MATHEMATICAL REQUIREMENTS FOR FIRST-YEAR BCOM STUDENTS AT NMMU

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MATHEMATICAL REQUIREMENTS FOR FIRST-YEAR BCOM STUDENTS AT NMMU

By

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SUMMARY

These studies have focused on identifying the mathematical requirements of first-year BCom students at Nelson Mandela Metropolitan University. The research methodology used in this quantitative study was to make use of interviewing, questionnaire investigation, and document analysis in the form of textbook, test and examination analysis. These methods provided data that fitted into a grounded theory approach. The study concluded by identifying the list of mathematical topics required for the first year of the core subjects in the BCom degree programme. In addition, the study found that learners who study Mathematics in the National Senior Certificate should be able to cope with the mathematical content included in their BCom degree programme, while learners studying Mathematical Literacy would probably need support in some of the areas of mathematics, especially algebra, in order to cope with the mathematical content included in their BCom degree programme. It makes a valuable contribution towards elucidating the mathematical requirements needed to improve the chances of successful BCom degree programme studies at South African universities. It also draws the contours for starting to design an efficient support course for future “at-risk” students who enter higher education studies.

KEY TERMS

Mathematical prerequisites; mathematics bridging course; business mathematics; mathematical literacy for business students; mathematics in the NSC and business; mathematics preparation courses.

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"Lord, you have assigned me my
portion and my cup;
you have made my lot secure.
The boundary lines have fallen for me
in pleasant places;
surely I have a delightful
inheritance."
Psalm 16:5, 6

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CHAPTER ONE: ORIENTATION AND OUTLINE OF THE STUDY

1.1 INTRODUCTION AND BACKGROUND TO THE STUDY

Some level of mathematics is part of the entry requirements for a BCom degree at most universities in South Africa. This requirement results in many school leavers being excluded from BCom studies based, solely on the fact that they do not meet the Mathematics entry criterion.

This study has undertaken an investigation into the mathematical requirements as stipulated by various universities, in order to be able to study towards a BCom degree. The researcher is attached to the Nelson Mandela Metropolitan University (previously the University of Port Elizabeth), and therefore has a keen interest in the relationship between candidates' mathematical skills prior to entering university and the actual mathematics required in the degree programme. This constituted the qualitative component of the research project conducted at this particular institution.

Up to 2007, matriculants in government schools in South Africa did the Senior Certificate. Learners doing the Senior Certificate could choose whether or not they wanted to do Mathematics as a learning subject in grades 10 to 12. The learners that chose to do Mathematics could choose to do it on either Higher Grade or Standard Grade. From 2006 a new school curriculum was phased in, called the National Senior Certificate (NSC). All Grade 10 to Grade 12 learners are now required to study either Mathematics or Mathematical Literacy.

The purpose of the "new" Mathematics in the NSC is to equip the learner with a functional knowledge of Mathematics in order to make sense of society, provide a suitable range of mathematical skills and knowledge to appreciate the discipline itself, and also to ensure access to extended studies in the mathematical sciences (National Curriculum Statement Grades 10 – 12 (General), Mathematics 2003). The purpose of Mathematical Literacy is to

equip learners with an understanding of the numerical and spatial information communicated in tables, graphs, diagrams and texts, as well as to develop the use of basic mathematical skills to critically analyse situations and solve everyday problems creatively (National Curriculum Statement Grades 10 – 12 (General), Mathematical Literacy 2003). Thus both the NSC Mathematics and Mathematical Literacy aim for a better conceptual understanding of the mathematical topics and skills needed to solve mathematically related problems.

As South African universities generally adhere to their own ideas of what constitutes mathematical preparedness to successfully study towards a higher education degree (in this case a BCom degree), they are now setting new entry requirements based on Mathematics and Mathematical Literacy in the NSC. It is thus important to determine whether the mathematical concepts included in the BCom degree are covered by Mathematics and Mathematical Literacy in the NSC. It is important to mention that Mathematical Literacy is not similar to Mathematics Standard Grade (a choice up to 2005).

In the light of this brief exposition, it now becomes clear that it was vital to undertake an in-depth study of the mathematical requirements needed for studying towards a BCom degree, as well as to explore to what extent the individual subjects, Mathematics and Mathematical Literacy, prepare the learners adequately for higher education courses.

1.2 DEFINITION OF TERMS

For the purpose of this study it is necessary that the researcher define those terms that will be used, so that the reader will have a clear understanding of what is being referred to.

Basic numeracy

Basic numeracy is an understanding of the Real Number system, including the ability to place numbers on the number line in terms of “bigger than” and “smaller than” other given numbers; the ability to perform basic

operations $(+;-;\times;\div)$ on whole numbers, decimal numbers and fractions, and also the ability to convert decimals to fractions and vice versa, all to be done without the use of a calculator.

Matriculants

Learners who are completing their final year, Grade 12, at South African schools are referred to as matriculants.

Senior Certificate

The Senior Certificate was the qualification that a learner attained after having successfully completed the Grade 10 to 12 school curriculum up to 2007. The phasing out of the Senior Certificate started in 2006.

Mathematics Higher Grade and Standard Grade

In the Senior Certificate, Mathematics was an optional subject for learners. Learners who chose to take Mathematics as a subject could choose between Higher Grade and Standard Grade, where a learning course offered on Higher Grade aimed for a deeper conceptual understanding, while Standard Grade focused more on procedural knowledge.

National Senior Certificate (NSC)

The National Senior Certificate (NSC) replaced the Senior Certificate at South African schools for Grades 10 to 12. The NSC was phased in from 2006, when it was introduced to learners in Grade 10. The first cohort of matriculants will complete the NSC at the end of 2008.

Mathematics in the NSC

From 2006, learners in South African Schools could choose whether to study Mathematics. The purpose of Mathematics in the NSC is to equip the learner with a functional knowledge of Mathematics in order to make sense of society, provide a suitable range of mathematical skills, and the necessary knowledge to appreciate the discipline itself, as well as to ensure access to extended studies in the mathematical sciences.

Mathematical Literacy in the NSC

From 2006, learners in South African schools were required to study Mathematical Literacy if they did not choose to do Mathematics. The purpose of Mathematical Literacy is to equip learners with an understanding of the numerical and spatial information communicated in tables, graphs, diagrams and texts, as well as to develop the use of basic mathematical skills to critically analyse situations and to be able to solve everyday problems creatively.

The Nelson Mandela Metropolitan University (NMMU)

The NMMU is a university located in Port Elizabeth in South Africa. NMMU opened its doors on 1 January 2005, subsequent to the merger between the University of Port Elizabeth (UPE), the Port Elizabeth Technikon (PET) and the Port Elizabeth campus of Vista University.

BCom degree

A business degree offered at South African universities.

Core subjects in the BCom degree programme

The subjects: Accounting, Business Management and Economics form the core of the BCom degrees at NMMU.

1.3 THE RESEARCH PROBLEM

The purpose of this study was to identify the mathematical requirements of BCom students at NMMU. In order to do this, it was decided to determine the mathematical concepts which are included as curriculum items in the core subjects in the NMMU (previously UPE) BCom degree programme.

The objectives of this study were to:

- Identify the mathematical concepts which are essential for BCom students to be able to cope with the mathematical content included in the core subjects in their BCom degree programme, (excluding BCom programmes majoring in Mathematics, Statistics and Computer Science and Information Systems).

- Determine whether Mathematics and Mathematical Literacy in the NSC would adequately prepare students for the mathematical concepts included in the BCom studies.

1.4 THE SIGNIFICANCE OF THE STUDY

In order to ensure that no students were excluded from BCom studies based on their high school mathematics mark, when in fact they should have been accepted, the mathematical concepts included in the BCom degree programme need to be determined. An additional reason for determining the mathematical concepts included in the BCom degree programme was to ensure that all students accepted for the BCom degree are equipped with enough mathematical knowledge to cope with the mathematical content in their BCom degree programme.

With the Senior Certificate a large component of school leavers have been excluded from the BCom studies, since Grade 12 mathematics for most BCom degrees at South African Universities was a prerequisite, although the exact mark needed differed from institution to institution. Only a small percentage of matriculants complied with this requirement.

In 2006 only 60% of the learners writing Grade 12 Senior Certificate examinations took Mathematics (Higher grade or Standard grade). Only 49% of the learners writing the Standard grade paper passed, while 70.5% of the learners writing the Higher grade paper passed, and in total 52.2% of learners who chose to take Mathematics on either Higher or Standard grade passed (Subject Results 2006).

Information about admission requirements for BCom studies at different universities was obtained by consulting the universities' web pages. A summary of the Grade 12 mathematics (Senior Certificate) required marks for different universities in South Africa is shown in Table 1.1. The discrepancies are evident. The University of Witwatersrand seems to have the strictest entrance requirements, with 50% Higher Grade or 60% Standard Grade, needed for the Grade 12 Mathematics (Senior Certificate) mark, while the

Universities of South Africa, Free State and the North do not have very strict entry requirements at all.

Table 1.1 Mathematics prerequisites for a Commerce degree with Senior Certificate

Excluding Degrees majoring in Accounting (CA) or Mathematical Sciences. (Information collected in 2000)

University	Mathematics Entry Requirements	Web Site
Fort Hare University	40% SG	www.ufh.ac.za
Potchefstroom University	40% HG or 50% SG	www.puk.ac.za
Rand Afrikaans University	40% HG or 40% SG	www.rau.ac.za
Rhodes University	30% HG or 40% SG	www.ru.ac.za
University of Cape Town	40% HG or 60% SG	www.uct.ac.za
University of Durban-Westville	50% SG or HG	www.udw.ac.za
University of Natal	BCom 40% HG or 60% SG, BBusSc 40% HG or 70% SG	www.unp.ac.za
University of Port Elizabeth	40% HG or 60% SG	www.upe.ac.za
University of Pretoria	40% HG or 50% SG	www.up.ac.za
University of South Africa	No specific requirements	www.unisa.ac.za
University of Stellenbosch	40% HG or 50% SG	www.sun.ac.za
University of Witwatersrand	50% HG or 60% SG	www.wits.ac.za
University of the Free State	SG Pass	www.uovs.ac.za
University of the North	Pass in Grade 12 Mathematics or Grade 12 Commercial subjects (Accounting, Economics, Business Economics)	www.unorth.ac.za
Vista University	No specific requirements	www.vista.ac.za

Grade 12 Mathematics (Senior Certificate) results are probably used by most universities, since no other, more accurate, criterion of success is available (*refer to section 2.3*). This critical value in Grade 12 Mathematics for success in general may be linked to the skills obtained through mathematics, rather than the specific content. A possible reason for maintaining Grade 12 Mathematics (Senior Certificate) as a prerequisite might be the maturity of

mathematical reasoning that can hopefully be expected from a person who has passed Grade 12 Mathematics (Senior Certificate). Doubt exists about whether a person who passed Grade 12 Mathematics (Senior Certificate) does actually possess these skills and competencies.

In 2009 the first matriculants who have passed the National Senior Certificate (NSC) will be entering universities. For the NSC, either Mathematics or Mathematical Literacy will be compulsory. It needs to be determined whether these two learning courses will prepare learners adequately to cope with the mathematical concepts in their BCom degree programmes.

Table 1.2 Mathematics prerequisites for a Commerce degree with NSC

Excluding Degrees majoring in Accounting (CA) or Mathematical Sciences (Information collected in September 2008)

University	Mathematics Entry Requirements	Web Site
Nelson Mandela Metropolitan University	Mathematics: 50 – 59% Mathematical Literacy: 70 – 79%	www.nmmu.ac.za
Rhodes University	Mathematics: 50 – 59% Mathematical Literacy: 60 – 69%	www.ru.ac.za
University of Johannesburg	Mathematics: 40 – 49% Mathematical Literacy: Not accepted For extended curriculum: Mathematics: 30 – 39% Mathematical Literacy: 60 – 69%	www.uj.ac.za
University of Pretoria	Mathematics: 40 – 49% Mathematical Literacy: Not accepted	www.up.ac.za
University of Stellenbosch	Mathematics 40 – 49% Mathematical Literacy: Not accepted	www.sun.ac.za
University of Western Cape	Mathematics: 40 – 49% Mathematical Literacy: 50 – 59%	www.uwc.ac.za
University of Witwatersrand	Mathematics: 50 – 59% Mathematical Literacy: Not accepted	www.wits.ac.za

Universities have set their entry requirements based on the NSC. A comparative table (Table 1.2) could be compiled based on information

available on websites. At the time when the information was collected, these were the only universities that had the requisite information available.

In comparing Table 1.1 and Table 1.2, it is clear that some of the universities have changed their names. This is due to a process that started in 2003 to merge some of the higher education institutions in South Africa. Nelson Mandela Metropolitan University is the result of the merger between the University of Port Elizabeth, the Port Elizabeth Technikon and the Port Elizabeth campus of Vista University. The University of Johannesburg is the result of the merger between the Rand Afrikaans University and Technikon Witwatersrand.

From Table 1.2 it is clear that some universities are willing to accept learners with Mathematical Literacy, while other universities will only accept learners with Mathematics. It is thus important to timeously determine whether there are shortcomings in either the Mathematics or the Mathematical Literacy curricula, in which case matriculants will need to be introduced to a bridging or introductory course to ensure a higher success rate.

1.5 DEMARCATION OF THE STUDY

For the purpose of this research an in-depth study of the BCom degree programme at NMMU was undertaken. In addition, only the South African school mathematics curriculum was considered.

Even though factors influencing success at higher education have been highlighted (*refer to section 2.4*), this research focuses on access to higher education and not success at higher education.

1.6 KEY ASSUMPTIONS

The first assumption

This study focused on the general BCom degree programmes, and not BCom degree programmes majoring in Mathematical Sciences (Mathematics, Statistics and Computer Science) or a degree in Chartered Accounting.

The second assumption

Only the first year of the BCom degree was considered, since this study focused on student preparedness when entering the BCom degree. Any modules teaching additional mathematical topics that the students might need for their further years of study should be included as part of the BCom curriculum itself.

The third assumption

The core subjects of all of the BCom degree programmes considered are Accounting, Business Management and Economics. At NMMU the content of the BCom degree programme has not changed much as it was previously offered at UPE before the merger. Hence the merger did not have any influence on this study.

The fourth assumption

This study focused on student entry into university, and not on student success at university.

1.7 RESEARCH METHODOLOGY

1.7.1 This research followed a grounded theory approach, where a grounded theory is one that is inductively derived from the study. The grounded theory approach is a method of qualitative research that uses a systematic set of procedures that lead to the development of a grounded theory about a phenomenon. A grounded theory is discovered, developed and verified through systematic data collection and analysis (Strauss & Corbin,1990:23), Glaser (2002).

1.7.2 A questionnaire was sent to all Accounting, Economics and Business Management lecturers at NMMU in 2001. The purpose of the questionnaire was to determine which mathematical topics a student needs in order to be able to complete their subjects successfully.

1.7.3 Following the analysis of this questionnaire, it was decided to interview lecturers from the departments of Accounting, Economics and Business Management. Lecturers were asked to answer an open question, namely what mathematics they feel the students need to obtain success in their subjects.

1.7.4 Next it was decided to do a thorough analysis of the textbooks and study guides for first-year Accounting, Economics and Business Management. The purpose was to ensure that all the mathematical topics had been identified.

1.7.5 Finally the examination papers for Accounting, Economics and Business Management were analysed to ensure that the list of mathematical knowledge topics was complete.

1.7.6 Once the mathematical requirements of BCom were identified, this list was compared with the content of Mathematics and Mathematical Literacy in the NSC. (This is known as a comparative method.)

1.7.7 To ensure validity, the list of topics identified was discussed with the lecturers previously interviewed.

1.8 PLAN OF STUDY

1.8.1 Chapter One: Introduction and outline of the study

The first chapter introduced the research problem area, and stated the research objectives. The significance, as well as the demarcations and the key assumptions of the research, were discussed. Thereafter, the research methodology was clearly outlined.

1.8.2 Chapter Two: Literature review

In this chapter a comprehensive literature review was undertaken.

1.8.3 Chapter Three: Research methodology

This chapter described how the qualitative research was conducted and focuses on data collection, analysis, interpretation and the conclusions.

1.8.4 Chapter Four: Questionnaires and interviews

In this chapter the questionnaires sent to BCom lecturers and the interviews held with BCom lecturers were discussed. An initial list of mathematical topics is presented.

1.8.5 Chapter Five: Textbooks and study guides

The in-depth analysis that was made of the first-year Accounting, Business Management and Economics text books and study guides were discussed in this chapter. A refined list of mathematical topics was presented.

1.8.6 Chapter Six: Test and examination papers

An analysis of test and examination papers for first-year Accounting, Business Management and Economics was discussed in this chapter.

1.8.7 Chapter Seven: Outcomes, comparison with NSC and lecturers' validation

In this chapter, the list of mathematical topics identified in the previous chapters was presented in the form of learning outcomes. These outcomes are then compared with Mathematics and Mathematical Literacy in the NSC. In addition, the process of obtaining lecturers' validation for the results was discussed.

1.8.8 Chapter Eight: Conclusion

An overview of the findings and areas for future research is given.

CHAPTER TWO: LITERATURE REVIEW

2.1 INTRODUCTION

After having presented the research design in Chapter One, Chapter Two focuses on a literature review of the problem that was investigated.

In order to collect data for this chapter, the following search methods were used: Searching online databases available at NMMU, namely EBSCOhost and NEXUS, consulting online journals available at NMMU and searching via Google Scholar.

The university librarian of NMMU assisted the researcher to locate sources. Further contributions from her included showing me how to use the online databases the university has access to and a general overview of methods to use in order to maximise the search.

This chapter will be constructed under the headings:

- Increased access to higher education,
- School mathematics as a criterion for success in higher education,
- Mathematics Anxiety as an aggravating factor to success in higher education,
- Student preparedness for higher education,
- Numeracy in higher education,
- Similar studies,
- University-support programmes.

2.2 INCREASED ACCESS TO HIGHER EDUCATION

In recent decades there has been a global increase in the numbers of students admitted to higher education institutions. In Europe, the demand for access to higher education has been growing since World War II (Gellert, 1996; More, 1996; Sebková, 1996). In Israel, as well as in the United Kingdom, there has been a similar growth in the demand for higher education (Guri-Rosenblit, 1996; Halsey, 1993).

In more recent years, South Africa has had a significant increase in student numbers in higher education (Bundy, 2005; Herman, 1995; Jansen, 2004). Generally, universities have moved from education for the elite towards mass education (Clancy, 1996; Leitner, 1996).

Mass education needs to be distinguished from mass access to higher education. Mass education has resulted in a significant growth in the numbers of students admitted to universities, yet mass access of students to university could create problems that are not in the interests of either the students or the university (Herman, 1995). In countries like Spain, Israel and Austria there is basically open access to some of the universities (Guri-Rosenblit, 1996; Leitner, 1996; More, 1996).

This means that once a learner completes their high-school career successfully they would be able to enrol at a university. Leitner (1996) warns that open access to universities can lead to overcrowding and often results in high drop-out rates and prolonged periods of study.

Not only did the increase in demand for higher education result in an increase in student numbers, but there is a greater distinction between different students' profiles. Herman (1995) argues that in South Africa methods of selection have become very important, yet critical evaluation of them is needed since there are stresses between the issues of access, equity and quality.

This argument is supported by Boughey (2003). The South African Minister of Education, Naledi Pandor, is quoted in Sapa (2008) as stating that many first-time entrants into university are quite unfamiliar with university life, the processes they will need to follow and the demands that will be placed upon them; and thus they struggle to cope. She continues by stating that since university expansion in 1994 there has been a significant growth in first-generation students.

These students carry the hopes of their families and communities, yet they are required to enter a challenging environment with little or no orientation.

Universities in South Africa can no longer ignore the fact that students entering university do not all fit the same profile (Harper & Cross, 1999). Universities need to cater for a wide range of students, and need to be able to accommodate all students that are accepted by universities. A student that is accepted for university must stand a fair chance of succeeding at university.

Factors that influence the students' success will be discussed later, but the university needs to ensure that it is able to accommodate all students that have met the entry requirements. This argument is supported by Herman (1995) who stated that universities must change in the areas of teaching, curriculum and their approaches to the problems of students who come from educationally disadvantaged backgrounds.

A university's access policies need to be supported by all the faculties in such a way that they will be able to support all students meeting those access criteria. For this reason universities need to keep up to date with curriculum changes at school level. This is supported by Lourens and Smith (2003) who state that changes in the school curriculum will have an impact on higher education institutions in terms of enrolment planning (*refer to section 1.4*).

