deep into the purpose of Mathematical Literacy as stipulated in the National Curriculum Statements and the characteristics drawn from the different definitions.

The literature reviewed interrogates “learner errors in Mathematics”; “type of errors for word problems”; the Radatz classification of errors which forms the foundation of the research questions of this study. Examples of learner errors on financial mathematics as cited on the chief marker’s report for 2012 also form part of this chapter, as the researcher brings into perspective common learner errors in Mathematical Literacy (in a South African perspective).

As a way forward, this study is aimed at gaining an understanding of learner errors, identifying the underlying factor structure of these errors which will be valuable to the educators’ understanding of the identified errors in order to assist in eliminating them at earlier stages of the FET-phase. It will also give a clear outline of the focus of the study; which is Error Analysis of Grade 10 Mathematical Literacy: a case of financial mathematics by means of the review of the different literatures on error analysis. The types of learner errors in Mathematics would be appropriate to form the focal essence of this study, and in conclusion the writer will elaborate on what the study brings to Mathematics Education.

2.1 CONCEPTUALIZATION OF MATHEMATICAL LITERACY AND MATHEMATICS

Mathematical Literacy and Mathematics are different subjects; they both deal with numbers and other mathematical knowledge and skills and are as a result interrelated even though they serve different purposes.

According to Brombacher (2007: 3), “Mathematics is designed to provide a tool of trade for professions such as mathematicians, engineers, physicists and economists. In their professions, mathematics enables them to solve problems that are mathematical in nature”.

For professional users of mathematics the priority is to use sophisticated mathematics in complex settings which require a deep understanding of the structure of the mathematics they use. Contrary to that, mathematical literate people use mathematics to make sense of their world, as their priority is to interpret and act on the day-to-day contexts that define their lives and the world in which they live.

Brombacher, (2007: 12) draws from the definitions brought into perspective by other writers, the following characteristics of Mathematical Literacy:

- It is a functional competency needed by individuals living in the twenty-first century
- It involves the confident and independent application of elementary mathematics in sophisticated and meaningful contexts
- It is developed through an interplay between mathematical content and relevant context
- It is different from mathematics, not in level or complexity, but rather in kind and purpose.

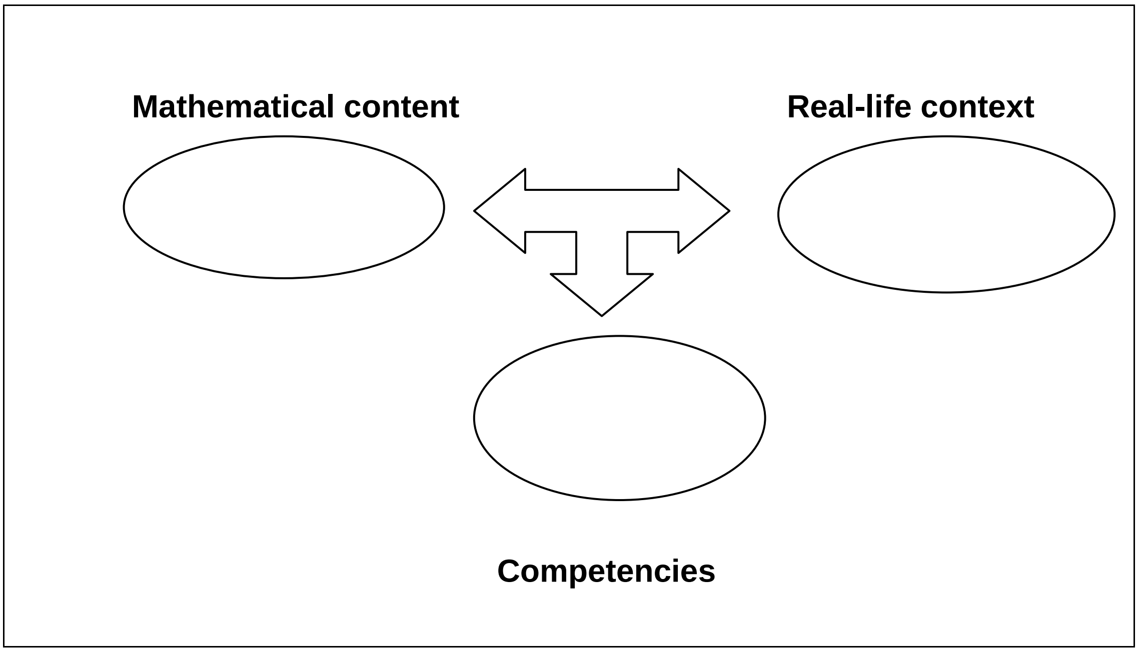

Figure 2.1: Summary of the purpose of the content and context of mathematical literacy (Adapted from Brombacher, 2007: 15)

The above sketch summarises the purpose of the content and context of mathematical literacy in developing life skills and competencies. Mathematical Literacy comprises the mathematical content fused in real-life contexts to develop the different competencies in the subject. According to Brombacher (2007: 15) elaborating, "those are competencies that the individual needs to participate in his/her world as a self-managing individual; as a contributing worker, as a life-long learner; and as a critical citizen".

Thus according to Gal (2009: 51), Mathematical Literacy curriculum is premised on the idea that an essential goal of an education of high-school graduates from all walks of life should be to prepare them for adult life after graduation. It emphasises that teaching of mathematical Literacy illustrates to students the relevance of learning knowledge to everyday life and its inherent linkage to and base in diverse real-world contexts, i.e. every day, societal, workplace life and task demands.

The primary aim of Mathematical Literacy is to equip learners with a set of skills that transcends both the mathematical content used in solving problems and the context in which the problem is situated. Learners develop the ability to devise and apply both mathematical and non-mathematical techniques. However this ability does not develop naturally but requires guidance from an educator. Learners pass through a number of challenges as they develop the confidence to solve problems. These challenges can be discerned by the errors they commit. The errors that learners commit according to Brodie (2005) are systematic and consistent across time and place, remarkably resistant to instruction, and extremely reasonable when viewed from learners' perspective.

2.2 FOCUS OF THE STUDY WITHIN A SOUTH AFRICAN CONTEXT

This study adds, in particular, to the small body of research in error analysis in ML. Its focus is the error analysis in Grade 10 Mathematical Literacy which is a school subject in a South African context but together with Mathematics, they are components of Mathematics education. That explains the writer's continuous reference to Mathematics education.

Strategies of more drill and practice have been replaced by regarding errors as a valuable source of learner thinking. Teachers find it difficult to escape from learners' mistakes so it is worthwhile finding out why learners make the mistakes. Mistakes can be entrenched on learners, so error analysis is the right step towards removing the causes of these errors in both mathematics and mathematical literacy.

Assessment in Mathematical Literacy is specifically focused on the Application Topics of finance, measurement, maps, plans and other representation of the physical world, data handling and probability. It is expected that the basic skills topics of interpreting and communicating answers and calculations, numbers and calculations with numbers and patterns, relationships and representations will be integrated throughout all topics (DoE, 2011: 96).

Formal assessment provides teachers with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject. During the process of administering assessment tasks, numerous errors could be identified. This study seeks to identify the underlying factor structure of the learner errors in financial mathematics. The study focuses on Grade 10 Mathematical Literacy but the author draws his influences from the Grade 12 Chief Markers' reports of the previous years which provide evidence that if untreated, these errors will persist until Grade 12.

With reference to the 2012 Mathematical Literacy Grade 12, paper 1 Chief Markers' Report cited numerous learner errors. These errors included the following:

Example 1: *Vuyo bought a chocolate cake that cost R43.60 (Incl. VAT).How much does the cake cost excluding Vat?*

Learners were given a Price that was VAT included and asked to calculate the amount excluding VAT. Grade 12 learners already know VAT is charged at 14% therefore decided to work-out 14% of the given amount and subtract the 14% from the given amount.

Incorrect method	**Correct method**
R43.60 × $\frac{14}{100}$ **= R6,10**	$\frac{R43,60}{1,14}$ **= R38.25**
Then R43.60 – R6.10 = R37.50 (Excl. VAT)	**OR**
	$\frac{R43,60 \times 100}{114}$ **= R38.25**

The learners ignored the fact that the amount given already had VAT, working out VAT from the given amount would be calculating VAT from the amount that already had VAT. This can be attributed to either: *Error due to the application of irrelevant rules and/or to deficient mastery of prerequisite skills*. The learner is able to work out VAT which is an indication that he/she has acquired the relevant skill but unfortunately could not ascertain that in this instance the procedure used was irrelevant as the amount was VAT inclusive. In considering the example illustrated above it is evident that by employing the irrelevant procedure, an incorrect answer was yielded to the problem posed.

Some learners employed the correct algorithms and it yielded ***38.245614*** as an answer. Learners were required to round off the answer to two decimal places. They could not correctly round off the final answer which should be rounded off correct to two decimal places. As learners were expected to round off the answer they committed all sorts of errors, some gave 38.24 and some gave 38.20 as a rounded-off answer while others left the answer as it was.

Example 2: *Below is an example extracted from the 2012 Grade 12 Mathematical Literacy Paper 1, Question 2.4 which formed part of the Chief Markers' Report.*

Kedibone has a cheque account with Iziko Bank charges a service fee up to a maximum of R31,50 (VAT included) on all transaction amounts.
TABLE 2.1 below shows five different transactions on Kedibone's cheque account.

NO.	DESCRIPTION OF TRANSACTION	TRANSACTION AMOUNT(in R)	SERVICE FEE (In R)
1	Debit order for car repayment	4 250.00	31.50
2	Debit order for cellphone contract	344.50	**A**
3	Personal loan repayment	924.00	14.59
4	Vehicle and household insurance	**B**	11.85
5	Cheque payment	403.46	8.34

2.4.1 Calculate the missing value A, using the formula:

Service fee (in rand) = 3.50 + 1.20% of the transaction amount

Learners are required to substitute the value of the transaction amount which is 344.50 in this instance.

Service fee = 3.50 + 1.20% × 344.50

Service fee = 3.50 + 0.012 × 344.50

Service fee = 3.50 + 4.134

Service fee = 7.634 ≈ 7.63

A learner would substitute the given formula correctly but then in 1.20%, they did not substitute the % sign (did not divide 1.20 by 100) as the % sign suggests. Error due to language difficulty and error due to application of irrelevant strategies can be attributed to this error. The learner deliberately ignored the percentage sign and that resulted in learners attaining an incorrect solution. Because the learner was working with money, the learner was supposed to round off the final answer to two decimal places. Some would incorrectly round off 7.634 to 7.65 instead of 7.63.

2.4.2. Calculate the missing value B, using the following Formula:

Amount (in Rand) = $\frac{service\ fee - 3.50}{1.2o\%}$

Amount (in Rand) = $\frac{11.85 - 3.50}{1.20\%}$

Amount (in Rand) = $\frac{8.35}{0.012}$

Amount (in Rand) = 695.8333333 ≈ 695.83

The same error arose in the above example. Learners would substitute correctly but did not substitute the correct value for the % sign (they divided by 1.20 instead of 0.012) and that would affect the final answer, which would be 6.95833333. They would round off correctly to two decimal places to 6.96 instead of 695.83.

Learners could not correctly substitute R344.50 on the given formula. In question 2.4.2 some learners were not able to substitute 11.85 divided by 1.20% and simplify (Chief Markers' Report 2012: 288).

Example 3: In Question 3.3.4 of Question 3 in the same paper, learners used the formula *A = P(1 + i)* even though they were required to calculate depreciation using the formula *A = P(1 – i)*. Language barrier is then evident in this instance as learners' responses illustrated that learners did not understand the meaning of "Depreciation". They perceived the minus sign (-) as an error in the formula and rectified it by replacing it with the plus sign (+). Some used the correct formula, substituted correctly but could not obtain the correct answer. This error could be an indication of incorrect calculator usage.

Rounding off was also highlighted as a challenge to learners as it is of great importance to financial mathematics where money is always given in two decimal places. In the aforementioned question, they were asked to give the answer to the nearest hundred.

If the few examples of errors cited by the Chief Marker in his/her report are considered, it is evident that there exists a huge demand for these kinds of errors to be eradicated at early stages of the Further Education and Training phase (FET-Phase) of the basic education. The researcher saw the need for an in-depth investigation into these kinds of errors as there are very few publications on error analysis in Mathematical Literacy.

I trust that if these kinds of errors can be detected at early stages (i.e. Grade 10 which is an entry level of the FET-phase), they could be rectified through remedial teaching intervention strategies.

From the afore-mentioned discussions, it can be asserted that learners are confronted by a challenge of the subject, its approach and the fact that it is more contextual based with more word problems. Sometimes teachers neglect to stress the importance of going through the work before submitting it in order to minimize errors made in tasks submitted. Teaching of and learning the studies including ML should be entrenched on the neuroscience and psychology of the learners.

2.3 NEUROSCIENCE AND PSYCHOLOGY OF TEACHING MATHEMATICS

The neuroscience of mathematics learning and psychology of mathematics learning have produced soundly conducted research, providing empirical evidence for learning theories and even common-sense ideas about what it means to learn mathematics (Soendergaard and Cachaper, 2008: 8). They also argue that although our brains are very much the same, every single person is neither the same as everyone else nor are they different. The cerebrum is divided into two hemispheres: the left and the right hemispheres, and is regarded as the seat of higher intellectual activities such as memory, reasoning, willpower and judgment. Both sides manipulate Arabic numbers and numerical quantities, however only the left side has access to linguistic connections and verbal memory of arithmetic tables.

Furthermore, the right hemisphere “approximates” while the left hemisphere calculates precisely. Therefore the inputs and the outputs of these areas must be integrated in order for people to estimate and calculate correctly. Lack of integration is one potential explanation for dyscalculia.

They substantiate it further by mentioning that children with dyscalculia (the difficulty that some children have in mathematics) need another type of teaching than other learners, as their brain is wired differently.

2.3.1 Mathematical thinking

Studies have shown that mathematical thinking can be described in terms of two distinct but interrelated components: (1) a non-verbal spatial understanding of quantity and (2) a verbal understanding that is related to language and symbolic reasoning. The aforementioned description of mathematical thinking can be closely correlated to the learning process of mathematical literacy.

Furthermore in the discussion Soendergaard and Cachaper (2008: 15) bring about a concept of “working memory” which they state is critical to mathematics learning because mathematics lessons demand that learners need to remember intermediate products of calculations in order to solve the problems. Good working memory has therefore been shown to be correlated with successful mathematics learning. Basic mathematical skills are vital to illustrate competency in the learning of mathematical literacy.

Goswami (2008: 282) states that: “small amounts of training can lead to rapid improvement in the strategic use of rehearsal, with accompanying improvement in recall”. The recall of the basic formula and the relevant algorithms is an important skill as by learning financial skills use of formula may be required.

2.3.2 Implications of cognitive neuroscience in teaching and learning

Ansari, Coch and De Smedt (2011) pointed out that people interested in learning and education might naturally want to know how results from cognitive neuroscience research could be applied in the classroom. Given that the brain is the 'organ of learning', it seems logical that knowledge about how the brain works should be able to inform education. The history revealed that implementation of research results to solve the problem is often indirect and rarely straightforward. Development of the field of Mind, Brain and Education and Collaboration will require much more complex than direct route from neuroscience laboratory to the classroom.

Based on the assumption that Mind, Brain, and Education should be framed in terms of interactions and based on mutually beneficial dialogue among participants with the knowledge of child development learning and teaching, will ensure that no knowledge hierarchy is created in which educators are merely the recipient of the information generated by neuroscientists. Cognitive Neuroscience should become a fundamental part of teacher education. This will assist teachers in a deeper understanding of child development and the biological constraints placed on the learning process. Educators might discuss with cognitive neuroscientists the different strategies they observed children using to solve a particular problem in the classroom and that could produce deep descriptions of classroom learning into the neuroscience realm.

As aforementioned teacher education programmes need to integrate cognitive neuroscience not only on the basic introduction of structural and functional brain development, but also on core domains of cognitive functions such as typical and atypical development of reading and mathematical skills, it should also discuss topics of relevance to education such as the effect of culture on brain function.

"In order to understand and better support human learning and development in their students, teachers need to know what science analysis, from multiple perspectives" (Ansari et al, 2011: 40) contributes. A deeper knowledge and understanding of cognitive neuroscience can assist a teacher to administer efficient and effective teaching in the classroom.

“Teachers who are able to critically evaluate the science to which they are exposed will not only avoid heeding advice based on inaccurate data and pseudoscience, but also will force the producers of education related literature on the brain to provide more sophisticated and accurate information” (Ansari et al, 2011: 41). This implies that teachers should also produce information on cognitive neuroscience as they are constantly in contact with learners. They need to be fully aware of how learners think.

Ansari et al (2011) ascertain that the interactions between education and neuroscience may also help to evaluate the relative benefits of arts and science education and thereby change the way in which we view educational priorities.

2.4 UNDERSTANDING IN THE LEARNING PROCESS OF MATHEMATICS

Understanding in the learning process of mathematics can be categorised in two, namely: (1) Instrumental understanding is demonstrated by someone who uses rules without understanding (rules such as to divide by a fraction you turn it upside down and multiply), (2) Relational understanding occurs when one has built up a conceptual structure of mathematics.

“Working memory is especially critical to mathematics learning because mathematics learning places frequent demands on working memory” (Cathercole et al. as cited in Soendergaard and Cachaper, 2008:15). Students must remember intermediate products of calculations in order to solve problems. Interconnected problems are more common in financial mathematics especially in the income, expenditure and taxation sections. Good working memory has therefore been shown to be correlated with successful mathematics learning.

“Relational understanding/thinking occurs when one has built a conceptual structure (Schema) of mathematics and therefore both know what to do and why when one solves a mathematical problem” (Soendergaard and Cachaper, 2008: 16). When dealing with simple and compound interest, the interest may be compounded monthly for three years; that then demands rational thinking of the fact that: three years is thirty six months in trying to find the value of *n*. Rational thinking needs to be developed through teaching and learning in the classroom and thus will play a major role in eliminating or reducing the errors committed by learners.

Soendergaard and Cachaper (2008: 19) further states that, “the concept of helping students monitor their own capacity to learn and engage in mathematical thinking has received little attention thus far. Teachers are typically not taught to enhance this skill.” This is the skill that can help equip learners to assist them in detecting and further eliminating the errors they commit during problem-solving.

Research in the last two decades showed a clear advantage to learning mathematics in a learner’s native language. For instance Adefula (1990) investigated the effect of presenting arithmetic word problems in learners’ native language or English to Nigerian learners. His findings indicated that the learners performed better when the word problems were presented in their native language.

There is a popular idea that all learners are special and different. However, evidence does not support this belief. Mathematics does not differ fundamentally across cultures, countries, or gender. The difference between the writer and one of his African ancestors of 100,000 years ago is not in the brain or genes, which are basically the same, but in the accumulated knowledge made possible by art, literature and technology. In elaborating on the afore-stated concepts on the learning process of mathematics, this study is based on the three theoretical frameworks expanded below.

2.5 THEORETICAL FRAMEWORK

The researcher was guided by three theoretical frameworks: Polya’s problem-solving techniques, threshold concepts and troublesome knowledge of Meyer and Land (2003) and Newman’s error analysis in deconstructing the concept of error analysis.

2.5.1 Polya's problem-solving techniques

Polya developed four basic principles that need to be considered during problem solving. Based on the principles, the four steps that need to be followed during problem solving were developed later. The researcher identified a problem which sought to solve that particular problem; namely that learners continue to commit errors in financial mathematics. This might be attributed to the learners' problem-solving techniques; Understanding of the aforementioned theory can assist in eliminating the errors. Polya's problem-solving techniques are a theory that describes four steps to be followed during problem-solving in mathematics education. These steps seem to be relevant also to the learning of mathematical literacy.

According to Polya (1945: 11) the following are the principles to be considered during problem solving:

First principle: Understand the problem

Learners might seem incompetent but maybe they did not understand the question fully. Polya states that teachers should ask learners the following questions:

- Do you understand all the words used in the problem statement?
- What are you asked to find?
- Can you restate the question in your own words?
- Think of the picture or diagram that might help you to understand the problem
- Is there enough information to enable you to find a solution?

Second principle: Devise a plan

There are many reasonable strategies to be employed in order to solve a particular problem. Polya (1945: 13) states that the skill of choosing an appropriate strategy is best learnt through solving many problems. According to Polya (1945: 13), the following are the strategies that a learner can choose in order to solve a particular problem:

- Guess and check
- Make an orderly list
- Use direct reasoning
- Solve a simpler problem than the complex one
- Work backward
- Use a formula

Third principle: Carry out the plan

This is considered to be easier than devising a plan as it only needs the patience of executing the devised plan. This will include for instance correct substitution if the chosen strategy was the use of a formula. According to Polya (1945: 14), "Consistency throughout the algorithms employed to arrive at the final answer is of the utmost importance in this step".

Fourth principle: Look back

Taking time to reflect on your work enables you to predict the relevant strategy for solving a future problem. If the devised plan does not work you will have to discard it and use another one until you arrive at the correct answer.

The study sought answering the question: why do learners commit errors in financial mathematics and the underlying factors related to the type of errors they commit. The objectives of the study are based on Polya's theory on problem-solving techniques. The study sought answers to the aforementioned research questions (cf Chapter 1), and to provide a solution for an identified problem.

If the teaching and learning process at schools could be influenced by the above-stated theory, both teacher and learners guided by the four-step principle, could dedicate more time to learners' work and thus reduce errors committed.

2.5.2 Threshold concepts and troublesome knowledge

Meyer and Land (2006: 1) argue that: “Threshold concept can be considered as akin to a portal, opening up a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress”. It changes the way learners perceive things, and the level of understanding of concepts is also improved. The acquisition of knowledge occurs through a process of gathering key concepts per particular subject. The concepts are entrenched in learners’ conceptual understanding therefore assist in the problem solving.

“A core concept is a conceptual ‘building block’ that progresses understanding of the subject; it has to be understood but it does not necessarily lead to a qualitative view of subject matter” (Meyer and Land, 2006: 4). As in the aforementioned discussion of Polya’s problem-solving techniques, problem solving does not solely depend on the acquisition of concepts but also depends on the choice of the relevant problem-solving technique. Application of irrelevant rules or strategies has been hypothesised as one of the underlying factors that contribute to learner errors in financial mathematics.

After discussions with practitioners in a range of disciplinary areas they came up with the following characteristics: threshold concepts are **transformative**, **irreversible**, **integrative, bounded** and **troublesome** (Meyer and Land 2006: 5).

Transformative: It brings about change in an individual’s conceptual thinking. The more the threshold concept develops, the more it brings a new perspective to things around an individual.

Irreversible**:** Once a learner understands the concept it is unlikely to be forgotten i.e. knowledge acquired is not easily to forgetten.

Integrative**:** Once a concept is mastered it is easy to link it to the already existing concepts and that would stimulate easy retention of the concepts as they are linked.

Bounded**:** A threshold concept is likely to be bounded in that ‘any conceptual space will have terminal frontiers, bordering with thresholds into new conceptual areas’ (Meyer & Land, 2006:6)

Troublesome**:** Mastery of a threshold concept might involve “common sense’ and intuitive understanding of it. It then becomes difficult and uncomfortable to reverse learners’ intuitive understanding.

Deficient mastery of prerequisite skills, facts and concepts has been hypothesised as one of the underlying factors that contribute to learners committing errors in financial mathematics (cf Chapter 1 section 1.7). If threshold concepts could be developed in learners, those could build their confidence and therefore reduce the level of errors committed in problem solving.

2.5.3 Newman’s Error Analysis

The researcher was guided by Newman’s Error Analysis technique in the error analysis of learners’ work. Newman’s Error Analysis (NEA) provided a framework for considering the reasons that underlie the difficulties students experienced with mathematical word problems and a process that assisted teachers to determine where misunderstandings occurred. NEA also provided By pinpointing the errors committed by learners in financial mathematics, teaching can be directed towards the correct procedure of solving the identified problem. directions for where teachers could target effective teaching strategies to overcome them (White, 2010: 129-148). In the search for underlying factors for learners committing the errors and finding the suitable strategies to overcome them, the researcher was guided by NEA.

The Newman’s error analysis and follow-up strategies have helped learners with their problem-solving skills, and teachers developed a much more consistent approach to the teaching of problem solving.

“Not only has it raised awareness of the language demands of problem solving, but through its systematic approach, teachers can focus on teaching for deeper understanding” (White, 2009: 37).

The afore-stated theory was also used in content analysis of the content-based questionnaire (cf Appendices A, B and C). Various studies have been published on learner errors, the following section of the study will investigate learner errors identified from the review of other studies.

2.6 THE ERROR ANALYSIS IN MATHEMATICS EDUCATION

Error analysis is more diagnostic to determine learners' procedural effectiveness; it also allows determining learners' lack of basic conceptual understanding. Peng (2009: 1) contends that, "mathematical errors are a common phenomenon in learners' learning of mathematics. Learners of any age irrespective of the performance in mathematics have experienced getting mathematics wrong". It is essential to consider that analyzing learners' mathematical errors is a fundamental aspect of teaching for mathematics teachers as it will allow them to develop corrective and preventative measures. Based on the aforementioned discussion the concept of error analysis (cf Chapter 4) was used in the content analysis of questionnaires A, B and C.

"Many teachers complain that learners find word problems in mathematics more difficult than straight computation and that many learners dislike and even fear word problems" (Murray, 2012: 55). Mathematical Literacy as described earlier is contextual based, therefore consists mostly of word problems in its nature which explains the existence of persisting learner errors therein.

Ryan and McCrae as cited by Sheinuk, (2010: 12) state that: "pre-service teachers who confront own mathematical errors, misconceptions and strategies in order to recognize their subject matter knowledge, have an opportunity to develop rich context knowledge".

Radatz (1979: 170) provides a good definition of error. Firstly, errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequences of definite processes whose nature must be discovered. Secondly, it seems to be possible to analyze the nature and underlying causes of errors in terms of the individuals' information-processing mechanisms. Thirdly, the analysis of errors offers a variety of points of departure for research into processes by which children learn mathematics. Mathematical Literacy is also based on the same principles as mathematics, only different in purpose and nature from mathematics. Error analysis in Mathematical Literacy is based on the same principles as in mathematics therefore the afore-stated Radatz definitions of errors are applicable to Mathematical Literacy.

"Error analysis in mathematics education has a long history that dates as far as 1925 by Buswell and Judd who cited more than 30 studies dealing explicitly with the diagnosis of Arithmetical errors" (Radatz, 1979: 163). Error analysis is of vital importance in addressing the careless errors that learners commit through their learning process.

Errors are an important part of any practice, because they illuminate what mechanisms need to be put in place to give access to the practice. Errors point to the demand of practice; while at the same time they are the points of leverage for opening access to the practice. To understand learner errors, one has to look at the methods or strategies that the learners use to arrive at the incorrect solutions. If teachers search for the ways to understand why learners may have made errors, they may come to value their thinking and find ways to work it into classroom conversations and bring preventative measures.

Errors can be the result of carelessness; misinterpretation of symbols or text; lack of relevant experience or knowledge related to the mathematical literacy topic/concepts; a lack of awareness or inability to check the answers given; or the result of a misconception.

By pinpointing learner errors in Mathematical Literacy, the teacher can provide instruction targeted to the learners' area of need. In general, learners who have difficulty learning mathematical literacy typically lack important conceptual knowledge for a number of reasons, including an inability to process information at the rate of the instructional pace; lack of adequate opportunities to respond; the lack of specific feedback from the teacher regarding the misunderstanding cited.

Hodes adapted the following table from Nolting (1998:1); which illustrates five types of errors for word problems.

Table 2.1: Types of errors for word problems

1. Read errors	**The learner cannot read a key word or symbol correctly.**
2. Comprehension errors	The learner reads all the words in the problem accurately but does not understand the overall problem or specific terms within the problem.
3. Transformation errors	The learner understands what the problem requires but is unable to identify the operation or the sequence of operations needed to solve the problem.
4. Procedural errors	These include: • Placement errors which is incorrect sequencing of digits or alignment of algorithms. • Incorrect steps which is use of steps that are not associated with any operations. • Missing steps where steps necessary to complete a procedure are missing.
5. Encoding errors	A learner solves the problem but does not write the solution in an appropriate form.

(Adapted from Nolting, 1988: 1)

The aforementioned types of errors have been used in the identification of learner errors in the content analysis of the three research questionnaires (Appendices A, B and C) which forms part of data analysis (cf Chapter 4).

Brodie (2005: 179) brought into the debate of learner errors “Situative” perspectives: Situative perspectives argue that what a learner says and does in the classroom make sense from the perspective of his/her current ways of knowing and being, his/her developing identity in relation to mathematics and to his/her previous experiences of learning mathematics, both in and out of school. After engaging with learners in class discussions of a particular topic, Brodie developed a coding scheme to categorize learners’ contribution (Brodie as cited in Khan & Chishti, 2011:656).