2.3 SCHOOL MATHEMATICS AS A CRITERION FOR SUCCESS IN HIGHER EDUCATION

Many quantitative studies have been done worldwide on those factors that influence students' success at university. Studies in Europe have shown that previous academic performance (high-school record) is the most significant predictor of university performance (Marques & Miranda, 1996; McKenzie & Schwietzer, 2001).

In South Africa this is supported by Lourens and Smith (2003) who argue that Grade 12 aggregate marks have a large influence on students' success at

Technikon Pretoria and that Grade 12 results are the only general measure of academic ability of first-time entering students. Studies at NMMU (Koch, 2002) have shown that Grade 12 marks are the best predictor of success of first-year BCom students.

More specifically, considering the mathematics done at school level, studies in the USA have shown that students who meet the mathematics requirements perform better in Economics than students who do not meet the prerequisite level of skills (O'Dea & Ring, 2008). Similarly, studies done in Canada show that calculus specifically, mastered at school affects the overall performance of students in an introductory Economics course (Anderson, Benjamin & Fuss, 1994).

Contrary to these studies, Siegfried and Fels (1979) and Williams, Waldauer and Duggal (1992) argue that mathematics preparation, and more specifically, calculus done at school, has no significant influence on performance in Economics at university level.

Research from Australia, USA, Malaysia and Hong Kong on students' performance in Accounting at university level reveals that school mathematics is a significant predictor of performance in first-year Accounting, (Auyeung & Sands, 1994; Eskew & Faley, 1988; Tho, 1994; Gul & Fong, 1993). Yet Bargate (1999) states that there is no correlation between the study of Grade 12 Mathematics and the passing of the first-year accounting programme at Natal Technikon (South Africa), and that success at tertiary level may be attributed to factors other than secondary school mathematics results.

Studies done at NMMU (Koch, 2002) and the University of the Western Cape (Jacobs, 2006) show that Grade 12 mathematics marks are good predictors of success in the first year of their BCom programme.

Cantwell, Archer and Bourke (2001) state that students entering university through non-traditional routes, who would not have been in university in other circumstances, have a performance at university that is only slightly lower

than students entering via traditional routes. It is rather the nature of the student that contributes to success.

Even though there are contradictions in the research literature, this researcher argues that performance in mathematics done at school must surely have some influence on success in BCom studies. It is assumed that mastering mathematics does give a learner a certain level of mathematical maturity that implies that such a learner should have strong analytical and problem-solving skills.

This is supported by Auyeung and Sands (1994) who stated that mathematical ability benefits accounting performance, probably because of the commonality in logic and analytical skills associated with performance in both subjects. In the case of South Africa, new research will be needed once students from the NSC have completed the first year of their studies (*refer to section 8.4.1*).

It will be interesting to see if there is a correlation between success at first-year level and the mathematics done at Grade 12 level. The links between first-year marks and both Mathematics and Mathematical Literacy will need to be investigated. Students will be entering a BCom degree at some universities with Mathematical Literacy instead of Mathematics. Will Mathematical Literacy have the same influence on success as Mathematics in the Senior Certificate?

2.4 MATHEMATICS ANXIETY AS AN AGGRAVATING FACTOR TO SUCCESS IN HIGHER EDUCATION

If the focus is specifically on studies at university that contain mathematical calculations, the influence of mathematical anxiety is a factor that cannot be ignored (Hembree, 1990). Joyce, Hassel, Montaño and Anes (2006) did a study in the UK and Spain and argue that there is evidence that suggests that students doing business studies have problems relating to mathematical anxiety.

In a similar study, Zanakis and Valenzi (1997) found that there are high levels of maths anxiety among business students doing a statistics course. Joyce et

al. (2006) define maths anxiety as, “Feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.”

They also state that maths anxiety is a barrier to skills development that would inevitably restrict academic performance. This implies that curriculum design, delivery and assessment need to be influenced by the fact that there are high levels of maths anxiety.

With the introduction of Mathematical Literacy in the school curriculum in South Africa it is hoped that learners who previously did not feel comfortable with the abstract nature of mathematics will now be doing Mathematical Literacy in Grades 10 to 12. Learners will be introduced to numbers and number operations in a real-world context. The ideal would be that this use of numbers in a real setting will assist learners to overcome their fear of mathematics.

The content of Mathematical Literacy is designed in such a way that students will have minimal contact with the abstract topics in the traditional mathematics curriculum, and focus instead on solving problems in a practical context. The success of Mathematical Literacy will depend on more than the content. Teaching and assessment will have a big influence on how the learners perceive the subject.

Baloglu and Koçak (2006) state that a student’s previous mathematical involvement has an influence on mathematical anxiety, since students who are more involved with mathematics courses show lower levels of mathematics anxiety.

There are many other factors that have an influence on success, for example low self-esteem, financial hardship, institutional integration, self-efficacy, teaching strategies, students’ motivation, students’ approach to studying, cultural expectations and a lack of preparedness (McKenzie & Schwietzer, 2001; Killen, 1994; Bennett, 2003; Byrne & Flood, 2005). **However, this**

specific study has demarcated access to higher education as the focus and therefore factors influencing success fall outside the ambit of the focus of this study (*refer to section 1.5*).

This is a possible topic for further research, seeing that the first national Grade 12 examinations will only be in November 2008; and therefore this can only be investigated after 2008. The cut-off date for this study is September 2008.

2.5 STUDENT PREPAREDNESS FOR HIGHER EDUCATION

Gransard (1996) suggested that many students entering Belgian universities with the required level of mathematics still struggle with university mathematics courses. She argued that the reason for this is that in school the focus is mainly on exams, and that the mathematics that is not used is quickly forgotten.

Hoyles, Newman and Noss (2001) agree that there is a gap between school and university mathematics in the UK and argue that students have not been adequately prepared. They highlight the fact that students lack fluency and reliability in numerical and algebraic manipulation and simplification. These areas are also highlighted by Heck and Van Gastel (2006), who argue that in recent years in the Netherlands, the mathematical abilities of incoming students have dropped significantly, and that many students are having problems making the transition from school to university.

Taylor and Morgan (1999) discuss a support programme that is offered to commencing engineering students in Australia. They state that students are less prepared than they were in previous years. This is evidenced by their lack of confidence in understanding algebraic manipulation and reduction of graphic skills. They argue that many students who had apparently mastered topics in the past were not able to recall or apply many of those topics.

Focussing specifically on BCom students, Pappanastos, Hall and Honan (2002) suggest that business students in the USA lack the basic mathematical skills needed to cope with business-related studies. They suggest that

students should be introduced to a refresher course in the relevant topics. It is important that the student should be able to see the relevance of a topic and that the relevance is grounded in the curriculum.

If this is not the case, the curriculum needs to be adjusted (Lonning, DeFranco & Weinland 1998). Lowe and Cook (2003) argue that the lack of preparation for higher education in the UK needs to be addressed by using different teaching and assessment styles and that new students' induction into university should be a process and not a singular event.

Goldsmith and Mark (1999) argued that in traditional mathematics education in the USA most of the time is spent on memorisation, rote learning and applications of facts and procedures. The approach should rather be on development of conceptual understanding and reasoning. If the learners are actively involved in building their own understanding, learning will be easier.

Hockman (2005) argues that the changes in quality and quantity of mathematics taught at secondary school in South Africa in the past decade have affected the students' mathematical readiness for tertiary studies. This being the case, the university mathematics departments should adapt to suit these changes. A possible topic for further research is investigating different tertiary mathematics departments and determining any efforts made to address this issue.

Lecturers at a number of South African Universities were interviewed in July and August 2008 and asked whether they felt that students entering university over the last number of years were less prepared than those who had entered higher education a few decades ago.

The universities included the Cape Peninsula University of Technology, the University of Kwazulu Natal, Rhodes University, the University of Johannesburg, the University of the Free State and NMMU.

Some of the interviews took place at the annual conference of the Association for Mathematics Education in South Africa (AMESA). The lecturers were unanimous in their agreement that students seemed much less prepared than in previous years. The lecturers from the University of Johannesburg and NMMU indicated that there was a greater demand for more and more students to complete a form of academic development before commencing their studies.

This view is supported by the South African Minister of Education, Naledi Pandor (Sapa 2008), who states that more and more South African universities will be moving to four-year, instead of three-year, degree programmes.

In the light of the comments above, the following question needs to be asked: If a student has met the minimum entry requirement (note that South African universities have different mathematical entry requirements for a BCom degree, *refer to section 1.4*), will the student then have the mathematical skills that are assumed to go with that minimum requirement? Even though there is little evidence of research into the students' serious lack of understanding of even the most basic mathematical skills needed for a course, (Pappanastos et al, 2002), it has become a serious concern amongst university staff.

Again, this is a point of concern at this stage, but it is beyond the ambit of this study.

Kendall and Williams (2004) state that at American colleges, a common student misconception is that meeting their high-school graduation requirements will prepare them for college. A possible reason why many students, meeting minimum mathematical requirements, still struggle with mathematical calculations required in their courses is suggested by Witten (2005).

He states that students do not naturally generalise what they have learnt to new situations. He refers specifically to science subjects, but this is true for

other disciplines as well. It could be argued that these students struggle to transfer the mathematical skills they have learned to the context in which these skills are applied. Transfer of learning refers to using ideas and knowledge learned in one context in another context (Evans 1999).

Smith (2002) argues that the failure to transfer knowledge could be due to the lack of knowledge in the field where the knowledge is applied. Unless close attention is paid to the secondary-school curriculum, together with what is expected of the students at university level, more and more students will not be able to cope with their university studies.

In the view of the above authors, it is thus evident that many feel that students meeting minimum requirements for entering universities still seem to struggle with sections of subjects where the mathematics that they have already had exposure to is used in an applied context. Universities can no longer depend on schools to prepare their learners for what they are teaching; rather it is up to the universities to adapt in order to accommodate students from diverse backgrounds.

Universities need to make the effort to determine what syllabus topics are covered in school and to what extent they cover what is actually required by higher education institutions.

2.6 NUMERACY IN HIGHER EDUCATION

There is concern from many countries about the level of numeracy at which students enter university (Chapman, 1999; Heck & Van Gastel, 2006; Joyce et al, 2006; Steen, 2000; Steen, 2001; Tariq, 2004; Wilson & MacGillivray, 2007). Terms like basic numeracy, numerical literacy, and quantitative literacy are all used to describe similar abilities of students. Wilson and MacGillivray (2007) describe numeracy as “an ‘at-home-ness’ with numbers and an ability to make use of the mathematical skills which enable an individual to cope with the mathematical demands of his everyday life.”

Feast, Kokkinn, Medlin and Frangiosa (1999) stated that being numerate is being able to situate, interpret, critique and use numbers in context.

For the purpose of this study, Basic Numeracy is defined as an understanding of the Real Number system, including the ability to place numbers on the number line in terms of “bigger than” and “smaller than” other given numbers; the ability to perform basic operations $(+;-;\times;\div)$ on whole numbers, decimal numbers and fractions, as well as the ability to convert decimals to fractions and vice versa, all without the use of a calculator.

These are skills that a student should have developed while at school in grades 1 to 7. If these skills were not practised on a continual basis, the student would not be able to use what knowledge had been learnt in these earlier grades.

Steen (2000) stated that in the USA high-school graduates are far below par as far as their quantitative skills are concerned; and they argue that there is little connection between the goals of quantitative literacy and what is taught in the traditional mathematics courses. This is supported by Howe (1999) who stated that successful completion of mathematics at college level is not evidence of a thorough understanding of elementary mathematics.

Tariq (2004) states that, in the past, performance in mathematics could be used as a measure of numerical ability, but that is not true anymore. Steen continues to state that only when logical thinking and numerical estimates are part of every course will graduates be numerate citizens. Yet Wilson and MacGillivray (2007) argue that a student will need to study further mathematics in order to master basic numeracy.

In another article, Steen (2001) states that looking at the importance of numbers in so many life contexts, it seems obvious that schools should focus more on numeracy, and that today’s students need both numeracy and

mathematics, where mathematics conveys the power of abstraction and numeracy conveys skills in practicality.

Since Basic Numeracy is highlighted by so many as an area where students are lacking, what are universities going to do to address this problem? Universities can no longer sit back and expect the schools to solve these problems; they need to put processes in place towards creating a practical solution. Strategies by some institutions to solve these problems will be discussed in section 2.6.

With the massification of higher education, more and more students from a diverse background are entering university. Since there are vast discrepancies across the whole spectrum of South African schools, in spite of there being one curriculum, universities cannot expect schools to prepare the students adequately for their success at higher education levels.

This research is specifically geared towards identifying what the gaps are between school education and higher education requirements with particular reference to the BCom degree programme at NMMU. One of the objectives of this study is to identify the mathematical concepts which are essential for BCom students to be able to cope with the mathematical content included in the core subjects in their BCom degree programme.

How then could the exact mathematics needs be determined, and how should the subject be presented so that it can be ensured that there is effective transfer to the practical context? What transpires in similar studies at some higher education institutions outside of South Africa, will be covered in the next section.

2.7 SIMILAR STUDIES

Two studies will be considered that aimed to determine mathematical prerequisites in selected courses. These studies were used as a starting block

in order to determine the research methodology followed in this research (*refer to chapter 3*). Both studies followed procedures that could be used in this research and are therefore valid for consideration.

2.7.1 Perceived Mathematical proficiencies required for First-Year study at the University of Southern Queensland

During March 1998, the Office of Preparatory and Continuing Studies (OPACS) at the University of Southern Queensland (USQ) released a report titled: Perceived Mathematical Proficiencies Required for First-Year Study at The University of Southern Queensland. This report was based on a survey that was done in 1997 amongst all academic staff at the university that lectured first-year courses.

The aim of the survey was to obtain the lecturers' opinions on the mathematical skills and content needed by students enrolling for their courses, to get the academics' perceptions of the skills the students possess, and to determine whether concerns about numeracy at USQ were widespread or isolated (Taylor, Galligan & Van Vuuren 1998). This study covered all faculties at the university.

Lecturers were asked to indicate the mathematical prerequisites for their courses. Valuable information was gathered from these lecturers' opinions, but the lecturers were not always aware of the content of the secondary school mathematics curriculum, and their opinions in regard to the prerequisites were sometimes based on incorrect information. Also, even though students had met the minimum requirements, this did not mean that they had no difficulties.

One cannot discard the lecturers' opinion, because these are the people who deal with the students. This implies that the lecturer of the subject should have a thorough knowledge of the areas where the students are lacking in certain skills. The report noted that students entering first year had a range of different abilities and levels of preparedness, and that these shortcomings become a major problem for the lecturers.

Overall, the biggest area of concern amongst the academics was the general numeracy skills that the students needed for university study. Taylor et al. (1998) highlighted the use of fractions, decimals, percentages and ratios, the ability to solve problems using a pencil and paper, the ability to use a calculator correctly, being able to interpret tables and graphs, and communicate in mathematical language information on the problem areas.

The following recommendations resulted from this study:

> "Academics require access to more information about commencing students' mathematical and general skills and knowledge and details of set prerequisites. OPACS should develop closer links with faculty staff so that they can work collaboratively to improve academic numeracy levels, in the first instance, and then develop strategies to address the problems of academic numeracy and academic literacy in a systematic way "(Taylor et al. 1998).

From this study it is evident that in order to determine the mathematics included in the BCom programme, the lecturers of the subjects need to be involved. Lecturers can give valuable input since they deal with the students on a regular basis, yet the amount of knowledge the lecturer has on the mathematics done at school could have an influence on the extent to which the lecturers' opinion could be used.

2.7.2 Numeracy demands in Graduate Diploma of Education programme at the University of Western Australia

Chapman (1999) described a project on the Graduate Diploma of Education programme at the University of Western Australia. There appeared to be a need for attention to students' numeracy skills in the context of particular units and of the broader programme; and since numeracy is increasingly emerging as an important issue linking quality education with the development of generic or transferable skills.

The aims of the project were to identify the numeracy demands of three units of the programme, to develop a framework for academic numeracy that can be used to identify and describe the numeracy demands of university courses, to devise and implement numeracy strategies appropriate to each of the three units, to evaluate the success of these strategies in enhancing student learning.

Initially, there was no clear framework for documenting the numeracy demands of the units, so the approach that was followed was to locate mathematical ideas and representation in texts and tasks, which was done by a research team. This was then verified by lecturers, tutors and students.

This project found that academic courses place considerable numeracy demands on university students. In addition, students did not necessarily have inadequate levels of numeracy, rather teachers felt that some students made better use than others of their mathematical knowledge and skills, to learn and achieve in what were seemingly non-mathematical tasks.

Even though the nature of the units that were considered for this project differed greatly with the current research, the methods used to determine the numeracy demands for each unit are helpful. The researchers in the project used the text and tasks included in the units to determine the numeracy demands of those units. This was then verified by the lecturers of the units.

These two studies indicate that in order to determine mathematical requirements in specific courses, valuable information can be gained from the lecturers of those courses, as well as from the text and tasks in the courses. This is supported by Witten (2005), who argues that an immediate requirement in designing a service course would be to determine the exact quantitative skills expected of the students in textbooks and lectures.
It is also important to verify the results with the relevant lecturers.

2.8 UNIVERSITY SUPPORT PROGRAMMES

Against the backdrop of what has emerged in the research undertaken for this literature review, it is clear that universities are faced with the imperative of designing and presenting support programmes to “at-risk” students, so as to enhance their abilities, knowledge and skills to achieve success at university. The purpose of this section, although outside the main ambit of this study, is to provide insight into the shortcomings identified, as well as how to address them in a support programme.

It will assist the researcher, once she gets to the stage of analysing the textbooks, on what the students are supposed to know and where the actual shortcomings “between the lines” lie (*refer to section 5.4*).

Once the exact mathematical requirements for BCom students at NMMU have been determined, they need to be presented in such a way that this will be useful for students and lecturers. This means that the support course needs to cover the required content, but attention to other factors, such as for example, skills of logic, skills of overcoming mathematics anxiety, and many other areas of importance need to be addressed as well.

To meet the mathematical requirements, students may need to be enrolled for an additional course to improve their mathematical skills, or before lectures commence they might be required to attend a workshop or refresher course.

Even though the purpose of this research is to identify the mathematical content included in the BCom degree programme, it is essential to mention here that there are more factors to developing a support module than content alone. In this section six innovative ways to teach mathematics as a support course, or the mathematics included in a course, will be discussed.

Once this research has determined the content of a support course, ideas highlighted in this section should be considered when developing the support module, (possibly as part of post-doctoral studies).

2.8.1 Preparation course in banking mathematics

Evans (1999) described in detail a preparation course in banking mathematics (BM) that was developed. This course was offered to employees of a major investment bank. Their aim was to contextualise what was done in the workplace in terms of academic mathematics, and in effect, probably challenge their position on academic mathematics. In the process of contextualisation they found that it would be necessary to challenge the traditional view of mathematics being decontextualised and abstract, and rather attempt to situate the mathematics in the environment in which it will be used in practice.

They found that in several ways the school mathematics course differs from the mathematics used in practice. Firstly, the terminology is different, and secondly, the standards of accuracy are distinctive. For example, in BM there is a tolerance of $25 allowed on large scale transfers. There are also certain well-known results that can be used to avoid calculations, for example the fact that a Treasury Bill will have a lower purchase price than a simple interest 'instrument' at the same rate of simple interest.

Further, in BM, graphs seem to be seen as representations of data rather than as a relationship between variables. An additional difference between academic mathematics and BM is the fact that in academic mathematics care is given to explain simple interest in detail before moving to compound interest, yet simple interest is relatively rare in practice.

In this course it has been shown how to use the differences and similarities between the academic and banking mathematics courses to enhance the development of the curriculum, and how with good research they are able to present a product that uses the practical context together with good teaching practices.

In developing this course valuable ways were found to use concepts in academic mathematics that could assist in the process of transfer, for example the concept of a function was used as a bridging concept. An attempt was made to develop a deeper mathematical understanding by setting

problems that might appear innocent in terms of BM, but were very significant in academic mathematics.

Evans argues that part of the success of the course was due to the fact that the students were already working in the industry in which they would be required to apply the information, so they were clear about the context in which they would be required to apply the knowledge.

In the case where the mathematical needs of BCom students at NMMU are addressed, the students will not necessarily know or understand the context in which the mathematics would be applied, so careful attention needs to be given to the development of the material students would use. A mathematics-based support course for Economics students at USQ was developed for students who lacked the basic mathematical skills needed in the Economics course.

This course was discussed in a paper presented at a Bridging Mathematics Network conference in Perth (Galligan & Taylor 2000).

2.8.2 A preparatory mathematics course for Economics

Galligan and Taylor (2000) stated that in Economics, students need certain basic mathematical skills, namely proficiency in arithmetic, the manipulation of equations and the interpretation of graphs. The students enrolling for the economics course were from diverse backgrounds. The Economics course had no mathematical prerequisite and had a high failure rate.

Experienced economics and mathematics support staff were of the opinion that the students struggled to understand the economics due to a lack of basic mathematical skills. They did note that the relationship between the completion of school mathematics and success in economics was not consistent.

In 1993 a pilot programme was launched at USQ in consultation with the economics staff. This programme consisted of an economics-based

mathematics workshop and voluntary on-campus support for students offered on a one-to-one basis. The objectives of the programme included providing students with knowledge of their mathematical readiness for their studies; providing an opportunity for students to refresh their mathematical knowledge needed for economics; assisting students with the topics in economics where mathematical understanding was required and thereby removing the fear of mathematics.

The programme was offered in the form of a six-hour workshop before the start of the semester. All economics students wrote a readiness test in the economics class and the relevant mathematics modules, and one-to-one support was made available to the students. In order to ensure the continued success and survival of the programme, Galligan and Taylor stated that continued evaluation was essential.

Following the success of the pilot programme, the support programme was started in 1998. It consisted of a mathematical readiness test for the economics students, followed by a general preparatory mathematics course for non-degree purposes. The programme was evaluated by staff and participants, and recommendations were made in order to improve the programme in the future.

These recommendations included an analysis of the content of economics in order to determine the prerequisite concepts, principles and skills; a way to determine students' skill levels and to identify at-risk students; the development of appropriate material and workshops; the need for follow-up academic support for students; an evaluation of the course, as well as review and further development.

2.8.3 Mathematics included in an engineering course

Otung (2001) describes an engineering course offered at a university in the UK that has undergone significant changes over time. Since there has been a decline in the number of students with the required skills and motivation in mathematics, and generally a decline in the mathematical ability of students

over the years, mathematics became the gatekeeper that prevented many students from studying engineering.

Otung states that even though there were mathematics centres available to the students, that there were too many problems associated with a belated attempt to impart mathematical skills that should have been inculcated in the student before entering university.

Otung suggests a minimal mathematics approach in the first year of study. Students are most at risk during their first year of study; hence it becomes essential to introduce students to the mathematics in the engineering course in such a way that the mathematics in the course would not become the reason for failing. The levels of mathematics could then be increased in later years when the students were more comfortable.

2.8.4 Statistics for non-statistics students

Similar to that described by Otung (2001), Freeman, Smith, Staniforth and Collier (2006) suggest that innovative ways to teach statistics to non-statistics students are needed in order to overcome the problem of students disliking the subject before starting it. Since it is common for students to dislike and under-perform in modules involving mathematics, numeracy or statistics, a redesign is called for.

2.8.5 A developmental mathematics course for students studying at a distance

Taylor and Mohr (2001) describe a developmental mathematics course which is part of the Tertiary Preparation Program (TPP) at USQ. It is offered to distance education students. TPP is a programme for students who want to enter tertiary studies, but who might not qualify for entry. What makes this course different from many other bridging type courses is that the course content is more than just mathematical topics essential for tertiary studies; it also includes topics dealing with students' attitudes towards mathematics and mathematics study skills.

This course is offered to distance students, many of who have forms of mathematics anxiety and a poor mathematics self-concept. Apart from the printed course, there were many other support strategies in place. It was found that students' confidence in everyday mathematics increased. Taylor and Mohr highlight previously developed strategies for teaching numeracy students, namely:

- Acknowledge and discuss student feelings about maths.
- Use activities that provide early experiences of success.
- Encourage risk taking.
- Value a range of approaches to problem solving.
- Include different teaching strategies.
- Teach in appropriate context.
- Help students develop their own strategies for overcoming math anxiety.
- Teach relevancy to adult students and use gender and culturally inclusive teaching strategies.
- Respond to the students' needs.
- Encourage learning through interaction and cooperation.
- Acknowledge differences between students in backgrounds and mathematical skills.
- Make learning an enjoyable experience.

2.8.6 Foundation mathematics

Taylor and Mander (2002) describe a foundation mathematics course developed, and they highlight some challenges facing the service mathematics course, namely:

- developing mathematical skills that are transferable to diverse disciplines,
- catering for uneven preparedness in a subject in which knowledge is hierarchical,
- developing self- management skills for success in university study, and
- addressing generic skills, especially those associated with problem solving and communication that are transferable to diverse disciplines.

The foundation course that they developed takes the form of a self-paced course with three entry points based on results in an entry test. The course has no formal lectures and is print-based study. The students have access to weekly problem-solving sessions and have consultation with lecturers.

2.8.7 Foundation Mathematics at UPE (later NMMU)

In an attempt to assist many applicants who do not satisfy the admission requirements, the University of Port Elizabeth (UPE) developed a Foundation Mathematics module in 1998. Departments at the University of Port Elizabeth were approached in 1998, and by means of a questionnaire, indicated which Mathematical topics they saw as essential for their subject.

The emphasis was mainly on computational skills. Students' needs when entering university and doing a mathematics course were also considered. This information was used in the development of the Foundation Mathematics module at UPE (later NMMU). This was used from 1999 to 2006 to prepare students enrolled for a one-year preparation course for Science and BCom degree studies. From 2007 NMMU moved away from a one-year preparation course towards an extended curriculum model.

The Foundation Mathematics module was adapted to be used for students in the extended curriculum.

In the development of the Foundation Mathematics course, the focus was on the content needed for different subjects, but presented in context. In addition to computational skills, skills such as problem solving, analyzing, and logical thinking were enhanced through the content. One problem with the Foundation Mathematics course was that the same module was presented to BCom and Science students.

The BCom and Science students were not in the same class, but all assessments were identical. Thus, to present the module in context was limited to the classroom and not extended to the assessment.

2.8.8 Mathematics support course for BCom students at NMMU

Various support programmes at universities have been discussed. Some of the programmes were intended for students who had direct access to university, and some were for students entering university through non-traditional routes. Whether the student was part of an enabling programme or gained direct access to university is not relevant in terms of the current study, yet the types of mathematical support offered to these students are relevant.

All the examples mentioned above have relevant ideas that could assist BCom students at NMMU who struggle with the mathematical content of their programme.

Evans (1999) suggested that the mathematics should be offered in context. This has also been highlighted as an important issue by Smith (2002), Witten (2005) and Lonning et al. (1998). This needs to be taken into account when designing a support course for BCom students at NMMU.

Galligan and Taylor (2000) emphasise that the lecturers from the course where support is required, need to be consulted when developing a support course. In addition to this, they do not only offer content knowledge, but attempt to remove the students' fear of mathematics. This is also shown in the distance education course described by Taylor and Mohr (2001), who also add mathematics study skills to the content of the course.

Otung (2001) suggests that in the course where students struggle with the mathematical content a minimal mathematics approach should be followed. Similarly, Freeman et al.(2006) state that innovative teaching strategies are needed in those courses where the mathematics is being used.

Taylor and Mander (2002) give a possible solution to the problem of students entering at different competency levels. They offer a course with different entry levels based on the students' performance in an entry test.

Once the mathematical requirements for BCom students at NMMU had been determined, they then needed to be formalised and presented in such a way that the student would be able to apply them to the context where needed.

There are students who are able to cope with the pace of first-year studies, and who are able to grasp the mathematics presented in the courses without much difficulty. There are also students who struggle with the mathematical concepts as used in a business context. Many students would benefit from being able to spend more time on the mathematical aspects of their courses without the pressure of finishing the academic content of the courses.

The only way this will be possible is to offer these students a course, or workshops, separate from the business content courses concentrating mainly on the mathematics that the students need, or the mathematics that would be beneficial to the students. It is very often not the degree of difficulty in Mathematics that is the problem, but the opportunity for the student to spend time in understanding the calculations.

Lack of understanding can result in the students rote learning procedures without a clear understanding of what is actually being done. For this course, or the workshops, to be truly beneficial for the students, they need to be relevant, and the examples need to be such that the students are able to transfer the mathematical knowledge to other subjects where it is also needed.

It will be essential to have close cooperation between the person presenting this course and the relevant first-year BCom lecturers. The first-year lecturers from the relevant BCom subjects need to have input in the content of the course and be part of the review process before the course is offered again. The evidence that such a support course is working will be in the students' performance in the mathematical components of the BCom subjects.

2.9 CONCLUSION

It is clear that in recent years, because of the massification of higher education, as well as the higher education policy making it possible for previously disadvantaged learners to study beyond school level, more and more students have entered higher education institutions. Universities are under pressure to adjust their entry requirements in order to ensure equity.

Yet the pressure to offer quality education is still there. Universities need to adapt the ways in which courses are offered to accommodate students from backgrounds with vastly different profiles.

There is evidence that the gap between high school and university is broadening and that universities can no longer assume that schools will produce learners that are ready for the challenges of university. Specifically, observations are made that students are not coping with the mathematics that they are encountering at university and there is evidence of decreasing levels of numerical abilities among students.

Even though the aim of this study is not to develop a support course for BCom students at NMMU, it needs to be emphasised that identifying the content of such a course would not be sufficient before offering the course. Thought will have to be given to the way the course is structured, lecturing styles, support tutorials and the methods of assessment.

Examples of support courses discussed in section 2.8 could give valuable input when designing such a support course.

In Chapter Three the qualitative research methodology used to collect data for this research will be discussed.

CHAPTER THREE: THE QUALITATIVE RESEARCH METHODOLOGY

3.1 INTRODUCTION

In order to investigate the research problem from a qualitative prospective, it was decided to make use of interviewing, questionnaire investigation, and document analysis in the form of textbook, test and examination analyses. These methods provided data that fitted into a grounded theory approach. In other words, the grounded theory was inductively derived from the study. The researcher's approach correlates with what several other researchers have depicted about grounded theory, namely that it uses a systematic set of procedures that lead to the development of a grounded theory about a phenomenon.

Thus a grounded theory is discovered, developed and verified through systematic data collection and analysis, (Giles, 2002; Glaser, 2002; Glaser & Strauss, 1967; Strauss & Corbin, 1990). Furthermore, it is a general method of comparative analysis, (Glaser & Strauss, 1967), where the constant comparative method of qualitative analysis is a method of joint coding and analysis with the aim to generate theory systematically. In grounded theory the constant comparative method is used jointly with theoretical sampling to collect new data.

The use of grounded theory procedures based on the constant comparative method results in the generation of a substantive or formal theory, (Glaser & Holton, 2004; Glaser & Strauss, 1967). Grounded theory is not rooted in the existing literature; the theory is rather grounded in the data themselves, (Giles 2002; Glaser & Strauss 1967; Strauss & Corbin, 1990).

Strauss and Corbin (1990) describe qualitative research as research that produces findings which have not been arrived at by statistical procedures or other means of quantification. The process of building theory implies that data

are interpreted by conceptualising, and the concepts form a theoretical rendition of reality.

3.2 THE ELEMENTS OF GROUNDED THEORY

The three elements of grounded theory are conceptual categories; their conceptual properties and the generalised relations between the categories and their properties, (Glaser & Strauss, 1967). Strauss & Corbin (1990) define a category as a classification of a concept, where this classification is discovered when concepts are compared with one another and appear to pertain to a similar phenomenon.

So, concepts are grouped together under a higher order, more abstract concept called a category.

Generating and developing categories, their properties and the generalised relationships among them constitute a process of joint collection, coding and analysis of the data. Grounded theory requires all three operations to be done together, as far as is possible, (Glaser & Strauss, 1967).

Data collection and data analysis are processes that must occur alternately because the analysis directs the sampling of data, (Strauss & Corbin, 1990). The data analysis for each case involved generating concepts through coding, where coding represents the process of breaking data down, conceptualising them and then putting them back together in new ways, (Strauss & Corbin, 1990). Glaser and Holton (2004) describe the basis of grounded theory as the conceptualisation of the data through coding.

The three major types of coding in grounded theory are open, axial and selective coding, (Strauss & Corbin, 1990). Open coding is the part of analysis that pertains specifically to the naming and categorising of phenomena through close examination of the data, (Strauss & Corbin, 1990). Axial coding is the set of procedures followed after open coding, to put the data back together in new ways by making connections between categories (Strauss &

Corbin, 1990). The process of integrating the categories in order to form a theoretical framework is called selective coding (Pandit, 1996).

Through selective coding the core category is selected and it is systematically related to all the other categories. The core category is the central phenomenon around which all other categories are integrated (Strauss & Corbin, 1990).

A grounded theory is complete when it is validated against the data, (Strauss & Corbin, 1990).

Grounded theory was chosen for this research since there was no hypothesis to test, rather something that needed to be discovered. At the start of the research it was not clear what the findings would be, so the research was open ended. Thus the process of joint collection, coding and analysis was ideal for this research.

3.3 THE PROCESS OF BUILDING GROUNDED THEORY ON THE MATHEMATICAL REQUIREMENTS FOR FIRST-YEAR BCOM STUDENTS

The four steps followed in this research were research design, data collection, data analysis and literature comparison. These steps were not sequential, as grounded theory is a process of joint collection, coding and the analysis of the data. The fact that these steps do not follow directly on each other gives the researcher the opportunity to collect, analyse and compare after each step in the research process, in order to determine whether additional data collection is still necessary.

3.3.1 The research design

As outlined in Chapter 1, the first objective of this research was to determine the mathematical requirements for first-year BCom students at NMMU. The sources of data were chosen based on theoretical sampling. Glaser and Strauss (1967) describe theoretical sampling as the process of data collection for generating theory whereby the analyst jointly collects, codes, analyses the data and decides what data to collect next in order to develop theory as it

emerges. Strauss and Corbin (1990) state that in a grounded theory study, sampling cannot be planned beforehand since sampling decisions will evolve during the research process.

Even though sampling is used it is not traditional form of sampling, as used in qualitative research, but rather it is sampling that is used when specifically doing grounded theory.

The first phase of the research was chosen to be a questionnaire sent to all Accounting, Economics and Business Management lecturers at NMMU. The purpose was to determine which mathematical topics a student requires in order to be able to complete his/her course successfully (*refer to section 4.2*).

It was decided that the experienced views of the lecturers would be more valuable than asking for retrospective views from senior students. The reason for this was that each student could speak only from his/her own experience, while the lecturers are able to give a more general view of what they have noticed regarding the courses they lecture.

A questionnaire was used since it is an effective tool to gather information from a large number of people. Doing interviews at this stage of the research would have been too time consuming. The feedback from these questionnaires was used to determine the further steps in the grounded theory procedure.

The second phase of the research included interviews with first-year lecturers from the departments of Accounting, Economics and Business Management (*refer to section 4.3*). The purpose of this selection was to see if the categories identified from the previous questionnaire could be extended or refined. The analysis of the questionnaires gave a very wide selection of mathematical topics. At this point in the research it was decided to focus only on the first year of the BCom degree programme, since this study focused on student preparedness when entering the BCom degree.

Modules teaching additional mathematical topics that the students needed for the further years of study should be included as part of the BCom curriculum. Thus the first-year lecturers in Accounting, Economics and Business Management were interviewed. It was decided that a face-to-face interview with each of the three lecturers would be done. The lecturers were asked an open question, namely what mathematics they felt the students needed to do in their course.

In contrast to the questionnaire, the lecturers were not provided with a list of mathematical topics from which to choose, they and had to rely on their own experiences gained from teaching the course.

After conducting the interviews, it was decided that the third phase of the research was to make a complete study of the first-year courses for Accounting, Economics and Business Management. This would be done in order to see how the mathematical topics are used in context (*refer to chapter 5*). This gave a clearer understanding of the mathematical topics students needed to be successful in their specific courses. No further questions were needed to gather additional information.

The fourth phase selected was to consider the examination papers the students wrote for these courses (*refer to Chapter 6*). This was undertaken to establish whether saturation level had been reached in the categories which had evolved during the first three phases, and which had been adapted and extended according to the new insights established by the researcher.

No new data were found from the examination papers, and the examination papers confirmed what was found in the first three phases. At this point theoretical saturation was reached, meaning that sufficient data had been found to answer the research question effectively, (Glaser & Strauss 1967).

In grounded theory the researcher is encouraged to have theoretical sensitivity and to develop it during the research. Theoretical sensitivity is a personal quality of the researcher which refers to an awareness of subtleties

in the meaning of data and the ability to recognise what is important in the data, (Strauss & Corbin 1990). The researcher originally did a BCom degree eventually majoring in Mathematics and Statistics. She is therefore well equipped to comply with the requirements of theoretical sensitivity.

3.3.2 Data collection and analysis

Since grounded theory is a method of joint data collection and analysis, these steps will be discussed in one section. Grounded theory encourages the use of different kinds of data in order to understand the categories, and develop their properties. These different views are referred to as slices of data, according to Glaser and Strauss (1967). The use of these multiple data sources enhances the validity and reliability of the study.

A questionnaire was sent to all Accounting, Economics and Business Management lecturers at NMMU in 2001. The purpose of the questionnaire was to determine which mathematical topics students needed to be able to complete their courses successfully (*refer to Addendum A*). Open coding was used to decide on the topics to include in the questionnaire, where open coding is that part of analysis that pertains specifically to the naming and categorising of phenomena through a close examination of the data, (Strauss & Corbin 1990).

The previous Grade 10 to Grade 12 School Mathematics syllabus was analysed in order to come up with a list of categories that seemed to cover most of the topics in the syllabus. The lecturers indicated which mathematical categories they saw as being essential, recommended or not considered to be important. This questionnaire was completed by six lecturers from Accounting, five lecturers from Economics and three lecturers from Business Management (*refer to section 4.2*).

After having analysed the questionnaire responses, it was decided to interview lecturers from the departments of Accounting, Economics and Business Management. The questionnaires were sent to all lecturers and not

just first-year lecturers. This resulted in a list of mathematical topics that covered the requirements for all the years of study.

In order to focus on students' entry to university, it was decided to focus only on the first year of the BCom degree programme. The interviews were held with first-year lecturers only, and they referred to the first-year courses that they lectured.

From each department a first year lecturer was interviewed face to face. The interview was conducted by the researcher. The lecturers were chosen because they had at least five years' lecturing experience. This was done in order to get an experienced view. The interviews were recorded and then transcribed. The interviews took place between 2004 and 2006. Lecturers were asked to answer an open question, namely what mathematics they felt the students needed to achieve success in their course.

The transcribed interviews were analysed and coded based on the same list of topics that had been used in the original questionnaire. Following on this analysis, the original categories were adapted (*refer to section 4.3*).

At this stage of the research it was clear that even though categories had been identified, there was not much correlation between the results from the questionnaire and those from the interviews. In order to further develop and adapt the categories, it was decided to do a thorough analysis of the textbooks and study guides for first-year Accounting, Economics and Business Management courses.

This analysis gave a very clear picture of the mathematical topics included in first-year Accounting, Economics and Business Management (*refer to chapter 5*).

In order to ensure that the list of topics could not be expanded or refined any further, the tests and examinations of first-year Accounting, Economics and Business Management courses were analysed. In all these subjects, the

assignments the students were required to do were very similar to the types of questions asked in the tests and examinations, hence the assignments as such were not considered (*refer to Chapter 6*).

3.3.3 The literature comparison

The final step in this phase of the research was to compare the emerging theory with the existing literature, and check for similarities and differences, (Pandit, 1996). (*Refer to section 8.3.*)

3.4 A COMPARISON BETWEEN THE MATHEMATICAL REQUIREMENTS FOR BCOM STUDENTS AND MATHEMATICS AND MATHEMATICAL LITERACY IN THE NSC

Once the mathematical requirements for Accounting, Business Management and Economics had been determined, these topics were compared with the content of Mathematics and Mathematical Literacy in the NSC (*refer to section 7.5*). The purpose of this was to reach the second research objective, namely to determine whether Mathematics and Mathematical Literacy in the NSC would adequately prepare students for those mathematical concepts included in BCom studies.

3.5 VERIFICATION: INTERVIEWS WITH LECTURERS

In order to verify the results of the research, the same lecturers that had been interviewed previously were interviewed again. The interviews were conducted by the researcher. The purpose of the interview was to determine whether the lecturers were in agreement with the findings of the research. This was done in order to ensure the reliability of the data.

The interviews were held in July 2008. During the interview, the lecturer was given the list of topics that had been identified; the researcher briefly discussed the findings and the lecturer had the opportunity to give feedback. The interviews were recorded and then transcribed (*refer to section 7.6*). Since a grounded theory is regarded as complete once it can be validated against the data, (Strauss & Corbin 1990), in the context of this study, this was achieved in this phase of the research.