Table 2.2: Brodie's coding scheme to categorize learners' contribution (Brodie, 2005: 177)

Basic Error	**An error not expected at the particular grade level, indicates that the learner is not struggling with the concepts that the task is intended to develop, but rather with the other concepts that are necessary for completing the task and have been taught in previous years**
Appropriate Error	An incorrect contribution expected at the particular grade level in relation to the task.
Missing Information	Correct but incomplete and occurs when the learner presents some of the information required by the task but not all of it.
Partial insight	Learner is grappling with an important idea, which is not quite complete, nor correct, but shows insight into the task.
Complete correct	Provides an adequate answer to the task or question.
Beyond task	Related to the task or topic of the lesson but go beyond the immediate task and/or make some interesting connections between ideas.

Riccomini (2005: 233) brought into perspective (1) *unsystematic errors*: unintended, non-recurring wrong answers which learners can readily correct by themselves; (2) *systematic errors*: though they are recurring wrong response methodologically constructed and produced across space and time, they are symptomatic of a faulty line of thinking that causes them to be referred to as misconceptions.

Elbrink (2008: 2) categorises learners' mathematical errors into three main categories: calculation errors, procedural errors and symbolic errors.

Table 2.3: Summary of the above-stated categories of learner mathematical errors:

Error Category	Description
1. calculation errors	• mistakes in addition, subtraction, multiplication and division.
2. procedural errors	• occurs when learner computes or applies an incorrect procedure and symbolic errors.
3. symbolic errors	• occurs when learners falsely relate mathematical problems that use similar symbols.

She elaborated each of the categories: (1) as an error of numbers which she attributes to carelessness and lack of attention and further suggested the possible solution to the calculating error is incorporation of an error checklist into a regular classroom routine and procedures. This will allow learners to assess themselves and identify repeated errors and mistakes in their work. (2) Learners are usually taught in drill and practice and so be automated to carry out specific mathematical tasks rapidly and effectively and can be confused for conceptual understanding. Therefore they cannot recognize the importance of applying and procedure correctly.

Procedural errors suggest that learners do not understand the concepts related to the procedure and are unable to build procedure from conceptual knowledge. She suggested the introduction of the concepts before the procedure, concrete manipulation and real-life application. In her elaborate discussion of procedural errors she brings up the importance of threshold concept which forms part of the theoretical framework of this study. Finally (3) learners try to create meaning in the patterns of mathematical symbols and signs that they see in front of them rather than trying to understand. The identification of errors in the content analysis is based on the aforementioned errors (cf Chapter 4). The errors described in Tables 2.1; Table 2.2 and Table 2.3 have been utilised to categorize the identified errors in this study. The procedural errors which were identified during content analysis could be eradicated from learners by means of teaching that is embedded on the threshold concept (cf Current Chapter, section 2.6.2). It is strongly associated with the errors due to incorrect association or rigidity of thinking which was stated as the second research question. The researcher chose to name this particular type of error as Radatz as that also describes it.

In the aforementioned discussions, a number of studies by different researchers have been reviewed; this study focused on the Radatz (1979) classification of errors which brings about the underlying factors that can be associated with learner errors. The types of errors discussed below form a fundamental part of the research propositions of this study.

One should, of course, acknowledge that errors are also a function of other variables in the education process which classifies errors according to information processing. Research questions and hypotheses of the study have been formulated based on the Radatz (1979: 164) classification of errors according to individual difficulties of learners. This section seeks to interrogate the five research questions (cf Chapter 1, section 1.6):

2.6.1 The underlying factors related to errors due to incorrect association or rigidity of thinking

This formed part of the research questions as research question 2 of this particular study (cf Chapter 1, section 1.6) and the findings would be interrogated fully in Chapter 4. Inadequate flexibility in decoding and encoding new information involves incorrect interaction between single elements. Radatz (1979: 167) states that, “Experience with similar problems will lead to a habit of rigid thinking; learners continue to use the cognitive operation they have developed even if the mathematical tasks have changed”.

Difficulties due to incorrect association or rigidity of thinking are also common areas of errors in mathematics. Pippig as cited in Radatz (1979) further classified this type of error into: (1) Errors of perseveration which is described as an error in which single elements of a task or problem predominates. Examples include: $9 \times 60 = 560$; $7 \times 50000 = 35000$

(2) Errors of association, involving incorrect interactions between single elements. Examples include: $56 + 12 = 67$; $6 \times 4 = 18$.

(3) Errors of interference, in which different operations or concepts interfere with each other. The example that follows displays interference between the algorithms for addition and subtraction:

$$
\begin{array}{lr}
 & 6845 \\
+ & 372 \\
+ & 35437 \\
\underline{+} & \underline{561} \\
 & 30375
\end{array}
$$

Here the learner added the digits in units' column, getting 15; added all but the top tens' and hundreds' column, getting 17 and 13 respectively and then subtracted to get the remaining two digits in the answer.

(4) Errors of assimilation, in which incorrect hearing is attributed to the causes of reading and writing. Those errors are a result of lack of attention and concentration (random or careless errors).

(5) Errors of negative transfer from the previous tasks, in which effects of erroneous impression from a set of exercises or word problems.

2.6.2 The underlying factors related to errors due to the application of irrelevant rules or strategies

This formed part of the research questions (cf Chapter 1, section 1.6) as research question 3 as well as proposition 3 of the study as stated in the background of the study. A number of underlying factors that are related to the aforementioned errors have been interrogated; the correlation between those factors was tested. It was concluded that errors committed by learners in financial mathematics are attributed to application of irrelevant rules and strategies. This type of error is described by the development of incorrect algorithms, and application of inadequate strategies in solving mathematical tasks. This type of error often stems from experiences in successfully applying comparable rules or strategies in other content areas. Learners often think of mathematics as an isolated game with peculiar sets of rules and no evident relation to reality.

"The pupils' understanding of mathematics and especially of arithmetic as a game with arbitrary rules, may provide the background for analyzing many causes of pupils' errors" (Radartz, 1979: 169).

Examples include 155 ÷ 5 = 301

The learner was introduced to first divide 150 by 5 which gives 30 and the divide 5 by 5 which gives 1. The answer is written as 30 and 1 which gives 301 instead of 30 + 1 which is 31. Elbrink and Nolting (1998) would classify this type of error as ‘procedural error’ as it is merely due to incorrect steps employed or incorrect strategy.

The aforementioned type of error may be similar to the procedural error as cited by Hodes. A kind of error referred to as placement error which is incorrect sequence of algorithms.

Example 4

Table 2.4: Comparison of learner’s steps and the correct steps followed when computing the aforementioned problem

STEPS	LEARNER’S RESPONSE	CORRECT ALGORITHMS
1	A = P(1+in)	A = P(1+in)
2	7854 = 5100 $(1+\frac{9}{100})^{n}$	7854 = 5100 $(1+\frac{9}{100}n)$
3	7854 = 5559 n	$\frac{7854}{5559} = 1 + \frac{9}{100}n$
4	$\frac{7854}{5559} = n$	$1.54 - 1 = \frac{9}{100}n$
5	= 1.4	$\frac{0.54 \times 100}{9} = n$
6	= 2 years	6 = n

The final answer would be: 6 years.

Learners were asked to find the number of years (period) it will take for the R5 100 to grow to R7 854; the learner used the relevant formula*: A = P (1 + in)* but could not work out the correct answer. The reason why they could not get the correct answer is that they ignored that the variable this time is *n.*

The respondent used the relevant formula but when substituting, changed the formula to the compound interest formula instead of making *n* the subject as the question required. If the respondent had followed the correct algorithm, continued to substitute correctly and followed the correct algorithms *n* would have remained the only unknown value. Therefore it would not be difficult to arrive to the correct solution.

2.6.3 The underlying factors related to errors due to deficient mastery of prerequisite skills, facts and concepts

This formed part of the research questions as research question 4 (cf Chapter 1, section 1.6) sought to address the factors related to learners committing this type of error. It formed part of the proposition 2 of this study and after an extensive interrogation it was concluded that errors committed in financial mathematics could not be related to deficient mastery of prerequisite skills, facts and concepts.

A number of factors were identified and fully discussed in Chapter 4 of this study. Deficit in basic prerequisites include ignorance of algorithms, inadequate mastery of basic facts, incorrect procedures in applying mathematical techniques, and insufficient knowledge of necessary concepts and symbols.

Mathematical knowledge can either be procedural or conceptual: (1) conceptual knowledge can be generalized and connectable, whereas (2) procedural knowledge is regarded as the competence of carrying out a mathematical task. Procedural knowledge is usually taught through drill and practice and so it can be automated to carry out specific mathematical tasks rapidly and efficiently. This type of error therefore refers to all deficits in the content and problem-specific knowledge necessary for the successful performance in the mathematical tasks.

When a learner does not possess the necessary prerequisite skills, facts, and concepts to solve a problem, he or she will be able to solve the problem correctly. For example if a learner does not know how to combine like-terms, a learner may face difficulty solving multi-step equations involving a combination of like-terms.

Example 5

Table 2.5: Comparison of learner's steps and the correct algorithms

STEPS	LEAERNER'S RESPONSE	CORRECT ALGORITHMS
1	A = P (1 + i)^n	A = P (1 + I)^n
2	9700 = P (1 + $\frac{9.5}{100}$)^3	A = 9700 (1 + $\frac{9.5}{100}$)^3
3	9700 = P 1.31	A = 9700 (1 + 0.95)^3
4	$\frac{9700}{1.31}$ = P	A = 9700 (1.312932395)
5	7404.58 = P	A = 12 735.44404

The final answer would be: R12 735.44

As indicated above the learner chose the correct and relevant formula but incorrect substitution of P instead of A in step 2. When working out the brackets, the learner rounded off 1.312932395 to two decimal places to 1.31. Then followed the correct algorithms and divided 9700 by 1.31. Because of the errors in step 2 and 3, the final answer was incorrect as they are supposed to round off the final answer. These type of errors have been identified in the content analysis of the questionnaires (cf Appendices A, B and C). Frequency tables 4.25 and 4.26 illustrated the summary of the responses related to rounding off that was included in questionnaire 4 (cf Appendix D) which sought to uncover the underlying factors related to the research questions (cf Chapter 4) for a detailed discussion.

2.6.4 The underlying factors related to errors due to language difficulties

The factors related to learners committing this type of error formed part of research question 5 as stated in the background of the study (cf Chapter 1). This was proposition 1 of this study which later could not prove any relationship between learner errors and language difficulties (cf Chapter 4). This particular study could not prove any correlation between the learner errors and language even though in the content analysis it could be identified as one of the underlying factors related to learner error in financial mathematics. It sound reasonable to assume that the learners' ability to understand the language of instruction and also their level of reading comprehension plays an important part in successful learning. In solving word problems learners should be familiar with the language used, mathematical concepts, symbols and the vocabulary but unfortunately for many of them English is foreign and a challenge in their learning. A misunderstanding of semantics of mathematical text could often be a source of learners' errors.

> *"For many pupils the learning of mathematical concepts, symbols, and vocabulary is a 'foreign language' problem. In solving word problems, pupils must refrain from using the manifold background of a word's meaning in natural language. A misunderstanding of the semantics of mathematical text is often the source of pupils' errors" (Radatz, 1979: 165).*

Poor language skills such as reading, writing and speaking are often associated with low attainment in mathematics and ML and in addition to that, mathematics has its own set of language patterns, symbols and vocabulary. That poses a challenge to mathematical literacy learners as they are struggling to grasp some of the mathematics terminology; they are also confronted by the language of instruction which is foreign to them.

"Learners may lack reading comprehension skills that are required to interpret the information needed to solve a problem. Learners may also have difficulty in understanding academic language required to solve a problem" (Baldwin and Yun, 2012: 24).

Murray (2012: 49) states that "the major part of developing an understanding of Mathematics involves learning to handle the set of mathematics language patterns, symbols and vocabulary in order to make connections between them".

Murray(2012: 55) brought into debate that there is a danger that functional literacy (including reading comprehension) can be interpreted too superficially, without taking into account the many factors that can prevent the learner from making sense of what he/she is reading. She listed the barriers to understanding word problems which mostly constitute Mathematical Literacy:

- *The mathematical structure of the problem*
- *The number size and kind of numbers involved*

- *The context used for the problem*
- *Learners are not familiar with the context*
- *The context has unpleasant connotations*
- *Limited context*
- *The problem has to be transformed or modelled by the learner before he/she can solve it*
- *The learners' beliefs about what is expected of them*
- *The teachers' beliefs about the nature of mathematics and how mathematics is best learnt.*

To enhance correct transformation of problem-solving statements into equivalent algebraic equations, for example, the problem solver needs to be equipped with adequate knowledge of mathematical words, symbols, notations and models. In questionnaire 1 (cf Appendix A) which sought to uncover research question 5 and proposition 1, learners needed to understand the concepts such as cost price, selling price, profit or loss illustrated in the table in order to answer the question.

Visualization of mathematical presentations, the use of diagrams, iconic instructions may sometimes pose a challenge to information processing and synthesis in some learners and may pose a demand that may be less content-specific than representational-specific. The table was intended to assist learners in determining the appropriate answers by filling in the spaces but that seemed to pose a challenge.

Radatz (1979: 165) states that a series of investigations showed that the iconic representation of mathematical situations can involve great difficulties in information processing and that perceptual analysis and synthesis often make greater demands on learners than does the mathematical problem itself.

> *Words, symbols, and graphs are powerful methods of communicating mathematical ideas and relationships. These tools allow students to express mathematical ideas to people. Moving*

from one representation to another is an important way to enhance mathematical concepts (Columba, 2012: 3).

This study is focused on financial mathematics, and seeks to identify the type of learner errors and their underlying factor structures. The afore-mentioned type of error is directly related to the geometry section of the curriculum.

Jasper, Polnick and Taube (2012: 33) assert, “Allowing time for students to discuss their thinking with their peers not only builds confidence, but also encourages intellectual curiosity to discover things and extend their thinking about spatial relations”.

This study carries forward an investigation and identification of the type of errors that learners commit that can be attributed to the aforementioned underlying factor.

2.6.5 The degree of predictability and hence strategies to underpin error analysis.

“…This curriculum aims to ensure that children acquire and apply knowledge and skills in ways that are meaningful to their own life” (DoE, 2010: 5). In ensuring that the aforementioned statement is a reality, teachers need to be aware of the underlying factors which influence learners in committing errors in a Mathematical Literacy classroom. The ability to prevent these errors can be achievable through the greater sense of the degree of their predictability and hence strategies can be employed in underpinning error analysis. Take time to figure what kind of errors learners were making and learn about their attitudes towards assessments and more importantly, their attitude towards reviewing their assessments before turning them in.

Hanson and Drews cited in Sheinuk, (2010: 12) assert that the notion of ‘misconception’ is rooted in an underlying confusion about a concept or it evidences itself when learners over- or under-generalize mathematical contexts without any conceptual understanding playing a role.

Smith and Roschelle (2011) alert us to the fact that when learners come to class, they bring with them preconceived conceptual ideas and beliefs which may conflict with the conceptual notions they subsequently experience in class. They further describe; “errors are characteristic of initial phases of learning because learners’ knowledge is inadequate and supports only partial understanding”.

Sheinuk, (2010: 15) alluded by stating that: “Teachers’ content knowledge is the basic platform from which to reason about learners’ thinking, particularly in terms of their errors and misconceptions. Procedural and conceptual knowledge are both acknowledged as type of knowledge that one draws when doing mathematics”.

Radatz (1979: 170) states: “Consideration of the diagnostic and causal aspects of errors could give specific help to mathematics teachers by allowing them to interrogate their knowledge of curriculum content with their knowledge of individual differences in children”.

Makonye, (2011: 16) asserts, “As learner errors and misconceptions are often consistent rather than random, this suggest instantiation or launching of necessary instructional strategies that can begin to resolve them”.

Brodie, Slonimsky and Shalem (2008) state that teachers need to reflect on learners’ performance in ways that do not blame the learners or themselves and which provide ways for them to work with learners or themselves and which provide ways for them to work with learner errors in order to transform those.

Errors also provide a useful focus because teachers orient towards errors in different ways. In more traditionally-oriented teaching, errors are either to be avoided or corrected, in the pursuit of correct mathematical knowledge. In more reform-oriented teaching, errors are to be embraced, as point of contact with learners’ thinking of conversation to generate discussions about mathematical ideas. In thinking about their own responses to errors in developing lesson plans and reflecting on teaching, teachers need to see how different systems of evaluation can constrain and different teaching approaches can help.

Error analysis provide a way of helping teachers to see learners as reasoning and reasonable thinkers and the practice as reasoned and reasonable, and bring these two into relationship. If the teachers seek to understand why learners may have made errors, they may come to value their thinking and find ways to work it into classroom conversation. Smith, Disessa & Roschelle (as cited in Brodie, et al, 2008) point out that a key theoretical understanding is that learner errors are a normal part of the learning process, are reasonable and make sense to the learning.Hart (as cited in Mahlabela, 2012: 20) points out that some methods used by learners lead to the error, which in turn does not automatically disappear through maturation. This implies that errors are products of incorrect strategies or products of incorrect use of correct strategies.

The research findings and recommendations may be useful to both teachers and the learners. If teachers are aware of the learner errors, that would transfer to the learners and promote self-reflection among learners. Learners will acquire skills to avoid committing the errors, predict the errors and rectify the errors committed. Watson, (2010: 2) argues that: “knowing how to engage critically with mathematics, communicated in various ways, leads to understanding how ‘errors’ are made and therefore reduce the need to learn about individual errors”.

Teachers should allow learners time to process information by making sure that their instructional pace allows time for learners to learn appropriately. Teaching and learning in the classroom should afford learners an opportunity to respond to the questions/classroom debates. Teachers should regularly give specific feedback to the learners on their class activity responses.

2.7 CONCLUSION

Chapter 2 reviewed the literature on conceptualization of Mathematical Literacy and Mathematics, followed by the focus of the study in a South African perspective, wherein the Chief Marker’s report for 2012 was brought into view.

Learner errors as cited in the Chief Marker's report were used to describe learner errors in financial mathematics.

Neuroscience also formed part of this chapter wherein its relationship with mathematics teaching which included the description of mathematics thinking as well as the implications of cognitive neuroscience to teaching and understanding of the learning process in mathematics formed part of the discussions in this chapter.

Three theories such as (1) Polya's problem-solving techniques; (2) threshold concepts and (3) Newman's Error Analysis formed part of the theoretical framework discussed in this section of the study. Lastly, the concept of error analysis in mathematics education with the literature on error analysis which is related to the research question and the hypothesised errors formed an integral part of the discussions in this chapter.

The findings of this study could also be transferred to the Higher Education Institutions; where error analysis in Mathematical Literacy could be included in their educator training curricular. That would serve as a long-term objective of the study and it would not only minimize learner errors but will have a direct effect on learner performance in the assessments of Grades 10-12.

CHAPTER 3

RESEARCH METHODOLOGY

3.0 INTRODUCTION

The study sought to interrogate the errors committed by Grade 10 Mathematical Literacy learners in a school in the Eastern Cape Province of South Africa while solving problems in financial mathematics. This chapter outlines the research methodology used to conduct this study. It focused on techniques and procedures but presented the research paradigm which Nieuwenhuis (2007: 47) defines as: "A set of assumptions or beliefs about fundamental aspects of reality which gives rise to a particular worldview; it addresses fundamental assumptions taken on faith, such as belief about the nature of reality (ontology), the relationship between knower and known (epistemology) and assumptions about methodologies". A detailed description of the research approach, research design, data-collection techniques of the study and data analysis forms part of this chapter.

3.1 RESEARCH PARADIGM

A positivist paradigm guided the researcher in formulating laws to account for the errors committed by Mathematical Literacy learners in their solving of financial mathematics problems, thus providing firm bases for prediction and control.

According to the *positivist* epistemology, science is seen as the way to get at truth to understand the world well enough so that it might be predicted and controlled. "The world and the universe are deterministic; they operate by laws of cause and effect that are discernible if we apply the unique approach of the scientific method" (Krauss, 2005: 760). A positivist paradigm which will include a *quantitative* approach was employed for the measurement of data which was used to discover and confirm causes and effects.

Healy and Perry (as cited in Krauss, 2005) point out that positivism predominates in science and assumes that science quantitatively measures independent facts about a single apprehensive reality. The data and analysis thereof are value-free and data do not change because they are observed. That is, researchers view the world through a “one-way mirror”. The paradigm is based on the notion that all knowledge should be based on practical experience or observations. Positivism may be characterised by its claim that science provides us with the clearest possible ideal of knowledge.

“Positivism implies a particular stance concerning the social scientist as an observer of social reality. The end-product of investigations by social scientists can be formulated in terms parallel to those of natural science” (Cohen et al, 2007: 9). It was imperative for the researcher to adopt the positivist paradigm as he sought to find the underlying factors related to errors committed by learners in financial mathematics.

3.1.1 The research approach

The researcher sought to uncover learner errors during problem solving of financial mathematics in Grade 10 Mathematical Literacy. The study sought to answer the research question (cf Chapter 1, section 1.6) by collecting numerical data which was analyzed using SPSS software to determine the frequencies, correlation among the variables and the analysis of the variance (cf Chapter 4).

The researcher maximised objectivity and minimised his involvement with the respondents during the progression of the study. All this is influenced by the principles of the positivist paradigm as indicated in the section above. The researcher is aware of the fact that he is part of the world and that poses a challenge in detaching himself from the research. Statistical analysis permits the researcher to discover complex causal relationships and to determine to what extent one variable influences another. The research results are relatively independent of the researcher as statistical significance is considered in the data-analysis section (cf Chapter 4).

The aforementioned statement drove the researcher to use the *Quantitative Approach* to address the research questions. “Quantitative methods are frequently described as deductive in nature, in the sense that inferences from tests of statistical hypothesis lead to general inferences about characteristics of a population” (Harwell, 2012: 149).

3.1.2 The advantages and disadvantages of the quantitative research approach

The researcher chose this type of a research approach as he is striving to eliminate bias from the study. This approach allowed the researcher to formulate the three propositions (cf Chapter 1) that supported him to speculate the outcomes before execution. This is a study that requires systematic data collection and analysis as its focus is on error analysis. It can be generalized for further research but the sample size might pose a challenge in that regard. Based on the research questions (cf Chapter 1) which sought answers to: (1) why do learners commit errors on given tasks in financial mathematics? and (2) the underlying factors related to the hypothesised errors (cf Chapter 1) the quantitative approach was found to be relevant for this particular study.

3.2 RESEARCH DESIGN

Cohen *et al.* (2007) point out that research methodology does not only focus on techniques and procedures used in the process of data-gathering but also describes approaches to kinds and paradigms of research. A case study was chosen as the relevant research design as the study sought to find out why learners commit errors on given tasks in financial mathematics and the underlying factors related to Grade 10 Mathematical Literacy learners committing errors.

Yin (as cited in Baxter and Jack, 2008: 545) points out that a case-study design should be considered when (a) the focus of the study is to answer “how” and “why” questions; (b) you cannot manipulate the behaviour of those involved in the study; (c) you want to cover contextual conditions because you believe they are relevant to the phenomenon under study; or (d) the boundaries between the phenomenon and context are not clear.

This study focused on a case of Grade 10 Mathematical Literacy learners from a conveniently chosen school and was studied by means of structured-interview questionnaires.

3.2.1 Case study

The selection of the case included one East London district school, as the research population consists of Grade 10 Mathematical Literacy learners. However, the respondents are selected using a *simple random sample* technique. The study intends to identify and describe learner errors as they solve problems in Mathematical Literacy financial mathematics.

Cohen et.al (2007: 170) describes a case study as a “naturalistic enquiry” that undertakes “an investigation into a specific instance or real phenomena in its real-life context”. The research design would therefore be a case study as the researcher seeks to provide an accurate and valid representation of the factors or variables that pertain to learner errors as depicted in the research questions.

“Criticism of case study methodology is frequently levelled against its dependence on a single case and it is therefore claimed that the case study is incapable of providing a generalizing conclusion” (Nieuwenhuis, 2007: 77).

The researcher needed to ensure that not only access is permitted but is, in fact practicable by conveniently choosing a secondary school in the East London district where he teaches. The researcher *purposively* chose the Grade 10 ML learners from a secondary school which the researcher *conveniently* chose in the East London district of the Eastern Cape Province in South Africa. The researcher considered the accessibility and the time frame when choosing this particular school.

3.2.2 Sample size (n) and Justification

The school currently has five Grade 10 ML classes with 186 learners as the research population (N). The population consists of 104 girls and 82 boys with an age range from 14 to 18 years. The researcher adopted the simplified formula by Yamane (as cited in Israel, 2009) for proportions to determine the sample size (n), where e is the level of precision.

Hence the sample size was 105 respondents, where, N is the population size and assuming that confidence level is 95% and the level of precision is .5 (Yamane 1967: 886).

According to Cohen *et al.* (2007: 97), “Determining the size of the sample will have to take account of attribution and respondent mortality, i.e. that some participants will leave the research or fail to return questionnaires”.

Israel (2009: 2) states that, “because a proportion of .5 indicates the maximum variability in a population, this is often used in determining a more conservative sample size, that is, the sample size may be larger than the true variability of the population attribute were used”.

After the sample size had been determined, the respondents were selected by a *simple random* selection method, in consideration of the reliability and trustworthiness of the results. The researcher compiled a list of the research population and next to each name a 3-digit number was assigned. The researcher selected the required number (i.e. 105) randomly from the list of the population. “One problem associated with this particular sampling method is that a complete list of population is needed and this is not always readily available” (Cohen *et al.* 2007: 100). This was not exactly the case in this study as the research site is a school and class lists were readily available so it was possible to compile them as one list.

The sample size was relatively large in order to cater for the ANOVA assumption and for the reliability of the study results.

3.2.3 Data-collection methods

This section will focus on how data was collected. A quantitative research was employed, hence structured interview questionnaires (cf Appendices A, B, C and D) and documentary studies were used to collect data. Oppenheim (as cited in Cohen *et al*, 2007: 247) states that highly structured questionnaires and closed questions are useful in that they can generate frequencies of response amenable to statistical treatment and analysis. The afore-mentioned statement guided the researcher in choosing the structured questionnaires as the data-collection instruments for the study.

The first three questionnaires (cf Appendices A, B and C) are content based where the respondents were expected to work out ML problems based on financial mathematics. The three questionnaires sought to illustrate the type of errors that Grade 10 ML learners commit in financial mathematics. These afforded the researcher an opportunity to identify the hypothesised errors (cf Chapter 1, section 1.4) based on the literature reviewed and theoretical framework. As a follow-up the fourth questionnaire (cf Appendix D) afforded the respondents an opportunity to reflect on the underlying factors related to the hypothesised error types. The fourth questionnaire sought to answer the research question (1) why do learners commit errors on given tasks in financial mathematics? The sampling technique (cf Chapter 3) was discussed above and data-collection instruments (cf Chapter 3) will be discussed below.

3.2.4 Questionnaire design

According to Cohen *et al*., (2007) highly structured, closed questions are useful in that they can generate frequencies of response amenable to statistical treatment and analysis. It enables patterns to be observed, comparisons to be made and the data to be processed and statistically calculated by Statistical Package for Social Sciences (SPSS(R)). The advantage of the questionnaire is that it is less time-consuming and the researcher is not able to influence the respondents' responses or opinions as may be the case in interviews.

Based on the aforementioned discussion and the anticipated data-analysis process, the researcher used the structured questionnaires.

The researcher chose the structured questionnaires to observe the patterns and compare the responses. Four structured interview questionnaires were administered to the respondents on separate occasions, to afford them an opportunity to uncover the hypothesized errors (cf *Appendices A, B,* C and *D*). Of the four sets of questionnaires used in this study, the first three questionnaires consisted of Mathematical Literacy questions on financial mathematics where respondent were expected to work out Grade 10 financial mathematics problems.

Those problems covered the following sub-topics of financial mathematics: (1) expenditure; cost price; selling price; profit/loss (2) income and expenditure and (3) simple and compound interest (DoE, 2011).

3.2.4.1 Questionnaire 1 (cf Appendix A)

It is a structured interview questionnaire which sought answers.

Research question 5: what are the underlying factors related to errors due to language difficulties?