The researcher was satisfied that she had used all the data she had collected during various phases of the research, had applied them each time to refine the emerging grounded theory (in this case the mathematical requirements of BCom students at NMMU), and that the theory had now reached saturation level and could not be refined any further in this study.

In the next chapter the focus will be on the questionnaire-based and the interview-based components of the qualitative research project undertaken for this study. The emphasis will be on providing an in-depth analysis and reflection of the data collected.

CHAPTER FOUR: THE QUESTIONNAIRE AND INTERVIEW RESULTS AND THEIR ANALYSIS

4.1 INTRODUCTION

The first step in collecting data to answer the research question was to send questionnaires to lecturers in the departments of Accounting, Business Management and Economics. (Read this section in conjunction with Section 3.3.2 where the methodology is explained.)

4.2 QUESTIONNAIRE

In 2001 questionnaires were sent to all 41 lecturers in the departments of Accounting (23 lecturers), Business Management (nine lecturers) and Economics (nine lecturers) at the University of Port Elizabeth. The purpose of the questionnaire was to determine which mathematical topics students needed to be able to complete their course successfully (*see Addendum A*).

The mathematical topics included in the questionnaire were topics included in the Senior Certificate Grade 10 to Grade 12 School Mathematics syllabus. The lecturers indicated which mathematical topics they saw as being essential, which ones they recommended, as well as those they did not consider as being important. Six lecturers from Accounting and five lecturers from Economics completed the questionnaire. The Business Management department sent in a collective response. The results are summarised in Table 4.1. The marked cells indicate the topics that the lecturers from the different departments felt were essential for their subjects.

Table 4.1 Summary of the results from the Questionnaire sent to departments in the Faculty of Economic Sciences

Mathematical Topic	Economics	Accounting	Business Management	School Syllabus 2003	NSC Mathematics 2006	NSC Mathematical Literacy 2006
Basic Trigonometry	✓			Grade 10	Grade 10	Grade 11
Applications of Trigonometry	✓			Grade 10	Grade 11	Not

Factorisation and Products of Algebraic Expressions	✓	✓	✓	Before Grade 10	Before Grade 10	Before Grade 10
Solutions of Linear and Quadratic Equations	✓			Grade 10	Grade 10	Grade 11
Systems of Equations	✓	✓	✓	Grade 11	Grade 10	Grade 11
Solving Linear Inequalities	✓			Grade 10	Grade 10	Not
Linear Programming	✓	✓	✓	Grade 11 HG only	Grade 11	Not
Solving Quadratic Inequalities	✓			Grade 11	Grade 11	Not
Exponents	✓			Before Grade 10	Before Grade 10	Before Grade 10
Logarithms	✓	✓		Grade 12	Grade 12	Not
Graphs of Exponential and Logarithmic Functions	✓			Not	Grade 10 (only exponential function)	Not
Differentiation	✓			Grade 12	Grade 12	Not
Differentiation - Advanced Applications	✓			Grade 12	Grade 12	Not
Graphs of Linear and Quadratic Functions	✓	✓		Grade 10	Grade 10	Grade 10
Absolute Values	✓		✓	Grade 11 HG only	Not	Not
Graphs of Absolute Value Functions	✓		✓	Grade 11 HG only	Not	Not
Interpreting of Graphs	✓	✓	✓	Grade 10	Grade 10	Grade 10
Formulas	✓	✓	✓	Grade 10	Grade 10	Grade 10
Percentages and Averages	✓	✓	✓	Before Grade 10	Before Grade 10	Before Grade 10
Ratio and Proportion	✓	✓	✓	Before Grade 10	Before Grade 10	Before Grade 10
Sequences and Series	✓	✓	✓	Grade 12	Grade 12	Not
Areas and Volume	✓	✓	✓	Before Grade 10	Before Grade 10	Before Grade 10

Most of the topics the lecturers indicated as being important were covered by the Grade 10 Mathematics syllabus in the Senior Certificate, but there were exceptions:

- Systems of Equations, from the Grade 11 syllabus (Senior Certificate);
- Linear Programming from the Grade 11 Higher Grade syllabus (Senior Certificate);
- Sequences and Series from the Grade 12 syllabus (Senior Certificate).

Systems of Equations are included in Grade 10 Mathematics in the NSC and in Grade 11 Mathematical Literacy, while both Linear Programming and Sequences and Series are covered in Mathematics in the NSC, but are not done in Mathematical Literacy.

The most interesting of the topics chosen by the lecturers as essential, was Linear Programming to which only students who took Grade 11 mathematics on the higher grade (Senior Certificate) would have been exposed. Most students admitted to a BCom degree programme at NMMU took Mathematics on Standard Grade only. In the NSC, Linear Programming is covered in Grade 11 of the Mathematics curriculum, but not done in Mathematical Literacy.

Linear Programming is a very useful tool to assist in solving practical problems, and it is fair to assume that a prospective BCom student would benefit from some knowledge on Linear Programming. Linear Programming is however not included as a topic in the required Mathematics service course for a BCom programme at NMMU (Parsons & Petersen 1999). This Mathematics service course has not been changed to date.

Another topic that all lecturers regarded as essential was the interpretation of Graphs. The school syllabus, however, includes only a small section on graph sketching. Particularly in Standard Grade Mathematics, the emphasis is usually on the skill of drawing the graph, and not on any interpretation of results. Yet, in a BCom degree a student would be expected to analyse and

interpret graphs in order to be able to find meaning in what they represent (*refer to sections 5.3.18 and 5.3.19*).

The questionnaire was sent to all the staff from the three departments and not only the first-year lecturers. After having analysed the results, the researcher came to the conclusion that the responses of first-year lecturers had to be separated from those of second and third-year lecturers. The reason was that some mathematical topics might only be needed in the second or third year of the BCom degree programme, but they could nevertheless be included in the degree programme for the first year. The best way to isolate the first-year lecturers' responses was to interview first-year lecturers from the departments.

In the next section the interview results will be presented discussed and analysed.

4.3 INTERVIEWS WITH FIRST-YEAR LECTURERS

It was decided that a face-to-face interview with the first-year lecturers from Accounting, Economics and Business Management would be done (*refer also to section 3.3.2*). The lecturers were asked an open-ended question, namely what types of mathematics they felt the students needed to do in their course. Contrary to the questionnaire, the lecturers were not provided with a list of mathematical topics from which to choose, and had to rely on their own experiences from teaching the course.

The reason for this approach was that lecturers do not always understand what a mathematical topic means, and they then respond by guessing whether a topic might be important or not. When the lecturer is asked what mathematics a student needs, they have an opportunity to describe the type of calculations required, rather than having to choose an area or topic into which it falls. In addition, the lecturer gets the opportunity to emphasise those areas where they feel the students are lacking.

4.3.1 A summary of the interview held with a Business Management first-year lecturer

A Business Management first-year lecturer was interviewed on 15 September 2004. The purpose of the interview was to get the lecturer's impression of what mathematical knowledge the students doing first-year Business Management needed. From the interview it followed that, as a lecturer, she perceived that students needed to be able to do the following mathematical calculations in first-year Business Management:

- **Calculating percentages and ratios**
- **Solving linear equations**
- **Using basic formulas**
- **Reading information from a graph.**

The lecturer indicated that because of time constraints they did not teach the students how to draw the break-even graphs; they only require them to read information from the graph.

4.3.2 A Summary of the interview held with a first-year accounting lecturer

A first-year Accounting lecturer was interviewed on 29 August 2005. The purpose of the interview was to get the lecturer's impression of what mathematical knowledge the students taking first-year Accounting needed.

From the interview it was clear that the lecturer felt that the most important factor in the student's success in first-year Accounting was their ability to do **elementary calculations without the use of a calculator.** Students needed to be able to do **simple additions, subtractions and multiplications** in a short period of time. The lecturer mentioned that even though the students could get the correct answers if they used their calculator, this was time consuming, and the students were not able to complete test and examination papers in the available time.

Students needed to have an in-depth understanding of **percentages** and be able to do complex calculations involving percentages. It was not enough to simply be able to find the percentage of an amount; students needed to be able to work back as well.

The lecturer added that some knowledge of **solving linear equations** would be a handy tool to use when certain values had to be calculated.

The lecturer expressed concern that students believed implicitly whatever value was displayed on the calculator, and did not seem to have the ability to judge whether it was a **reasonable answer** to the calculation.

It was clear from the interview that the lecturer was aware of all the mathematical skills the students needed, and that the lecturer felt very strongly about the importance of strong **numerical abilities**.

Hence the essential mathematics highlighted by the lecturer is:

- **basic numeracy,**
- **percentage calculations,**
- **evaluating answers,**
- **solving linear equations**

Note that evaluating answers is not really a mathematical topic, but a skill that should be incorporated into all areas of mathematics.

4.3.3 A summary of the interview held with an Economics first-year lecturer

An Economics first-year lecturer was interviewed on 5 April 2006. The purpose of the interview was to get the lecturer's impression of what mathematical knowledge the students taking first-year Economics needed.

From the interview, the lecturer seemed to have a good understanding of what mathematical concepts the students needed in their course. He perceived that students needed the following mathematical skills in first year:

- **Basic numerical skills**
- **Calculating percentages and ratios**
- **Understanding straight line graphs**
- **Using formulas**

The lecturer indicated that graphs form a very important part of Economics, and students should be **comfortable working with graphs**. The lecturer did mention that very little time is spent on explaining graphs, but that there is a section in the notes on it, and that it is assumed that students acquired the mathematical skills they would need from school. The lecturer noted that generally there are concerns with the students' level of mathematical understanding.

4.3.4 Combining information from the questionnaire with the interviews

After the interviews were analysed there seemed to be fewer topics that were highlighted by the first-year lecturers than those topics seen as being essential for the subjects by all lecturers that completed the questionnaires. In addition two important aspects arose from the interviews. Firstly, it seems that because of time constraints lecturers were removing some of the mathematical aspects from their courses, especially those referring to graph sketching. The lecturer assumed that since this was a topic done at school level that the student would be able to cope with the sections on graph sketching.

The second important observation was that the accounting lecturer indicated that an important skill for the students was to be able to estimate answers effectively. This in itself is not a mathematical topic, yet how much time is spent in school on developing this skill?

The question that arose at this stage of the research was whether only what the lecturers said in the interviews should be considered, or whether these topics should be combined with the topics from the questionnaire. To avoid missing topics that might be essential, it was decided to combine the topics

from the questionnaire with the topics from the interviews. These topics are listed in Table 4.2. In grounded theory the term “categories” is used most often, but since this research is considering mathematical topics, the term “topics” has replaced the term “categories” in this research.

Table 4.2 Summary of results from the Questionnaire combined with the interviews

Mathematical Topic	Questionnaires	Interviews
Basic Numeracy		✓
Ratio and Proportion	✓	✓
Percentages and Averages	✓	✓
Basic Trigonometry	✓	
Applications of Trigonometry	✓	
Factorisation and Products of Algebraic Expressions	✓	
Solutions of Linear Equations	✓	✓
Solutions of Quadratic Equations	✓	
Systems of Equations	✓	
Solving Linear Inequalities	✓	
Linear Programming	✓	
Solving Quadratic Inequalities	✓	
Exponents	✓	
Logarithms	✓	
Formulas	✓	✓
Graphs of Exponential and Logarithmic Functions	✓	
Differentiation	✓	
Differentiation - Advanced Applications	✓	
Graphs of Linear Functions	✓	✓
Graphs of Non-Linear Functions	✓	
Absolute Values	✓	
Graphs of Absolute Value Functions	✓	
Interpreting of Graphs	✓	✓
Sequences and Series	✓	
Areas and Volume	✓	

From Table 4.2 it is clear that there was little correlation between the information gathered from the questionnaire and that from the interviews. So it was decided that more data would be needed to answer the research question satisfactory.

The following chapter discusses the next step in the research, namely analysing textbooks and study guides used in the core subjects of the BCom programme.

CHAPTER FIVE: TEXTBOOKS AND STUDY GUIDES

5.1 INTRODUCTION

At NMMU students wanting to do a BCom degree have a choice of 11 programmes, each with a slightly different curriculum and different major subjects (Prospectus: Faculty of Business and Economic Sciences 2006:10). The aim of this study is to find the mathematical requirements of BCom students at NMMU, excluding students wanting to major in Mathematics, Statistics and Computer Science and Information Systems.

In addition to the data gathered, as described in Chapter 4, a detailed analysis was made of the course content the students will encounter, as described by Witten (2005:3). As the subjects: Business Management, Accounting and Economics form the core of all these programmes, attention was given to the first year of these courses (*refer to section 3.3.2*).

First year Business Management is divided into two semester modules with codes EB101 and EB102. The textbook used is Business Management, 2nd edition, written by S. Marx, D.C. Van Rooyen, J.K. Bosch, and H.J.J. Reynders, printed in 1998 by JL van Schaik in Pretoria. Each semester has its own study guide. The guides are written by JK Bosch, SM Van Eeden and M Tait, and the 2004 versions were used. They are titled: Introductory study guide Business Management Course code: EB101 and Introductory study guide Business Management Course code: EB102.

First year Accounting uses a textbook called Introductory Accounting for both semesters (R101 and R102). The textbook is written by L De Villiers, B Prinsloo and J Rowlands. The version printed in 2004 was used.

First-year Economics is divided into Microeconomics (EC101) in the first semester and Macroeconomics (EC102) in the second semester and the textbook Economics written by M Parkin is used. The 7th edition was considered.

For each of the subjects the textbooks (as well as study guides in the case of Business Management) were analysed. (A lecturing schedule was available for each of the courses, so only the chapters included in the curriculum were considered.) In the analysis of the books, each time a mathematical concept was used or referred to, it was noted and linked to the list of topics identified in the previous chapter.

5.2 SUMMARY OF THE ANALYSES

The list of topics identified in Chapter 4 was used as a basis for the analysis of the textbooks and study guides. Every time a mathematical calculation was observed, it was compared to the topics in the list to determine where it would fit in.

Table 5.1 contains a summary of the information that was gathered from the textbooks and the study guides of Business Management, Accounting and Economics. The marked cells indicate the topics that were used in the different subjects.

Table 5.1 Summary of the analyses of the first-year content of Economics, Accounting and Business Management

Mathematical Topics	Economics	Accounting	Business Management
Basic Numeracy	√	√	√
Ratio and Proportion	√	√	√
Percentages and Averages	√	√	√
Basic Trigonometry			
Applications of Trigonometry			
Factorisation and Products of Algebraic Expressions			
Solutions of Linear Equations	√		√
Solutions of Quadratic Equations			
Systems of Equations			
Solving Linear Inequalities			
Linear Programming			
Solving Quadratic Inequalities			
Exponents	√		√
Logarithms			
Formulas	√	√	√
Graphs of Exponential and Logarithmic Functions			
Differentiation			
Differentiation - Advanced Applications			
Graphs of Linear Functions	√		√

Non-Linear Graphs	√		√
Absolute Values			
Graphs of Absolute Value Functions			
Interpreting of Graphs	√		√
Sequences and Series			
Areas and Volume			

In addition to these topics it was found that Tables are also used in Business Management and Economics. A sample of examples where these topics are used in Accounting, Business Management and Economics, follows. This sample represents all the mathematical topics that were found to be used. It is not a complete list of all the examples, but does highlight how those mathematical topics are used in context.

5.3 EXAMPLES OF USES OF MATHEMATICAL TOPICS IN ACCOUNTING, BUSINESS MANAGEMENT AND ECONOMICS

Examples were chosen from the textbooks and study guides of first-year Accounting, Business Management and Economics that represented all the mathematical topics found in the documents. There is at least one example included of each of the topics identified, yet to avoid repetition all the examples were not given. These examples were thus selected to be an adequate illustration of the mathematical topic identified.

5.3.1 Basic numeracy

First-year BCom students will need **basic numerical skills**. When looking at the different textbooks, it becomes immediately evident that a BCom student will need to be very confident of their ability to **work with numbers** and not be intimidated by texts and examples that include numbers and calculations. The student will not be required to do any calculations without the use of a calculator, but will need a basic knowledge of numbers to **use the calculator correctly,** to **round their answers off** to the correct number of decimal places and to be able to **judge whether the answer obtained is a realistic answer** for the question.

It often happens that a student sees an answer on the calculator, and then writes it down without even thinking of the question this answer relates to, or whether it is a realistic answer.

George Polya is renowned among mathematicians for his ideas on problem solving, and his general Principles for Problem Solving (Polya 1957:5) are:

1. Understand the problem
2. Think of a plan
3. Carry out the plan
4. Look back

It is the fourth step that learners often disregard, resulting in learners not considering their answers and the feasibility of their answers. Smith (2001:34) states that using common sense about the answer you obtain is part of problem solving. A student should be able to consider an answer and decide whether it is reasonable in terms of the question asked. For example, if a student is asked to calculate the cost of a loaf of bread, an answer of R25 000 would not be reasonable, and similarly an answer of R0.03 is also not reasonable.

The ability to **estimate realistic answers** to problems is a skill that is developed as early as Grade 1, according to the Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics (2002:28), yet if it is not further developed as one's knowledge increases, the student will lose the ability to be able to estimate reasonable answers. The question needs to be considered that even though it is included in the curriculum, how much time is actually spent on this topic.

Consider the following extract form the EB102 textbook Marx et al. (1998:179). This is included in the book to give the reader an indication of how to put costs into perspective:

> "A product is sold for R50,00. The necessary expenditure (costs) of the product amounts to R35,00, but the actual sacrificed means of production have a value of R60,00. In other words, sacrifices = costs (R35,00) + waste (R25,00) = R60,00.

> Is this loss-making production? The answer is no, because the product yields a profit of R15,00 (R50,00 – R35,00). However, the profit of R15,00 is entirely cancelled by inefficient production methods. In the end, the enterprise concerned will suffer a loss of R10,00 (R50,00 – R60,00) on the relevant product, but this in itself is not sufficient reason to withdraw the product from the market. The solution may be the improvement of production methods and increased productivity."

The student will require a basic knowledge of **addition and subtraction**, as well as the ability to perform these operations. Without being comfortable with numbers, the student might find it difficult to follow this example. The student needs to be able to make sense of the numbers in the given context, and be able to work with the large amount of data that are provided. In order to do that, the student needs a level of logical mathematical reasoning that will ensure that they will not be intimidated by examples containing many calculations.

In the first chapter of the accounting textbook, students are introduced to the accounting equation. All transactions need to satisfy this equation:

> "Assets = Liabilities + owner's equity" (De Villiers, Prinsloo & Rowlands 2004:12)

Examples from De Villiers et al., (2004:12) of how students use information to show that a transaction will satisfy this equation are:

> Purchased office furniture and equipment for R2 500, paying R1 500 by cheque and receiving 3 months credit for the balance owing.
> Assets: + R2 500 (Office furniture and equipment)
> − R1 500 (Bank)
> Liabilities: + R1 000 (Accounts payable)
> Hence: Assets = Liabilities + owner's equity"

Students will need to understand the role of an **equals sign** in an equation and what it means for an equation to be satisfied. Students are introduced to equations from as early as Grade R level (Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics 2002:14), and work with equations throughout their school career. It will be essential that the student should be able to transfer the knowledge from the mathematics classroom to this new use of equations.

The students will need to be able to understand and apply statements like the following from De Villiers et al. (2004:15):

> "Debit entries must always:
> Increase asset values, or decrease liability values or, decrease owner's equity (capital)
>
> Credit entries must always:
> Decrease asset values, or increase liability values, or increase owner's equity (capital)"

These two statements are a summary of what the student has learned about assets, owner's equity and liabilities, together with the concepts of debit and credit. This information could be confusing to students who are not comfortable with **basic numerical concepts** like increasing, decreasing and balance, yet to the student that knows the theory very well, it should not be too difficult.

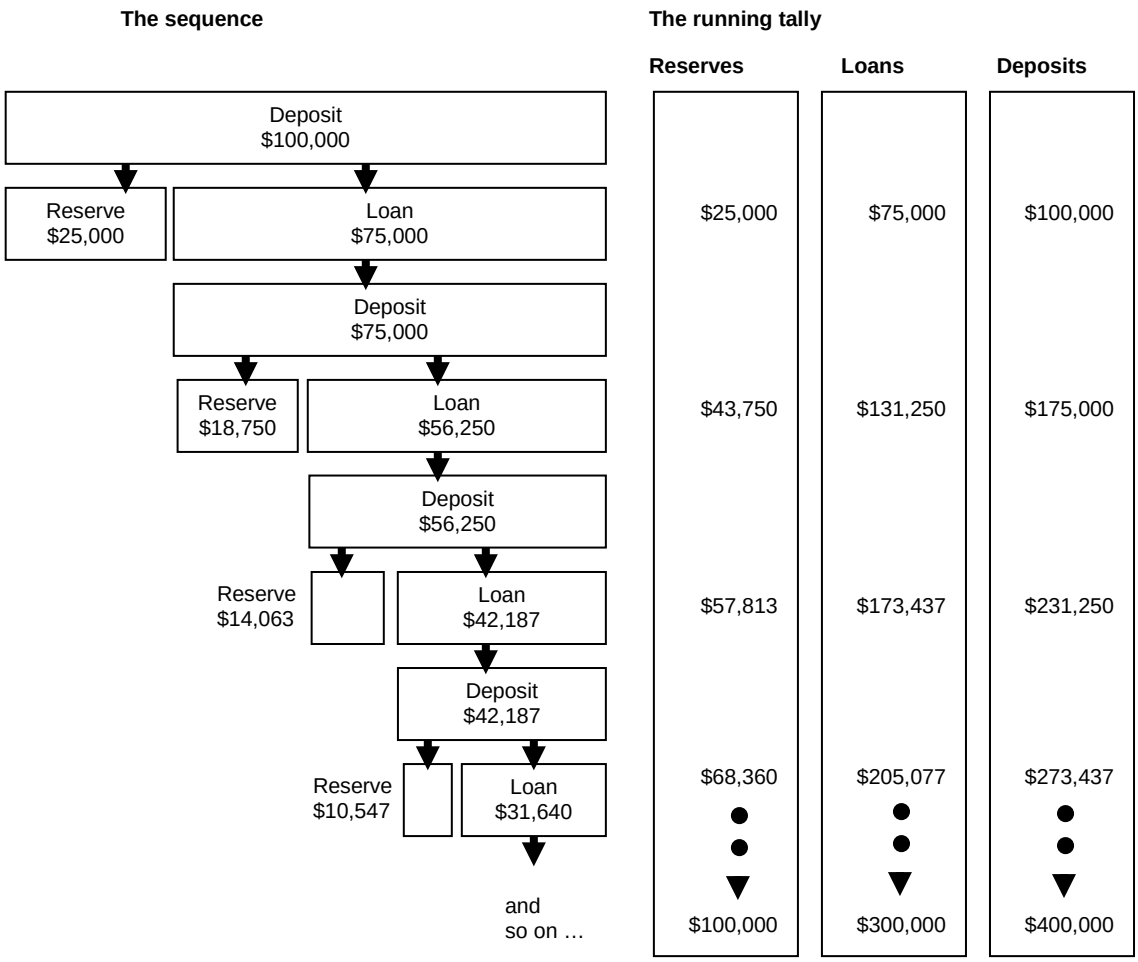
In Parkin (2005:614) there is a very detailed description of how the banks create money. Students need to be able to follow this example and not be intimidated by the **large number of calculations**.

> "The fraction of a bank's total deposits that are held in reserves is called the reserve ratio. The reserve ratio changes when a bank's customers make a deposit or withdrawal. Making a deposit increases the reserve ratio, and making a withdrawal decreases the reserve ratio.
>
> The required reserve ratio is the ratio of reserves to deposits that the banks are required, by regulation, to hold. A bank's required reserves are equal to its deposits multiplied by the required reserve ratio.
>
> Actual reserves minus required reserves are excess reserves. Whenever banks have excess reserves, they are able to create money. To see how, we'll look at a model banking system.
>
> In the model banking system that we'll study, the required reserve ration is 25 percent. That is, for each dollar deposited, the bank keeps 25c in reserves and lends the rest. Figure 26.2 is going to keep track of what is happening in the money-creation process, which begins when Art decides to decrease his currency holding and put $100,000 on deposit. Art's bank now has $100,000 of new deposits and $100,000 of additional reserves. With a required reserve ratio of 25 percent, the bank keeps $25,000 on reserve and lends $75,000 to Amy. Amy writes a check for $75,000 to buy a copy-shop franchise from Barb. At this point, Art's bank has a new deposit of $100,000, new loans of $75,000, and new reserves of $25,000. You can see this situation in Figure 26.2 as the first row of the running tally.

For Art's bank, that is the end of the story. But it is not the end of the story for the entire banking system. Barb deposits her check for $75,000 in another bank. Its deposits and reserves increase by $75,000. This bank puts 25 percent of its increase in deposits ($18,750) into reserve and lends $56,250 to Bob. Bob then writes a check to Carl to pay off a business loan. The current state of play is seen in Fig. 26.2. Now total reserves of the banking system have increased by $43,750 ($25,000 plus $18,750), total loans have increased by $131,250 ($75,000 plus $56,250), and total deposits have increased by $175,000 ($100,000 plus $75,000).

When Carl takes his check to the bank, its deposits and reserves increase by $56,250, $14,063 of which it keeps in reserve and $42,187 of which it lends. This process continues until there are no excess reserves in the banking system. But the process takes a lot of further steps. Figure 26.2 shows one additional step. It also shows the final tallies: Reserves increase by $100,000, loans increase by $300,000, and deposits increase by $400,000."

FIGURE 26.2 The Multiple Creation of Bank Deposits



The figure in the example above from Parkin (2005:614) gives the student a visual explanation to assist them in their understanding. The calculations are

limited to **addition, subtraction and multiplication**, yet the student who is not confident in working with **large volumes of numbers** can be intimidated by this example. If a student is not comfortable in doing these calculations, irrespective of the level of difficulty, it would be very difficult to come to grips with or understand this example. The student will therefore need basic numerical skills to be able to make sense of the numbers, as well as a level of logical mathematical reasoning which will give the student the confidence to deal with questions with many calculations without getting frightened by the numbers.

If the following extract below, from the EB101 textbook, (Marx, Van Rooyen, Bosch & Reynders 1998:53) is to have any value to the student, he needs an understanding of the **magnitude of the size of numbers**. The extract gives an example of an opportunity in the market environment.

> "From Beijing (Peoples Republic of China) it is reported that approximately 600 million Chinese do not brush their teeth, according to an official newspaper. China's population of 1,2 billion on average buy less that two tubes of toothpaste per person per annum. At least five tubes of toothpaste are required annually to maintain oral hygiene. This implies an additional annual market of 3,6 billion tubes of toothpaste."

A student must have the ability to comprehend big numbers. Students will know that a million is a large number, but does the fact that one million is 10^6 and one billion is 10^9 have much meaning when relating these figures to everyday events? Smith (2001:37) gives the examples:

> "If we were to count one number per second, non-stop, it would take us about 278 hours or approximately 11,5 days to count to a million. If you gave away $1 000 per day, it would take you more than 2 700 years to give away a billion dollars."

Thus, for the student to realise that there is an opportunity in the market, the relative size of 3.6 million tubes of toothpaste per year must have meaning. On 7 June 2005, a 100ml tube of Colgate Regular Toothpaste had a selling price at Pick 'n Pay supermarkets of R4.95. If 3.6 million tubes of toothpaste are to be sold in a year, the total sales would amount to R17.82 billion per year. Once students are able to put thought into the size of the numbers in the example, they would be able to appreciate what a good example it is.

Students are required to determine the cost of inventory on hand using a prescribed method. De Villiers et al. (2004:111) list the different methods as follows:

> "a) First-in-first-out (FIFO)
>
> b) Last-in-first-out (LIFO)
>
> c) Weighted average
>
> This method determines the cost of inventory on hand at the average cost of goods during the period. This method is used when inventory records are adjusted on a periodic basis and the average is computed only once at the end of the financial year.
>
> d) Moving average
>
> This method involves adding the cost of each purchase to the cost of units on hand and dividing the total cost by the total quantity on hand to find the average cost. A new average is computed after each purchase. This method is a modification of the weighted average method, and is used when perpetual inventory records are kept."

There are no difficult calculations involved, but the students need to keep track of what inventory was purchased at what price, and according to the given method, at what cost the inventory was sold. The students will need to be able to **think in a systematic way**, and be able to **organise given information** in such a way that it is easy to use. The last two methods require the **calculation of averages**. Calculating an average is covered in Grade 7, according to the Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics (2002:88).

The economics student will need to understand **what concepts like lower, higher, increase, decrease, minimum and maximum mean**. These terms are used repeatedly during the course and understanding is essential to be able to understand the theory that is being explained. This is clearly shown by the following two extracts from the textbook:

> "A price floor is a regulation that makes it illegal to trade at a price lower than a specified level. When a price floor is applied to labour markets, it is called a minimum wage. If a minimum wage is set below the equilibrium wage, the minimum wage has no effect. The minimum wage and market forces are not in conflict. If a minimum wage is set above the equilibrium wage, the minimum wage is in conflict with market forces and does have some effect on the labour market." Parkin (2005:128)

> "When demand and supply increase, the quantity increases and the price might increase, decrease or remain the same.

> When both demand and supply decrease, the quantity decreases and the price might increase, decrease, or remain the same.
> When demand decreases and supply increases, the price falls and the quantity might increase, decrease or remain the same.
> When demand increases and supply decreases, the price rises and the quantity might increase, decrease or remain the same." Parkin (2005:72)

This is an example where a calculator will not assist the student. General questions can be asked where the answer is simply "price increased". The student will need to have an **understanding of numbers**, and be able to relate the **sizes of numbers**. It will be essential for the student to know the theory in order to answer such questions.

These examples show that the first-year BCom student must be able to:

- understand the basic properties of numbers,
- perform basic operations on numbers,
- understand the magnitude or size of numbers.

In addition, generally the student must be able to:

- use a calculator effectively,
- round answers off to the correct number of decimal places,
- estimate if answers are realistic,
- think in a systematic way,
- deal with large volumes of calculations.

The topic "Basic numeracy" is included in the list. Now there is a clearer understanding of which aspects of numeracy are important in Accounting, Business Management and Economics.

5.3.2 Ratio and Proportion

According to Smith (2001:289), ratios are a way of comparing two numbers or quantities. An example of a ratio is 1: 3. This implies that the second number is 3 times the size of the first number. This ratio can also be written as a fraction, $\frac{1}{3}$.

Some properties of ratios:

- A ratio is in its simplest form if all numbers in the ratio are integers that are relatively prime numbers. For example: 12:18 = 2 : 3 and $1\frac{1}{2} : 4 = 3 : 8$. (Two numbers are relatively prime numbers if their greatest common divisor is 1.)
- The statement 'one in ten' can be written as a ratio 1 : 10.

Understanding ratios is an important part of EB101. An example of the possible disregard of consumer rights and dubious business ethics containing a ratio is from Marx et al. (1998:53):

> "South Africa is slightly behind Australia as the country with the highest incidence of skin cancer in the world. Not one local suntan lotion bears the mark of the South African Bureau of Standards (SABS). Every summer more than a million South Africans soak up the sun. At least 40 000 – one in 25 – will develop skin cancer, sometimes only ten or twenty years later. Malignant melanoma, the most dangerous skin cancer, is usually fatal. Nevertheless, not one South African manufacturer has applied for SABS approval since the bureau formulated its standards for sunscreen lotions in 1992."

Without any understanding of ratios, the student will not be able to see the impact of the 'one out of 25' ratio that is mentioned. Knowledge of ratios would improve the students' understanding of this example.

A very specific type of ratio which students would be required to understand is **exchange rates.** These are defined as the **rate (ratio)** at which the currency of one country can be exchanged for that of another (Marx et al., 1998:63). Students would not be required to do exchange rate calculations, but in order to be able to understand any examples containing exchange rates, a good understanding of ratios is needed.

The last chapter in the accounting textbook deals with ratio analysis. Students are required to **calculate ratios** based on given financial information. Some of the ratios from De Villiers et al. (2004:450) are:

"Gross margin – $\frac{\text{Gross profit}}{\text{Sales}}$

Net margin $= \frac{\text{Profit}}{\text{Sales}}$

Return on assets $= \frac{\text{Profit}}{\text{Total assets}}$

Return on equity $= \frac{\text{Profit}}{\text{Total equity}}$"

Calculating these ratios is very straightforward, as students **use a calculator**.

It is essential for accounting students to understand how to **work with ratios**. For example, when the students learn to account for partnerships, they will be faced with information, as in this extract from De Villiers et al. (2004:296):

> "Beam, Farlow and Krebbs have, for a number of years, operated a trading partnership in which they shared profits and losses equally.
>
> Towards the end of 20x1, the partners had agreed that Beam should assume sole responsibility for the routine administration of the business, and that from 1 January 20x2 the profit sharing ratio would be Beam 3/6, Farlow 2/6 and Krebbs 1/6. It was also agreed that the estimated fair market values of the assets should be taken into account when the new profit sharing arrangement came into effect, and on all future occasions when the profit-sharing ratio is changed."

Students must be able to **divide profits according to the ratio**, make the needed adjustments when the ratio changes, (e.g. a partner retires) and adjust capital accounts accordingly. This process can get very complicated, and there are many things to account for, as shown in De Villiers et al. (2004:298):

"WORKINGS: CAPITAL ADJUSTMENT CALCULATIONS

1. Change in profit ratio	Beam	Farlow	Krebbs
Old partnership	2/6	2/6	2/6
New partnership	3/6	2/6	1/6
Gain/(loss) in share of unrealised and hence future profits			
	+1/6	NIL	−1/6

Krebbs is entitled to be compensated for the 1/6 share of the EXISTING UNREALISED PROFITS which he is relinquishing to Beam i.e. 1/6 × (10 000 + 2 000) = 2 000

∴ Krebbs' capital account is credited and Beam's is debited.

2. Retirement of Krebbs

Change in profit ratio:	Beam	Farlow	Krebbs
Old partnership	3/6	2/6	1/6
New partnership	3/5	2/5	NIL
Ratios compared:			
Old partnership	15/30	10/30	1/6
New partnership	18/30	12/30	NIL
Gain/(loss) in share of unrealised and hence future profits			
	+3/30	+2/30	−5/30

Krebbs is entitled to be compensated for the remaining 1/6 (5/30) share of the existing unrealised profits which he relinquished to Beam (3/30) and Farlow (2/30) i.e. 5/30 ×(10 000 + 2 000) = 2 000.

∴ Krebbs' capital account is credited with R2 000, Beam's debited with R1 200 and Farlow's debited with R800."

The students will need to understand how to **use ratios**, but will also need an in-depth understanding of the accounting principles applied.

Economics students will need to understand **what ratios are**, and **how a change in one variable in a ratio can affect the other variable**.

Parkin (2005:33) defines the opportunity cost of an action as the highest-valued alternative foregone. The following explanation then follows in Parkin (2005:33):

> "Opportunity cost is a ratio. It is the decrease in the quantity produced of one good divided by the increase in the quantity produced of another good as we move along the production possibility frontier."

The use of the Production Possibility Frontier graph will help students to understand and explain these kinds of scenarios. By knowing the explanation of opportunity cost, a student should not have much trouble with the calculation. Since opportunity cost is calculated between two variables, the following statement indicates that students will need to **understand the concept of inverse**.

> "Because opportunity cost is a ratio, the opportunity cost of producing an additional CD is equal to the inverse of the opportunity cost of producing an additional pizza." Parkin (2005:34)

This concept is then further explained with an example which gives a clear explanation of what is meant by the inverse.

Consider the following example from Marx et al. (1998:209):

> "ABC Company is involved in the production of domestic irons. The main components of each iron are raw materials such as alloys and synthetic resin, and component parts like switches, thermostats, an electric cable and couplings.
>
> Using scientific methods, the standard manufacturing cost price per iron is made up as follows:
>
> - 3 kg of alloy (standard quantity)
> @ R8,00 (standard price) per kg = R24,00
> - Other variable costs (per unit) = R 6,00
> - Fixed costs per iron at normal capacity = R30,00
> - Standard manufacturing cost price = R60,00
> - Normal capacity = 1000 irons per month
> - Standard manufacturing time per iron = 1 hour
>
> The fixed costs consist of the salaries of full-time employees, the depreciation of the machines and the rental of the building. These costs are budgeted at R30 000 per month and are calculated by means of a labour hour tariff. The normal labour hours amount to 1000 per month. The labour time allowed per iron is one labour hour.
>
> The total standard manufacturing costs therefore amount to
> R60 000 (1000 × R60) per month."

The student must be able to do **conversions**, for example: Price per unit to total price, cost per hour to total cost, price per unit to total price.

In addition to understanding what ratios are, these examples show that the first-year BCom student must be able to:

- divide a number into a given ratio,
- explain how a change of one variable in a ratio will affect the other variables,
- solve elementary proportion questions,
- convert units.

The topic "Ratio and Proportion" is included on the list. Now there is a clearer understanding of which aspects of ratios and proportion are important in Accounting, Business Management and Economics.

5.3.3 Percentages and Averages

The mathematical topic that seems to be most used in EB101 is percentages. A percent, according to Smith (2001:868), is the ratio of a given number to 100. This means that the percent is the numerator of the fraction whose denominator is 100. Some properties of percentages are listed below:

- A percentage can be written as a fraction, for example: 25% = $\frac{25}{100} = \frac{1}{4}$
- $\frac{a}{100} = a\%$, the percentage symbol simply implies "divided by 100", it is a notation used to simplify the presentation of fractions with 100 as a denominator.
- To calculate a percentage of an amount, multiply the amount with the percentage in fraction form.
- $\frac{1}{100} = 1\%$, therefore the percentage symbol can be replaced by a denominator of 100 or vice versa.
- 1 is the multiplicative identity, so

  $a = a \times 1 = a \times \frac{100}{100} = (a \times 100) \times \frac{1}{100} = a \times 100\%$, hence to convert a number to a percentage it can be multiplied by 100%, and the value will not change.
- To convert a fraction to a percentage, multiply the fraction by 1, in the form $\frac{100}{100}$, or 100%:

  $$\frac{a}{b} = \frac{a}{b} \times \frac{100}{100} = \frac{a}{b} \times 100 \times (\frac{1}{100}) = \frac{a}{b} \times 100\%$$

Marx et al. (1998:62) define the gross domestic product (GDP) as the total value of finished goods and services produced in a given period, usually one year, within the borders of a country. The following example (Marx et al. 1998:62), deals directly with GDP.

> "South Africa's GDP has a low growth rate – too low to employ all the unemployed people. During the years 1990, 1991 and 1992 there was a decline of respectively -0,3%, -1,0% and -2,2% in the GDP. During 1993-95, 3,3% was recorded. The growth rate for 1996-97 was approximately more

than 6% to employ all its job-seekers. Agricultural production contributes a great deal to the growth in the GDP."

The example given above shows that a student needs to understand what the percentage symbol represents in order to fully understand the situations described in the examples.

During the Microeconomics course, students do not do many **percentage calculations**, yet the student will have to be totally aware of what a percentage is, and how to calculate it. The following extracts from the textbook clearly show the need for the understanding of percentages:

> "What we produce changes over time. Sixty years ago, almost 25 percent of Americans worked on farms. That number has shrunk to less than 3 percent today. Over the same period, the number of people who produce goods – in mining, construction, and manufacturing – has shrunk from 30 percent to 20 percent. The decrease in farming and manufacturing is reflected by an increase in services. Sixty years ago, 45 percent of the population produced services. Today almost 80 percent of working Americans have service jobs." Parkin (2005:3)

> "The 20 percent of people with the lowest incomes earn 4 percent of total income, while the richest 20 percent earn 49 percent of total income. So on average, people in the top-20 percent earn more than 12 times the income of those in the bottom 20 percent." Parkin (2005:4)

A basic **understanding of percentages** should be enough for the students to cope with the information that contains percentages.

EB101 students would be required to not only **understand what is meant by percentages**, but also to **do percentage calculations**. The first is to calculate the rate of return of an enterprise, where Marx et al. (1998:119) define it as:

$$\frac{\text{Profit}}{\text{Sales income}} \times \frac{\text{Sales income}}{\text{Total assets}} \times \frac{100}{1} = \text{Profit margin on sales} \times \text{total asset turnover}$$

An example from Marx et al. (1998:119) is:
(Note that the mistake in the example when calculating percentages will be discussed after the example.)

"Total assets employed during the past year	R4 000 000
Ordinary share capital (equity capital)	R3 000 000

Borrowed capital (debt)	R1 000 000
Interest paid on borrowed capital (10%)	R100 000
Sales for the past year	R10 000 000
Profit before interest and taxation (sales minus total costs, interest included)	R1 000 000

No dividends were paid for the past year.