Questionnaire 1 was more focused on financial mathematics errors and the researcher tried to minimize the content knowledge demands. It sought to evaluate the learners' understanding of the concepts. It covered content topics such as *Expenditure; Cost price; Selling price; Profit/loss*. It consisted of one question with two sub-questions: sub-section (a) comprised a table where the respondents were required to use given cost price and selling price to determine whether the sale made profit/loss. The first requirement of the question was the language demand which also formed part of the research question. In the second part of the table, they were required to determine either the selling price or cost price and whether it was profit or loss. This challenged flexibility on the acquired mathematical procedures. Sub-section (b) comprised two questions where they had to determine the profit in justifying whether profit or loss was made.

The respondents were given the first set of questionnaires in class for learners to complete and afterwards collected by the subject teacher. The researcher marked and by means of Newmans' Error Analysis (NEA) pointed out errors and compiled a list of all identified errors (cf Chapter 4). The questionnaire had no time restrictions so as to allow learners to attempt all questions.

3.2.4.2 Questionnaire 2 (cf Appendix B)

The second questionnaire *(Appendix B)* covered *Income and Expenditure*. The applicable questions sought to uncover the employment of relevant algorithms in working out problems.

Questionnaire 2 attempted to answer **research question 4: what are the underlying factors related to errors that are committed due to deficient mastery of prerequisite skills, facts and concepts?**

This questionnaire required the four basic operations in order to solve the problems and consisted of three questions with the last question sub-divided into three:

- The first question required the respondents to determine the income of an hourly-paid worker. This was supposed to be the easiest question which only required straight-forward algorithms.
- The second question required a multi-step procedure that included both division and multiplication. It sought to unearth the mastery of prerequisite skills, with its underlying factor structure related to the errors committed in the set question.
- The third question introduced the percentage concept used at different instances

This questionnaire was administered and collected by the subject teacher. The researcher marked and conducted NEA for content analysis with an opportunity to interrogate the errors committed by learners (cf Chapter 4).

3.2.4.3 Questionnaire 3 (cf Appendix C)

The third questionnaire (cf *Appendix* C) consisted of five questions that covered simple and compound interest. The respondents were required to employ relevant algorithms, the use of formula, and flexibility in the use of formula.

- The first question required the respondents to calculate simple interest where a number of approaches could be used including the use of formula. The researcher would not prescribe the approach to be used to avoid any bias in the study.

- The second question required the respondents to determine compound interest.
- The third question required calculation of the value of the investment with simple interest. In this question respondents were confronted with a challenge of converting years to months or vice versa.

Flexibility can be demonstrated by the use of formula even in not so similar problems. In this instance; errors due to incorrect association or rigid thinking can pose a challenge as in the case of error due to (1) rigid thinking, (2) application of irrelevant rules or strategies, (3) deficient mastery of prerequisite skills and (4) language difficulties. The third set was collected, marked and content-analysed using NEA and the respondents were given the fourth questionnaire where they were expected to reflect on how they had handled the questions previously handed to them.

3.2.4.4 Questionnaire 4 (cf Appendix D)

The fourth set of questionnaires (cf *Appendix* D) consisted of three sections:

- (Section A) Short biographical data of the respondents, which helped to uncover the academic background (school history) of the respondents
- (Section B) sought honest reflections on how they handled tests and other forms of assessment in general, as it sought to uncover the underlying factors related to the afore-stated learner errors and
- (Section C) required respondents' reflection on how they interacted with their assessment tasks in Mathematical Literacy.

In sections A, B and C of the fourth questionnaire, the researcher used the rating-scale questions (i.e. Likert scale) providing a range of responses to avoid dichotomous questions as in most instances humans have a tendency to agree with the statement rather than disagree with it. These according to Cohen *et al*., (2007) are very useful devices for the researcher, as they build in a degree of sensitivity and differentiation of responses whilst still generating numbers. The aforementioned statement suggests that dichotomous questions might build in respondents' bias. The researcher chose Likert scale where the respondents indicate on the scale by circling the relevant scale. This fourth questionnaire was used to collect data from the respondents on their responses to the first set of questionnaires.

3.3 DATA ANALYSIS

The researcher sought to answer the questions; what, how and why? The researcher sought to analyze the trends in learners' thinking and related known characteristics of learners to commit errors.

3.3.1 Content analysis using the Newman's Error Analysis (NEA)

NEA procedure was used in the analysis of the content-based questionnaires (Questionnaires 1, 2 and 3). In reading responses it is critical to comprehend what learners wrote to determine learners' thinking in formulating their responses. "It is necessary to infer the most appropriate/effective mathematical strategy that learners have used to formulate their answers" (White, 2010: 130). NEA was used to identify the type of errors learners commit by their responses to Questionnaires 1, 2 and 3. A list of the different types of errors with a short description of each error identified per research question (cf Chapter 4) was compiled.

3.3.2 Analysis of the research questions by means of correlation analysis

Quantitative analysis with statistics which described the distribution, the relationship among variables and the variability by use of frequencies was used to analyze the fourth questionnaire. Statistical Package for Social Sciences (SPSS(R)) was used for correlating coefficient analysis, to measure the relationship between variables of each research question.

Correlation analysis was used on learners' responses to the fourth questionnaire (cf *Appendix D*) of the study to investigate whether a number of variables were linearly related to a small number of unobserved factors.

Preliminary Analysis

SPSS is used for data screening, i.e. look for inter-correlation between variables by creating the *correlation matrix.* In this instance the relationship between learner errors and the underlying factors will be observed. As indicated in the propositions of the study, there are three types of errors that the study seeks to uncover as well as their underlying factors which influenced the errors. As aforementioned in the first chapter the following underlying factors have been hypothesised: (1) language difficulties, (2) deficient mastery of prerequisite skills, facts and concepts and (3) application of irrelevant rules or strategies.

3.3.3 Hypotheses Testing using the Analysis of Variance (ANOVA)

In testing the hypotheses Analysis of Variance (ANOVA) was used taking into account the following assumptions. This was based on the set of questions and themes posed (cf background and research questions).

3.3.3.1 Assumptions of ANOVA

- **Homogeneity of variances**

  ANOVA assumes that the variance between the groups should be equal; the cases are independent from each other and the cases should not show any pattern. This means that each of the groups should have the same variance. Therefore it was used to control the undesirable types of variances. Standard deviations illustrated in the Descriptive statistics table have been used (cf Chapter 4). The largest value of Standard deviation divided by the smallest value give an answer which is not greater than 2.

- **Normality**

  As one of the ANOVA assumptions the following was taken into account in considering the normality of the research population:
  The research population was controlled in order to create a normal environment for the experiment as the normal population has a common variance. Within each population, the response variable is normally distributed. According to Garson (2012: 17), "A common rule-of-thumb test for normality is to run descriptive statistics to get skewness and kurtosis. Skewness should be within +1 to -1". The skewness from Descriptive statistics tables of each research question was used (cf Chapter 4).

- **Sample**

  The research sample was randomly chosen by means of a simple random sampling technique to give each subject a fair and equal chance.

- **Sample size**
  A sample size of 105 respondents was adequate as a large sample size approximates normality by the Central Limit Theorem (which recommended a sample size > 50). The effect size was calculated to find the degree to which the null hypothesis is false. After running an ANOVA the researcher noticed that there was no statistical significance in some groups which therefore suggested that the ANOVA should be run with a larger sample.

Garson (2012: 22) states that, "Analysis of Covariance (ANCOVA) has the same assumptions as any linear model except that there are two important additional considerations: (1) independence of the covariate and treatment effect, and (2) homogeneity of regression slopes". The covariate should not be different across the groups. The aforementioned assumptions were not violated in any regard.

3.4 RELIABILITY OF THE STUDY FOR INFERENTIAL ANALYSIS

TYPE I error is a type of error that occurs in the data-analysis stage where a researcher rejects the null hypothesis when in fact it is true, whereas TYPE II error is committed where a researcher accepts the null hypothesis when it is in fact not true. The researcher was mindful of the place and significance of test, not forgetting the problem of the Hawthorne effect operating negatively or positively on students who have to undertake the tests (Cohen et al, 2007: 117). The Hawthorne effect is the phenomenon in which participants alter their behaviour as a result of being part of the study.

The researcher ensured standardized procedures in administering the structured interview questionnaires. At the data-analysis stage, the researcher used SPSS to determine the correlation coefficient analysis where the degree of freedom was tested in order to avoid TYPE I and / or TYPE II errors by presenting the data without misrepresenting its meaning. The researcher ensured that invalidity was minimized as much as possible throughout the study by instituting a pilot study before the commencement of the study.

According to Macmillan and Schumacher (2006: 134) validity refers to truthfulness of findings and conclusion. Generally it refers to whether the measurement measured what it was suppose to measure. Validity of the study cannot be achieved by one set of structured questionnaires only, but the results of the three different questionnaires used should be analyzed concurrently.

The researcher ensured that the variables were isolated and controlled in the Sampling stage; the sample was randomly selected to avoid any influence on the study. To test the reliability and validity of the instruments the questionnaires were developed and administered as a pilot study. Variables of each research question were tested in the pilot stage of the study. "For research to be reliable it must be carried out on a similar group of respondents in a similar context (however defined), then similar results would be found" (Cohen et al, 2007: 118).

Correlation coefficient analysis was used to unlock the underlying factors that cause the learners to commit errors when working through financial mathematics problems.

3.5 ETHICAL CONSIDERATIONS OF THE STUDY

A code of ethics is an essential principle of any profession that deals with human life, it is therefore important to outline the ethical considerations of this study guided by the Research Ethics Framework of the University of Fort Hare.

A research portfolio (including a protocol checklist) was submitted to the University Research Ethics Committee for approval (cf Appendix F). It was approved and the Ethical Clearance Certificate was issued and the permission to continue with the study was granted (cf Appendix K).

Permission was requested from the East London education district as the research site fell under its periphery (cf Appendix I). A letter requesting permission to conduct a study in school X was written and delivered to the school (cf Appendix I). The permission was granted with no conditions attached (cf Appendix J).

Consent from parents or guardians were sought as some of the participants were minors. Informed consent forms included a brief outline of the nature of the study, a description of what was the participants' involvement, the duration of the study, the researcher's name and contact details, signature and date of the letter of agreement (cf Annexure E).

The data collected was treated with confidentiality and be protected. The data will be kept under strict care of the researcher, the supervisor and the academic staff of the institution. The rights of the participants were protected through anonymity, confidentiality and privacy. To maintain anonymity, confidentiality and privacy; participants' biographical information and the name of the school were hidden by use of the unique 3-digits codes for participants and the school was assigned a pseudo-name. To ensure that the participants were not exposed to any harm, the right to withdraw at any stage of the research was ensured.

3.6 CONCLUSION

This chapter outlined the research design and methodology used for this study. An explanation for the choice of methodology and the paradigm of the study was presented. It included reliability and trustworthiness of the study and the data analysis was acknowledged. The next chapter will present detailed analysis of the data collected. Although the researcher does not claim that the results would be applicable to all South African learners, there are commonalities between this group and others from a similar background. By providing the research report, other teachers or researchers would be able to examine the findings, and find points of commonality that might be applicable in other situations.

CHAPTER 4

DATA PRESENTATION AND ANALYSIS

4.0 INTRODUCTION

The current chapter focuses on the presentation and analysis of the data collected by means of four sets of structured questionnaires (cf Appendices A, B, C and D) described under questionnaire design in chapter 3. This study on error analysis in Grade 10 Mathematical Literacy was conducted in a High School in the East London district. Four sets of questionnaires were distributed to 165 Grade 10 learners and 105 valid questionnaires were collected.
The researcher sought answers to the following research questions:

1. Why do learners commit errors on given tasks in financial mathematics?
2. What is/are the underlying factor(s) related to the errors due to incorrect association or rigidity of thinking?
3. What is/are the underlying factor(s) related to the errors due to the application of irrelevant rules or strategies?
4. What is/are the underlying factor(s) related to the errors due to deficient mastery of prerequisite skills, facts and concepts?
5. What is/are the underlying factor(s) related to the errors due to language difficulties?
6. What degree of predictability and hence strategies underpin error analysis in questions 1-5?

Four research questionnaires were administered, seeking answers to the afore-mentioned research questions and identifying the underlying factors of the identified errors. Three questionnaires consisted of content-based questions that sought to afford respondents a platform to display the hypothesised errors. The fourth questionnaire sought the rationale behind the learners' committing errors and to identify the factors related to the errors.

Composition of data analysis techniques

The following are the data analysis techniques used in this section of the study:

> Newmans' Error Analysis (NEA) provided a framework for considering the reason that underlay the difficulties learners experienced with mathematical word problems and a process that assisted teachers to determine where misunderstandings occurred. NEA also provided directions for where teachers could target effective teaching strategies to overcome them (White, 2010: 129).

1. *Newman's error analysis* was used during the analysis of the content by reading questions and learner responses to the questions. White (2008: 2) states that: "Newman's Error Analysis was designed as a simple diagnostic procedure. Newman (1977, 1983) maintained that when a person attempted to answer a standard, written, mathematical word problem then that person had to be able to pass over a number of successive hurdles". The researcher critiqued and comprehended learner responses to determine their thinking while engaging with the questions and how they came to their responses. By repeating intensive analysis the researcher was able to identify and categorically group the different learner errors. This method is used for the content-based questionnaires (i.e. Questionnaires 1, 2 and 3).
2. *Quantitative analysis* with descriptive statistics describing the distribution, the relationship among variables and variability through the use of frequencies and means were used to analyze the fourth questionnaire. Statistical Package for Social Sciences (SPSS) version 21 was used for correlation coefficient analysis to measure the relationship between variables of each of the afore-stated research questions.
3. *Analysis of variance* (ANOVA) was used for the hypotheses of the study. It was used to test the degree of freedom and the levels of significance of each of the variables for research questions 3, 4 and 5. Based on the ANOVA results the researcher was able to accept or reject the null and alternative hypotheses.

4.1. THE DEMOGRAPHICS OF THE RESPONDENTS

The first questionnaire contained the demographics of the respondents, background information such as gender, age and the number of years the learner had been at that particular school.

4.1.1 Gender distribution of the study sample

Based on the table 4.1 below which is a summary of the gender composition of the study sample, 32.4% of the respondents were male and 67.6% were female. This question was included in order to clarify gender disparity description of the study sample as it does not form part of the research questions but is vital in the sample description.

Table 4.1: gender composition

Gender	Frequency	Percent (%)
Male	34	32.4
Female	71	67.6
Total	105	100.0

4.1.2 Age distribution in the study sample

Table 4.2 illustrates the age distribution in the research sample. The dominant age groups are 15-16 and 17-18 with the latter representing the majority in the sample. In the study sample 27.6% of the respondents were aged 15-16 with the majority (72.4%) of them aged 17-18 years.

Table 4.2: Age distribution in the research sample

Age	Frequency	Percent (%)
15-16	29	27.6
17-18	76	72.4
Total	105	100.0

4.1.3 The number of years the respondents had spent at that particular school

This question forms part of section A of questionnaire 4. This looked to establish the time each of the respondents spent at that particular school. The number of years the learner spent at a school does not form part of the research questions but assisted the researcher to recognize the learner's experience in the subject or the grade. The number of years the respondents spent at the school ranged from 1 year to 5 years. One year was represented by 1.9% who were newcomers from another school and were in Grade 10 for the first time, 2-years was represented by 9.5%, 3-years by 16.2%, 4-years by 49.5% and 5-years by 26.9% who were definitely repeaters. The pie chart below illustrates the representation of the number of years learners spent at the school.

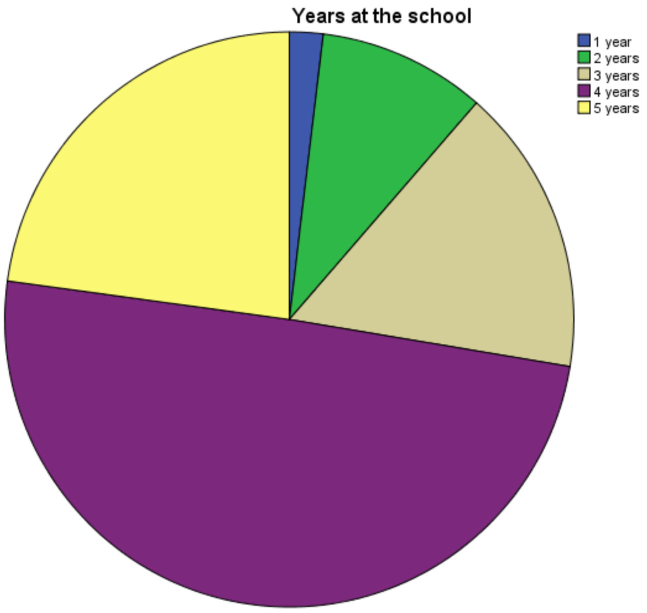

Figure 4.1 Pie chart of the number of years respondents spent at the school

All the afore-stated questions were not part of the research questions but the researcher used these questions as introduction and background information that can be useful in describing the sample. The questions were used to introduce the respondents to the questionnaire and the data collected is used to describe the demographics of the sample. Based on the afore-mentioned data, the majority of the respondents were female, majority of respondents (17-18years of age) and had been at the school for 4 years.

Discussion of the different questionnaires in addressing each of the research questions is presented below:

4.2 RESEARCH QUESTIONS AND THE FINDINGS

The study focused on the five research questions listed (cf Chapter 1, section 1.6) by the researcher as he sought answers to the questions and had to interrogate the learners' rationale as they did the financial mathematics assessment tasks.

4.2.1 Why do learners commit errors in given tasks in financial mathematics?

Seven variables were tested in search of the underlying reasons related to research question 1. Those formed part of section B of questionnaire 4 and are listed in the table below. The frequency tables for each of the variables are illustrated below:

Table 4.3: I want to do my best in my tests

	Frequency	**Percent (%)**
Never	2	1.9
Rarely	4	3.8
Sometimes	19	18.1
Always	80	76.2
Total	105	100.0

Most (n = 80; 76.2%) always want to do their best in tests (cf Table 4.3). The above variable has a mean of 3.69 which is the highest mean in the group. It has according to Field (2012) a moderate positive correlation where $r = .39$ with variable 7 (reviewing homework after having done it). According to this response learners always want to do their best in a test and there is a moderate relationship between wanting to do their best in a test and reviewing their homework.

Table 4.4: I feel confident when I submit my test because I know I have done my best

	Frequency	Percent (%)
Never	2	1.9
Rarely	7	6.7
Sometimes	59	56.2
Always	37	35.2
Total	105	100.0

Almost half of the study sample (n =59; 56.2%) sometimes (cf Table 4.4) feel confident when submitting their tests. Learners did not display confidence when they submit their test as they were not certain that they had done their best in the test. This is a clear indication that there was always a lack of confidence which could have been caused by learners not being well prepared for the test or did not understand the questions in the test due to other factors. The above variable (feeling confident when submitting their test) has a weak correlation where r =.09 with the first variable (I want to do my best in my test). Based on the afore-stated frequency table ambiguity was visible from the learners' responses in responding to this variable. They could not confidently state whether they felt confident when they submitted their test scripts. This could be caused by the nature of the question itself or learners were not honest in their response.

Table 4.5: I feel rushed when I am writing a test

	Frequency	Percent (%)
Never	27	25.7
Rarely	19	18.1
Sometimes	50	47.6
Always	9	8.6
Total	105	100.0

Almost half of the research population respondents (n =50; 47.6%) could not confidently answer this question but 'sometimes' (cf Table 4.5) they felt rushed when writing a test. Uncertainty was also displayed in the learners' responses to this variable. It could have been caused by the fact that they did not want to give any negative response. They could have interpreted it as negative reporting on their teacher and needed to stay neutral.

Table 4.6: I want to be the first one done on the test

		Frequency	Percent (%)
Valid	Never	48	45.7
	Rarely	16	15.2
	Sometimes	30	28.6
	Always	11	10.5
	Total	105	100.0

The afore-stated sub-variable ‘wanting to be the first one to finish’ (n = 48; 45.7%) accounted for never wanted to finish first and therefore the majority of the respondent implied that they did not rush to be the first ones to finish when writing the test (cf Table 4.6). The variable indicated a negative weak correlation where $r = - .04$ with variable 5 (I go back and read through what I have written before handing in the test script) and $r = - .056$ with variable 6 (I take time to answer test questions). Even though 45.7% of the respondents could not attribute their errors to their wanting to finish first in the test, that variable does not correlate with going through their answers before submitting it and taking time to answer questions. This means the relationship between the afore-stated variables with wanting to do their best in test could not be established.

Table 4.7: I go back and read through what I wrote before handing in the test script

		Frequency	Percent (%)
Valid	Never	8	7.6
	Rarely	10	9.5
	Sometimes	37	35.2
	Always	50	47.6
	Total	105	100.0

For the afore-stated sub-variable almost half (n = 50; 47.6%) of the respondents always went back and read through what they had written before handing in their test scripts (cf Table 4.7). The above-stated variable has a negative weak correlation where $r = -.12$ with variable 3 (I feel rushed when I am writing a test). That indicates that there is no significant relationship between the two variables.

Table 4.8: I take my time to answer the test questions

		Frequency	Percent (%)
Valid	Never	5	4.8
	Rarely	8	7.6
	Sometimes	47	44.8
	Always	45	42.9
	Total	105	100.0

For the afore-stated sub-variable almost half (n = 47; 44.8%) of the respondents sometimes took time to answer the test questions whereas almost an equal number (n = 45; 42.9%) always read the test questions (cf Table 4.8). The above-mentioned variable has a weak correlation (where $r = .09$) with variable 5 (I go back and read through what I have written before handing the test script). That indicates that there is no significant relationship between the two variables.

Table 4.9: I review my homework after I have done it

		Frequency	Percent (%)
Valid	Never	12	11.4
	Rarely	7	6.7
	Sometimes	48	45.7
	Always	38	36.2
	Total	105	100.0

Table 4.9 illustrates that almost half of the research population (n = 48; 45.7%) sometimes reviewed their homework whereas just below half (n = 38; 36.2%) always reviewed their homework (cf Table 4.9). This indicates that only 36.2% of the respondents in the sample reviewed their homework. The above-mentioned variable has no correlation where $r = .00$ with variable 6 (I take time to answer the test questions).

Variables 3 and 4 displayed the lowest mean compared to the other five variables. The ambiguity of the responses to the two variables was also obvious from the frequency tables displayed in the afore-stated data presentation. Correlation between each of the afore-stated variables was tested and all variables displayed weak correlation with only two variables, variable 1: ‘I want to do my best in my tests’ and variable 7: ‘I review my home work after doing it’ displaying a moderate correlation where $r = .39$. The majority of the variables in the afore-stated research question showed no evidence of any correlation amongst them.

Even though there is no relationship among the variables, the frequencies of the responses suggest the respondent account for the variables as underlying factors for learners committing errors.

Makonye (2011: 144) ascertains that: “It is necessary to infer the most effective mathematical strategy the learner used in order to arrive to their answers”. The researcher coded learner errors in order to categorize them. The coding and categorizing was a repeated process before the type of errors learners commit could be identified.

Summary of main findings of research question 1

Based on the analysis of the frequencies of the variables in research question 1 it was revealed that the majority (76.2%) of the respondents always wanted to do their best in a test but from the sample only 35.2% of the respondents always felt confident when submitting their test scripts, whereas 56.2% sometimes felt confident.

Only 8.6% attested that they always felt rushed when writing a test when 47.5% sometimes felt rushed but about 28.6% of the respondents sometimes wanted to be the first ones to finish writing whereas 10.5% always wanted to be the first to finish writing.

The majority which is 83.1% of the respondents claimed to go back and read through what they had written even though the study revealed that only 47.6% always did that.

From the summary of the responses it was ascertained that 87.7% of the respondents took time to answer the test questions with only 42.9% always doing that.

The majority which is 81% of the respondents reviewed their homework but only 36.2% always did that, which means 45.5% occasionally did it.

4.2.2 What is/are the underlying factor(s) related to the errors due to the incorrect association or rigidity of thinking?

Questionnaire no. 1 (cf Appendix A), consisted of five questions based on simple and compound interest; Questions 1 and 2 wanted to give respondents an opportunity to commit errors that are related to the afore-mentioned research question. Out of 105 questionnaires, 87 learners (i.e. 83%) used formulae in answering these questions whereas Curriculum Assessment Programme Statements (CAPS) is silent on the use of formulae. CAPS encourage use of multi-step procedures that involve the basic algorithms (addition, subtraction, division and multiplication) in working out simple and compound interest problems. The researcher checked through each of the questions identifying errors, engaged in determining the learners' reasoning in the process of working out the solutions to the questions. A number of errors were identified, ranging from use of incorrect or rather irrelevant formula to incorrect substitution.

1. *Calculate the interest on R4 500 loan at 14% interest per year over a period of 6 years.*

This question required learners to work out the interest on simple interest; then again language was an attributive challenge as learners had different interpretations. Some worked out the future value (amount to be paid back after 6 years), therefore used the Formula: **A = P(1 + *in*)**. Others worked out the interest as the question required, therefore using the formula: **SI = P × *I* × *n***. For those who used formula whether relevant or not it may be attributed to the recalled prior knowledge acquired in Grade 9.

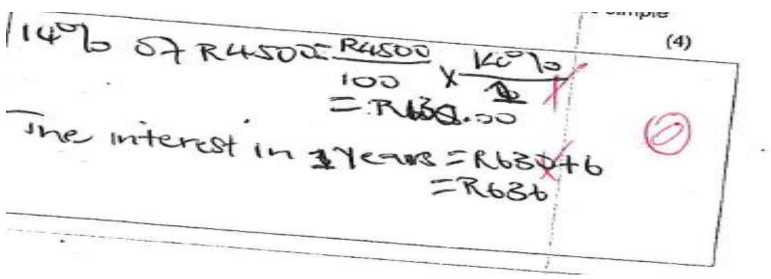

Figure 4.2: Response of learner no.16 to question 1

Learner 16 identified the correct and relevant method to use in answering the afore-stated question but instead of multiplying by 6 added 6. Elbrink (2008) and Nolting (1998) would classify this type of error as a *Procedural error* but the researcher regards this type of error as *error due to incorrect association*. Nolting (1988) states that the procedural error includes an incorrect step which is use of steps not associated with any operations (cf Chapter 2). In this instance the learner did have a clue of the steps to be followed but incorrectly used addition instead of multiplication. Here the researcher aligns himself with Radatz (1979) who describes it as incorrect interactions between single elements. In this instance the single element is the use of the addition (+) sign instead of the multiplication (x) sign. The aforementioned error was hypothesised (cf Chapter 1 section 1.4) in the background of the study. The learner demonstrated an error of association, which indicates that the learner knew the correct algorithms to employ but confused the multiplication sign (×) with the addition sign (+).

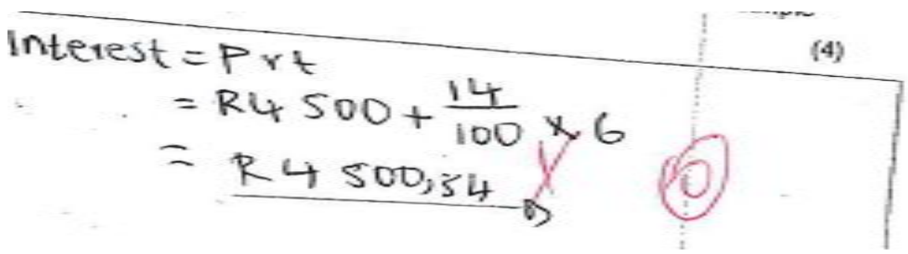

Figure 4.3: Response of learner no.59 to the afore-stated question 1

Learner 59 committed a similar error in that the learner again confused the multiplication sign (×) with the addition sign (+). The difference is that learner 16 was able to calculate 14% of R4 500 but instead of multiplying by 6 decided to add 6. Learner 59 knew that 14% is $\frac{14}{100}$ but instead of multiplying decided to add 14%. They both confused the multiplication sign for the addition sign but in different parts of their steps. Or maybe because they were supposed to add the percentage, they decided to directly add it without working it out first, then adding it.

To arrive at the correct solution they should have worked out the interest as:

$$Interest = R4\ 500 \times \frac{14}{100} \times 6 = R3\ 780$$

They both arrived at incorrect solutions but committed errors in different parts of their answers.

The majority (n = 84; 80%) used a formula but an incorrect formula. This type of error according to Nolting (1998) can be classified as *Transformation error* as the learners understood the concept of simple interest but could not identify the correct operation to use in order to arrive at the correct solution.

The minority (n = 20; 19%) understood the question and employed the correct procedure to arrive at the correct solution, but 5 of them (5% of the sample) proceeded and added the interest to the loan amount (i.e. *R4 500*) as if they were asked to determine the value after it had accumulated interest.

The majority of the respondents employed irrelevant procedures; as a result they could not arrive at the correct answer to the question. This may be attributed to a number of factors such as (1) the concept threshold developed in the lower grades; learners used the formula (2) error of assimilation, they knew the correct algorithm but confused multiplication with addition.

2. *Determine the value of R5 000 loan at 9% interest per year compounded monthly over a period of 4 months.*

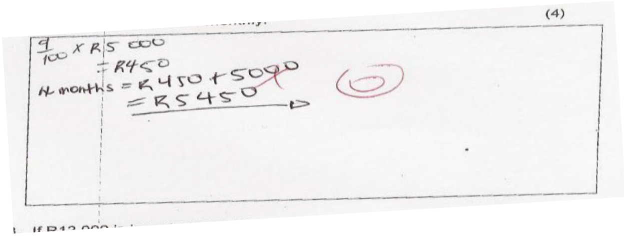

Figure 4.4: Response of learner no. 47 to question 2

Learner 47 calculated 9% of R5 000 correctly but did not consider the fact that 9% interest is compounded monthly for 4 months. 58 learners out of 105 respondents (55% of the sample) did exactly as learner 47 i.e. only calculated 9% of *R5 000,* but did not consider the fact that the interest is compounded monthly for *4 months.*

The respondent should have used the following method:

9% ÷ 12 = $\frac{3}{400}$ as the interest is compounded monthly

R5 000 + (R5 000 × $\frac{3}{400}$) = R5 037.50 for the first month

R5 037.50 + (R5 037.50 × $\frac{3}{400}$) = R5 075.28 for the second month

R5 075.28 + (R5 075.28 × $\frac{3}{400}$) = R5 113.35 for the third month

R5 113.35 + (R5 113.35 × $\frac{3}{400}$) = R5 151.70 for the fourth month

The respondent clearly missed that step. This type of error is classified as *Procedural error of missing steps* as learners missed the steps in their solution to the problem.

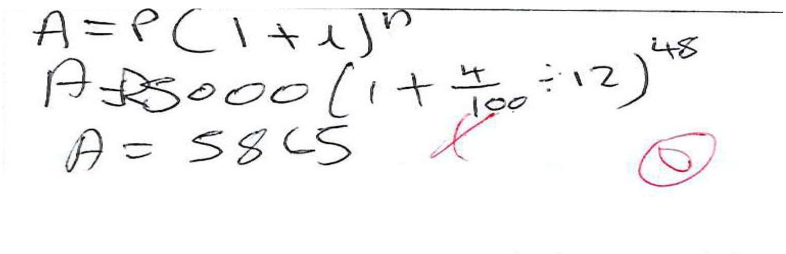

Figure 4.5: Response of learner no. 94 to question 2

Learner 94 decided to use the formula even though the use of formula is irrelevant, and also incorrectly substituted the formula. Interest rate is 9% but substituted 4%, the *n*-value is 4 not 48. This question required learners to work out the value with interest compounded monthly.

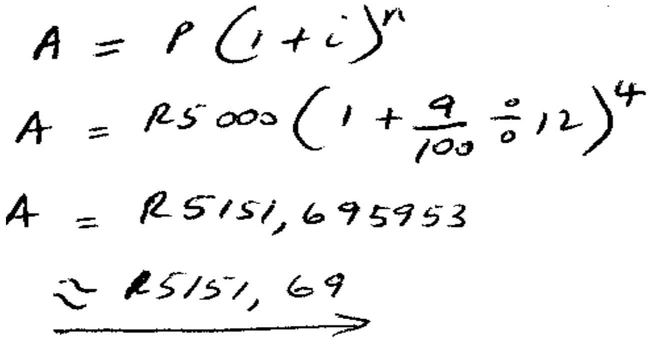

Figure 4.6: Response of learner no.89 to the afore-stated question 2

Learner 89 used the correct formula, correctly substituted the formula and arrived at the correct solution to the problem. The relevance of the formula can stimulate a debate as CAPS is not clear on the use of the formula. The learner could have recalled the *Threshold concept* entrenched in the lower grades or the educator taught them the use of formula.

Learner 17 and learner 47 committed a similar error; that of calculating 9% correctly but both did not divide the interest by 12 then multiply by 4.

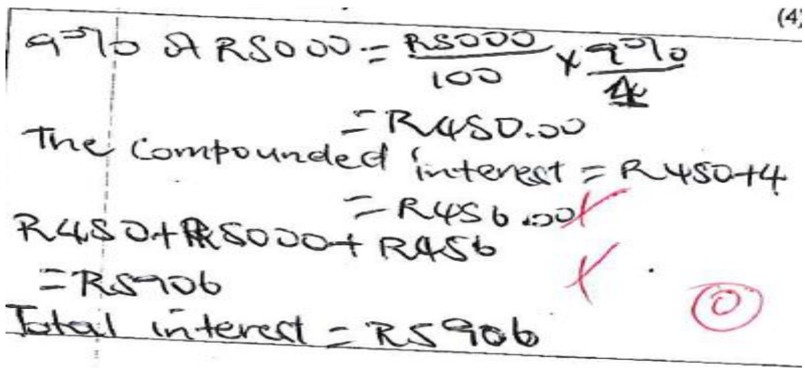

Figure 4.7: Response of learner no. 17 to question 2

Learner 78 used an algorithm similar to learner 94 but unlike learner 94; did not divide 9% by 12 even though it was compounded monthly and the *n*-value was given as 4.

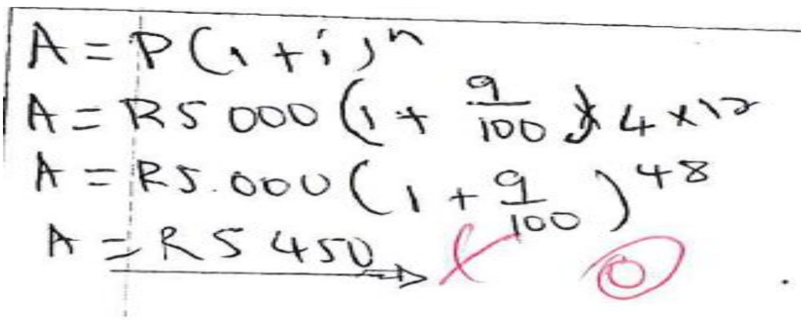

Figure 4.8: Response of learner no. 78 to question 2

Some learners used an incorrect formula i.e. **A = P (1 + *in*)** instead of **A = P(1 + *i*)*ⁿ***. This according to Brodie (2005) is classified as *Partial Insight* as learners show that they have grappled with an idea which is incorrect but show insight in the task.

Some would substitute correctly while others would substitute an incorrect value, as ***i*** (interest rate) must be divided by 12 if the interest is charged monthly. Others did divide the interest by 12 but then multiplied *n* by 12 as if the period was given in years. There was no need to multiply by 12 as the period was already given in months (therefore no conversion was necessary).

Twenty-seven learners out of 105 (25.7%) respondents used the formula; 13 (12% of the sample) could not round off correctly (i.e. *Placement error)*. Nolting (1998) classifies this type of error as an *Encoding error* as learners solved the problem but did not write the solution in an appropriate form. The questions; as well as the study focused on financial mathematics, money which should be presented in 2-decimal places.

Twenty learners (20) out of 105 respondents (19% of the sample) used the formula, substituted correctly except for the value of *n* as 12× 4 = 48. That is the method used when *n* value is given in years and the interest is compounded monthly, it then requires conversion of ***i*** *(interest rate)* by dividing by 12 and converting *n* by multiplying by 12. They overlooked the fact that *n* was already given in months; therefore there was no need for conversion. This type of error confirms the underlying factor related to an error due to *Incorrect Association or Rigidity of Thinking.* Learners did not allow flexibility in their thinking and took notice of the fact that *n (period)* was already given in months. This can also according to Nolting (1998) be classified as *Transformation error* as incorrect steps are followed in solving the problem.

Table 4.10: Frequency table summarizing learner performance per question

Question No.	No attempt	%	Correct Response	%	Incorrect	%	Total
1.	16	15%	43	41%	46	44%	**105**
2.	11	11%	40	38%	54	51%	**105**
Total	27	13%	83	39%	100	48%	**105**

n = 105

Table 4.10 summarizes the analysis of the learner responses to questionnaire 1. About 48% of the sample gave incorrect responses to questions 1 & 2 whereas only 39% correctly answered those questions. About 13% of the respondents did not attempt to answer the afore-stated questions.

Three variables were tested in search of the underlying factors related to the research question. Below are the tables that show the frequency tables of each of the variables related to underlying factors for learner errors:

Table 4.11: I confuse addition with multiplication

		Frequency	Percent (%)
Valid	Never	26	24.8
	Rarely	12	11.4
	Sometimes	63	60.0
		4	3.8
	Total	105	100.0

Respondents could not give a confident response to the above variable as 60% of the sample responded as sometimes. Ambiguity of the responses to the question could be attributed to a number of factors as the respondent might have owned responsibility in committing the stated error. The above-mentioned variable has a mean of 2.43.

Table 4.12: I forget to write units

		Frequency	Percent (%)
Valid	Never	31	29.5
	Rarely	21	20.0
	Sometimes	45	42.9
	Always	8	7.6
	Total	105	100.0

From the selected sample 42.9% of the respondents gave ‘sometimes’ as their response to the above variable, which indicates that they could not confidently account for committing errors based on the above variable.

Table 4.13: I write down an incorrect number

		Frequency	Percent (%)
Valid	Never	66	62.9
	Rarely	8	7.6
	Sometimes	29	27.6
	Always	2	1.9
	Total	105	100.0

The majority of the respondents (i.e. 62.9% of the sample) confidently responded as ‘never’ to writing down the incorrect number. Only 2% admitted to ‘always’ writing an incorrect number. The above-mentioned variable has the lowest mean of 1.69. The researcher categorized each identified error with the related underlying factor.

Table 4.14 that follows summarises the identified errors with the identified factors related to the errors committed.

Table 4.14: Identified learner errors and the associated underlying factors

Question No.	Identified errors	Related underlying factor of identified error
1.	Use of formula even though not required; use of incorrect formula: A = P (1+*in)* instead of *SI* = *P* × *I* × *n* as the relevant formula if the question required the use of formula in calculating Interest.	Application of irrelevant rules or strategies
2.	Use of formula even though not	Incorrect association or rigidity

	required; use of incorrect formula: A = P (1 + *in*) instead of $A = P(1 + i)^n$ as the relevant formula if the question required the use of formula in calculating compound interest.	of thinking

Summary of main findings of research question 2

Based on the analysis of questionnaire 4 which sought to answer research question 2 (what are the underlying factors related to errors due to incorrect association or rigidity of thinking?), it was revealed that 63.8% of the respondents admitted to sometimes confusing the addition sign with the multiplication sign but 60% only did that occasionally as against 3.8% who admitted to always making that error.

From 105 respondents 50.5% sometimes forgot to write units whereas only 7.6% always committed such an error, 42.9% admitted to occasionally committing such an error.

Learners would write an incorrect number or digit when transcribing from their rough work, 29.5% of the respondents attested to sometimes committing this type of error with 1.9% always doing that.

4.2.3 What are the underlying factors related to errors due to the application of irrelevant rules or strategies?

Questionnaire 2 consisted of three questions with question 3 having three sub-questions (cf Appendix B). The first question demanded respondents to determine Jabu's income and were given his hourly pay of R44.50, the number of days per week, hours per day. A number of errors ranging from forgetting to write a comma expressing money to writing money in one decimal place could be identified.

1. *What is Jabu's income if he works for 6 days, 7 hours per day and is paid R44.50 per hour?*

To arrive at the correct answer learners were supposed to multiply the hourly pay by the hours worked then by the number of days worked.

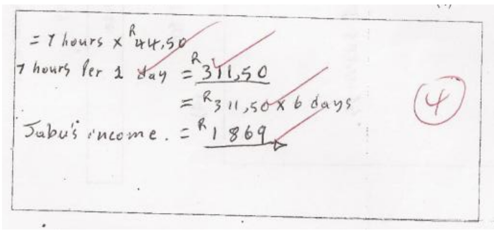

Figure 4.9: Correct response to question 1 by learner no. 94

Some would only multiply the hourly pay by number of hours worked and number of days worked not considered or vice versa. 81% of the respondents got question 1 correct and the errors identified on the 25% of the respondents who got incorrect answers were mostly *careless mistakes*.

2. *Portia travels 900km per month to get to work. The fuel consumption of her car is 12km per litre and she pays R10.80 per litre for fuel. Determine Portia's transport expenses for a month.*

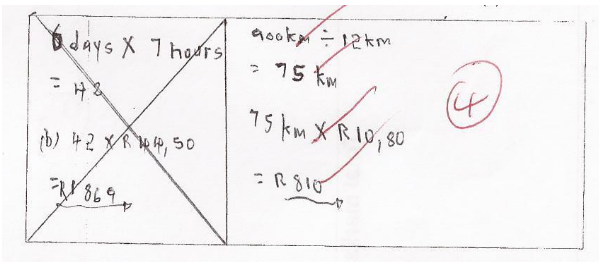

Figure 4.10: Correct response to question 2 by learner 18

In question 2, respondents were expected to determine transport expenses for a month taking into account the distance travelled per month, fuel consumption of the car and fuel price. They were expected to divide the distance travelled per month by fuel consumption then multiply by fuel price.

Thirty five percent (35%) of the respondent incorrectly answered question 2. They decided to multiply distance travelled by fuel consumption and fuel price. This is attributed to the fact that in the previous question all the components were multiplied. *Incorrect association and rigidity thinking* surfaces again in this question as it was identified in questionnaire 1.

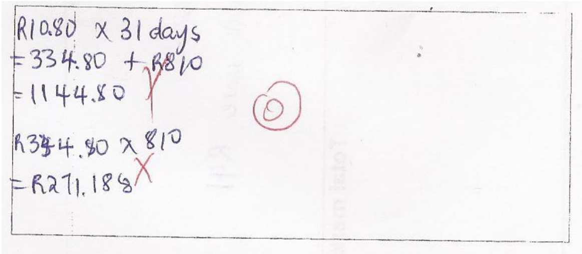

Figure 4.11: Incorrect response to question 2 by learner 24

3. *Reginald earns a salary of R5 700 per month. From his salary 15% is deducted for income tax, R850 for medical aid fund and R1 406 for his motor vehicle instalment.*

(a) How much is left for Reginald to cover his other expenses?

In this question, respondents were given Reginald's earnings per month, his deductions; income tax given as a percentage, therefore they had to work it out. They we required to determine the amount left after deductions. Instead of working out 15% as the income tax then subtract it from the salary; they would subtract 15 from the salary.

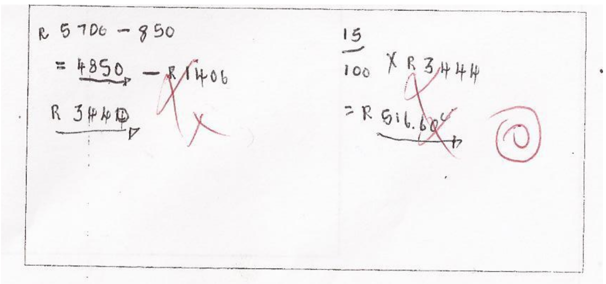

Figure 4.12: Incorrect response to question 3(a) by learner 12

Learner 12 worked out the problem correctly but did not show all the steps that led to the correct answer. Nolting (1998) classifies this type of error as the *Procedural error*. The respondent 7 just worked out the total expenses R3111 using a calculator and correctly subtracting that from R5700 which was the monthly salary. If the respondent had made a mistake in adding the total expenses that would have affected the final answer and would not have been credited.

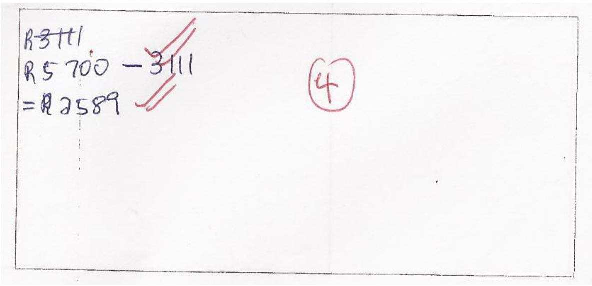

Figure 4.13: Correct response to question 3(a) by learner 7

(b) *If all his other necessities amount to R1 480, what is left for entertainment and savings?*

This is a sub-question to the above, therefore in order to arrive at the correct answer to this question the respondent should find the correct answer from above. It required simple mathematical procedure of subtracting the given amount from the amount that was left (R2589 which was calculated as the amount left from the previous question).

Figure 4.14: Correct response to question 3(b) by learner 31

(c) *If Reginald's payment to the medical aid fund increase by 8% and he spends R150 on entertainment, how much is he able to save?*

This is the confirmation of the afore-stated question; therefore the respondents were expected to work on the values from the previous question. The medical payment was given and the instruction was given that medical aid payment should be increased by 8%. Learner 16 calculated 8% of the R150 for entertainment because the learner only focused on this question as an isolated question. The learner only focused on the values presented in the sub-question not looking at the question as a whole.

This type of error is regarded as *Comprehension error* as the learner read the words correctly but could not make sense of the actual problem to be solved.

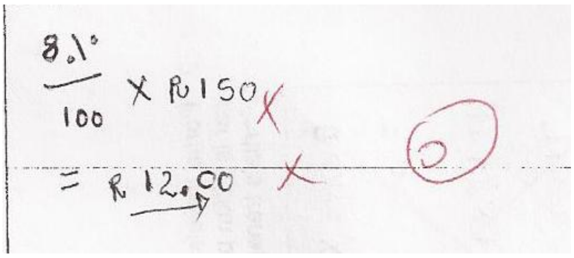

Figure 4.15: Incorrect response to question 3(c) by learner 16

The respondent calculated 8% of R150 which is the amount spent on entertainment, instead of calculating the increase by 8% to the medical aid contribution of R850. In this instance the respondent was able to identify the relevant method to apply but applied it incorrectly.

The subsequent table 4.15 summarises the frequencies of learner performance regarding the afore-stated questions; correct and incorrect responses.

Table 4.15: Learner performance per question

Question No.	Not attempted	%	Correct Responses	%	Incorrect Responses	%	Total
1.	0	0	46	44%	59	56%	**105**
2.	0	0	37	35%	68	65%	**105**
3.(a)	6	6%	40	38%	59	56%	**105**
(b)	5	5%	40	38%	60	57%	**105**
(c)	4	4%	41	4%	60	63%	**105**

n = 105

Below are the frequency tables for the six questions from questionnaire 4 which sought to uncover the underlying factors influential on learners' rationale in committing errors.

Table 4.16: I lose marks on my work for not showing my workings

		Frequency	Percent (%)
Valid	Never	17	16.2
	Rarely	13	12.4
	Sometimes	66	62.9
	Always	9	8.6
	Total	105	100.0

Respondents could not give an unambiguous response on this question, 62.9% of the respondents provided 'sometimes' as the response to the question. They claim to lose marks for not showing their working as an event that only occurred sometimes not regularly.

Table 4.17: I lose marks on my work for not completing all the problems

		Frequency	Percent (%)
Valid	Never	17	16.2
	Rarely	26	24.8
	Sometimes	48	45.7
	Always	14	13.3
	Total	105	100.0

Only 45.7% of the respondents accounted for 'sometimes' as the response to the question which indicated that they did not consistently lose marks due to incomplete work. Only 13.3% always lost marks for not completing their work.

Table 4.18: I do not show all my steps

		Frequency	Percent (%)
Valid	Never	19	18.1
	Rarely	12	11.4
	Sometimes	70	66.7
	Always	4	3.8
	Total	105	100.0

Most respondents (i.e. 66.7% of the sample) chose 'sometimes' as the relevant answer to the question, which indicated that it only occurred sometimes not regularly. 'Always' accounted for 3.8% which indicated that it was the minority that always missed steps in their workings.

Table 4.19: I make basic computation errors

		Frequency	Percent (%)
Valid	Never	13	12.4
	Rarely	25	23.8
	Sometimes	57	54.3
	Always	10	9.5
	Total	105	100.0

Only 9.5% of the sample confirmed to making basic computation errors (like rounding off, addition) in their workings, 54.3% claimed to only committing these errors 'sometimes'.

Table 4.20: I show all my steps but do not find the correct answer

		Frequency	Percent (%)
Valid	Never	21	20.0
	Rarely	13	12.4
	Sometimes	64	61.0
	Always	7	6.7
	Total	105	100.0

Respondents could not give a certain answer to the question, (n = 64, 61%) of the respondents claimed to 'sometimes' find an incorrect answer even though they showed all the steps in their workings. Only 6.7% of the sample accounted for always showing their steps and getting an incorrect answer to the question.

Table 4.21: I show my steps but not all of them

		Frequency	Percent (%)
Valid	Never	12	11.4
	Rarely	15	14.3
	Sometimes	66	62.9
	Always	12	11.4
	Total	105	100.0

From the study sample (n = 66; 62.9%) that percentage of respondents associate themselves with not always but 'sometimes' showing their steps but not all of them and 11.4% regarded themselves as 'always' showing their steps but not all of them.

Out of 46 learners who answered question 1 correctly 17 learners (36%) did not get the full 4 marks on the question. They multiplied the hours worked by the rate per hour (i.e. *R44.50 × 7 hours = R311.50).* When writing the answer, they wrote *3115 instead of R311.50,* and that indicated the incompetence in rounding off and they were not sure of the place values. When they continued with the problem, they multiply by a number of days (i.e. *3115* × *6 = 18 690*).

Table 4.22 summarizes the errors identified during the content analysis using Newman's error analysis. These are listed, described from each question and associated with underlying factors based on research questions.

Table 4.22: Identified learner errors and the associated underlying factors

Question no.	**Identified errors from incorrect responses**	**Underlying factor associated with error committed**
1.	Multiplied hours worked by days (i.e. 7×6 and given an incorrect answer 36 instead of 42. Then 36×R44.50=R1602	Application of irrelevant rules or strategies
2.	Multiplied distance travelled by price of fuel (i.e. 900×10.80 = 9 720). Others multiplied the fuel price by number of day in a month (10.80 × 31 days = 334.80 + 810). R810 is a product of 75 litres × R10.80.	Application of irrelevant rules or strategies
3. (a)	5700 ÷ 15 = 380 380 + 850 = 1230 as the amount left after expenses	
(b)	It was then impossible to subtract the other necessities from the amount calculated in the previous question, as a result learners decided to work it out as (R1 480 – R1 230 = R250).	Deficient mastery of prerequisite skills, facts and concepts
(c)	• R850 + 8% = R858	

n = 105

Summary of the analysis of research question 3 above in terms of main errors committed.

When confronted with mathematical problems respondents sometimes felt confused about the type of the mathematical algorithm to employ in order to arrive at the correct answer. That is evident from the content analysis of their responses in trying to solve the afore-stated problems where they seemed undecided about whether to multiply which value, and when. They just used the value found in the problem statement without even looking at the relevance. The majority admitted to losing marks for sometimes not showing their workings or for incomplete work. Only 9.5% admitted to marking the basic computation errors even though those were evident in their work. This could be because they did not understand what the computation error was or maybe it was not clearly explained in the questionnaire. The proper sequence of the algorithms assisted in arriving at the correct answer.

Based on the analysis of questionnaire 4 which sought to address research question 3 (what are the underlying factors related to the errors due to the application of irrelevant rules or strategies?), it was revealed from content analysis that about 71.5% lost marks for not showing all the steps. However, only 8.6% admitted to always commit such errors, with the majority (which is 62.9%) occasionally committing such errors. From a sample 54.3% lost marks for not completing the work assigned, only 8.6% admitted to always committing such errors and 45.7% occasionally committing such errors. It was established that 63.8% made basic computation errors and only 9.5% always made such errors. A number of respondents, about 61% showed all the steps but sometimes could not arrive at the correct answer when 6.7% admitted they always did not find the correct answer even if they showed all their steps. From the sample 62.9% admitted to occasionally showing their steps but not all of them.

4.2.4 What are the underlying factors related to errors due to deficient mastery of prerequisite skills, facts and concepts?

The following question formed part of the content based questionnaire and is followed by the examples of learner responses.

3. *If R12 000 is invested at 9.5% simple interest per year, calculate the value of the investment after 4 years and three months.*

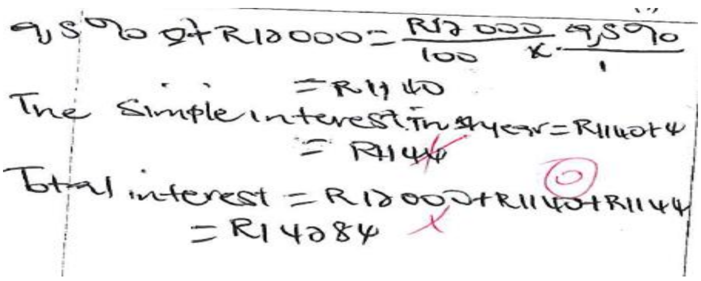

Figure 4.16: Response of learner no. 15 to question 3

The learner worked out 9.5% of R12 000, then added 4 to the answer instead of multiplying by 4 but even then the period of investment was 4 years and 3 months not 4. Calculated total interest was then added to the invested amount which was R12 000 in this regard. Elbrink (2008) classifies this type of error as a *calculation error* as the learner mistakenly used addition instead of multiplication. Nolting (1998) classifies this type of error as a *procedural error* as the learner employed incorrect steps.

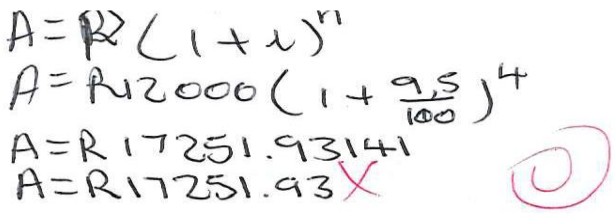

Figure 4.17: Response of learner no. 37 to question 3

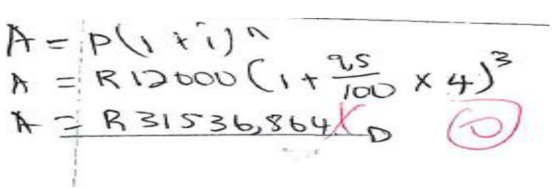

Figure 4.18: Response of learner no. 19 to question 3

The learner calculated 9.5% of R12 000, multiplied the answer by 4 and missed the fact that the period of investment was 4years 3 months. Learner 19 committed the exact same error as learner 59.

Learner 19 used a formula *A = P (1 + in)* but then incorrectly substituted the formula, which is according to Elbrink (2008) classified as the *calculation error*. The learner substituted 4 for *n*, and this can also account for *procedural error* as *incorrect steps* were followed and *missing steps* identified. The errors identified in the afore-stated learner responses can be classified as *errors due to incorrect association or rigidity of thinking.*

Incorrect association is justified by the use of incorrect steps, where learners add instead of multiply. The incorrect steps might be attributed to the threshold concept where learners had been taught a particular method and tended to use it even in irrelevant situations.

4. *How long will it take R5 100 invested at 9% simple interest per year to yield an amount of R7 854?*

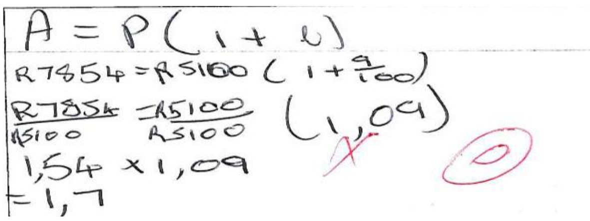

Figure 4.19: Response of learner no. 32 to question 4

Learner 32 wrote an incomplete formula, even though the learner substituted the formula correctly, but there was a missing component in the formula. The learner was required to determine how long it would take the invested amount to yield a given value (i.e. find the value of *n*).

Expected Correct method

A = P(1 + in)

R7 854 = R5 100(1 + $\frac{9.5}{100}$***× n)***

$\frac{R7\ 854}{R5\ 100}$***= 1 + 0.09 × n***

1.54 – 1 = 0.09 × n

$\frac{0.54}{0.09}$***= n***

6 = n

Therefore it will take 6 years for the invested amount to yield R7 854.

When comparing the method used above it is clear that the learner did not write the formula correctly, and as such could not arrive at the correct answer. This type of error could be classified as an *error due to incorrect association or rigidity of thinking*. The fact that a component of the formula was missing would be classified as a *missing step error*.

5. *Calculate the value of R9 700 invested at 9.5% per annum compound interest for a period of 3 years.*

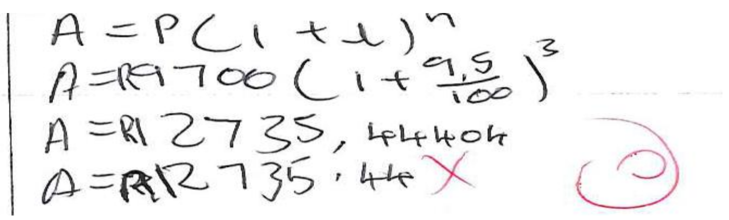

Figure 4.20: Response of learner no. 36

The formula used by learner 36 is correct but as the learner proceed with the step towards solving the problem decided to hide the figures. The step before the final answer or the final answer could not be clearly distinguished whether is R112735.44 or A R735.44 which is the indication of the lack of confidence.