(a) *Total rate of return of the enterprise*

$$\left[\frac{\text{Profit before interest and tax}}{\text{Sales income}} \times \frac{\text{Sales income}}{\text{Total assets}} \times \frac{100}{1}\right]$$

$$= \frac{1\,000\,000}{10\,000\,000} \times \frac{10\,000\,000}{4\,000\,000} \times \frac{100}{1}$$

= 25%

(b) *Rate of return on equity capital*

$$= \frac{\text{Profit before tax}}{\text{Shareholders' interest}} \times \frac{100}{1}$$

$$= \frac{900\,000}{3\,000\,000} \times \frac{100}{1}$$

= 30%”

The student will be required to do calculations similar to these, but these examples contain a slight mathematical error. The mathematical flaw in this calculation is that by multiplying the fraction by 100, you are changing the value. Hence the formula suggested is incorrect. When converting the ratio to a percentage, you are not changing the value; you are simply rewriting it in a different format. The formula should read as follows:

$$\frac{\text{Profit}}{\text{Sales income}} \times \frac{\text{Sales income}}{\text{Total assets}} \times 100\%$$

First-year accounting students need to be very comfortable with the concept of a percentage, and must be able to do **percentage calculations**. According to the Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics (2002:41), students are expected to start working with percentages at Grade 6 level.

The following statement from De Villiers et al. (2004:33) can be very confusing to the student who does not understand percentages:

> "If goods are marked up on cost, then on a percentage basis, the cost equals 100% and the selling price equals 100% plus the % mark-up."

The example that follows in De Villiers et al. (2004:33) further emphasises the need for an understanding of percentages.

> "Wingbat Wholesalers mark all their goods at cost plus 50%. Goods are sold for R600.
> Calculate the cost price of the goods.
> As the % mark-up is on cost price, then on a percentage basis:
> Cost price = 100% Gross Profit = 50% Selling price = 150% (R600)

Cost Price (CP)	100%		
+Profit (P)	50%	P ÷CP = 50% ($\frac{1}{2}$)	(Given)
=Selling price (SP)	150%	P ÷SP = $33\frac{1}{3}$% ($\frac{1}{3}$)	(Calculated)

$\therefore$ $\frac{1}{3}$ of SP = Profit

$\frac{1}{3}$ of R600 = R200

CP = SP – P = R600 – R200 = R400"

There are many occasions where the students will need to do percentage calculations. These are not limited to calculating the percentage of an amount, but very often they will involve **manipulating percentages,** as in the note that follows another example in De Villiers et al. (2004:34) shows.

> "A mark-up of 50% on cost gives the same result as a mark-up of $33\frac{1}{3}$% on selling price and a mark-up of 25% on selling price gives the same result as a mark-up of $33\frac{1}{3}$% on cost."

De Villiers et al. (2004:135) state that the term "depreciation" as it is used in accounting refers to the process whereby the acquisition cost of a non-current asset is allocated to the depreciation expense over the accounting periods making up the estimated useful lives of the assets. In doing **depreciation calculations**, students will be required to do a multiple of percentage calculations, as well as being able to follow from one calculation to the next without getting confused.

This implies that the student needs to be comfortable with repeated calculations, and be able to systematically complete the question in an organised way in order not to get confused.

The following example from De Villiers et al. (2004:137) illustrates what types of calculations a student must be able to perform given the relevant information.

"The west Cape Construction Company purchased a new machine on 1 January 20x0 for R22 500. The company estimates the machine to have a useful life of six years, and a salvage value at the end of six years of R1 500.

Required:

Prepare a depreciation schedule to show

(i) the net book value of the machine at the beginning of the year

(ii) the annual depreciation expense

for each year of the estimated six-year life under the following alternative depreciation methods:

- Straight line
- Sum-of-the-years-digits
- Reducing / diminishing balance at a rate of $33\frac{1}{3}\%$ p.a."

From the solution (De Villiers et al., 2004:138), it can be seen what a large amount of work is involved in doing the depreciation calculations.

"Depreciation Schedule

	Straight line		Sum-of-the-years-digits		Reducing balance	
Year	**Book value at beginning of year**	**Annual depreciation**	**Book value at beginning of year**	**Annual depreciation**	**Book value at beginning of year**	**Annual depreciation**
1	R22 500	R3 500	R22 500	R6 000	R22 500	R7 500
2	19 000	3 500	16 500	5 000	15 000	5 000
3	15 500	3 500	11 500	4 000	10 000	3 333
4	12 000	3 500	7 500	3 000	6 667	2 222
5	8 500	3 500	4 500	2 000	4 445	1 482
6	5 000	3 500	2 500	1 000	2 963	998
		R21 000		R21 000		R20 525

Depreciation calculations

Straight line = $\frac{22\,500 - 1500}{6}$ = R3 500 per annum

Sum-of-the-years-digits

Sum = $\frac{n(n+1)}{2} = \frac{6(7)}{2} = 21$

Amount to be depreciated = 22 500 – 1 500 = R21 000

Year	Proportion used to calculate depreciation	Annual depreciation
1	6/21 x 21 000 =	R6 000
2	5/21 x 21 000 =	5 000
3	4/21 x 21 000 =	4 000
4	3/21 x 21 000 =	3 000
5	2/21 x 21 000 =	2 000
6	1/21 x 21 000 =	1 000
		R21 000

Reducing balance

Year	Rate		Book Value	Annual depreciation
1	$33\frac{1}{3}\%$		R22 500	R7 500
2	$33\frac{1}{3}\%$	(22 500 - 7 500)	15 000	5 000
3	$33\frac{1}{3}\%$	(15 000 - 5 000)	10 000	3 333
4	$33\frac{1}{3}\%$	(10 000 - 3 333)	6 667	2 222
5	$33\frac{1}{3}\%$	(6 667 - 2 222)	4 445	1 482
6	$33\frac{1}{3}\%$	(4 445 - 1482)	2 963	998
				R20 525”

In the previous example all depreciation calculations were limited to assets purchased at the beginning of the accounting period. This is not always the case, so students must be able to calculate depreciation for assets that were purchased during the accounting period. There is no separate example for this case, but it forms part of a bigger example the students have to work through. The relevant part of the example from De Villiers et al. (2004:153) reads:

> “A new vehicle costing R12 000 was purchased on 1 July 20x4. The financial year ends on 31 December. Depreciation is provided for on the straight line basis at 20% per annum.”

In order to complete the journal entries correctly, the depreciation of this vehicle needs to be calculated:

$$\text{Depreciation at 31 December 20x4} = \text{R}12\,000 \times \frac{20}{100} \times \frac{6}{12} = \text{R}1\ 200$$

In this case during the first year, the asset was purchased with six months remaining in the accounting year. This can change from one example to the next, and the student needs to be able to do the correct calculations. It is only by repeated exercise that students will be confident in their ability to do depreciation calculations correctly.

When students start with company accounting, there are many new terms and concepts that they need to get used to. Dealing with shares is new for the students, so they need to understand what shares are, as well as to be able to grasp the calculations that are involved with shares. This can easily get very confusing if the students are not confident in their understanding of how shares are used, as shown if the following example from De Villiers et al. (2004:348).

> "A company has an issued capital of 50 000 9% cumulative preference shares of R1 each and 100 000 ordinary shares of R1 each and has been unable to declare or pay any dividends for three years (it has not earned enough profit or was short of cash). If dividends were to be declared and paid for year 4 the preference shareholders would have to receive R18 000 (50 000 x 9/100 x 4 years) before any ordinary dividend could be declared or paid."

The following example from De Villiers et al. (2004:367) shows how complex share calculations can get:

> "Redeemable preference shares issued at par redeemable at a premium:
>
> Rhebok limited issued 10 000 R2,7% redeemable preference shares at par on 1 January 20x2. The preference shares are redeemable on 31 December 20x4 at a premium of 10%.
>
> The substance of the transactions:
>
> | | |
> |---|---|
> | 1 January 20x2 | : Borrowed R20 000 |
> | 20x2 – 20x4 | : Paid interest on the loan |
> | 31 December 20x4 | : Repaid loan of R20 000 |
> | | Additional payment of R2 000 (R20 000 x 10%) |
>
> The additional payment is the cost of borrowing and should be accounted for as such. The effective rate of interest for this loan is therefore greater than 7%. Using a **financial calculator**:

PV	:	20 000
PMT	:	−1 400
FV	:	−22 000
n	:	3
COMP	:	i (answer 10.0205% p.a.)

Redemption schedule at 10.0205% p.a.

	Payment	Interest	Capital	Balance
01/01/20x2				20 000
31/12/20x2	1 400	2 004	(604)	20 604
31/12/20x3	1 400	2 065	(665)	21 269
31/12/20x4	1 400	2131	(731)	22 000
	22 000	–	22 000	NIL
	26 200	6 200	22 000”	

In this example the students are required to **use a financial calculator** to calculate the interest rate. Students should be able to use the financial calculator easily if they are instructed clearly. It simply requires entering the correct values in the correct places. Students need to use the interest rate they calculated to work out the redemption schedule. These are straight-forward **percentage calculations**, which should be easy for the students to do if they understand the theory.

In addition to understanding what percentages are, these examples show that the first-year BCom student must be able to:

- do percentage calculations to calculate cost, profit and other financial concepts,
- do depreciation calculations,
- use a financial calculator.

The topic “Percentages and Averages” is included in the list. Now there is a clearer understanding of which aspects of percentages and averages are important in Accounting, Business Management and Economics.

5.3.4 Basic trigonometry and applications of trigonometry

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.5 Factorisation and products of algebraic expressions

Refer to section 5.3.6, 5.3.14 and 5.3.18.

5.3.6 Solutions of linear equations

A linear expression is an expression in the form ax + b, where $a; b \in \square$, and a $\neq 0$. An equation is formed when two expressions are set equal to each other. A linear equation is an equation in which the highest power of the variable is 1, for example: ax + b = cx + d, or ax + b = 0.

Students will need to understand what linear expressions and equations are, how to construct them and how to form an equation from expressions and then solve the equation.

To explain how to choose between manufacturing and outsourcing, Marx et al. (1998:212) give this example:

> "The BETA motor company could manufacture brake discs or outsource the production to a specialist brake disc manufacturer. In the case of manufacturing, the fixed costs of the plant amount to R3 million per month, while the variable cost per brake disc amounts to R320,00. The lowest tender price of a specialist brake disc manufacturer is R560,00 per break disc. BETA requires 10 000 brake discs per month.
>
> Question: Should BETA manufacture the brake discs or outsource the manufacturing?
>
> Solution: Establish at which quantity of brake discs the manufacturing costs would equal the cost of outsourcing.
>
> Equate the number of brake discs to Q (quantity). With outsourcing, the total costs = 560 Q. With manufacturing, the total costs amount to 3 000 000 + 320 Q. To calculate the critical quantity, the two cost equations are equated.
>
> 560 Q = 3 000 000 + 320 Q
> 240 Q = 3 000 000
> ∴ Q = 12 500 brake discs

All other factors being equal:

- If the number of break discs required is less than 12 500: outsource
- If the number of break discs required equals 12 500: point of no difference
- If number of break discs required exceeds 12 500: manufacture”

In this example, students need to be able to derive the linear expressions for the manufacturing and outsourcing of the brake discs (3 000 000 + 320 Q and 560 Q). In order to do this the student needs to know the relationship between data and equations, and how to use data to derive an equation. The student then needs to set these expressions equal to each other to form an equation from which the variable (Q) can be solved.

The difficult part for the student would be to decide whether to outsource or manufacture when the number of brake discs is less than 12 500, and similarly when the number of brake discs is more than 12 500. In the example it states that if the number of break discs required is less than 12 500, the company should outsource, and similarly that if the number of brake discs required exceeds 12 500 the company should manufacture. This is not so easy to derive from the information given, since the only information derived directly from the calculation is the critical quantity.

The student will need a confident understanding of **linear expressions and equations** to be able to derive this information. That understanding can only come if the student has had a lot of exposure to working with linear expressions and equations and linear graphs. An alternative is that a general assumption can be made to outsource if the quantity needed is less than the critical quantity, and to manufacture if the quantity needed is more than the critical quantity. This is then something the student will have to memorise, without having much insight into why this is true.

This assumption cannot always be true, so the assumption will only be true for specific examples. A method that would require no assumptions would be for the student to **substitute** the required quantity into the two different cost functions and compare the answers to see where the cost would be less. This

method would be preferred. It is therefore not advisable that the lecturer suggest that the students simply make an assumption.

The Economics student will need to have an **understanding of what linear equations** are, and how to **solve linear equations**, as is shown in the following example from Parkin (2005:77):

> "Calculating market equilibrium, assuming demand and supply curves are straight lines:
> The demand for ice-cream cones is
> $P = 800 – 2Q_D$.
> The supply of ice-cream cones is
> $P = 200 + 1Q_S$.
> The price of a cone is expressed in cents, and the quantities are expressed in cones per day.
> To find the equilibrium price (P^*) and equilibrium quantity (Q^*), substitute Q^* for Q_D and Q_S and P^* for P. That is,
> $P^* = 800 – 2Q^*$
> $P^* = 200 + 1Q^*$
> Now solve for Q^*:
> $800 – 2Q^* = 200 + 1Q^*$
> $600 = 3Q^*$
> $Q^* = 200$.
>
> And
> $P^* = 800 – 2(200)$
> $= 400$.
>
> The equilibrium price is $4 a cone and the equilibrium quantity is 200 cones per day."

In this example the assumption is made that demand and supply are linear. Students will not be expected to do algebraic calculations of equilibriums for any non-linear examples. In these cases only the graphs are used.

Students are not required to draw the graph for this example, but to be able to **represent these equations graphically** could assist the students in evaluating whether the answer is correct.

These examples show that the first-year BCom student must be able to:

- perform basic operations on linear expressions,
- solve linear equations.

5.3.7 Solutions of quadratic equations

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.8 Systems of equations

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.9 Solving linear inequalities

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.10 Linear programming

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.11 Solving quadratic inequalities

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.12 Exponents

Refer to section 5.3.14.

5.3.13 Logarithms

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.14 Formulas

According to Hornby (1995:465), a formula is a rule, principle or law expressed by means of letters and symbols. Mathematically, a formula is an equation where the variables represent physical quantities. If the values of all but one of the quantities are known, the formula can be used to determine the value of the unknown quantity. This process can be made easier by first

rewriting the formula so that the unknown quantity is the subject of the formula.

An EB102 student will be required on numerous occasions to be able to remember formulas, as well as **substitute values into the formulas**. Here are some examples of these formulas:
(Note that the mistake in the calculating percentages example will be discussed after the examples.)

- "$\text{Profitability} = \frac{\text{Net Profit}}{\text{Capital invested}} \times 100$" (Bosch, Van Eeden & Tait 2004b:20)

- "Cost price = $\frac{\text{total costs}}{\text{production quantity}}$" (Marx et al 1998:199)

- "Rate of return on total capital =

  $R_{TC} = \frac{\text{profit before interest and taxation(PBIT)}}{\text{total capital employed}} \times 100$" (Marx et al 1998:596)

- "Gross profit margin = $\frac{\text{gross profit}}{\text{net sales income}} \times 100$", Marx et al (1998:599)

- "EOQ = $\sqrt{\frac{2 \times A \times O}{P \times C}}$

  EOQ = the most economical order quantity or optimal quantity to be ordered each time
  A = expected annual consumption in units
  O = cost per order
  P = the purchase price per unit
  C = the inventory-carrying cost expressed as a percentage of the inventory investment
  2 = the assumption that half the inventory remains on hand – thus an average stock",
  Marx et al. (1998:419)

The student needs to learn these formulas and be able to calculate the required quantity from the given information. The student will therefore be required to use a list or a table to determine which figures to use. This will only be possible if the student has a good understanding of the theory. The student must then substitute the values into the correct formula and **use his calculator** to do the calculation. The student must have the ability to discern

whether the answer he gets from the calculation is a reasonable answer for the question.

It is important to note that the above-mentioned formulas that calculate percentages are incorrect. The percentage symbol is omitted in all of them. The formulas should read as follows:

- $\text{Profitability} = \dfrac{\text{Net Profit}}{\text{Capital invested}} \times 100\%$
- $R_{TC} = \dfrac{\text{profit before interest and taxation (PBIT)}}{\text{total capital employed}} \times 100\%$
- $\text{Gross profit margin} = \dfrac{\text{gross profit}}{\text{net sales income}} \times 100\%$

It is required of the students to know how to **change the subject of a formula**, as shown in this formula concerning interest calculations (Marx et al. 1998:625):

"$F_n = P(1 + k)^n \Rightarrow P = \dfrac{F_n}{(1+k)^n} = F\left[\dfrac{1}{1+k}\right]^n$

P = present value

F_n = Future value at time n

k = interest rate (for calculating future values) or discounting rate (if calculating present value)

n = term of investment"

Students are required to do **interest calculations**, yet it is possible to do these calculations without changing the subject of the formula. The student could simply substitute the values and then solve for the unknown. In that case, the student will need to be able to solve equations. The student needs to be comfortable with **basic operations, addition, subtraction, multiplication and division, as well as having knowledge on the order of operations**. The student will also need to have knowledge of **exponents** and the ability to do calculations involving exponents. The student must know how to **use brackets correctly**. All of these calculations are done using a calculator, so the student needs to know how to **use a calculator**. These calculations could also be done on a financial calculator, which will remove the use of any of the formulas. Again it is important for the student to have the

ability to judge whether or not the answer they have is a reasonable answer to the question.

In addition to substituting into the formulas, the student is required at times to understand how to **manipulate a formula**. This means that the student needs to know what effect certain changes have on the formula, and what changes to make to get a desired effect. An example from Marx et al. (1998:439) reads as follows:

> "$\text{Productivity} = \dfrac{\text{Product quantity (output)}}{\text{Resource quantity (input)}}$
>
> The greater the figure above the line compared to the figure below the line, the greater the productivity.
>
> Suppose a factory operates for eight hours a day, ten persons work in the factory and the factory has one machine. The factory therefore has 80 labour hours and eight machine-hours available per day. If the factory produces 400 products per day:
>
> $\text{Labour productivity} = \dfrac{400}{80} = 5 \text{ products per labour hour}$
>
> $\text{Capital productivity} = \dfrac{400}{8} = 50 \text{ products per machine hour}$
>
> The productivity of an enterprise can be improved in one of five ways:
>
> - proportionally more products are manufactured using more resources
> - more products are manufactured with the same quantity of resources
> - more products are manufactured using less resources
> - the same number of products are manufactured using proportionally less resources
> - less products are manufactured using proportionally less resources"

Here the student needs to understand the different components of the formula, as well as be able to know how the result would be affected by **changes to any of the variables**. The student needs to have an understanding of **ratio and proportion** to be able to make the conclusions needed in this example. A proportion is found when two ratios are set equal to each other. Mathematically, to improve the productivity, the ratio a : b needs to be considered as a fraction $\frac{a}{b}$. To increase the size of the fraction, the student needs to understand that this is done by either increasing a, decreasing b, increasing a and decreasing b or increase a proportionally more

than b. This knowledge the student will have if he is comfortable and familiar with working with ratios and fractions. The student will not be required to work with proportions, or do calculations involving proportions, but knowledge on proportions will improve the student's understanding.

In the macroeconomics course there are many formulas that the students will have to remember. If they have an understanding of the theory, this does not have to happen by rote learning. The first set of formulas as they are introduced in Parkin (2005:480) is:

> "Consumption expenditure (C)
> Investments (I)
> Government purchases (G)
> Net taxes (T)
> Exports (X)
> Imports (M)
> Net exports (X – M)
> Aggregate income (Y)
> Private savings (S)
>
> Then: Y = C + I + G + X – M.
>
> GDP equals aggregate expenditure and equals aggregate income.
>
> Notice that households' income is consumed, saved, or paid in taxes. That is,
> Y = C + S + T.
> But you've seen that Y also equals the sum of the components of aggregate expenditure. That is,
> Y = C + I + G + X – M.
> By using these two equations, you can see that
> I + G + X – M = S + T.
> Now subtract G and X from both sides of the last equation and add M to both sides to obtain
> I = S + (T – G) + (M – X)
> In this equation, (T – G) is the government budget surplus and (M – X) is borrowing from the rest of the world."

Students are required to **know these formulas**, and be able to **do calculations** when different values are given. It will also be expected of the students to explain **what will happen to one variable if another variable changes**. These same variables are later used to explain further concepts, and provide more formulas, as seen in Parkin (2005:680):

"The marginal propensity to consume (MPC) is the fraction of a change in disposable income that is consumed. It is calculates as the change in consumption expenditure (ΔC) divided by the change in disposable income (ΔYD) that brought it about. That is,

$$MPC = \frac{\Delta C}{\Delta YD}.$$

The marginal propensity to save (MPS) is the fraction of a change in disposable income that is saved. It is calculated as the change in saving (ΔS) divided by the change in disposable income (ΔYD) that brought it about. That is,

$$MPS = \frac{\Delta S}{\Delta YD}.$$

You can see that these two marginal propensities sum to 1 by using the equation:

$\Delta C + \Delta S = \Delta YD$

Divide both equations by the change in disposable income to obtain

$$\frac{\Delta C}{\Delta YD} + \frac{\Delta S}{\Delta YD} = 1.$$

$\frac{\Delta C}{\Delta YD}$ is the marginal propensity to consume (MPC) and $\frac{\Delta S}{\Delta YD}$ is the marginal propensity to save (MPS), so

MPC + MPS = 1."

None of these formulas are very complicated, and a good understanding of the theory will assist the students in remembering the formulas. In order to understand how to derive some of these formulas, the students will need to have an understanding of **solving linear equations**, but if the students choose to rather memorise the formulas, it will not be necessary to be able to derive them.

Another example of **changing the subject of the formula** is found in Parkin (2005:622):

"The size of the money multiplier depends on the magnitudes of the required reserve ratio and the ratio of currency to deposits. To see how these two ratios influence the size of the money multiplier, call required reserves R, the required reserve ratio r, currency C, the ratio of currency to deposits D, the quantity of money M, and the monetary base B.

Required reserves R = rD and currency C = cD.

The quantity of money is M = C + D, or,

M = (1 + c)D. (1)

The monetary base B = R + C, or,

B = (r + c)D. (2)

Divide equation (1) by equation (2) to get

$$\frac{M}{B} = \frac{(1+c)}{(r+c)}$$

or

$$M = \frac{(1+c)}{(r+c)} \times B \text{ .”}$$

In the explanation of where the formula comes from, the author changes the subject of a formula (where factorising by taking out a common factor was used). Again, students will not be able to follow the explanation if they are not comfortable with the techniques of changing the subject of the formula.

These examples show that the first-year BCom student must be able to:

- substitute into formulas,
- change the subject of formulas,
- explain how a change in the value of one variable in a formula affects the other variables in the formula,
- understand and use exponents,
- factorise by taking out a common factor,
- solve linear equations.

The topic “Formulas” is included in the list. Now there is a clearer understanding of which aspects of formulas are important in Accounting, Business Management and Economics.

5.3.15 Graphs of exponential and logarithmic functions

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.16 Differentiation

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.17 Differentiation - advanced applications

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.18 Graphs of linear functions and interpreting of graphs

Graphs form a very large part of the Microeconomics course. Graphs are often used to assist in explanations and to illustrate the different properties of the market. One lecture in the Microeconomics course is dedicated to discussing graphs before any Microeconomic content that requires graphs is covered.

The objectives of this section of the textbook are to:

> "Make and interpret a time-series graph, a cross-section graph, and a scatter diagram
> Distinguish between linear and non-linear relationships that have a maximum and a minimum
> Define and calculate the slope of a line
> Graph relationships among more than two variables" (Parkin 2005:17)

The explanations in the textbook are very clear and well laid out. Students who are unsure on any part of this section should be able to refer to the textbook to develop an understanding. The different types of graphs are clearly explained and examples of all of them are included. Misleading graphs are explained and a warning is included that correlation in scatter diagrams could sometimes be a coincidence. A very valuable part of this section is the part dedicated to graphs used in Economic Models (Parkin 2005:20). It is emphasised that students should aim to look for **patterns, trends and features in the graphs**, for example variables that move in the same or opposite directions. These are clearly explained including examples and graphs.

A section of EB102 deals with break-even analysis. "Break-even analysis is a graphic or algebraic representation of the relationship between the production quantity, costs, sales income and profit of an enterprise" (Marx et al. 1998:200). There are many different mathematical calculations involved in break-even analysis that students need to consider.
The following information is given in Marx et al. (1998:201):

"Assume that the SUN Company manufactures and sells electric light bulbs and that the following information applies:

Selling price per bulb (P)	R15,00
Total fixed costs (FC)	R200 000
Variable costs per bulb (V)	R10,00
Total capital invested	R500 000

The number of bulbs manufactured and sold by the enterprise can vary between zero and 80 000 and is represented by the symbol Q."

The break-even graph is drawn by substituting different production quantities into equations derived from the information given. The students are not often required to draw the graphs, but they need to be able to **interpret and understand the graph** (see Figure 5.1 below).

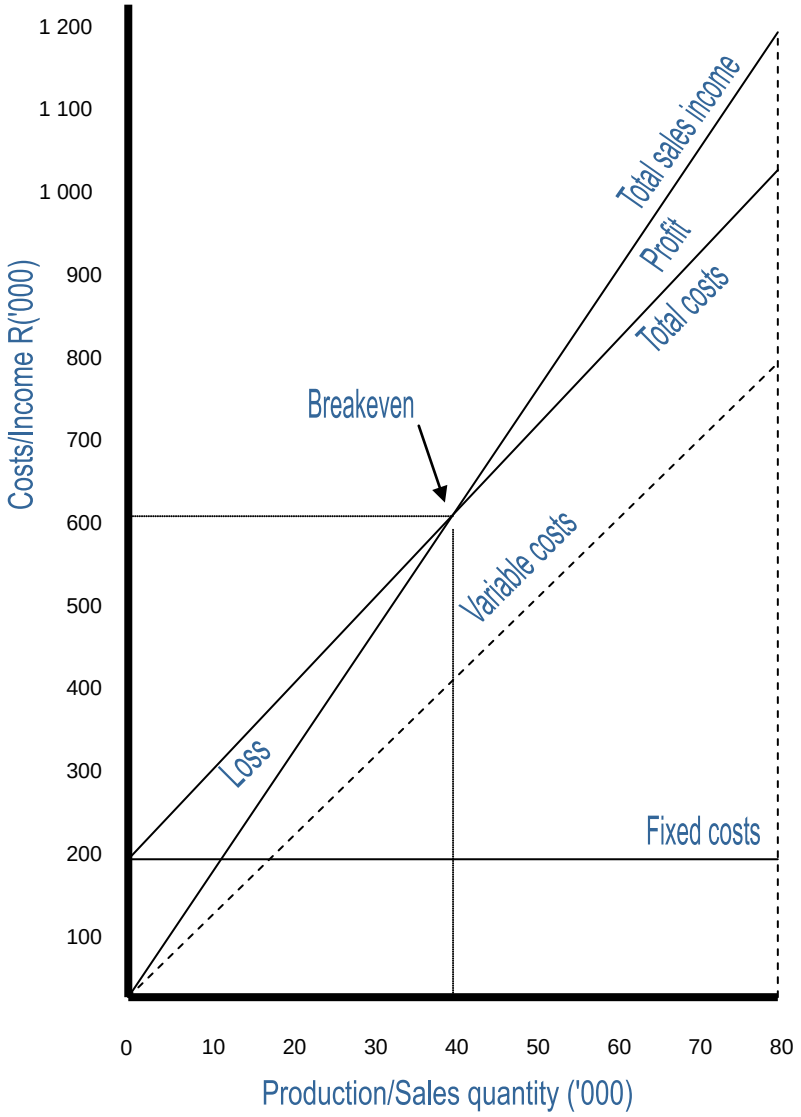


Figure 5.1 Break-even Graph (Marx et al. 1998:203)

Students should be able to derive information from the graph. For example, for the graph given in Figure 5.1 the following information can be derived:

- "Up to a production quantity of 40 000 bulbs, the total costs exceed the sales income, and a loss is consequently incurred.
- At a production quantity of 40 000 bulbs, costs equal income. No profit is made, nor is any loss incurred. This production quantity is known as the break-even quantity (critical production quantity).
- With a production quantity exceeding 40 000 bulbs, a profit is made. The projected profit at a maximum capacity amounts to R200 000.
- Since the fixed income component serves as a leverage, the rate of return could more than double if sales increased by 20 000 bulbs from 50 000 to 70 000 units. Because the increase in profit and rate of return is more than proportional to an increase in sales quantity, SUN will attempt to maximise the profit and rate of return by increasing sales. This more than a proportional increase in profit and rate of return is known as operating leverage and SUN will strive to utilise this benefit. However, SUN might not be able to sell as much as the enterprise can manufacture.

In the short term (with a maximum production capacity of 80 000 units in the example given), the break-even analysis is a simple but effective projection of the enterprise's total sales income, total costs and profit and loss expectations at different production quantities." (Marx et al. 1998:202)

Students should be able to **read information from a graph** and although it is not required for students to derive any formulas from the graphs, knowledge of the **relationship between straight line graphs and their formulas** would assist the students in their explanations and understanding of break-even analysis.

In addition to understanding break-even graphs, students need to be able to **calculate the break-even quantity algebraically**. From Marx et al. (1998:204) we get the following:

"Q = number of units manufactured and sold
FC = total fixed costs (R200 000 for the SUN company)
V = variable cost per unit = R10,00
P = selling price per unit = R15,00
Q_B = break-even quantity
VQ = total variable costs

It is known that:
Profit = total sales income – total costs

$= PQ – (VQ + FC)$

$= PQ – VQ – FC$

$= Q(P – V) – FC$ ……..……(1)

At the break-even quantity the profit is zero. To determine the break-even quantity, Q_B , equation (1) is equated with zero and solved for Q_B .

$Q_B(P – V) – FC = 0$

$$Q_B = \frac{FC}{P – V} \quad \text{……(2)}$$

$$= \frac{\text{total fixed costs}}{\text{selling price per unit} – \text{variable cost per unit}}$$

Break-even quantity for SUN:

$$Q_B = \frac{R200\,000}{R15 – R10}$$

$= 40\ 000$ units

To calculate the break-even in monetary value, the selling price (P) is multiplied by the break-even quantity (Q_B) (R15,00 × 40 000 = R600 000). This can also be calculated algebraically as follows:

$$\text{Break-even in rand sales} = PQ_B = \frac{P(\ FC)}{P – V}$$

$$= \frac{FC}{1 – \frac{V}{P}}$$

$$= \frac{R200\,000}{1 – \frac{10}{15}}$$

= R600 000”

To be able to understand and do these calculations, a student will need to have a clear **understanding of formulas and linear equations**. The student must be able to **use formulas, by substituting** into them. **Changing the subject of the formula** is also required. The student must have the ability to **manipulate a formula** if the given conditions change. The student will be required to **solve a linear equation**, **add or subtract algebraic expressions** and in the process **factorise by taking out a common factor**.

If student was required to draw the break-even graph, it could be done by firstly deriving equations from the given information. The following equations will be derived for this example, where Q refers to the quantity:

FC = 200 000	(Fixed costs)
V = 10Q	(Variable costs)
TC = 10Q + 200 000	(Total costs)
TS = 15Q	(Total sales)

These are fairly simple straight lines to draw, and shouldn't take the student too long. There are many benefits in letting the students **draw the graph**. The first of which is that it would simplify the calculation of the break-even quantity. Break-even quantity is the quantity where total costs are equal to total sales. Hence solving for Q in the following:

$$TC = TS$$
$$10Q + 200\,000 = 15Q$$
$$200\,000 = 15Q - 10Q$$
$$200\,000 = 5Q$$
$$Q = 40\,000$$

The study guide (Bosch et al. 2004b:53), indicates that the student should be able to answer questions referring to the break-even analysis For example:

> "What would be the individual effects on the break-even volume and profit should the:
> Selling price be decreased?
> Fixed costs increase?
> Variable costs decrease?
> Fixed costs increase stepwise?
> Variable costs increase progressively?"

These questions could be answered using either the graph or the equations, but the student will need a clear understanding of the tool he chooses to answer the question. If the students were required to draw the graph themselves, these questions could be answered by simply moving some of the individual graphs and seeing what happens.

In Parkin (2005:700), there is a section dedicated to the algebra of the Keynesian Model. In this section, the known formulas are used together with the theory to derive new formulas using simple algebraic techniques, as is shown below:

"**Aggregate Expenditure Curve**

Use the consumption function and the import function to replace C and M in the AE equation. That is,

$AE = a - bT_a + b(1 - t)Y + I + G + X - mY.$

Collect the terms on the right side of the equation that involve Y to obtain

$AE = (a - bT_a + I + G + X) + (b(1 - t) - m)Y.$

Autonomous expenditure (A) is $(a - bT_a + I + G + X)$, and the slope of the AE curve is $[b(1 - t) - m]$, So the equation for the AE curve, which is shown in Fig. 1, is

$AE = A + [b(1 - t) - m]Y.$

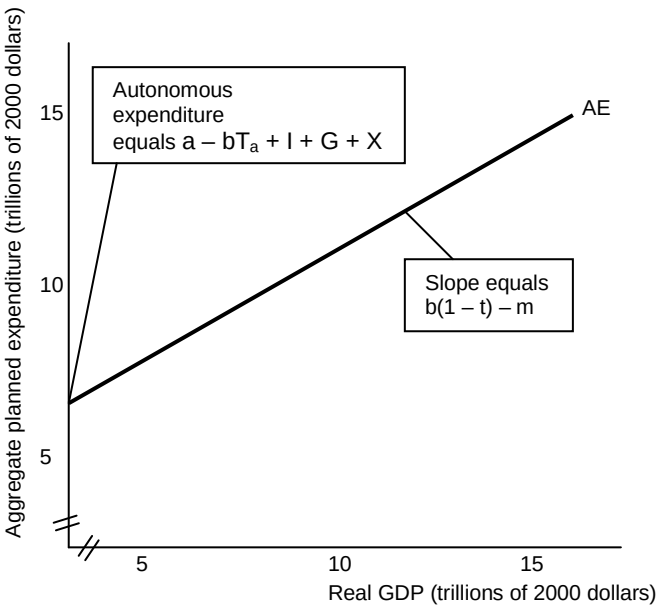


Figure 1 The AE curve

Equilibrium Expenditure

Equilibrium expenditure occurs when aggregate planned expenditure (AE) equals real GDP (Y). That is,

$AE = Y.$

In Fig. 2, the scales of the x-axis (real GDP) and the y-axis (aggregate planned expenditure) are identical, so the 45° line shows the points at which aggregate planned expenditure equals real GDP.

Figure 2 shows the point of equilibrium expenditure at the intersection of the AE curve and the 45° line.

To calculate equilibrium expenditure, solve the equations for the AE curve and the 45° line for the two unknown quantities AE and Y. So starting with

AE = A + [b(1 – t) – m]Y
AE = Y,

Replace AE with Y in the AE equation to obtain

Y = A + [b(1 – t) – m]Y.

The solution for Y is: $Y = \frac{1}{1-[b(1-t)-m]}A$. ”

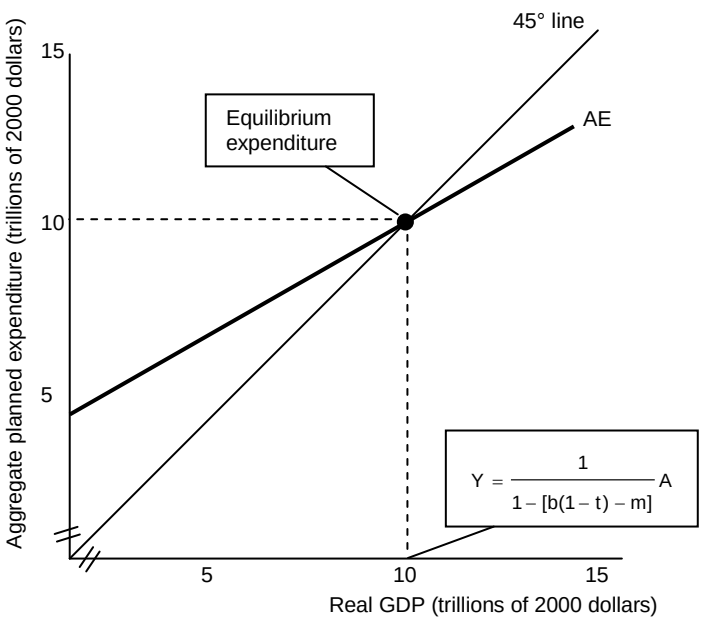


Figure 2 Equilibrium expenditure

Students will have to understand what **linear equations** are, and how to **represent them graphically**. The student also needs to know how to **change the subject of a formula**. All these concepts are explained clearly in the textbook.

These examples show that the first-year BCom student must be able to:

- sketch a straight line graph,
- read information from a graph,
- find equations of straight line graphs,
- perform basic operations on algebraic expressions,
- solve linear equations.

The topic “Graphs of Linear functions” is included in the list. Now there is a clearer understanding of which aspects of linear graphs and the interpretation of linear graphs are important in Accounting, Business Management and Economics.

5.3.19 Non-linear graphs and the interpretation of graphs

At Grade 7 level, a learner must be able to describe a situation by interpreting the graph of the situation, or be able to draw a graph from the description of a situation (Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics 2002:76). This is then extended in the grades that follow. A lot of time is spent teaching learners to draw graphs, but it seems that interpreting graphs in real-life situations often takes a back seat due to time constraints. Drawing and interpreting graphs is a skill that needs constant practice. It cannot be taught once and then expected to be part of the learner’s knowledge base.

In the first chapter of the Economics textbook it is stated that students should look for relationships between variables (Parkin 2005:21). They need to understand the difference between a **positive and a negative relationship**. They will not necessarily be expected to state the kind of relationship, but they will need to know **how a change in one variable will influence the other**. The following statements from Parkin (2005:71) form the basis for understanding demand and supply:

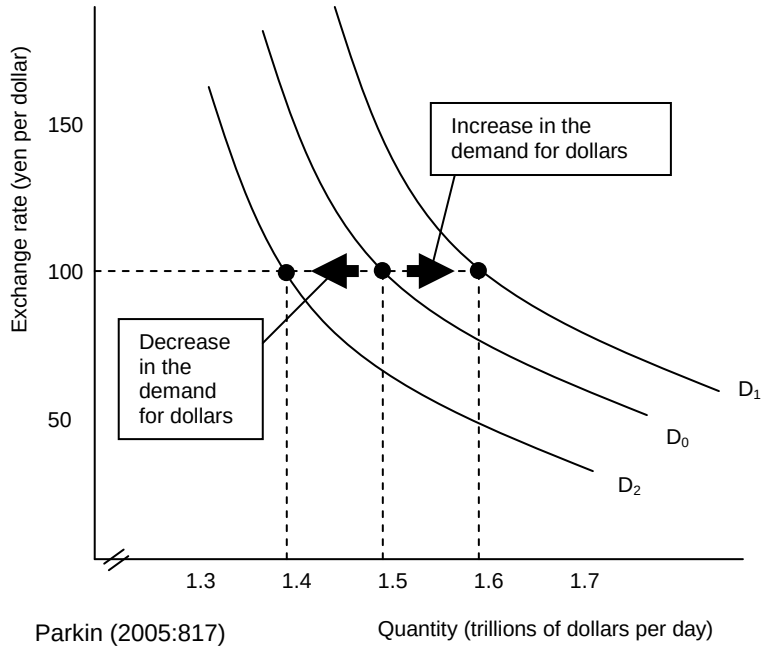
> ”When demand increases, both the price and the quantity increase.
> When demand decreases, both the price and the quantity decrease.
> When supply increases, the quantity increases and the price falls.
> When supply decreases, the quantity decreases and the price rises.”

It would be easier to remember this if the student could draw the demand and supply curves and use them to determine the change.

Students doing macroeconomics will need to have a good understanding of the **relationship between two variables in an equation or formula**. They will need to be able to **predict how a change in one variable would affect another variable**, as shown below:

> "A change in any influence on the quantity of dollars that people plan to buy, other than the exchange rate, brings a change in the demand for dollars.
>
> The demand for dollars
>
> Increases if:
>
> - The U.S. interest rate differential increases
> - The expected future exchange rate rises
>
> Decreases if:
>
> - The U.S. interest rate differential decreases
> - The expected future exchange rate falls"

FIGURE 34.5 Changes in the Demand for Dollars



Parkin (2005:817)

This is another example that shows that knowing what the related graph looks like could assist students in predicting any changes. The graphs are not always given, so a student will need to be able to reproduce a rough version of the graph to assist them in their understanding.