Table 4.23 below illustrates the summary of respondents' performance per question, with the majority in each answering incorrectly. In the questions listed above 53.5% of the respondents gave incorrect responses while 36.8% gave correct responses.

Table 4.23: Frequency table for learner performance per question

Question No.	Not attempt	%	Correct Response	%	Incorrect Response	%	Total
1.	6	15%	43	41%	46	44%	**105**
2.	1	11%	40	38%	54	51%	**105**
3.	5	5%	38	36%	62	59%	**105**
4.	3	12%	36	34%	56	53%	**105**
5.	6	6%	36	34%	63	60%	**105**
Total	**51**	**9.7%**	**193**	**36.8%**	**281**	**53.5%**	**525**

n = 105

Frequency tables have been drawn up for the statistical frequencies for each variable in this research question 4.

Table 4.24: I forget to read the instructions

		Frequency	Percent (%)
Valid	Never	67	63.8
	Rarely	10	9.5
	Sometimes	26	24.8
	Always	2	1.9
	Total	105	100.0

This frequency table illustrates that, 63.8 % of the respondents ‘never’ forget to read the instructions, whereas 24.8% of them ‘sometimes’ forget and only 1.9% of them always forgot to read the instructions.

Table 4.25: I round off the answer into 2 decimal places

		Frequency	Percent (%)
Valid	Never	11	10.5
	Rarely	9	8.6
	Sometimes	52	49.5
	Always	33	31.4
	Total	105	100.0

The respondents do not always round of their answers to 2 decimal places. That is explained by their choice of response which is ‘sometimes’ and accounted for 49.5% of the respondents while 31.4% admitting to ‘always’ round off the final answer to 2 decimal places. Few respondents accounted for ‘never’ (i.e. 10.5%) and ‘rarely’ (i.e. 8.6%) as the response to the above-stated question.

Table 4.26: I do round off but incorrectly

		Frequency	Percent (%)
Valid	Never	29	27.6
	Rarely	17	16.2
	Sometimes	53	50.5
	Always	6	5.7
	Total	105	100.0

The majority of the respondents chose ‘sometimes’ and that accounted for 50.4% of respondents in the sample to round off but incorrectly and only 5.7% always rounded off incorrectly. Some respondents chose ‘never’ (i.e. 27.6%) and ‘rarely’ (i.e. 16.2%) as responses to the question.

Table 4.27: When using a calculator I forget to write down the correct answer

		Frequency	Percent (%)
Valid	Never	67	63.8
	Rarely	16	15.2
	Sometimes	20	19.0
	Always	2	1.9
	Total	105	100.0

Most respondents claimed that they never forgot to write down the answer. That is justified by their response to ‘When using a calculator I forget to write down the correct answer’, 63.8% of the respondents chose ‘never’ in a 4-Likert scale with a mean of 1.59.

Table 4.28 illustrates the types of errors identified from the responses together with the underlying factor structure which could have influenced the errors. According to McHugh (2008), a standard error is a measure of the variability of the sampling distribution. When looking at the standard error illustrated above the sample was fairly distributed.

Table 4.28: Identified learner errors and the associated underlying factors

Question No.	Identified errors	Underlying factors of the identified errors
1.	Use of incorrect formula: A = P (1+*in)* instead of $SI = P \times I \times n$ as the question required calculation of simple interest.	Application of irrelevant rules or strategies
2.	Use of incorrect formula: A = P (1 + *in*) instead of using formula: $A = P(1 + i)^n$ as the question required calculation of compound interest.	Incorrect association or rigidity of thinking
3.	Use of the correct formula but the components of the formula incorrectly substituted.	
4.	could not comprehend the meaning and effect of the question to the relevant formula	Language difficulties
5.	Employed a relevant formula but could not substitute the components correctly	Deficient mastery of prerequisite skills, facts and concepts

Summary of the analysis of the aforementioned research question 4 in terms of main errors committed.

In the afore-stated research question the respondent illustrated errors attributed to their prerequisite skills, facts and concepts that were gained in the previous grades. The majority of the respondents as illustrated in the afore-stated frequency tables used the formulae in working out the simple and compound interest problems. The formulae were drawn from the previous knowledge as those are not part of the teaching and learning programme of the Curriculum and Assessment Programme Statements (CAPS).

Some would use the formula and arrive at the correct answer but some would use the incorrect formula but then arrive at an incorrect answer. Others would use the correct formula but incorrectly substitute the formula and as a result arrive at an incorrect answer. The use of the correct formula could not guarantee the correct answer as some would not round off correctly, as the final answer is supposed to be rounded off to two decimal places. Only 31.4% admitted to always rounding off the final answer to decimal places, but 10.5% admitted they had never rounded off their final answer.

About 2% of the research sample admitted to forgetting to write down the correct answer as displayed by the calculator and that indicated negligence.

Based on the analysis of questionnaire 4 (cf Appendix D) where frequencies on each variable of research question 4, it could be established that very few learners (24.8%) of the sample agreed to sometimes forgetting to read the instructions but about 49.5% of them maintained that they sometimes rounded off the answer to 2 decimal places whereas 31.4% claimed to always round off the final answer to 2 decimal places. About 56.2% of the sample asserted that they rounded off but incorrectly, with 5.7% declaring that they always committed such an error. About 20.9% of the respondents claimed to forget to write down the correct answer as displayed in a calculator when it was used.

4.2.5 The underlying factors related to errors due to language difficulties.

Questionnaire 1 consisted of only one question sub-divided into three; the first sub-question was a table with five columns (cf Appendix A). Respondents were expected to be able to differentiate the following concepts: *cost price, selling price, profit and loss.* If learners understood the meaning of those concepts they would then be able to calculate the amount of profit as well as of loss when given the cost price and selling price.

In this question respondents demonstrated: *Errors due to incorrect association or rigidity of thinking (research question 1)* as well as *Errors due to language difficulties (research question 4).* As it has been proposed in chapter 1 learners lack the understanding of concepts such as cost price, selling price, profit or loss as well as the interrelationship between the concepts and the basic operations to employ when working out each of the concepts.

Learner no. 16 seemed to follow and understand the meaning of the concepts and the relevant basic operations to use when answering this question. This learner became unsure when working out the cost price of the dining-room suite shown by writing illegibly to confuse the assessor. Learner no. 16 was among the 37 learners (35%) who answered question 1(a)(i) correctly.

In order for learners to be able to answer the above question, clear understanding of the concepts: variable, occasional, low-profile and high-profile is essential. Errors committed in this question are clearly attributed to *language difficulties*.

Item	Cost price	Selling price	Profit/loss	Amount of profit/loss
Leather Lounge Suite	R9 999,95	R8 000,00	17,999,45	R35,9949
Bedroom Suite	R4 999,95	R4 375,00	R9374,95	R35,999
Home Theatre System	R1 199,95	R1400	profit	R200,05
Dining room Suite	R3,999,95	R6 500,00	loss	R2 499,95
Wall Unit	R3900,05	R3 550,00	profit	R350,05

Figure 4.22: Response of learner no.11 to question 1(b)

Some learners, 68 of them (65%) answered incorrectly because they could not establish the relationship between the concept and the relevant algorithms to employ in order to arrive at the correct answer. Learner no. 11 is one of the learners who identified profit as a result of adding the cost price and the selling price.

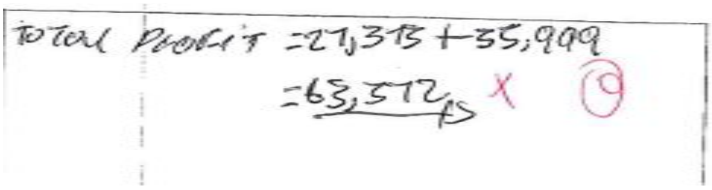

Figure 4.23: Response of learner no. 31 to question 1(c)

In the second sub-question respondents were expected to state whether Peter made profit/loss. Learners could check the table to find the number of items on which Patrick made profit/loss. They could add the amounts of profit and also add the amounts of loss and compare them. If the sum of amounts of profits was greater than the sum of losses, they could conclusively state that Peter made a profit. 95% of the respondents got the answer correct, only 5% gave an incorrect answer. The incorrect responses to this question may be attributed to an *error due to language difficulties.* Learners could not state whether Peter made profit/loss if they do not know the meaning of the concepts.

The third sub-question only demanded the respondents to calculate the total profit and loss. They were supposed to add all the amounts listed as profits and add all the amounts listed as losses to answer this question. 38% of the learners answered the question correctly whereas 67% got incorrect answers. Learner error can be attributed to both: *incorrect association or rigidity of thinking and language difficulties.*

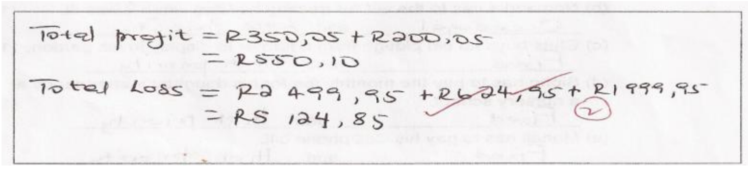

Figure4.24: Response of learner no. 52 to question 1(c)

The following table 4.29 summarises frequencies of the performance of the respondents for each question.

Table 4.29: Frequency table for learner performance per question

Question No.	No attempt	%	Correct responses	%	Incorrect responses	%	Total
1.	18	17%	22	21%	65	62%	105
1(b)(i)	0	0%	37	35%	68	65%	105
(ii)	0	0%	32	30%	73	70%	105
(iii)	0	0%	66	63%	39	37%	105
(iv)	3	3%	39	37%	63	60%	105
(v)	0	0%	42	40%	63	60%	105
1(c)(i)	0	0%	66	63%	39	37%	105
(ii)	18	17%	36	34%	51	49%	105

n = 105

Incorrect responses accounted for 54% whereas 43% of the respondents gave the correct responses.

Table 4.30 below illustrates the summary of the identified errors with the underlying factors associated with the errors committed. This was constructed by means of Newman's error analysis steps during content analysis of the questionnaires.

Table 4.30: Identified learner errors per question

Question no.	Identified errors	Underlying factors associated with error committed
1.(a)	If the Cost price is greater than the Selling price that indicates that it is a loss. Learners could not identify the loss; which indicated the lack of conceptual understanding. Working out the amount of loss: **Amount of loss = Cost price – Selling price**	Incorrect association and rigidity of thinking
1.(b)(i)	Learners could not differentiate between "loss" and "profit".	Language difficulties
(ii)	They were asked to find the "total" profit or loss that led the learners to adding the values under the column labelled the amount of profit/loss.	Incorrect association, rigidity thinking and language difficulties

Table 4.31 below gives the summary of the identified errors with the underlying factors associated with the errors committed. This was constructed by means of Newman's error analysis steps during content analysis of the questionnaires. Each underlying factor is associated with the relevant research question.

Table 4.31: Identified learner errors and the underlying factors related to each error

Question no.	Identified errors from incorrect responses	Underlying factors associated with errors committed
1.	R4 500 + $\frac{14}{100}$ × 6 = R4 500.84.	Incorrect association or rigidity of thinking
2.	$\frac{9}{100}$ × R5 000 = R450 R450 + R5 000 = R5 450. R5 450 × $\frac{4}{12}$ = R1 816	Application of irrelevant rules or strategies
3.	The period of investment was 4 years three months. Learners decided to ignore the three months on their workings, they only used 4 years.	
4.	This time learners were required to find the period not the value of investment.	Language difficulties

	9.5% of R9 700 compounded for 3 years but they employed the simple interest procedure instead of the compound interest one. Some used a formula SI = P × I × n instead of $A = P(1 + i)^n$ which is the formula for compound interest.	Deficient mastery of prerequisite skills, facts and concepts

Table 4.32 below illustrates the frequency table of the responses to provide the variable: respondents forgot to indicate answers as expected.

Table 4.32: I forget to indicate my answers as expected

		Frequency	Percent (%)
Valid	Never	50	47.6
	Rarely	12	11.4
	Sometimes	37	35.2
	Always	6	5.7
	Total	105	100.0

'I forget to indicate my answers as expected' is a variable which was tested by the 4-Likert scale of 'never', 'rarely', 'sometime' and 'always'. 'Never' accounted for a frequency of 47.6%, with 'sometimes' accounting for 35.2%. The above-mentioned variable accounts for a mean of 2.07 and a standard error of 0.135.

Table 4.33: I do not follow instructions even though I read them

		Frequency	Percent (%)
Valid	Never	58	55.2
	Rarely	11	10.5
	Sometimes	35	33.3
	Always	1	1.0
	Total	105	100.0

'I do not follow instructions even though I read them', never accounted for 55.2% and sometimes accounted for 33.3% frequency. The above variable gave a negative weak correlation of -0.019 towards 'I forget to indicate my answer as expected' (variable 1) and 'If I do not understand what is asked I write any answer' (variable 4) of this group of variables. The above-mentioned variable accounts for a mean of 1.80 and a standard error of 0.092.

Table 4.34: When given a task to complete I do not understand the instruction

		Frequency	Percent (%)
Valid	Never	29	27.6
	Rarely	12	11.4
	Sometimes	61	58.1
	Always	3	2.9
	Total	105	100.0

'When given a task to complete I do not understand the instruction', accounted for 58.1% of the respondents. This also displayed a negative weak correlation of -0.02 which indicated that there was no relationship between the two variables 'when given a task to complete I don't understand the instructions' and 'I forget to indicate my answer as expected'. The above-mentioned variable accounts for a mean of 2.36 and a standard error of 0.090.

Table 4.35: If I do not understand what is asked I write any answer

		Frequency	Percent (%)
Valid	Never	24	22.9
	Rarely	17	16.2
	Sometimes	51	48.6
	Always	13	12.4
	Total	105	100.0

'If I do not understand what is asked I write any answer', sometimes as the response accounted for 48.6% of the respondents. This also gave a negative weak correlation of -0.09 compared to 'I forget to indicate my answer as expected' (variable 1). The above-mentioned variable accounts for a mean of 2.50 and a standard error of 0.096.

Summary of the analysis of research question 5 (what are the underlying factors related to errors due to language difficulties).

Based on the analysis of the frequencies on variables of the research question, 40.9% forgot to indicate the answer as expected with 35.2 % occasionally committing such an error and 5.7% who always committed such an error. About 34.3% did not follow instructions even though they read them and 61% did not understand the instructions of a given task, while 58.1% claimed to experience such occasionally.

According to the response 61% just wrote any answer if they could not understand what was asked, while 12.4% always did that and 48.6% claimed to encounter that occasionally.
To comprehend the meaning of the question, language competency is vital as it allows learners to employ the relevant algorithms. Conceptual understanding plays an important role in guiding learners to the correct answers. From the afore-stated frequency tables it could be ascertained that learners sometimes read the instructions but could not understand what the instructions meant. Learners sometimes write any answer when they do not understand what is asked.

4.2.6 What degree of predictability and hence strategies underpin error analysis in questions 1-5?

The importance of the coefficient of determination is that by the use of the Pearson *R*-statistic and the standard error of the estimate, the researcher can construct a precise estimate of the interval in which the true population correlation will fall. Here the correlation among those can be tested by means of the SPSS, the Analysis of Variance (ANOVA) where the Degree of Freedom was used to test the significant difference among the sample means.

Research question 1: Why do learners commit errors on given tasks?

Table 4.36 illustrates the Pearson correlation and significance (*p*-value) of the seven variables of this particular research question.

Table 4.36: Summary of the correlation analysis of each variable of research question 1

		A	B	C	D	E	F	G
A	Correlation	1	.095	.014	.031	-.040	.235	.385
	Sig. (2-tailed)		.337	.889	.751	.682	.016	.000
B	Pearson Correlation	.095	1	-.002	.188	-.015	.079	
	Sig. (2-tailed)	.337		.981	.055	.879	.425	.196
C	Pearson Correlation	.014		1	.032	-.124	.056	.034
	Sig. (2-tailed)	.889	.981		.749	.207	.573	.727
D	Pearson Correlation	.031	.188	.032	1	-.038	-.056	.007
	Sig. (2-tailed)	.751	.055	.749		.699	.570	.944
E	Pearson Correlation	-.040	-.015	-.124	-.038	1	090	.105
	Sig. (2-tailed)	.682	.879	.207	.699		.359	.286
F	Pearson Correlation	.235*	.079	.056*	-.056	.090*	1	.003*
	Sig. (2-tailed)	.016	.425	.573	.570	.359		.979
G	Pearson Correlation	.385**	.127	.034**	.007	.105**	.003	1**
	Sig. (2-tailed)	.000	.196	.727	.944	.286	.979	

NOTE: the variables of research question 1 were labelled A - G for the writer's convenience in constructing the following table which summarises the correlation and the significance of the stated variables.

A – I want to do my best in tests
B – I feel confidence when submitting test
C – I feel rushed when writing a test
D – I want to finish first
E – I read through my work before submitting
F – I take time to answer questions
G – I review homework after it has been done

A correlation analysis was conducted to examine the relationship between learners wanting to do their best in the tests (A) and learners taking time to answer questions (B). The analysis was not significant, p = .337 (r = +.09) which illustrates a weak correlation among the afore-stated variables.

There was no significance, when the correlation analysis was conducted, to examine the relationship between learners wanting to do their best in the tests (A) and learners reviewing homework when done (C), p = .889 (r = +.01) which illustrated a weak correlation among the afore-stated variables.

No significance could be established from the correlation analysis conducted to examine the relationship between learners wanting to do their best in the tests (A) and learners wanting to finish first (D), $p = .751$ ($r = +.03$) which illustrated a weak correlation among the afore-stated variables. Learners wanting to do their best in the tests (A) and learners reading though any work before submitting (E) illustrated non-significance with a weak negative correlation, where $p = .682$ ($r = -.04$).

A correlation analysis which was conducted to examine the relationship between learners wanting to do their best in the tests (A) and learners reviewing homework when done (G) was significant where $p < .001$, ($r = +.39$), with a moderate correlation between the two variables. A correlation analysis to examine the relationship between learners feeling confident when submitting a test (B) and learners wanting to do best in tests (A) illustrated a weak correlation with non-significance, where $p = .337$, ($r = +.09$).

A correlation analysis was conducted to examine the relationship between learners feeling confident when submitting a test (B) and learners reviewing homework when done (C). The analysis was not significant, $p = .981$ ($r = -.002$) which illustrated a negative weak correlation. A correlation analysis was conducted to examine the relationship between learners feeling confident when submitting a test (B) and learners wanting to finish first (D). The analysis was significant, $p < .05$ ($r = +.19$) which illustrated a weak correlation.

A correlation analysis which was conducted to examine the relationship between learners feeling confident when submitting a test (B) and learners reviewing homework when done (E) was not significant, where $p = .879$ ($r = -.02$) with a negative weak correlation between the variables.
A correlation analysis was conducted to examine the relationship between learners feeling confident when submitting a test (B) and learners taking time to answer questions (F) illustrated non-significance, where $p = .425$ ($r = +.08$) with a weak correlation between the variables. The relationship between learners feeling confident when submitting a test (B) and learners reviewing homework when done (G) illustrated a non-significance, where $p = .196$ ($r = +.13$) with a

weak correlation between the variables.

The relationship between learners wanting to do their best in the tests (D) and learners reviewing homework when done (C) illustrated a non-significance and a weak correlation between the variables, where $p = .749$ ($r = +.03$).
A correlation analysis which was conducted to examine the relationship between learners feeling rushed when writing a test (C) and learners reading through before submitting (E) was not significant, $p = .207$ ($r = -.12$) with a weak negative. The relationship between learners feeling rushed when writing a test (C) and learners taking time to answer questions (F) was not significant, $p = .573$ ($r = +.06$) which illustrated a weak correlation between variables. The relationship between learners feeling rushed when writing a test (C) and learners reviewing homework when done (G) was not significant, $p = .727$ ($r = +.13$) which illustrated a weak correlation.

There was a negative weak correlation illustrated when correlation analysis was conducted to examine the relationship between learners wanting to finish first (D) and learners reading through before submitting (E) with no significance, as $p = .699$ ($r = -.04$). The relationship between learners wanting to finish first (D) and learners taking time to answer (F) was not significant, as $p = .570$ ($r = -.06$) which illustrated a negative weak correlation. The correlation analysis was not significant, $p = .944$ ($r = +.01$) which illustrated a weak correlation when conducted to examine the relationship between learners wanting to finish first (D) and learners reviewing homework when done (G). A correlation analysis was conducted to examine the relationship between learners reading through before submitting (E) and learners taking time to answer questions (F) which was not significant, $p = .359$ ($r = +.09$) and illustrated a weak correlation.

A correlation analysis was conducted to examine the relationship between learners reading through before submitting (E) and learners reviewing homework when done (G) which was not significant, as $p = .288$ ($r = +.11$) and illustrated a weak correlation. A correlation analysis was conducted to examine the relationship between learners taking time to answer questions (F) and learners reviewing homework when done (G) which was not significant and illustrated a weak correlation, as *p* = .979 (*r* = +.003).

Table 4.37 is the SPSS Outputs that illustrates the Standard Deviation and the Skewness statistics for each sub-variable of each of research question 1. It has been utilised in Testing of the Normality and Homogeneity as discussed earlier (cf Chapter 3).

Table 4.37 Descriptive statistical analysis of Research question 1

	N	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Std. Error
I want to do my best in my tests	105	.640	-2.299	.236
I feel confident when I submit my test because I know I have done my best	105	.662	-.725	.236
I feel rushed when I am writing a test	105	.966	-.271	.236
I want to be the first one done on the test	105	1.082	.433	.236
I go back and read through what I have written before handing in the test script	105	.912	-1.09	.236
I take my time to answer the test questions	105	.797	-1.08	.236
I review my homework after I have done it	105	.943	-.976	.236
Valid N (listwise)	105			

Normality: The first sub-variable (I want to do my best in the test) did not meet the Assumption of Normality as the Skewness is not within the range of -1 and +1 (-2.23). All the other sub-variables met the Assumption as their Skewness Statistics are within the range (cf Table 4.55).

Homogeneity: When testing the Homogeneity using the Standard Deviation values as illustrate in Table 4.55 this Assumption was met (1.082 ÷ 0.640 = 1.690625 which is not greater than 2).

Summary of the findings of research question 1

There is a statistically significant relationship between learners wanting to do their best and learners reviewing their homework, the degree of freedom illustrated that only 1% of the findings could be incorrect. Learners feeling confident when submitting their test scripts, correlated with those wanting to finish first with a 5% probability that it could be incorrect.

The afore-stated results illustrated a moderate correlation between a pair of variables (A) and (G) and a weak correlation between two pairs of variables (A) and (F); (D) and (B). Therefore only 43% of the variables correlated and they do not have a significant effect on the research question (cf page 118).

Research question 2: What are the underlying factors related to the errors due to incorrect association or rigidity of thinking?

Table 4.37 below illustrates the Pearson correlation (*r*) and significance (*p*-value) of the four variables of this particular research question.

Table 4.38: Summary of the correlation analysis of each variable of research question 2

		A	B	C
A	Pearson Correlation	1	.045	.069
	Sig. (1-tailed)		.325	.242
B	Pearson Correlation	.045	1	.171
	Sig. (1-tailed)	.325		.040
C	Pearson Correlation	.069	.171	1
	Sig. (1-tailed)	.242	.040	

NOTE: the variables of research question 2 were labelled A - C for the writer's convenience in constructing the following table which summarises the correlation and the significance of the stated variables.

A – I confuse addition with multiplication

B – I forget to write units

C– I write down an incorrect number

The correlation analysis was conducted to examine the relationship between learners forgetting to write units (B) and learners writing down an incorrect number/figure (C).The analysis was significant, *p* = .04 (*r* = +.17) which illustrated a weak correlation between the afore-stated variables. Learners confusing addition with multiplication (A) and learners writing down an incorrect number (C) illustrated non-significance and weak correlation, as *p* = .242 (*r* = +.07).

The correlation analysis was conducted to examine the relationship between learners confusing addition with multiplication (A) and learner forgetting to write units (B) was not significant, *p* = .325, (*r* = +.05) which illustrated a weak correlation.

Table 4.39 Descriptive statistical analysis of Research question 2

	N	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Std. Error
I confuse addition with multiplication	105	.908	-.648	.236
I forget to write units	105	.978	-.101	.236
I write down an incorrect number	105	.944	.812	.236
Valid N (listwise)	105			

Normality: All the sub-variables of research question 2 have met the Normality assumption as all the Skewness Statistic value are within the range of -1 and +1 (cf Table 4.39).

Homogeneity: The afore-stated research question has met the Homogeneity Assumption as the highest value of the Standard Deviation 0.978 ÷ 0.908 = 1.077092511 which is not greater than 2.

Summary of the findings of research question 2

The afore-mentioned results illustrated a weak relationship between learners forgetting to write units and those writing down an incorrect number when transcribing from the rough paper, and a significance $p<.05$, which represent only a third (33.3%) of the variables of research question 2. All the other variables illustrated no significance, a weak correlation and cannot be generalized to a broader population. The fact that the correlation of variables in this research question accounted for 33.3% indicated that they have no significant effect on the research question.

Research question 3: What are the underlying factors related to the errors due to the application of irrelevant rules or strategies?

Table 4.38 below illustrates the Pearson correlation and significance (*p*-value) of the six variables of this particular research question.

Table 4.40: Summary of the correlation analysis of each variable of research question 3

		A	B		D	E	F
A	Pearson Correlation	1	.298	.113	.179	-.030	-.011
	Sig.(2-tailed)		.011	.003	.087	.753	.914
B	Pearson Correlation	.298**	1**	.023**	.102**	.247**	.080**
	Sig.(2-tailed)	.011		.814	.302	.011	.418
C	Pearson Correlation	.113	.023	1	.197	.273	.274
	Sig.(2-tailed)	.003	.814		.045	.005	.005
D	Pearson Correlation	.179	.102	.197	1	.134	.179
	Sig.(2-tailed)	.087	.302	.045		.172	.067
E	Pearson Correlation	-.030	.247	.273	.134	1	.169
	Sig.(2-tailed)	.753	.011	.005	.172		.084
F	Pearson Correlation	-011	.080	.274	.179	.169	1
	Sig.(2-tailed)	.914		.005	.067	.04	

NOTE: the variables of research question 3 were labelled A – F for the writer's convenience in constructing the following table which summarises the correlation and the significance of the stated variables.

A – I lose marks for not showing workings

B - I lose marks for not completing all problems

C – I do not show all steps

D – I make basic computation errors

E – I show all steps but do not find the correct answer

F – I show my steps but not all of them

The correlation analysis which was conducted to examine the relationship between learners losing marks for not showing workings (A) and learners not showing all steps (C) illustrated significant, p = .003 (r = +.11) with a weak correlation. The relationship between learners losing marks for not showing workings (A) and learners making basic computation errors (D) was not significant, where p = .087 (r = +.18) and illustrated a weak correlation.

The relationship between learners losing marks for not showing all workings (A) and learners showing all the steps but not finding the correct answer (E) was not significant, p = .753 (r = -.03) with negative weak correlation. The relationship between learners losing marks for not showing all workings (A) and learners showing steps but not all of them (E) illustrated a negative weak correlation with no significance, where p = .914 (r = -.01).

The correlation analysis was conducted to examine the relationship between learners losing marks for not completing all the problems (B) and learners not showing all the steps (C). The analysis was not significant, p = .814 (r = +.02) which illustrated the weak correlation.

After a correlation analysis was conducted to examine the relationship between learners losing marks for not completing all the problems (B) and learners making basic computation errors (D), there was no significance, as p = .302 (r = .10) and illustrated the weak correlation. The correlation analysis was conducted to examine the relationship between learners losing marks for not completing all the problems (B) and learners showing all the steps but not finding the correct answer (E) was significant, as p = .011 (r = +.25) and illustrated a weak correlation.

The correlation analysis was conducted to examine the relationship between learners losing marks for not completing all the problems (B) and learners showing steps but not all of them (F). The analysis was not significant, p = .418, (r = +.08) which illustrated the weak correlation.

The correlation analysis was conducted to examine the relationship between learners not showing all the steps (C) and learners making basic computation errors (D) was significant, as p = .045, (r = +.20) which illustrated a weak correlation.

The correlation analysis was conducted to examine the relationship between learners not showing all the steps (C) and learners showing all the steps but not finding the correct answer (E) was significant, as p = .005, (r = +.27) which illustrated a weak correlation.

The relationship between learners not showing all the steps (C) and learners showing steps but not all of them (F) was significant, as p = .005, (r = +.27) which illustrated a weak correlation between the two variables. The correlation analysis was conducted to examine the relationship between learners making basic computation errors (D) and learners showing all the steps but not finding the correct answer (F) was not significant, as p = .172, (r = +.13) which illustrated a weak correlation between the two variables.

After a correlation analysis was conducted to examine the relationship between learners showing steps but not all of them (F) and learners making basic computation errors (D), it was not significant, as p = .067, (r = +.18) which illustrated a weak correlation between the two variables.

The correlation analysis was conducted to examine the relationship between learners showing all the steps but not finding the correct answer (E) and learners showing steps but not all of them (F) was not significant, as p = .084, (r = +.17) which illustrated the weak correlation between the two variables.

Table 4.41 Descriptive statistics of Research question 3

	N	Std. Deviation	Kurtosis	
	Statistic	Statistic	Statistic	Std. Error
I lose marks on my work for not showing my workings	105	.856	-.120	.467
I lose marks on my work for not completing all the problems	105	.919	-.727	.467
I do not show all my steps	105	.831	-.229	.467
I make basic computation errors	105	.826	-.260	.467
I show all my steps but do not find the correct answer	105	.888	-.561	.467
I show my steps but not all of them	105	.809	.374	.467
Valid N (listwise)	105			

Normality: All the sub-variables of research question 3 have met the Normality Assumption as illustrated Skewness within the range of -1 and +1 (cf Table 4.41).

Homogeneity: They have also met the Homogeneity Assumption as the highest value of the Standard Deviation 0.919 ÷ 0.809 = 1.122100122 which is not greater than 2.

Summary of the findings of research question 3

Learners could lose marks for not showing all the steps but fortunately they were not penalised for missing steps. From the afore-stated correlation results learners did not show all the steps and there was a relationship between not showing all the steps and making basic computation errors. They could show the steps but do not find the correct answer.

From the six pairs of variables, four illustrated a weak and moderate correlation. It therefore accounts for 66.7% of the variables of this research question with illustrated significance to research question 3 even though the correlation between the variables was weak

Research question 4: What are the underlying factors related to the errors due to deficient mastery of prerequisite skills, facts and concepts?

Table 4.42 below illustrates the Pearson correlation and significance (*p*-value) of the four variables of this particular research question.

Table 4.42: Summary of the correlation analysis of each variable of research question 4

		A	B	C	D
A	Pearson Correlation	1	.146	.305	.119
	Sig. (2-tailed)		.182	.002	.281
B	Pearson Correlation	.146	1	.137	.035
	Sig. (2-tailed)	.182		.455	.726
C	Pearson Correlation	.305**	.137**	1**	.208**
	Sig. (2-tailed)	.002	.455		.019
D	Pearson Correlation	.119	.035	.208	1
	Sig. (2-tailed)	.281	.726	.019	

NOTE: the variables of research question 4 were labelled A - D for the writer's convenience in constructing the following table which summarises the correlation and the significance of the stated variables.

A – I forget to read instructions

B – I do not round off the answer to 2 decimal places

C – I do round off but incorrectly

D – I do not write the answer as shown on a calculator

The correlation analysis was conducted to examine the relationship between learners forgetting to read the instructions (A) and learners rounding off answers to 2 decimal places (B), and the results were non-significant and illustrated a weak correlation, as $p > .05$ ($r = +.15$). The relationship between learners forgetting to read the instructions (A) and learners forgetting to write down the answer shown by the calculator (D) was not significant and a weak correlation, as $p = .281$ ($r = +.12$). The relationship between learners rounding off answers to 2 decimal places (B) and learners rounding off but incorrectly (C) illustrated non-significant results and a weak correlation, where *p* = .455 (*r* = +.14).

The correlation analysis conducted to examine the relationship between learners forgetting to read the instructions (A) and rounding off but incorrectly (C), was significant, as *p*<.01 (*r* = +.31) but illustrated a weak correlation between the afore-stated variables.

The correlation analysis between learners rounding off answers to 2 decimal places (B) and learners forgetting to write down answers shown by the calculator (D) revealed non-significance, *p* = .726 (*r* = +.04) but illustrated a moderate correlation between the variables. Examining the relationship between learners rounding off but incorrectly (C) and not writing answers as shown by the calculator (D) revealed significance, where *p*<.05 (*r* = +.21) which illustrated a weak correlation between the two variables.

Table 4.43 Descriptive statistical analysis of Research question 4

	N	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Std. Error
I forget to read the instructions	105	.920	.915	.236
I round off the answer into 2 decimal places	105	.909	-.899	.236
I do round-off but incorrectly	105	.949	-.328	.236
When using a calculator I forget to write down the correct answer	105	.863	1.089	.236
Valid N (listwise)	105			

Normality: All the sub-variables have met the Normality assumption they have skewness that is within the range of -1 and +1 (cf Table 4.43). Sub-variable D (When using a calculator I forget to write the correct answer illustrates skewness of 1.089 which is slightly above the range by 0.89 which does not have much effect.

Homogeneity: They have also met the homogeneity as the highest value of the Standard Deviation 0.949 ÷ 0.863 = 1.099652375 which is less than 2.

Summary of the findings of research question 4

From the afore-stated correlation analysis there was a relationship between learners forgetting to read the instructions and rounding off incorrectly. There was a reasonable confidence that the relationship may be stronger also in another research population. Rounding off correctly and not writing the answer as shown by the calculator also illustrated a degree of confidence with a probability of 5% errors when tested in another research population.

From the four pairs of variables two proved to be significant and illustrated weak correlations. Therefore 50% of the variables of research question 4 showed a correlation and significance to the research question.

All the above displayed variables indicated a weak correlation among the variables. This could be predisposed by a number of factors such as the sample size, sample distribution, the relevance of the questions and/or the respondents' interpretation of the questions.

Research question 5: What are the underlying factors related to errors due to language difficulties?

Table 4.44 provides a summary of the correlation analysis results and the significance of the four variables tested for research question 5.

Table 4.44: Summary of the correlation analysis of each variable of research question 5

		A	**B**	**C**	**D**
A	Pearson Correlation	1	-.019	-.019	-.094
	Sig. (2-tailed)		.832	.849	.503
B	Pearson Correlation	-.019	1	.150	-.035
	Sig. (2-tailed)	.832		.126	.884
C	Pearson Correlation	-.019	.150	1	.062
	Sig. (2-tailed)	.832	.473		.473

D	Pearson Correlation	-.094	-.035	.062	1
	Sig. (2-tailed)	.340	.884	.473	

NOTE: the variables of research question 5 were coded for the writer's convenience in constructing the following table which summarises the correlation and the significance of the stated variables.

A – I don't indicate answers as expected

B – I don't follow instructions

C – I don't understand instructions

D – I write any answer

In examining the relationship between learners who do not follow instructions (B) and learners who do not understand instructions (C) by means of correlation analysis, it revealed non-significant results and a weak negative correlation, as p = .832 (r = -.02). The correlation analysis conducted to examine the relationship between learners not following instructions (B) and learners writing any answer (D) non-significant results and a weak negative correlation, as p = .503 (r = -.09) between the two variables appeared.

The correlation analysis between learners not understanding instructions (C) and learners not following instructions (B) was not significant, p = .473 (r = +.15) which illustrated a weak correlation between the two variables. The relationship between learners not following instructions (B) and learners writing any answer (D) revealed no significance, as p = .884 (r = -.04) which illustrated a weak negative correlation between the two variables.

Examining the relationship between learners who do not understand instructions (C) and learners who write any answer (D) revealed no significance, as p = .473 (r = +.06) which illustrated a weak correlation between the variables.

Table 4.45 Descriptive statistical analysis of Research question 5

	N	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Std. Error
I forget to indicate my answers as expected	105	1.409	3.452	.236
I do not follow instructions even though I read them	105	.945	.483	.236
When given a task to complete I do not understand the instruction	105	.921	-.562	.236
If I do not understand what is asked I write any answer	105	.982	-.355	.236
Valid N (listwise)	105			

Normality: Sub-variable A (I forget to indicate my answers as expected) illustrated Skewness of 3.452 which is not within the range; therefore it did not meet the Normality Assumption. All the other three sub-variables are within the range of -1 and +1(cf Table 4.45), therefore they have met the Assumption.

Homogeneity: Research question 4 met the Homogeneity Assumption as 1.409 ÷ 0.921 = 1.529858849 which is not greater than 2.

Summary of the findings of research question 5

From the four pairs of variables of research question 5, none of them illustrated any correlation. Therefore 0% of the variables showed any correlation, as all showed negative correlation. The variables have no significant effect on research question 5 and that could be attributed to a number of factors.

Cohen, L., *et al*. (2007: 197) state that: "Statistical significance varies according to the size of the population in the sample. In order to be able to determine the significance we need to have the two factors in our possession: the size of the sample and the co-efficient of correlation". The sample size was worked out as 165 but only 105 questionnaires were returned and tested, and that might have affected the results.

4.3 RESEARCH HYPOTHESES AND STATISTICAL FINDINGS

Based on the six research questions the following hypotheses were formulated and a statistical analysis was performed above. The Analysis of Variables (ANOVA) was used in this section of the data analysis. “When ANOVA is conducted we look at the overall relationship between the outcome (dependent variable) and the covariate” (Field, 2012: 1). Statistical science provided an objective procedure for distinguishing whether the observed difference connotes any real difference among group variables.

HYPOTHESIS 1

H0: Errors committed by learners in financial mathematics are due to language difficulties.
H1: Errors committed by learners in financial mathematics are not impacted on by language difficulties.

Significance of four variables of research question 5 was tested using a one-way ANOVA test where the following results were illustrated in tables showing the degree of freedom and the levels of significance (*p*-values) of each variable. Tables 4.46 – 4.49 illustrate the ANOVA results of the variances related to Proposition 1 of the study.

Table 4.46: I do not indicate my answers as expected

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	1.770	3	.590	.291	.832
Within Groups	204.763	101	2.027		
Total	206.533	104			

The effect of the afore-stated variable on the hypothesis could not illustrate any significance as the results shown that, $F(3,101) = .291$ and $p = .832$ ($r = -.02$). it therefore could not be included as the factor which related to learners committing errors in financial mathematics.

Table 4.47: When given a task to complete I do not follow instructions

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2.157	3	.719	.843	.473
Within Groups	86.091	101	.852		
Total	88.248	104			

Learners not following instruction when given a task to complete could not illustrate any significance as the ANOVA test illustrated that, $F(3,101) = .843$ and $p = .473$ ($r = -.04$).

Table 4.48: If I do not understand what is asked I write any answer

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2.295	3	.765	.789	.503
Within Groups	97.953	101	.970		
Total	100.248	104			

The afore-stated variable did not illustrate any significance in this study as the ANOVA test illustrated that, $F(3,101) = .789$ and $p = .503$ ($r = +.20$). If learners do not understand what is asked, write any answer prove not to be a reason for learners committing errors.

Table 4.49: I do not follow instructions even though I read them

	Sum of Squares	Df	Mean Square	Sig.
Between Groups	1.062	4	.266	.884
Within Groups	91.738	100	.917	
Total	92.800	104		

Learners not following instructions even though they have read them, as illustrated by the ANOVA test results was not significant, $F(3,101) = .290$ and $p = .884$ ($r = -.09$). Thus cannot be related to the factors that contribute to learners committing the kind of errors they commit in financial mathematics.

Based on the ANOVA research question 5 has little or no effect on the variables tested above, that is shown by the *F*- values and the significance levels that appear to be greater than .05. We therefore accept the null hypothesis and drop the alternative hypothesis as the significance levels of all variables tested is greater than .05. All the variables have significance $p > .05$ which therefore points the researcher to accept the null hypothesis and to reject the alternative hypothesis. Errors committed by learners in financial mathematics are not impacted on by language difficulties.

HYPOTHESIS 2

H0: Errors committed by learners in financial mathematics are due to prerequisite skills, facts and concepts.

H1: Errors committed by learners in financial mathematics are not due to prerequisite skills, facts and concepts.

Significance of four variables of research question 4 was tested using a one-way ANOVA test where the results were illustrated in tables showing the degree of freedom and the levels of significance (*p*-values) of each variable.

The correlation analysis was conducted to examine the relationship between learners forgetting to read the instructions (A) and learners rounding off but incorrectly (C). The analysis was significant, $p = .002$ ($r = .31$). Tables 4.50 – 4.53 illustrate the ANOVA results of the variances related to Hypothesis 2 of the study.

Table 4.50: I forget to read the instructions

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	4.113	3	1.371	1.652	.182
Within Groups	83.849	101	.830		
Total	87.962	104			

Learners forgetting to read the instructions proved not to be significant, as *F*(3,101) = 1.652 and *p* = .182 (*r* = .31). This indicates that forgetting to read the instructions cannot be related to the afore-stated hypothesis.

Table 4.51: I do not round off the answer to 2 decimal places

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2.185	3	.728	.878	.455
Within Groups	83.777	101	.829		
Total	85.962	104			

Learners not rounding off the answer to 2 decimal places did not have any significance on errors committed in financial mathematics, ANOVA illustrated, *F*(3,101) =.878 and *p* = .455(*r* = .20).

Table 4.52: I do round off but incorrectly

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	8.751	3	2.917	3.470	.019
Within Groups	84.906	101	.841		
Total	93.657	104			

Analysis of the Hypothesis through ANOVA test illustrated a significance, where *F*(3,101) = 3.470 and *p* = .019(r = .31), revealed less than 5% Type II error. That indicated that there is an effect of learners does rounding off but doing so incorrectly impacting upon the type of errors learner commit in financial mathematics.

Table 4.53: When using a calculator I forget to write down the correct answer

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2.860	3	.953	1.292	.281
Within Groups	74.531	101	.738		
Total	77.390	104			

The afore-stated variable does not have any effect on the hypothesis 2, that was revealed by the ANOVA test results which indicate that ‘learners forgetting to write down the correct answer when using a calculator was not significant, *F*(3,101) = 1.292 and *p* = .281(r = .21).

ANOVA results on research question 4 testing hypothesis 2, one variable ‘forget to read the instructions’ illustrated a significance level where $p < .05$. All the other variables illustrated significance levels where $p > .05$ which indicated non-significance to the research questions. Based on the illustrated results I had to drop the alternative hypothesis and accept the null hypothesis. I then concluded that the errors learners commit in financial mathematics are not due to the prerequisite skills, facts and concepts.

HYPOTHESIS 3

H0: Errors committed by learners in financial mathematics are due to the application of irrelevant rules and strategies.
H1: Errors committed by learners in financial mathematics are not due to the application of irrelevant rules and strategies.

Significance of six variables of research question 3 was tested using a one-way ANOVA test where the following results were illustrated in tables showing the degree of freedom and the levels of significance (*p*-values) of each variable. Tables 4.54 – 4.59 illustrate the ANOVA results of the variances related to Proposition 3 of the study.

Table 4.54: I lose marks on my work for not showing my workings

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	7.938	3	2.646	3.912	.011
Within Groups	68.310	101	.676		
Total	76.248	104			

Learners losing marks on their work for not showing the workings illustrated significance, where $F(3,101) = 3.912$ and $p = .011$. The ANOVA test results revealed less than 1% Type I error the afore-stated variable has effect on Hypothesis 3 which illustrated that errors committed by learners in financial mathematics are due to the application of irrelevant rules and strategies.

Table 4.55: I lose marks on my work for not completing all the problems

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	11.350	3	3.783	4.995	.003
Within Groups	76.498	101	.757		
Total	87.848	104			

Learners losing marks for not completing all the problems was significant, table 4.55 above illustrated that $F(3,101) = 4.995$ and $p = .003$ ($r = .30$). ANOVA test result illustrated less than 5% of Type II could be revealed in the above table (cf

Table 4.55). Thus, revealed that learners not completing all the problems have an effect on the errors committed in financial mathematics.

Table 4.56: I do not show all my steps

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	9.299	3	3.100	5.005	.003
Within Groups	62.549	101	.619		
Total	71.848	104			

Learners not showing all steps illustrated significance as illustrated in the ANOVA test results (cf Table 4.56), $F(3,101) = 5.005$ and $p = .003$ ($r = .11$). The results indicated less than 5% Type II error as illustrated above (cf Table 4.56). It indicated that learners not showing all their steps in their working have an effect on the errors committed in financial mathematics.

Table 4.57: I make basic computation errors

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	4.457	3	1.486	2.255	.087
Within Groups	66.533	101	.659		
Total	70.990	104			

Learners admitting to making basic computation errors did not illustrate any significance but it could not be completely declared as non-significant as the ANOVA test results illustrated, $F(3,101) = 2.255$ and $p = .087$ ($r = .18$).

Table 4.58: I show all my steps but do not find the correct answer

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	.966	3	.322	.401	.753
Within Groups	81.091	101	.803		
Total	82.057	104			

ANOVA test results illustrated non-significance of learners showing all steps but not find the correct answer, $F(3,101) = .401$ and $p = .753$ ($r = .27$). It has no effect on the type of errors committed in financial mathematics.

Table 4.59: I show my steps but not all of them

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	5.179	3	1.726	2.773	.045
Within Groups	62.878	101	.623		
Total	68.057	104			

Learners showing steps but not all of them illustrated a significance, where $F(3,101) = 2.773$ and $p = .045$ (r = .27). It revealed less than 5 % of Type II error on the test results of the particular variable so results were regarded as significant.

The ANOVA results, after testing the effect of research question 3 on the six variables, revealed that ‘making basic computation error’ as the only variable with little or no significance. All the other five variables illustrated the significant levels where $p < .05$. The researcher therefore dropped the null hypothesis (H0) and accepts the alternative hypothesis (H3). That brings a conclusion that errors committed by learners in financial mathematics are due to the application of irrelevant rules and strategies.

Summary of the ANOVA results

In this section three hypotheses were tested and yielded results that errors committed by learners in financial mathematics could not be attributed to language difficulties even though language proved to be a barrier to ML learning. This was revealed by content NEA conducted during content analysis of the questionnaires 1, 2 and 3 (cf Appendices A, B and C).

Errors committed by learners in financial mathematics could not be directly related to the prerequisite skills, facts and concepts even though it was revealed when NEA was conducted with learners' responses to content-based questionnaires.

It was ascertained that errors committed by learners in financial mathematics were due to the application of irrelevant rules and strategies.

4.4 CONCLUSION

This chapter presented the data analysis and the findings of this study regarding the underlying factors related to errors committed in Grade 10 financial mathematics. Different types of errors were identified and grouped according to their underlying factors which will make it easier for teachers to identify relevant remedial instruction. The findings of the study indicated that the errors committed by learners are attributed to a number of factors such as the application of irrelevant rules or strategies, not showing all the steps, making basic computation errors, incorrect place values, inaccurate transfer of values shown on a calculator, reviewing the work before submission. Furthermore, content analysis of the questionnaires illustrated much support for the incorrect association or rigidity thinking as an underlying reason for learners to commit errors. The language of instruction and assessment is English which is an additional language for the study sample' therefore language becomes a challenge, which was established in content analysis.

CHAPTER 5

DISCUSSION OF RESEARCH FINDINGS

5.0 INTRODUCTION

This chapter discusses in detail the findings illustrated in Chapter 4 (Data presentation and Analysis) of this particular study. The chapter addresses the research questions taking into account the literature review and theoretical framework of Chapter 2. The first section of this chapter addresses the findings of each research question, followed by the theoretical framework and its relation to the results of each research question.

5.1 FINDINGS OF EACH RESEARCH QUESTION:

5.1.1 Why do learners commit errors in given tasks in financial mathematics?

This research question specifically sought the reasoning behind learners committing the kind of errors they do when dealing with financial mathematical problems in Mathematical Literacy. A number of variables were tested for this research question, and they ranged from interest in doing their best in a test, confidence when writing tests, wanting to be the first to finish writing, not going back to read through what was written. In describing the errors Radatz (1979: 170) states that: “Errors in learning of mathematics are not simply the absence of correct answers or results of unfortunate accidents. They are the sequence of definite processes whose nature must be discovered”. The research question sought to analyse the nature and the underlying causes of errors in terms of the individual information processing mechanism.

The majority of respondents (76.2%) always wanted to do their best when writing the tests and they also felt confident when they submitted their test as they knew they had done their best. It was evident in the correlation results which showed a moderate correlation between the two variables. Even though there was a moderate correlation between the two variables, learners could not unanimously agree on the confidence when submitting.

The majority (56%) were indecisive and stated that they did not always feel confident. The fact that they did not feel confident was because they felt rushed when writing a test. A minority (10.5%) sometimes wanted to be the first ones to finish writing. Learners lacked confidence when submitting the tests and that was illustrated in the results of this specific variable. It is attributed to the following:

- Lack of relational understanding (cf Chapter 2, page 40) which according to Saoendergaard and Cachaper (2008) occurs when one has a built-in conceptual structure of mathematic. Meyer and Land (2006) brought into perspective threshold concept which forms part of the theoretical framework of this study and which they describe as a conceptual understanding that is the building block of the understanding of the subject.
- Preparation before any assessment task helps build conceptual understanding of the questions and/or subject content itself. It takes time to build a threshold concept but, as has been argued, it can be bounded and is therefore not easy to lose it.

Taking into cognisance the aforementioned attributions to lack of confidence, learners need to be introduced to a concept of monitoring their own capacity to learning. A small group of learners admitted to not going back and read their test scripts before submitting. There was no clear distinction between those who always took time to answer test questions (42.9%) and those who sometimes (44.8%) took time when answering test questions. Taking time to answer questions showed a weak correlation with going back and read through what they had written, which indicated that according to the correlation results there was no relationship. Even though they took their time to answer a question that does not mean they spent some time reading what they had written. As part of the theoretical framework of this study, Polya's problem-solving techniques were discussed (cf Chapter 2 section 2.6.1) where the fourth principle: 'review' formed part of the four problem-solving steps that learners needed to follow.

Among 105 respondents only 47.6% attested that they always went back and read what they had written before handing in their scripts. It also illustrated a negative weak correlation to the other variables when tested which indicated that it had no relationship with those variables.

Only 42.9% of the respondents attested to always taking time to answer the test questions, which left 57.1% who never or sometimes did so.

Learners had divided opinions on whether they reviewed their homework as 45.7% did not always review their homework whereas 36.2% always reviewed their homework. The uncertainty of the learners' responses was evident in the correlation results which all indicated weak or no correlation between the variables.

Wanting to do their best in the test and reviewing homework before submission were statistically significant with $p<.01$ with a moderate correlation of $r = .40$ between them. To answer the afore-stated research question, a number of factors could be attributed to learners committing errors. Based on the previous statistical test results (cf. Chapter 4, section 4.3.1), frequency tables that summarise learner responses and the correlation analysis between variables led to the researcher drawing the following conclusions:

- Learners always wanted to do their best when writing a test and that was revealed by the statistical test results and evident in the frequency tables illustrated in the aforementioned chapter. When a correlation test was run between wanting to do the best and reviewing homework before submission it was found to be moderately correlated. Therefore there is no strong relationship between wanting to do the best and reviewing homework. This indicates that even though learners want to do their best they are not doing enough in terms of going through their work, identifying those errors.
- Sometimes learners felt rushed when writing a test and therefore they panicked and rushed to finish writing without spending enough time in answering test questions. Statistical tests and recorded frequencies revealed that some learners felt rushed when writing a test even though the results could not significantly support that, but based on frequencies

it is worth considering, 47.6% of the sample considered that as an underlying factor related to committing errors.

- Sometimes learners did not go back and read through what they had written, and that was apparent from the careless mistakes obvious in their work. This revealed a negative correlation which indicated according to Field (2012) *r* = .00 which is a zero or no correlation. Therefore there was no relationship between going back to read through the work and wanting to do their best in a test. This could be attributed to a number of factors as the respondents were Grade 10 learners or it could have been the relevance of the question.
- The majority of learners did not always review their homework after completing it. This could be attributed to the time allocated to do the homework and conditions under which the learners did their homework.

To avoid committing errors learners could be taught the four steps to follow when given a Financial Mathematics task.

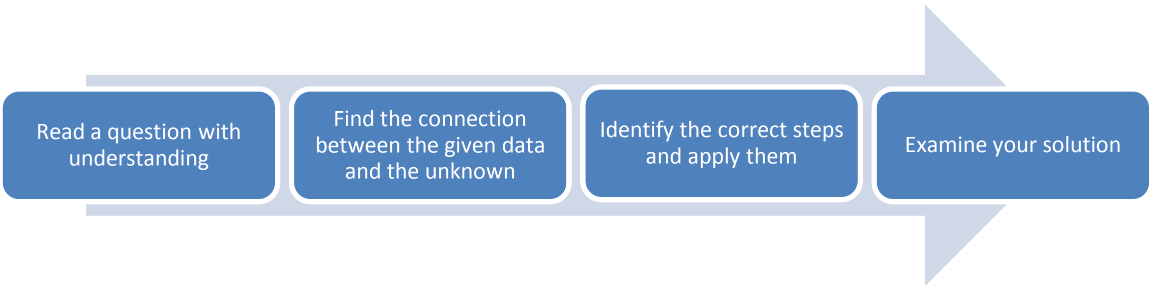

Figure 5.1: Steps to follow when working out a Financial Mathematics problem

(Adapted from Polya's problem solving techniques, 1945)

5.1.2 The underlying factors related to the errors due to incorrect association or rigidity of thinking

According to Radatz (1979: 167), "Inadequate flexibility in decoding and encoding new information often means that experience with similar problem will lead to habitual rigidity of thinking." Learners develop cognitive operations and continue to use them even if those are no longer relevant.

Respondents were given questionnaire no. 1 (cf Appendix A) which consisted of five simple or compound interest questions, which was analyzed by marking and content analysis. When given simple and compound interest problems, 83% of the respondents applied a formula which was either relevant or irrelevant. Drawing from the theoretical framework of the study; threshold concept is known to be irreversible. Once a learner understands the concept it is unlikely to be forgotten. The use of formula, even if not taught, is the result of the afore-stated characteristic of threshold concept. They may have drawn the formulae from Grade 9 threshold concept, without noticing that they were expected to apply a multi-step procedure to find the correct answer. Radartz (1979) describes this as related to incorrect association or rigidity of thinking. Some errors identified in the content analysis, where a learner would confuse addition with multiplication, are classified by Nolting (1998) as *Transformation error* (cf Chapter 2, Table 2.1). Some would add instead of multiply even though they followed the correct steps and that, according to Nolting (1998) and Elbrink (2008), is classified as *Procedural error*. The researcher classifies this type of error as an error due to incorrect association as the respondents demonstrated error of assimilation because they knew the correct algorithms but confused the multiplication sign with the addition sign.

Based on the frequency table (cf Chapter 4, Table 4.11) which summarises learner responses, it could be ascertained that the majority (n =63, 60%) of learners admitted to sometimes confusing addition with multiplication. This was also evident in the responses to the content-based questionnaires where a number of errors related to the afore-stated factor were identified. Learners would add instead of multiply.

Only 7.6% admitted to always forgetting to write the units in their final answers whereas 42.9% sometimes made that error. Based on the correlation analysis previously illustrated (cf. Chapter 4, section 4.4.6) learners forgetting to write units was significant but with a weak correlation to writing an incorrect value. It is the only variable out of three (33.3%) of the variables for the stated research question that was revealed to be significant. Forgetting to write units in the final answer in financial mathematics is a common error committed and is evidently supported by the frequency tables illustrated in the previous chapter (cf.

Chapter 4, section 4.3.2) of the current study. Currency symbols should be used to illustrate the currency value to be worked out.

Content analysis revealed that learners were introduced to the use of simple and compound interest formula in the previous grades. In their attempts to work-out simple and compound interest problems, they would use the formula. The majority of learners (n = 89, 84.8%) used formulae to answer the simple and compound interest problems. In using the formula, others would use an incorrect formula and that according to Brodie (2005) is *Partial Insight* (cf Chapter 2, Table 2.2) described as the learners demonstrating that they grappled with an idea but showed insight in the task.

The respondents strongly disagreed with writing down an incorrect value or digit when working out financial mathematics problems. Only a few (n = 2, 1.9%) admitted to always committing that error and it was not related to any of the variables indicated.

The following errors were identified from content analysis:

- Use of formula even though it was not appropriate. Learners were supposed to follow a multi-step procedure without the use of the formula when working out the final amount in simple and compound interest. As discussed in the previous sections this was attributed to the irreversible characteristic of threshold concept.
- Use of an incorrect formula, where learners used a formula for calculating the final amount (A) with simple interest instead of using only the simple interest (SI) formula.
- Incorrect substitution of the formula. According to Nolting (1998) that is regarded as transformation error and he describes it as occurring when a learner understands what is required but is unable to employ the sequence of operations needed to solve the problem. The majority of the learners substituted the value of i (interest rate) incorrectly not taking into account that i is a percentage.
- Learners converted the period and the interest rate to months even if one of them was already given in months. That is also attributed to the drill and practise method used by educators where “you divide i (interest rate)

by 12 and multiply *n* (period) by 12 if the interest is compounded monthly". Learners apply the rules entrenched in the drill and practise method in class. In elaborating on the phenomenon Soendergaard and Cachaper (2008) brought into perspective a concept instrumental understanding which they describe as demonstrated by someone who uses rules without understanding.

- Even though some employed the correct algorithms they incorrectly rounded off the final answer, the researcher classifies that type of error as a *Placement error* which is classified as a particular type of procedural error. Nolting (1998) describes this type of error as incorrect sequencing of digits or alignment of algorithms.

The errors identified above can be classified into four groups as in accordance with the previous studies. Figure 5.2 summarises the errors identified in the above:

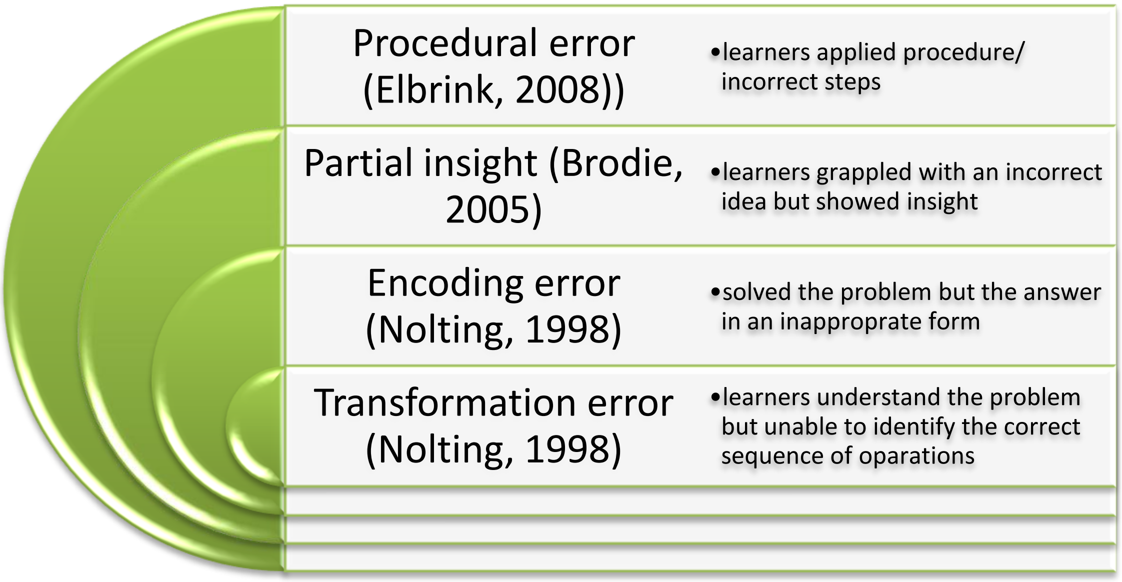

Figure 5.2: Identified types of errors related to incorrect association and rigidity of thinking

The aforementioned classification of errors identified, describes the types of errors related to research question 2: (underlying factors related to the errors due to incorrect association or rigidity of thinking) which was drawn from the previous studies. These are the errors identified by means of content analysis.

5.1.3 The underlying factors related to the errors due to the application of irrelevant rules or strategies

"...This kind of error often stems from experiences in successfully applying comparable rules or strategies in other content areas" (Radatz, 1979: 168). Learners used incorrect rules which led them to incorrect algorithms they employed to solve mathematical tasks. The majority of learners (n= 66, 62.9%) admitted to sometimes losing marks for not showing their algorithms while 45.7% did not complete their work. The majority (n = 70, 66.7%) admitted that they did not show all their steps which led to the final answer. The correlation analysis results of learners losing marks for not showing their workings illustrated significance where $p < .05$.

Only 54% admitted to making computation errors and 61% showed all the steps but were unable to find the correct answer. This then confirmed that learners did make computation errors as the steps were present but did not produce the correct answer. The correlation analysis revealed the significance where $p < .05$ but a weak correlation with showing all steps but not finding the correct answer. The following errors were identified by content analysis:

- In multiplying 7 by 6 an incorrect answer of 36 instead of 42 was given.
- Learners would not know when to multiply or add. In one question they were expected to determine travel cost for the month, given the fuel consumption, price of fuel per litre.
- Learners would treat sub-questions as independent questions (separate questions) and ignore the fact that those continued and were related to the main question. Some of the values determined in the main question were used in the sub-question.
- When expected to increase by percentage, they would just add the percentage to the value increased.
- Learners did not show all the steps to be followed in order to arrive at the final answer.

Some would correctly follow the expected algorithms but not find the correct final answer due to error committed during the steps. Learners showed the steps they followed in working out the problem but sometimes they did not show all their steps with the result that they lost the trend of the logic in their calculations.

The correlation analysis justified the significance where $p < .05$ with a weak correlation with the learners not finding the correct answer.

The following underlying factors related to errors due to the application of irrelevant rules or strategies:

- Learners lost marks for employing incorrect algorithms, irrelevant formula with intent of arriving to the correct final answer.
- Learners lost marks for not completing all the questions on a task.
- Learners did not show all the steps that would lead them to the correct answer.

Figure 5.3 illustrates that the underlying factors could be ascertained after an intense contentment analysis, guided by Newman's error analysis, in resolving the factor related to the application of irrelevant rules or rigidity of thinking.

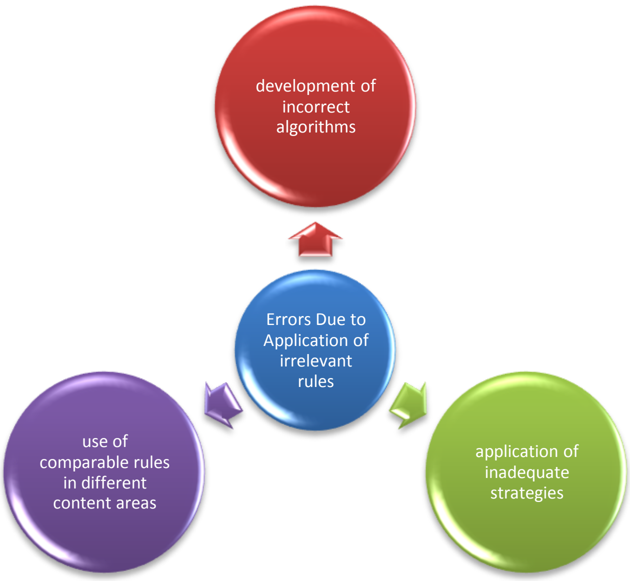

Figure 5.3: Description of the errors due to application of irrelevant rules (Adapted from Radatz, 1979)

The aforementioned figure illustrates the description of the errors due to application of irrelevant rules. Errors due to application of irrelevant rules are attributed to the following:

- Learners develop the incorrect algorithms when they continue following those until they reach the final answer. If only learners would be taught problem-solving techniques, the relevance and the importance of each of the four steps that need to be followed in problem solving, they will benefit. The fourth principle advises learners to take time to reflect on their work (i.e look back, read through what you have written).
- Learners would identify a correct strategy to employ in solving a particular problem but as illustrated in the above description of learner errors, use it incorrectly/inadequately.
- Learners acquire rules, adhere to those rules and sometimes apply them in irrelevant situations (cf Current Chapter, section 5.2.2). NEA, which is also a theoretical framework of this study, if employed, can introduce learners to analyze their work, pinpoint the type of errors they commit and enable them to avoid those errors.

The research question 3 was also identified **as proposition 3** where the alternative hypothesis was accepted based on the ANOVA results (cf Chapter 4). It was ascertained that errors committed by learners in financial mathematics were due to application of irrelevant rules and strategies.

5.1.4 The underlying factors related to the errors due to the deficient mastery of prerequisite skills, facts and concepts

The types of errors that are related to the afore-stated underlying factors include the deficits in content and problem-specific knowledge for successful performance in mathematical tasks. This is attested by ignorance of algorithms, inadequate mastery of basic facts, application of incorrect procedures and insufficient conceptual understanding. Learners did not admit to sometimes forgetting to read the instructions. Only 24.8% admitted to that. The majority of the learners (63.8%) never forgot to read the instructions.

Based on the results of the study learners always remembered to round off the final answer to 2 decimal places. Only a few (n = 11, 10.5%) never rounded off their final answer. Even though they rounded off their final answer, many (n= 53, 50.5%) sometimes rounded off incorrectly.

The majority of the respondents claimed they never forgot to write down the correct answer when using a calculator.

The following errors were identified by content analysis:

- The use of formula: learners were not encouraged to use any prescribed formula for both simple and compound interest. Due to previously acquired knowledge, learners would recall previously taught formulae and employ those to calculate simple and compound interest. Herein the entranced threshold concept proved to be irreversible as described previously (cf Chapter 2 section 2.6.2).
- Learners would use the formula but incorrectly. They would use a formula to find a final amount (A) when asked to find simple interest (SI).
- When the correct formula was used, the components of the formula were incorrectly substituted. This could be associated with a number of factors such a lack of working memory as the learner needed to remember intermediate products of calculations and the sequence of steps to be followed in order to arrive at the appropriate answer.

Figure 5.4 summarises the prerequisite skills from the GET-phase. Those skills could be useful but could also sometimes be problematic. In this instance learners drew skills acquired from the threshold concept which might be irrelevant.

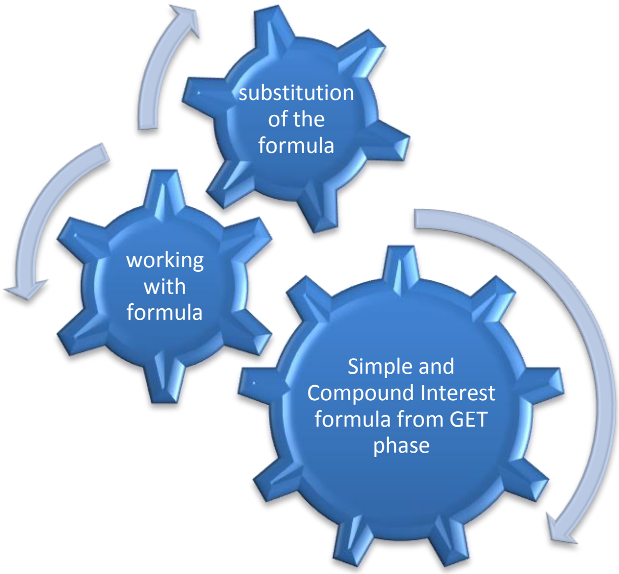

Figure 5.4: Prerequisite skills which learners acquired in the GET-phase

In the previous grade learners were introduced to the use of formula when working out some mathematical problems. It seemed though they acquired those skills, they showed no mastery of those skills. It is evident because they sometimes used an incorrect formula, substituted the formula incorrectly and rounded off the final answer incorrectly.

The aforementioned research question 4 is related to **Proposition 2**, based on the ANOVA result H0 which was supported. Therefore errors committed by learners in financial mathematics are not due to prerequisite skills, facts and concepts.

5.1.5 The underlying factors related to the errors due to language difficulties

"For many pupils the learning of mathematical concepts, symbols, and vocabulary is a foreign language problem. In solving word problems, pupils must refrain from using the manifold background of a word's meaning in natural language" (Ratadz, 1979: 165). A number of learners (n = 50, 47.6%) never forgot to indicate the answers as expected, while others (n = 37, 35.2%)

sometimes forgot. Some learners sometimes did not follow the instructions even though they read them.

Some learners (n = 61, 58.1%) sometimes did not understand the instructions when given a task. Salman (2007) points out that for a word problem to be meaningfully and conceptually interpreted there should be cognitive interaction with the concepts featuring in the problem. Some learners (n= 51, 48.6%) when they did not understand what was asked, admitted to just writing any answer.

No evidence of significance could be revealed by the correlation and significance test as all variables had $p > .05$ and all revealed a negative weak correlation. That suggested that there was no relationship among all variables on the afore-stated research question. Based on the frequency tables illustrated in the previous chapter (cf. Chapter 4, section 4.3.5) learners just wrote anything when they did not understand the question. They did not attempt to take time to consider the question before answering.

The following errors were identified by means of the content analysis:

- Learners could not comprehend the meaning of some phrases like “loss”, “profit”, “cost price”, etc used in the question.
- Learners employed incorrect strategies and therefore could not arrive at the correct answer.

Figure 5.5 below illustrates the three steps they needed to follow to arrive at a correct final answer:

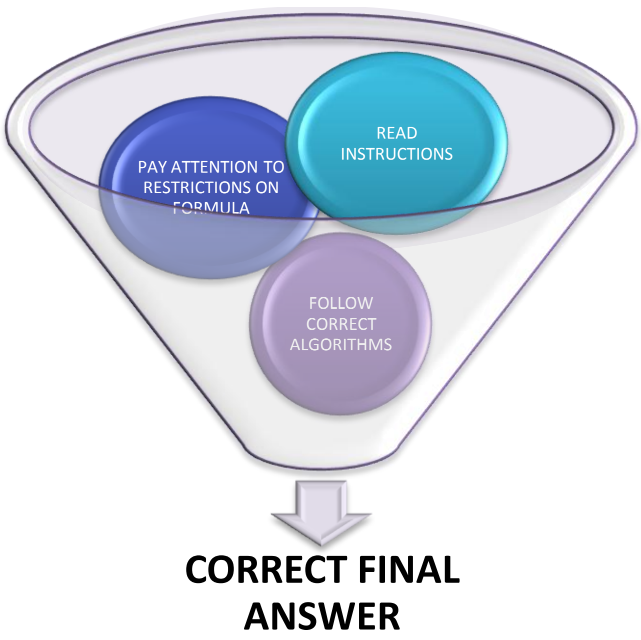

Figure 5.5: Three steps to follow to arrive at the correct answer

In order to arrive at the correct final answer in financial mathematics, word-problems seemed to be the issue. Therefore language posed the challenge and language could prove to be a barrier to arriving at the correct answer. Figure 5.5 illustrates the three steps that need to be taken into account to avoid committing errors of language difficulties.

As afore-stated learners should:

- Carefully read the instructions, as instructions provide a guide to what the question requires of the learner to do. It is not easy to answer the question without following the instructions.
- Pay attention to the restrictions of the formula as each formula has its own limitations, so learners need to be aware of those. Learners need to be absolutely aware of when and how to use that particular formula.
- Follow the correct algorithms; once the learner understands the question, the learner needs to employ the appropriate algorithm that will lead to finding the correct answer.

The aforementioned research question is related to **proposition 1**, based on the ANOVA used for the hypothesis, when it was tested it was ascertained that language difficulties did not impact on errors committed by learners.

5.1.6 The degree of predictability and hence strategies to underpin error analysis in question 1 – 4.

Based on the correlation coefficient analysis and the significance test, the degree of freedom of each variable of the research question was worked out by means of SPSS. This was conducted to establish the probability that the results may be incorrectly presented and may be misleading (cf Chapter 3, section 3.5).

5.1.6.1 Research question 1: why do learners commit errors on given tasks in financial mathematics?

- Learners wanting to do their best in the test and learners reviewing homework showed significance and a moderate correlation to indicate that there was a relationship between the two.
- Learners feeling confident when submitting tests and wanting to finish first proved to be significant but illustrated a weak correlation between the two.

Out of 7 variables tested for the underlying factors of the research question; 43% correlated and did not show any significance.

5.1.6.2 Research question 2: What are the underlying factors related to the errors due to incorrect association or rigidity of thinking?

- Learners forgetting to write units and writing down incorrect digits showed significance and a weak correlation between the two variables.

Out of 3 variables in the afore-stated research question only the above proved to be significant (33.3% significant) and illustrated a weak correlation.

5.1.6.3 Research question 3: What are the underlying factors related to the errors due to irrelevant rules or strategies?

- Learners losing marks for not completing all the problems and showing all the steps but did not find the correct answer, illustrated significance and a weak correlation between the two variables.
- Learners not showing all the steps and making basic computation errors, illustrated significance and a weak correlation between the two variables.
- Learners not showing all the steps and showing all the steps but did not find the correct answer proved to be significant and illustrated a weak correlation between the two variables.
- Learners showing all the steps and learners not showing all the steps proved significant and illustrated a weak correlation between the two variables.

From 6 pairs of variables of research question 3, 4 illustrated weak and moderate correlation and therefore account for 66.7% to be significant.

5.1.6.4 Research question 4: What are the underlying factors related to the errors related to deficient mastery of prerequisite skills, facts and concepts?

- Learners rounding off but incorrectly showed significance with a moderate correlation to learners forgetting to read the instructions.
- Learners not writing answers as shown by the calculator illustrated significance with a weak correlation to learners rounding off but incorrectly.

5.1.6.5 Research question 5: What are the underlying factors related to the errors related to language difficulties?

- All the variables of the afore-stated research question were not significant with the majority displaying a weak correlation to each other. That indicates that all the variables have no significant effect on the research question. It may be attested to a number of factors like the honesty of the respondent, the sample size etc.

5.2 THEORETICAL FRAMEWORK AND THE RESEARCH QUESTIONS

In conducting the study the researcher was guided by three theoretical frameworks: Polya's problem-solving techniques, Threshold Concepts and Troublesome Knowledge and Newman's Error Analysis. In addressing the research question the theoretical frameworks were taken into account.

5.2.1 Polya's Problem-solving Techniques

Learners seem to struggle with problem solving, simple because they do not understand the problem; they do not have a plan on how to solve the problem.

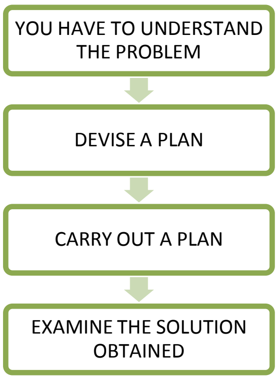

Figure 5.6: Polya's problem-solving steps (Adapted from Polya, 1945)

If the four steps illustrated in figure 5.6, are followed correctly it will reduce the learner errors in financial mathematics. If learners understand the problem, they can devise a plan to work out the problem. A plan can only be effective by proper execution and when the solution is found it has to be examined to be certain that it is the expected solution.

5.2.2 Threshold Concept and Troublesome Knowledge

"A core concept is a conceptual building block that progresses understanding of the subject; it has to be understood but it does not necessarily lead to a qualitatively different view of subject matter" Meyer and Land, (2007: 2). The subsequent diagram illustrates the characteristics of a Threshold Concept:

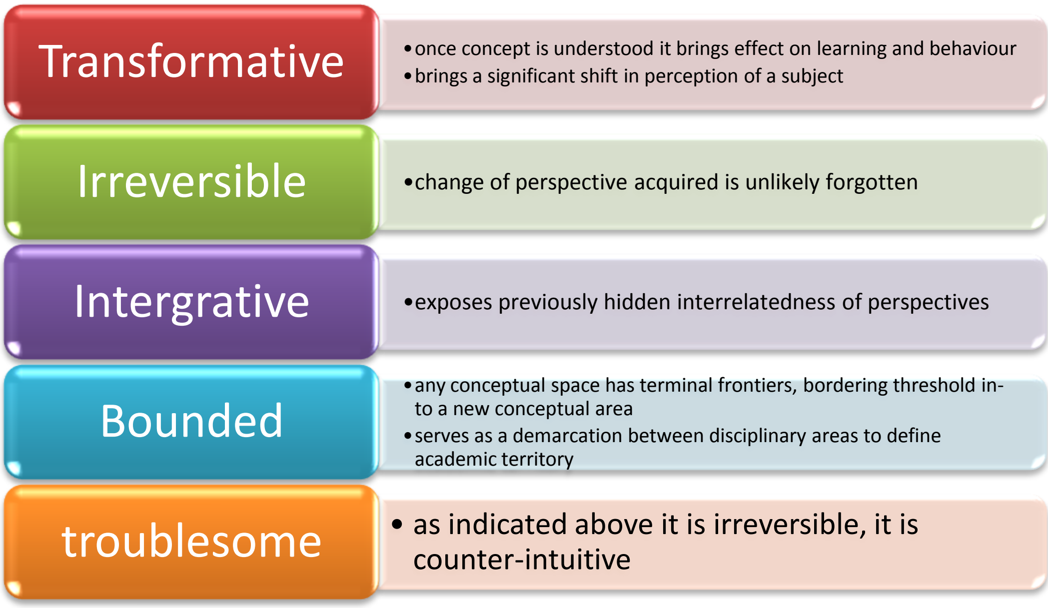

Figure 5.7: Characteristics of Threshold Concept

Throughout the school year, learners acquire knowledge, retain the knowledge and it forms the threshold concept. Figure 5.7 illustrates the characteristics of the threshold concept which was identified in research question 4, and can lead to errors if not mastered.

5.2.3 Newman's Error Analysis

"The Newman's error analysis and follow-up strategies has helped students with their problem-solving skills, and teachers have developed a much more consistent approach to the teaching of problem-solving" (White, 2009: 37).

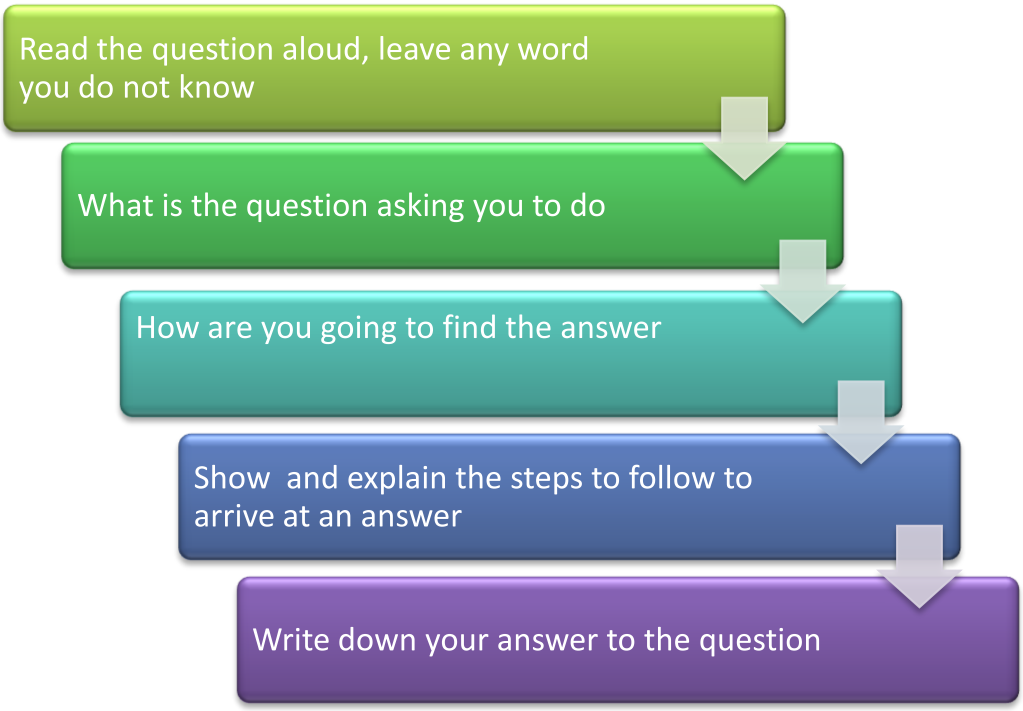

Figure 5.8: Newman's Error Analysis Interview Prompts (Adapted from Polya, 1945)

Newman's Error Analysis was used in the content questionnaire analysis and it could be effective if used in the classroom to identify learner errors. It affords learners an opportunity to reflect on their errors and helps them to identify their own errors. Figure 5.8 summarises the interview prompts used during Newman's error analysis.

5.3 CONCLUSION

This section presented the detailed discussion of the research findings per each research question including the hypotheses. Each of these research questions were discussed in relation to the theoretical framework and the review of literature used in this study.

CHAPTER 6

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.0 INTRODUCTION TO THE CHAPTER

This chapter provides a summary of the main ideas, findings of the study, conclusion and recommendations for further studies.

6.1 SUMMARY OF WHAT HAS BEEN DONE IN TERMS OF RESEARCH QUESTIONS

Chapter 1 provided the intent and the background of the study, where Errors, Mathematics, Mathematical Literacy and Financial Mathematics were clearly defined. This was followed by the statement of the research problem, the research questions and the hypotheses which were developed. The significance, the scope and limitations of the study also formed part of this section. A brief outline of the literature review and theoretical framework was followed by research methodology and the ethical considerations of the study.

Chapter 2 focused on review of the literature on error analysis and theoretical framework of the study. This section dealt with the conceptualization of Mathematical Literacy and Mathematics; the focus of the study in a South African perspective; Neuroscience and psychology of mathematics teaching which included mathematical thinking and implications of cognitive neuroscience on teaching and learning; understanding in the learning process of mathematics and the classification of learner errors in mathematics education. Throughout the review of the literature underlying factors that are related to the errors committed by learners were identified and discussed and that assisted in the formulation of the hypotheses of the study. Polya's problem-solving techniques, Threshold Concepts and Newman's Error Analysis (NEA) formed the theoretical framework of the current study.

Chapter 3 described the research methodology, with an elaborate description of the research paradigm followed by the research approach and design. The selected research design which included the case study, sampling technique, sample size, data-collection methods and the clear description of data-collection instruments were addressed. An outline of the data analysis and interpretation also formed part of this section and a reliability test for inferential analysis, and ethical consideration of the study were also discussed in this section.

Chapter 4 focused on the presentation and analysis of the data collected. This chapter began with the research questions and was followed by, data-analysis techniques, the demographics of the respondents, presentation of the frequency tables of different variables of each research question, and the correlation analysis results. This section elaborated on the research findings based on the hypotheses of the study which tested by means of the Analysis of Variance (ANOVA).

Chapter 5 addressed the research finding given in Chapter 4 with a focus on the following:

- Findings of each of the five research questions and hypotheses, their relation to the theoretical framework and previous literature.
- Polya's problem-solving techniques
- Threshold concept and troublesome knowledge
- Newman's Error Analysis

Chapter 6 summarised the main ideas of the study, summaries of the findings, a conclusion and recommendations of further studies.

6.2 MAIN FINDINGS OF RESEARCH QUESTION AND HYPOTHESES

NEA was used for content analysis where learner errors were identified and statistical analysis by frequencies, correlation analysis and ANOVA were also used to identify the underlying factors related to the following research questions tested.

The following are the results:

1. **Why do learners commit errors on given tasks in financial mathematics?**

    - Although learners wanted to do their best in assessment tasks, there was a weak relationship with reviewing their homework.
    - The study revealed that learners sometimes felt rushed, therefore panicked, when they wrote tests and made mistakes.
    - Learners felt confident when submitting their test scripts, which was also related to them finishing first and as a result committing errors in their work.

2. **What are the underlying factors related to errors due to incorrect association or rigidity of thinking?**

    - Learners forgot to write units in the final answer.
    - Learners wrote down incorrect values when transcribing their rough work. This could be attributed to the fact that they panicked and felt rushed.

3. **What are the underlying factors related to the errors due to application of irrelevant rules or strategies?**

    - Learners did not show all the steps to be followed in order to arrive at the final answer.

- Sometimes they showed all the steps but did not find the correct answer.
- Sometimes they showed steps but not all of them.
- Learners made basic computation errors.

4. **What are the underlying factors related to the errors due to the deficient mastery of prerequisite skills, facts and concepts?**
    - Learners forgot to read the instructions; this could be related to the fact that they felt rushed in tests, or to language difficulties.
    - When required to round off the answer; they did round off but incorrectly. Rounding off is supposed to be acquired and mastered earlier in their school years.
    - When using a calculator they incorrectly transcribed the value displayed by the calculator. A relationship with rounding off incorrectly was revealed by the correlation test.

5. **What are the underlying factors related to errors due to language difficulties?**
    - If learners could not understand what was asked they just gave any response.

All the variables of the research question 5 afore-stated were not significant with the majority displaying a weak correlation to each other. That indicates that all the variables had no significant effect on the research question. That could be attested to a number of factors like the honesty of the respondent, the sample size etc.

For Hypothesis 1: the results of the ANOVA presented that errors committed by learners were not impacted on by language difficulties. Out of the 4 variables none illustrated any statistical significance, all contributed $p>.05$.

For Hypothesis 2: the ANOVA results presented that errors committed by learners in financial mathematics were not due to prerequisite skills, facts and concepts. Out of 4 variables, only 1 illustrated statistical significance of $p<.05$.

Therefore the majority (75%) showed non-significance of the variables to research question 2.

For Hypothesis 3: results illustrated that errors committed by learners in financial mathematics were due to the application of irrelevant rules and strategies. Research question 3 included six variables and four (66.7%) illustrated the significance levels where $p<.05$

6.3 CONCLUSIONS OF THE STUDY

The study was conducted by means of a case study: Grade 10 Mathematical Literacy learners. Primary data was collected by structured questionnaires from 105 respondents determined by a simple random technique. The study sought to answer five research questions (cf Chapter 1, section 1.6). Based on the ANOVA the three proposition were tested and the aforementioned findings indicate that, errors committed by learners in financial mathematics were not impacted on by language difficulties, errors committed by learners in financial mathematics were not due to prerequisite skills, facts and concepts, errors committed by learners in financial mathematics were due to the application of irrelevant rules and strategies. Four of the six variables of the errors due to the application of irrelevant rules and strategies illustrated a relationship between the variables.

6.4 RESEARCH IMPLICATIONS

6.4.1 Theoretical implication of the study

The study was guided by three theoretical frameworks:

- Polya's problem-solving techniques which guided the researcher in identifying, planning, execution of the plan and reviewing the identified factors related to learner errors.

- Threshold concepts formed part of the research problem as it was related to **research questions 2, 3 and 4.** The threshold concept forms a fundamental part of error analysis as most of the errors were associated with entrenched knowledge from the previous grades. This was evident in the content analysis (cf Chapter 4).
- Newman's Error Analysis guided the researcher in the content analysis of questionnaires 1, 2 and 3 that sought to identify the type of errors committed by learners and the underlying factors related to those errors. It gave learners an opportunity to reflect on the errors.

6.4.2 Practical implication of the study

Error analysis may be incorporated in the teacher training curriculum as it will assist in reducing or eliminating learner errors. It will assist educators to be able to identify learner errors, assist learners in eliminating those errors and encourage learners to review the work before submission. Understanding learners' rationale when going through their work can, also assist teachers to institute remedial lessons. Educators need to incorporate error analysis in their lesson designs, as knowledge of why learners commit errors is valuable to the educators as it will help strategies.

Learners should be taught to apply Polya's problem-solving techniques. That will train them in applying the techniques to make sure they understand the question before attempting to answer it; to plan before answering; to answer and then review what was written to make sure that they reduce the errors committed.

6.5 RECOMMENDATION OF FURTHER RESEARCH IN FINANCIAL MATHEMATICS ERROR ANALYSIS

- Further research studies could be conducted in error analysis in financial mathematics but the focus should be on higher grades (Grades 11 and 12) as learners continue to commit these kinds of errors even in those grades. The study population could be increased to a number of schools (5 or more schools) to increase the reliability and validity of the research findings. Error analysis is a topic that has not yet been researched much in South Africa especially in both Mathematics and Mathematical Literacy. More studies need to be conducted so it can provide recommendations to assist educators in their lesson designs in order to assist learners in avoiding the identified errors. That could increase the learner performance in Mathematics and Mathematical Literacy.

- The use of formula in working out financial mathematics problems in Mathematical Literacy. Educators and learners still perceive the use of formula relevant and convenient when working out simple and compound interest problems. The study should be focused on Grades 10 and/or 11.

- The educators' understanding of the Curriculum Assessment Programme Statement (CAPS) in Mathematical Literacy. Most educators seem not to understand the Curriculum Assessment Programme Statements, even though this has already been implemented in Grades 10 and 11. It is scheduled to be introduced to Grade 12 in 2014. Educator training has been rolled out in the past three years (2010 – 2013) throughout the provinces but teachers do not appear to understand the content and the purpose.

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ACRONYMS:

Analysis of Covariance – ANCOVA

Analysis of Variance – ANOVA

Department of Education – DoE

Further Education and Training - FET

Grade 10 - G10

Mathematical literacy - ML

National Curriculum Statements - NCS

Newman’s Error Analysis - NEA

Statistical Package of Social Sciences - SPSS

Value Added Tax - VAT

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Appendix A

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Research Question 5: What are the underlying factors related to Errors Due to Language Difficulties?

Instructions to the respondents:

1. Answer all questions on the question paper provided.
2. Show all your workings.
3. Use the space provided to answer questions.
4. Non-programmable calculator may be used unless stated otherwise.
5. Write neatly and legible.

1. Peter bought some furniture, but then had to sell it again because his employer sent him to work in Norway for a couple of years.

(a) Complete the table below:

Item	**Cost price**	**Selling price**	**Profit/loss**	**Amount of profit/loss**
Leather Lounge Suite	R9 999,95	R8 000,00	__________	___________
Bedroom Suite	R4 999,95	R4 375,00	__________	___________
Home Theatre System	R1 199,95	____________	profit	R200,05
Dining room Suite	____________	R6 500,00	Loss	R2 499,95
Wall Unit	_____________	R3 550,00	profit	R350,05

(b) Taking all five items into account:

(i) Did Patrick make a profit or a loss? (1)

____

(ii) What is Patrick's total profit and loss? 2+2=4

Total = 12

XolaniKhalo Ethics Human 2011

<<Approved >> OFFICE USE ONLY

Ref:	Date:

Appendix B

University of Fort Hare
Together in Excellence

Structured Interview Questionnaire

The questions that follow cover the Income and expenditure in grade 10 financial mathematics.

Instructions to the respondents:

1. Answer all questions on the question paper provided.
2. Show all your workings.
3. Use the space provided to answer questions.
4. Non-programmable calculator may be used unless stated otherwise.
5. Write neatly and legible.

Research question 3: What are the underlying factors related to errors due to the application of irrelevant rules or strategies?

1. What is Jabu's income if he works for 6 days, 7 hours per day and is paid R44.50 per hour? (4)

2. Portia travels 900km per month to get to work. The fuel consumption of her car is 12km per litre and she pays R10.80 per litre for fuel. Determine Portia's transport expenses for one month. (4)

3. Reginald earns a salary of R5 700 per month. From his salary 15% is deducted for income tax, R850 the medical aid fund and R1 406 for his motor vehicle instalment.

(a) How much is left for Reginald to cover his other expenses? (4)

(b) If all his other necessities amount to R1 480, what is left for entertainment and savings? (2)

(c) If Reginald's payment to the medical aid fund increase by 8% and he spends R150 on entertainment, what is he able to save? (4)

Total marks = 18

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Ref:	Date:

Appendix C

University of Fort Hare
Together in Excellence

Structured Interview Questionnaire

The questions that follow cover the simple and compound interest in grade 10 Financial mathematics.

Instructions to the respondents:

1. Answer all questions on the question paper provided.
2. Show all your workings.
3. Use the space provided to answer questions.
4. Non-programmable calculator may be used unless stated otherwise.
5. Write neatly and legible.

Research question 2: What are the underlying factors related to the errors due to the incorrect association or rigidity thinking?

1. Calculate the interest on R4 500 borrowed for 6 years at 14% simple interest per year. (4)

2. Determine the value of R5 000 after 4 months if it is increased at 9% interest per year compounded monthly. (4)

Research question 4: What are the underlying factors related to errors due to deficient mastery of prerequisite skills, facts and concepts?

3. If R12 000 is invested at 9.5% simple interest per year, calculate the value of the investment after 4 years and three months. (4)

4. How long will it take R5 100, invested at 9% simple interest per year to amount to R7 854? (4)

5. Calculate the value of R9 700 after 3 years if it is invested at 9.5% compounded annually. (4)

Total mark 20

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Ref:	Date:

Appendix D

University of Fort Hare
Together in Excellence

A QUESTIONNAIRE FOR GRADE 10 MATHEMATICAL LITERACY LEARNER

Instructions:

1. This questionnaire consists of **2 SECTIONS** (i.e. Section A and B). You are requested to answer all of questions.
2. Your responses to the questions will receive serious consideration as they will provide assistance towards minimizing or eliminating errors committed in Mathematical Literacy: Financial mathematics.
3. Honesty will be highly appreciated.

SECTION A

1. General Information about yourself.
Please circle the correct number relevant to your personal information.

1.1. Gender

Male	Female
1	2

1.2. Age

11-12	13-14	15-16	17-18
1	2	3	4

1.3. How long have you been at this school?

1year	2years	3 years	4 years	5 years
1	2	3	4	5

SECTION B

Please give an honest response to each of the following statements by circling the relevant response:

1= never, 2= rarely, 3= sometimes and 4= always

2. **Research Question 1: Why do learners commit errors on given tasks in financial mathematics?**

2.1	I want to do my best in my tests.	1	2	3	4
2.2	I feel confident when I submit my test because I know I have done my best.	1	2	3	4
2.3	I feel rushed when I'm writing a test.	1	2	3	4
2.4	I want to be first one done on the test.	1	2	3	4
2.5	I go back read through what I have written before handing in the test script.	1	2	3	4
2.6	I take my time to answer the test questions.	1	2	3	4
2.7	I review my homework after I have done it.	1	2	3	4

SECTION C

These questions are specifically referring to how you handled the questions in your Mathematical Literacy tasks.
Please answer the questions by circling the number referring to you relevant answer.
1 = never, 2 = rarely,3 = sometimes and 4 = always

3. **Research question 2: What are the underlying factors related to the Errors due to incorrect association or rigidity of thinking?**

3.1 I confuse addition with multiplication	1	2	3	4
3.2 I forget to write units	1	2	3	4
3.3 I write down an incorrect figure (number)	1	2	3	4

4. Research question 3: What are the underlying factors related to the Errors due to the application of irrelevant rules or strategies?

4.1 I lose marks for not showing workings (steps)	1	2	3	4
4.2 I lose marks for not completing all the problems	1	2	3	4
4.3 I do not show all steps	1	2	3	4
4.4 I make basic computation errors	1	2	3	4
4.5 I show all steps but do not find the correct answer	1	2	3	4
4.6 I show all my steps but not all of them	1	2	3	4

5. Research question 4: What are the underlying factors related to the Errors due to mastery of prerequisite skills, facts and concepts?

5.1 I forget to read the instructions	1	2	3	4
5.2 I do not round off the answer to 2 decimal places	1	2	3	4
5.3 I do round off but incorrectly	1	2	3	4
5.4 I do not write the answer as shown in a calculator	1	2	3	4

6. Research question 5: What are the underlying factors related to Errors due to language difficulties?

6.1 I don’t indicate answers as expected	1	2	3	4
6.2 I do not follow instructions	1	2	3	4
6.3 I don’t understand instructions	1	2	3	4
6.4 I write any answer if i don’t understand the question	1	2	3	4

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Ref:	Date:

Appendix E

University of Fort Hare
Together in Excellence

PARTICIPANT INFORMATION LEAFLET AND CONSENT FORM FOR USE BY PARENTS/LEGAL GUARDIANS

TITLE OF THE RESEARCH PROJECT: *Error analysis in Grade 10 Mathematical Literacy: Case of Financial mathematics*

REFERENCE NUMBER: BAY01 1SKHA01

PRINCIPAL INVESTIGATOR: Xolani Khalo

ADDRESS: P. O Box 4033

Duncan Village

East London

5200

CONTACT NUMBER: 073 3036 658

Your child is being invited to take part in a research project. Please take some time to read the information presented here, which will explain the details of this project. Please ask the researcher any questions about any part of this project that you do not fully understand. It is very important that you are fully satisfied that you clearly understand what this research entails and how your child could be involved. Also, your child's participation is **entirely voluntary** and you are free to decline to participate. If you say no, this will not affect you or your child negatively in any way whatsoever. You are also free to withdraw him/her from the study at any point, even if you do initially agree to let him/her take part.

This study has been approved by the **University Research Ethics Committee at the Universityof Fort Hare** and will be conducted according to the ethical guidelines and principles of the international Declaration of Helsinki, South

African Guidelines for Good Clinical Practice and the Medical Research Council (MRC) Ethical Guidelines for Research.

What is this research study all about?

- *The research is focused on Grade 10 Mathematical literacy learners.*
- *It is aimed at analysing the errors(mistakes) that learners do when working out Mathematical literacy problems (Financial mathematics)*
- *Learners will be given questions to answer, their answers to the questions will be marked and analysed.*
- *The analysis will help identify the errors they commit and help them to avoid the errors and that will help reduce the errors in their work therefore bring an improvement in their performance in the subject.*

Why has your child been invited to participate?

- *Your child is a Grade 10 Mathematical literacy learner, therefore is the targeted group for the study.*

What will your responsibilities be?

- *Allow your child to participate in the interview sessions and complete and return the questionnaires.*

Will your child benefit from taking part in this research?

- *Yes, the researcher is a grade 10 Mathematical literacy teacher who will provide remedial or support lessons to learners. The support lessons will focus on the identified errors and provide ways and means of avoiding them.*

Are there any risks involved in your child taking part in this research?

- *There are no risks involved in your child participation in the research study.*

If you do not agree to allow your child to take part, what alternatives does your child have?

- *The participation is voluntary therefore each child is free not to participate those who do not participate will not suffer any consequences.*

Who will have access to your child's records?

- *The information collected will be treated with confidentiality and protected. If it is used in a publication or thesis, the identity of the participant will remain anonymous. The information will be kept under strict care of the researcher, the supervisor and the academic staff of the institution.*

What will happen in the unlikely event of your child getting injured in any way, as a direct result of taking part in this research study?

- *The research as it has been explained above will not pose any treats to the participants' physical or psychological well-being as it only involves answering question on financial mathematics.*

Will you or your child be paid to take part in this study and are there any costs involved?

You or your child will not be paid to take part in the study, but out-of-pocket expenses will be covered for each study visit. There will be no costs involved for you if your child does take part.

Is there anything else that you should know or do?

- You can contact the Chairperson of the University Research Ethics Committee if you have any concerns or complaints that have not been adequately addressed by the researcher.
- You will receive a copy of this information and consent form for your own records.

Assent: Children with an age of 7 and above must give assent to participate in research

Declaration by parent/legal guardian By signing below, I *(name of parent/legal guardian)* ………………………………..……. agree to allow my child (name of child) ………………………………..… who is ………. years old, to take part in a research study entitled *Error analysis in Grade 10 Mathematical Literacy: Case of Financial mathematics.* **I declare that:**
• I have read or had read to me this information and consent form and that it is written in a language with which I am fluent and comfortable. • If my child is older than 7 years, he/she must agree to take part in the study and his/her ASSENT must be recorded on this form. • I have had a chance to ask questions and all my questions have been adequately answered. • I understand that taking part in this study is **voluntary** and I have not been pressurised to let my child take part.

- I may choose to withdraw my child from the study at any time and my child will not be penalised or prejudiced in any way.
- My child may be asked to leave the study before it has finished if the study doctor or researcher feels it is in my child's best interests, or if my child does not follow the study plan as agreed to.

Signed at (*place*) ....…........……………….. on the (*date*) …………....……………….

Signature of parent/legal guardian   Signature of witness

Declaration by investigator

I *(name)* ……XolaniKhalo………………………………………..……… declare that

I explained the information in this document to …………………………………..

I encouraged him/her to ask questions and took adequate time to answer them.

I am satisfied that he/she adequately understand all aspects of the research, as discussed above

I did not use a interpreter (i*f a interpreter is used, then the interpreter must sign the declaration below).*

Signed at (*place*) ....…........……………….. on (*date*) …………....………..

Signature of investigator:…………………………………………

Declaration by interpreter (Only complete if applicable)
I *(name)* ……………………………………………..……… declare that:

I assisted the investigator (*name*) ………….………………………. to explain the information in this document to (*name of parent/legal guardian*) ……..……………………….... using the language medium of Afrikaans/Xhosa.
We encouraged him/her to ask questions and took adequate time to answer them.
I conveyed a factually correct version of what was related to me.
I am satisfied that the parent/legal guardian fully understands the content of this informed consent document and has had all his/her questions satisfactorily answered
Signed at (*place*) ....…........……………….. on (*date*) ……………...……………
Signature of interpreter: **..**

Appendix F

University of Fort Hare
Together in Excellence

UNIVERSITY RESEARCH ETHICS COMMITTEE

The completed form must be submitted with the application. An incomplete checklist form will result in the return of the whole application to the originator

CHECKLIST-GENERAL
Section A. To be completed by Applicant and checked by GMRDC Office

PROTOCOL TITLE: Error analysis in Grade 10 Mathematical literacy: case of Financial mathematics

PROTOCOL NUMBER		**PROTOCOL VERSION**		**PROTOCOL DATE**	

	CV (max 2 pages)	**Investigator Declaration**	**Conflict of Interest statement signed.**	**Admin Office Comments**
PRINCIPAL INVESTIGATOR:				
SUPERVISOR:				
CO-INVESTIGATORS				
1.				
2.				

3.				
4.				
5.				
6.				
OTHERSTAFF				

	Applicant	Comments	Admin Office
	Y /N / NA		
Applicant Signature	Y		
Supervisor Signature	Y		
HOD Signature	Y		
Protocol synopsis	Y		
Full protocol	Y		
Page numbers on protocol?	Y		
Budget	Y		
Informed Consent Form	Y		
Questionnaires	Y		
Other measuring tools/instruments	Y		
Recruitment material/	N/A		

Advertisement(s)			
DoH or other letters of approval to conduct research	**N/A**		
Material Transfer Agreement	**N/A**		

Section B: *To be completed by Applicant. The Reviewer will cross check*

INFORMED CONSENT FOR RESEARCH CHECKLIST

	Yes / No or NA (PI)	***Yes/No/ NA****(Reviewer)*
1. That consent is being sought from the participant to participate in research.	Yes	
2. The purpose of the research and where it will be conducted.	Yes	
3. The expected duration of the participant's involvement in the research.	Yes	
4. The total number of participants that will be involved at this site and/or South Africa and worldwide.	Yes	
5. A description of **all** the processes and procedures to which the participant will be subjected,	Yes	
6. The principal investigator's name and contact details.	Yes	
7. Explanation of participants' responsibilities.	Yes	
8. Explanation of any randomization process if applicable).	N/A	
9. Circumstances that may result in the project being terminated or the participant being withdrawn.	N/A	
10. A description of foreseeable risks and discomforts.	N/A	
11. A description of benefits to the participant or others both during and after the research. If there are no expected benefits, the participant must specifically be made aware of this.	Yes	

12. Disclosure of alternative procedures and course of treatments available if applicable	N/A	
13. Description of extent to which confidentiality will be maintained and protected.	Yes	
14. Statement that sponsors of the study, study monitors or auditors or UREC members may need to inspect research records.	N/A	
15. Statement that the UREC has approved the research.	N/A	
16. Contact details of the committee.	Yes	
17. Explanation of how research related injury will be managed and details of insurance if applicable.	N/A	
18. Explanation as to whom to contact in the event of research related injury.	N/A	
19. Participation in the study is entirely voluntary	Yes	
20. Participants are free to withdraw at any point without explanation or any negative consequences.	Yes	
21. Participants must be informed of their rights to be told any new relevant information that arises during the course of the trial and the ICF should be revised, where appropriate to incorporate this information.	Yes	
22. That the study will be conducted according to the International Declaration of Helsinki and other applicable international ethical codes for research on human subject.	N/A	
23. Any expense to which the participant may be liable.	N/A	
24. Explanation regarding payment for participation or out of pocket expenses	N/A	
25. Identity of the funder, where applicable and any potential conflict of interests.	N/A	
26. Where appropriate, the participant should also be requested/advised to inform his general practitioner and life insurance company or medical aid of his/her participation. ☐ Not considered appropriate/necessary	N/A	

27. Simple, clear language has been used (Maximum Grade 8 reading level) and all medical and technical terms have been explained.	Yes	

Section C. To be completed by <u>Applicant</u>

	Yes(PI), NA	**Yes/No/NA (Reviewer)**
1. Does the study have relevance and scientific or clinical value and applicability to the proposed research population?	Yes	
2. Does the protocol include an adequate literature review?	Yes	
3. Is the selection of subjects equitable and appropriate; adequate consideration and protection of vulnerable research populations.	Yes	
4. Is the design and methodology appropriate to answer the research question?	Yes	
5. Is the methodology clearly described, in sufficient detail?	Yes	
6. Is the statistical analysis plan, including sample size calculations, clearly outlined and justified?	Yes	
7. Are the inclusion and exclusion criteria clearly defined and appropriate?	Yes	
8. Have risks been minimized and is there an acceptable balance between potential risks and benefits?	Yes	
9. Does the PI have the necessary qualifications, expertise, facilities, and time and support staff, to carry out the proposed research?	Yes	

10. Has a section on 'Ethical Considerations' been included in the protocol?	Yes	
11. Has the informed consent process been clearly explained in the protocol?	Yes	
12. Are issues relating to protection of privacy and confidentiality of data adequately addressed, especially if the study involves a retrospective review of clinical records?	Yes	
13. Has a waiver of informed consent been requested if the study involves a retrospective review of clinical records?	N/A	
14. Does the study involve collection of DNA/RNA and, if so, has consent been adequately sought for this?	N/A	

OFFICE USE ONLY

Ref:	Date:

University of Fort Hare
Together in Excellence

LETTER OF PERMISSION TO THE DEPARTMENT OF EDUCATION

Enquires: X. Khalo **Cell no: 073 3036 658**

Email: x.khalo@gmail.com

University of Fort Hare

Private Bag x 1314

Alice

5700

21 February 2013

The Superintendent-General

Eastern Cape Department of Education

Private Bag X 0032

Bisho

5608

Dear Sir

RE: Request for permission to conduct research in an East London district school

I am a student and pursuing a Master of Education (M.Ed) degree at the above mentioned institution and currently planning a research project for a full dissertation in fulfilment of the requirements for the M.Ed qualification.

The title of my proposed research is: *"ERROR ANALYSIS IN GRADE 10 MATHEMATICAL LITERACY: CASE OF FINANCIAL MATHEMATICS".* This is a quantitative study which will involve a sample of about 165 Grade 10 mathematical literacy learners from a school in the East London district.

Learners continue to commit similar errors in their work when assessed even with the best teaching and learning strategies. To be able to reduce and/or eliminate these errors both learners and educators need to be able to:

- Identify the errors
- Understand why learners continue to make them
- Then be able to minimize/avoid them.

It is anticipated that the study will take two weeks direct interaction with learners and a month interaction with collected data. Data collection will be conducted through questionnaires and will make every effort to ensure minimal use of school time. It is hoped that the study will not in any way harm the image of the department, nor the school where conducted, or violate any laid rules of conduct expected of the researcher.

Furthermore, every effort will be made to ensure that the anonymity of the concerned respondents (i.e. learners) and that of their school, as well as confidentiality regarding information that will be provided, are maintained.

Participation in this study is voluntary and at any point during this research the respondent does not feel comfortable to continue will be free to withdraw from the study without any negative consequences.

All data collected during this study will be kept confidential until the research is over. It is hoped that the findings from this study will help both Educators and learners to deal amicable with the errors in financial mathematics and devise methods to minimize or eliminate them.

If you would like to query anything about the study, you may contact the researcher (contact detail provided) or my supervisor at the University of Fort Hare, Faculty of Education (East London campus).

Thanking your co-operation in advance.

Yours Faithfully

XolaniKhalo (**Student No: 200444867**)

XolaniKhalo Ethics Human 2011

<<Appendix H >> OFFICE USE ONLY

Ref:	Date:

University of Fort Hare
Together in Excellence

LETTER OF PERMISSION TO THE DEPARTMENT OF EDUCATION (East London District)

Enquires: X. Khalo

Cell no: 073 3036 658

Email: x.khalo@gmail.com

University of Fort Hare

Private Bag x 1314

Alice

5700

21 January 2013

The District Director

Private Bag X 9007

EAST LONDON

5200

Dear Sir

RE: Request for permission to conduct research in an East London district school

I am a student and pursuing a Master of Education (M.Ed) degree at the above mentioned institution and currently planning a research project for full dissertation in fulfilment of the requirements for the M.Ed qualification. The title of my proposed research is: *“ERROR ANALYSIS IN GRADE 10 MATHEMATICAL LITERACY: CASE OF FINANCIAL MATHEMATICS”.* This is a quantitative study which will involve a sample of about 165 Grade 10 mathematical literacy learners from a school in the East London district (i.e.

Kusile Comprehensive School). Learners continue to commit similar errors in their work when assessed even with best teaching and learning strategies. To be able to reduce and/or eliminate these errors both learners and educators need to be able to identify the errors, understand why learners continue to make them and then be able to minimize/avoid them.

It is anticipated that the study will take two weeks direct interaction with learners and a month interaction with collected data. Data collection will be conducted through questionnaires and will make every effort to ensure minimal use of school time. It is hoped that the study will not in any way harm the image of the department, nor the school where conducted, or violate any laid rules of conduct expected of the researcher.

Furthermore, every effort will be made to ensure that the anonymity of the concerned respondents (i.e. learners) and that of their school, as well as confidentiality regarding information that will be provided, are maintained.

Participation in this study is voluntary and at any point during this research the respondent does not feel comfortable to continue will be free to withdraw from the study without any negative consequences.

All data collected during this study will be kept confidential until the research is over. It is hoped that the findings from this study will help both Educators and learners to deal amicable with the errors in financial mathematics and devise methods to minimize or eliminate them.

If you would like to query anything about the study, you may contact me or my supervisor at the University of Fort Hare, Faculty of Education (East London campus).

Thanking your co-operation in advance.

Yours Faithfully

XolaniKhalo (**Student No: 200444867**)

XolaniKhalo Ethics Human 2011
<<Appendix I >> OFFICE USE ONLY

Ref:	Date:

University of Fort Hare
Together in Excellence

LETTER TO THE SCHOOL PRINCIPAL AND THE SCHOOL GOVERNING BODY (Kusile Comprehensive School)

Enquires: X. Khalo **Cell no: 073 3036 658**

Email: x.khalo@gmail.com

University of Fort Hare

Private Bag x 1314

Alice

5700

21 February 2013

The Headmaster

Kusile Comprehensive School

P.O Box 4033

Duncan Village

5200

Dear Sir

RE: Request for Permission to use Grade 10 Mathematical Literacy learners in a research study.

I am a Master of Education student at the University of Fort Hare conducting a quantitative study in Mathematical Literacy, and would like to formally request permission to use Grade 10 mathematical Literacy learners in the research study. The title of the study is: "*ERROR ANALYSIS IN GRADE 10 MATHEMATICAL LITERACY: CASE OF FINANCIAL MATHEMATICS".*

I will distribute four sets of Questionnaires to about 165 Grade 10 learners; they will complete and return to the researcher. The researcher may request one on one interview for clarity where necessary. The data collected will solely be used for the purposes of an academic research project and participants will not be identified by name in any report, therefore the research endeavours to observe all forms of research ethical codes and confidentiality.

I trust that you will give the support accordingly in facilitating this project.

Yours faithfully

XolaniKhalo

(**Student no: 200444867)**

APPENDIX J

KUSILE COMPREHENSIVE SCHOOL

Mr. H. F. Gayiza
Mrs. B. Bottoman

Tel.: 043 733 8515
043 733 4690
Fax: 043 733 8515

OFFICE OF THE:-
Principal
Kusile Comprehensive School

P.O. Box 4033
DUNCAN VILLAGE
5216

Est. Let there be light 1990

2 May 2013

Mr. X. Khalo
Kusile Comprehensive School
Msimango Street
Duncan Village
EAST LONDON
5216

Dear Sir

RE: PERMISSION TO DO RESEARCH

Regarding a request you have extended to do a research towards your studies with some of our class groups, as head of this school I accept your request and thus grant you permission to do the research.

In my capacity as head of this school I also wish you all the best in this wonderful journey of self-enrichment and developing yourself so that you are well equipped to face the challenges of the world we live in.

Good luck with your studies.

Yours faithfully

KUSILE COMPREHENSIVE SCHOOL P.O. BOX 4033, DUNCAN VILLAGE 5216 TEL: 043 733 8515 PRINCIPAL

2/05/2013

H.F.Gayiza
PRINCIPAL

Appendix K

University of Fort Hare
Together in Excellence

ETHICAL CLEARANCE CERTIFICATE

Certificate Reference Number:	BAY01 1SKHA01
Project title:	**Error analysis in Grade 10 Mathematics Literacy: A case of financial mathematics.**
Nature of Project:	Masters
Principal Researcher:	Xolani Khalo
Supervisor:	Dr A Bayaga
Co-supervisor:	

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above-mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

The Principal Research must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
  - Any unethical principal or practices are revealed or suspected
  - Relevant information has been withheld or misrepresented
  - Regulatory changes of whatsoever nature so require
  - The conditions contained in the Certificate have not been adhered to
- Request access to any information or data at any time during the course or after completion of the project.

The Ethics Committee wished you well in your research.

Yours sincerely

Professor Gideon de Wet
Dean of Research

21 February 2013