Price elasticity of demand is defined as follows:

> ”The price elasticity of demand is a units-free measure of the responsiveness of the quantity demanded of a good to a change in its price when all other influences on buyers’ plans remain the same.” Parkin (2005:82)

It is important to note that when a price changes, the change in your expenditure on a good depends on your elasticity of demand (Parkin 2005:87):

> “If your demand is elastic, a 1 percent price cut increases the quantity you buy by more than 1 percent and your expenditure on the item increases.
> If your demand is inelastic, a 1 percent price cut increases the quantity you buy by less than 1 percent and your expenditure on the item decreases.
> If your demand is unit elastic, a 1 percent price cut increases the quantity you buy by 1 percent and your expenditure on the item does not change.”

There are many cases where students must be able to explain **what will happen to one variable if another changes**. In each case the student must either **know the formula** that contains the variables, or **know what the relevant graph looks like**. In order to determine the changes, the student must have a good **understanding of graphs,** as well as a clear **understanding of equations**.

Figure 5.2 compares the cost of corrective maintenance with that of preventative maintenance. Marx et al. (1998:459) discusses it as follows:

> “Figure 13.3 shows that the cost of preventative maintenance rises continuously because machinery and equipment are serviced and repaired at regular intervals according to a given programme, whether or not they have broken down. The cost of corrective maintenance falls to a certain point because the machinery and equipment are repaired only after they have broken down. The extent of the maintenance is consequently less.”

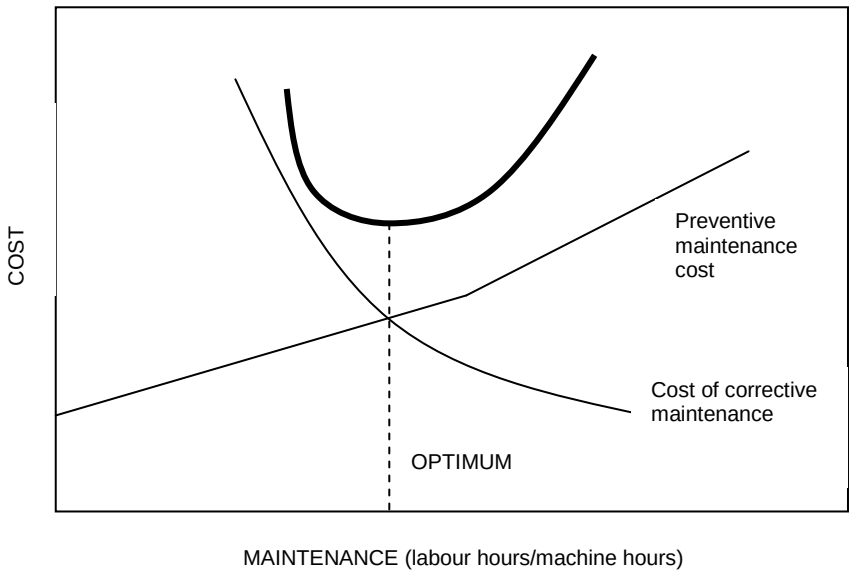


Figure 5.2: Cost of corrective maintenance compared with that of preventative maintenance (Marx et al. 1998:459)

It is important that the student understands the statement and is also aware that the graph supports what is stated.

Bosch et al. (2004b:49) requires the student to be able to **use graphs to assist explanations**. There is the option for the student to simply memorise the graph and then reproduce it, but if the student understands the graph, it can assist with the explanation. An example of this is: “Describe fixed costs in totality and per unit. Draw graphs to illustrate your answer.” (Bosch et al. 2004b:49). The student will need to draw the graphs in Figures 5.3 and 5.4 to assist their explanation:

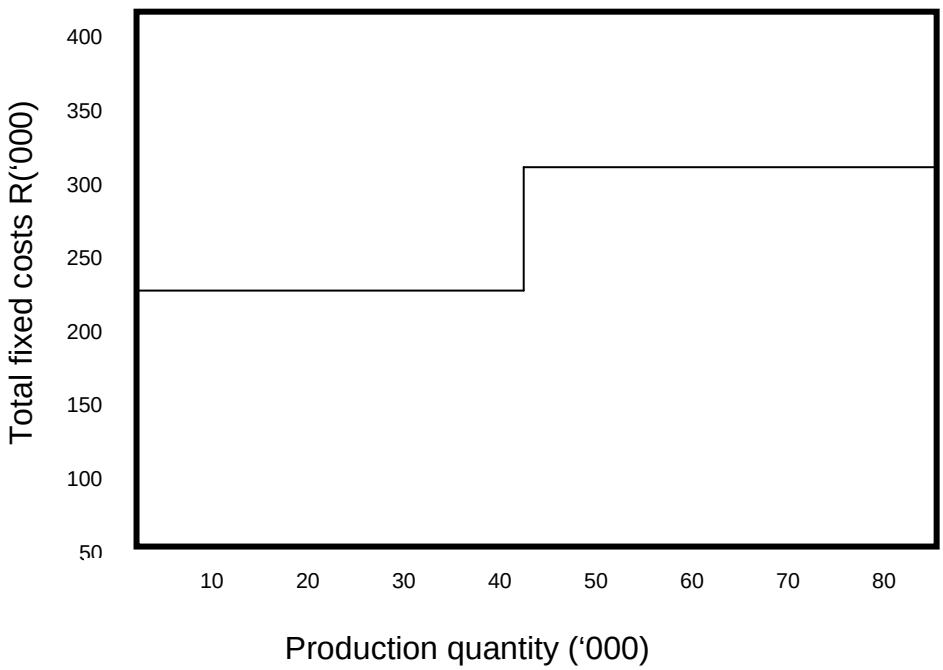


Figure 5.3 Total fixed costs versus production quantity (Marx et al. 1998:185)

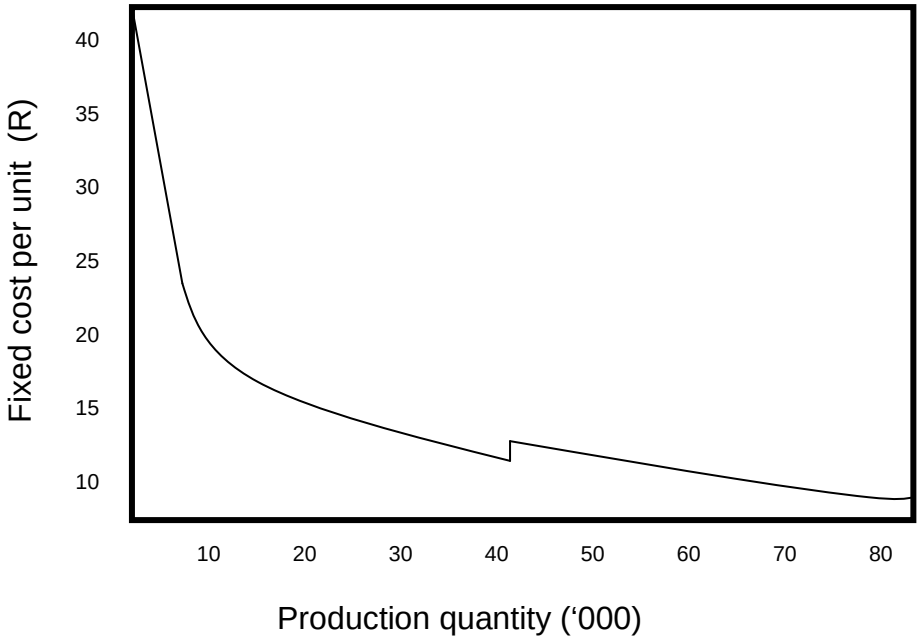


Figure 5.4 Fixed cost per unit versus production quantity (Marx et al 1998:185)

These graphs are not difficult to draw if they are memorised, and with a good understanding of the Cartesian Plane**, a student should find great benefit in using**

these graphs to help with the explanation, rather than reproducing the graphs after the explanation is finished.

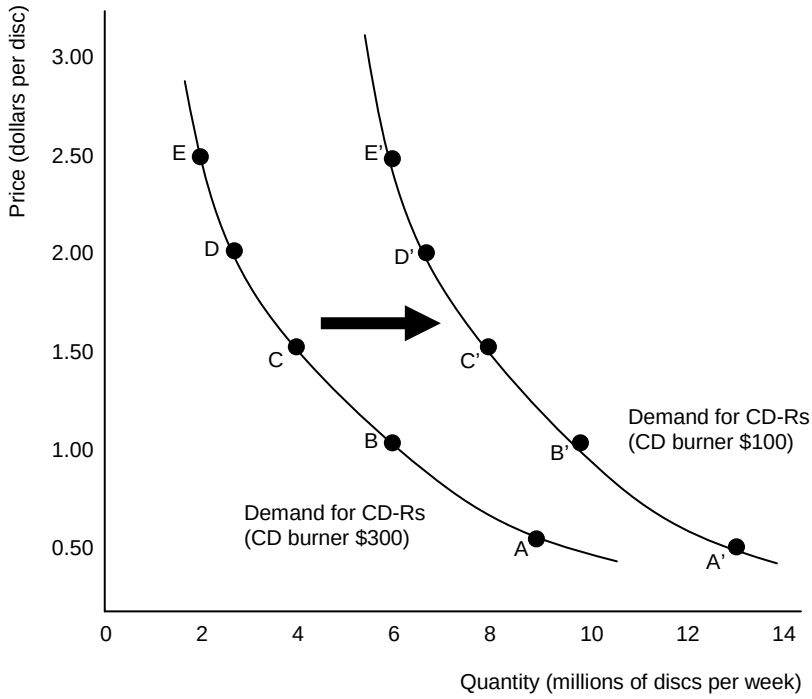
Demand and supply are concepts that Economics students need to understand. They form the base for many economical concepts that need to be understood by the students. Parkin (2005:59) states the following:

> "The law of demand states: Other things remaining the same, the higher the price of a good, the smaller is the quantity demanded; and the lower the price of a good, the greater is the quantity demanded."

If the concept of demand is represented graphically, changes in demand can be explained by using the graph. This is shown below in an extract from Parkin (2005:61):

> "Figure 3.2 illustrates an increase in demand. When demand increases, the demand curve shifts rightward and the quantity demanded is greater at each and every price."

FIGURE 3.2 An Increase in Demand

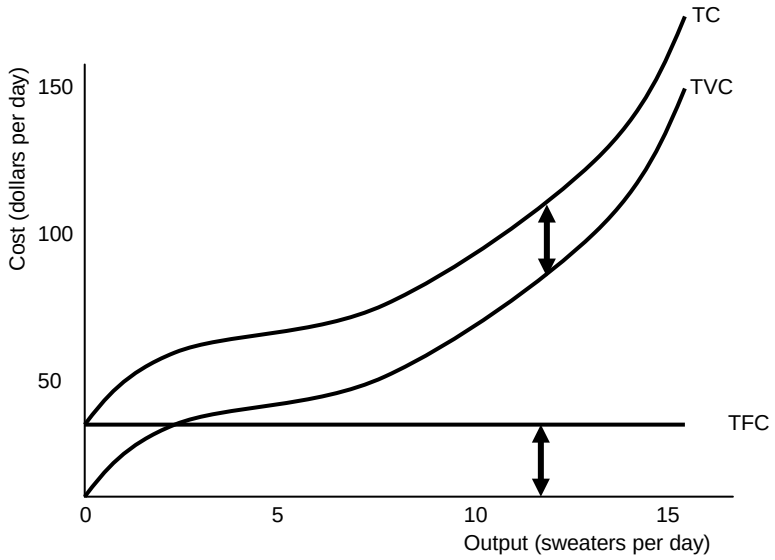


> "A change in any influence on buyers' plans other than the price of the good itself results in a new demand schedule and a shift of the demand curve. A change in the price of a CD burner changes the demand for CD-Rs. At a price of $1.50 a disc, 4 million discs a week are demanded when a CD burner costs $300 and 8 million CD-Rs a week are demanded when a CD burner costs $100. A fall in the price of a CD burner will increase the demand for CD-Rs. The demand curve will shift rightward."

Students can choose whether they want to use the graph to assist them in understanding the theory, or if they prefer to use the theory to assist them in understanding the graph. This will be determined by the individual student. It can be argued that once the graph is given, it becomes be easier to explain what happens to the price and the quantity.

The following example from Parkin (2005:223) shows how to relate the formula for total cost to the graph:

> "A firm's total cost (TC) is the cost of all the factors of production it uses. We divide total cost into total fixed cost and total variable cost.
> Total fixed cost (TFC) is the cost of the firm's fixed inputs.
> Total variable cost (TVC) is the cost of the firm's variable inputs.
> Total cost is the sum of total fixed cost and total variable cost. That is,
> TC = TFC + TVC.
>
> Figure 10.4 shows an example of Total Cost Curves.
>
> FIGURE 10.4 Total Cost Curves

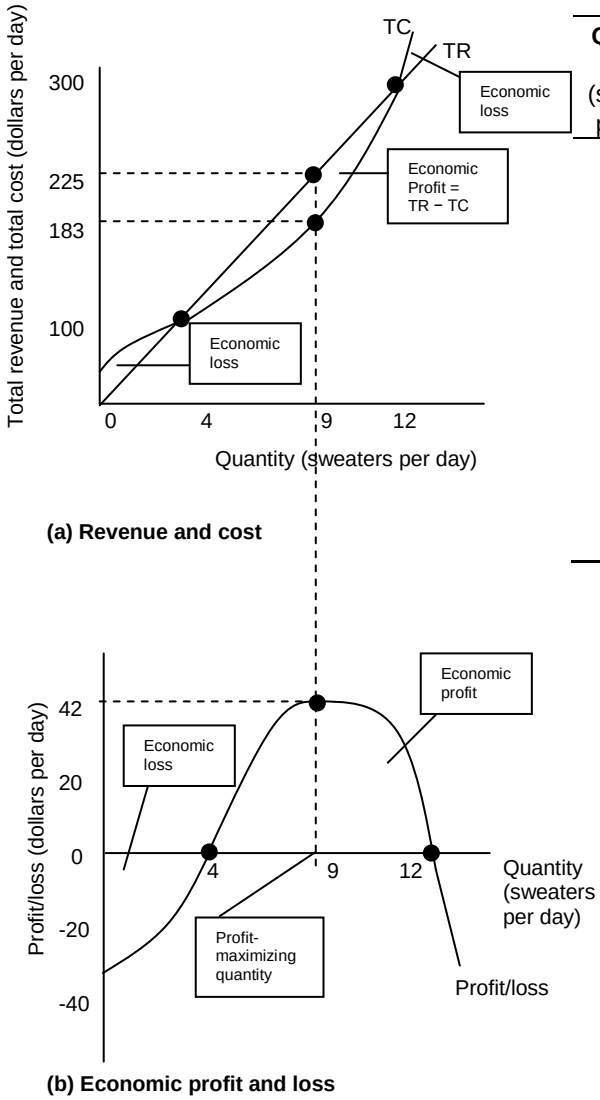


> Total fixed cost (TFC) is constant – it graphs as a horizontal line – and total variable cost (TVC) increases as output increases. Total cost (TC) increases as output increases. The vertical distance between the total cost curve and the total variable cost curve is total fixed cost, as illustrated by the two arrows."

A student will benefit from a good understanding of graphs, including what the **graph of a constant function** looks like. It would also be valuable for a student to understand **how graphs shift** if a constant is added, which relates to the **transformation of graphs**.

An example of using the graphs for doing calculations is from Parkin (2005:241):

FIGURE 11.2 Total Revenue, Total Cost, and Economic Profit



Quantity (Q) (sweaters per day)	Total Revenue (TR) (dollars)	Total cost (TC) (dollars)	Economic Profit (TR - TC) (dollars)
0	0	22	-22
1	25	45	-20
2	50	66	-16
3	75	85	-10
4	100	100	0
5	125	114	11
6	150	126	24
7	175	141	34
8	200	160	40
9	**225**	**183**	**42**
10	250	210	40
11	275	245	30
12	300	300	0
13	325	360	-35

"The table lists Cindy's total revenue, total cost and economic profit. Part (a) graphs the total revenue and total cost curves. Cindy's makes maximum economic profit, $42 a day ($225 − $183), when it produces 9 sweaters – the output at which the vertical distance between the total revenue and the total cost curves is at its largest. At outputs of 4 sweaters a day and 12 sweaters a day, Cindy makes zero economic profit – these are break-even points. At outputs less than 4 and greater than 12 sweaters a day, Cindy incurs an economic loss. Part (b) of the figure shows Cindy's profit curve. The profit curve is at its highest when economic profit is at a maximum and cuts the horizontal axis at the break-even points."

A student who is able to **relate the formulas to the graph** would have a better understanding of the concept. For example, understanding that economic profit is the greatest where the distance between the total cost and

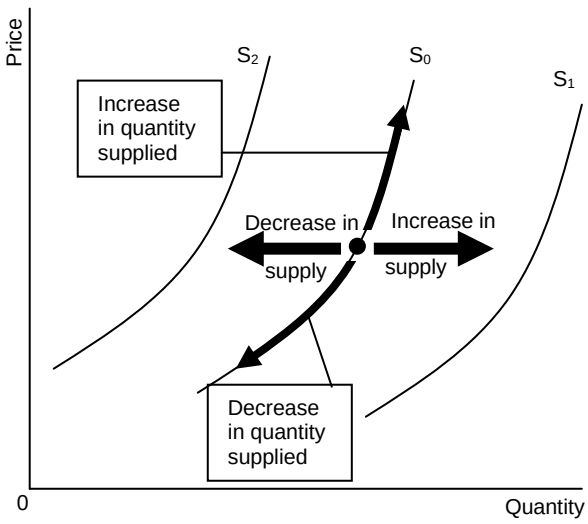
total revenue curves is the biggest, or that economic profit is the greatest at the maximum point of the profit/loss curve.

Students are not required to draw any graphs from formulas, but could rather use a given **table of points to plot the graph**. There are also many cases where students simply need to **draw the general form of the graph**, to assist them with the explanations.

The student will need to understand the difference between a **graph shifting** and **a point moving on a graph**. This is illustrated using a supply curve (Parkin 2005:67):

> ”When the price of the good changes there is movement along the supply curve and a change in the quantity supplied, shown by the arrows on the supply curve S_0. When any other influence on selling plans changes, there is a shift of the supply curve and a change in supply. An increase in supply shifts the supply curve rightward (from S_0 to S_1), and a decrease in supply shifts the supply curve leftward (from S_0 to S_2).

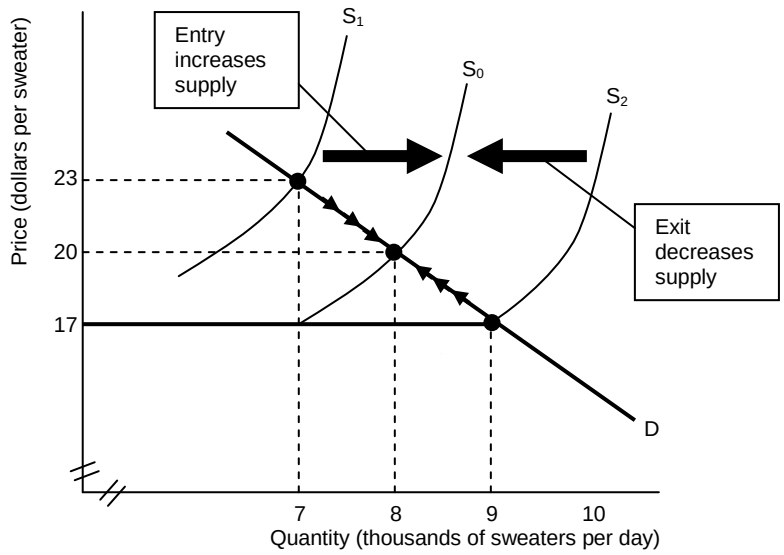
FIGURE 3.6 A Change in the Quantity Supplied Versus a Change in Supply



Similarly for demand, Parkin (2005:63) states:

> ”When the price of the good changes, there is a movement along the demand curve and a change in the quantity demanded. When any other influence on buyers’ plans changes, there is a shift of the demand curve and a change in demand. An increase in demand shifts the demand curve rightward. A decrease in demands shifts the demand curve leftward.”

Finding an equilibrium price is usually easier when using a graph rather than by calculating it algebraically. Parkin (2005:68) states that equilibrium price is the price at which the quantity demanded equals the quantity supplied. Using the demand and supply curves, this will be the **point where they intersect**. Students will need to explain what happens to the equilibrium price if certain variables change. This is illustrated in the following example from Parkin (2005:247):



"When new firms enter the sweater industry, the industry supply curve shifts rightward, from S_1 to S_0. The equilibrium price falls from \$23 to \$20, and the quantity produced increases from 7,000 to 8,000 sweaters.

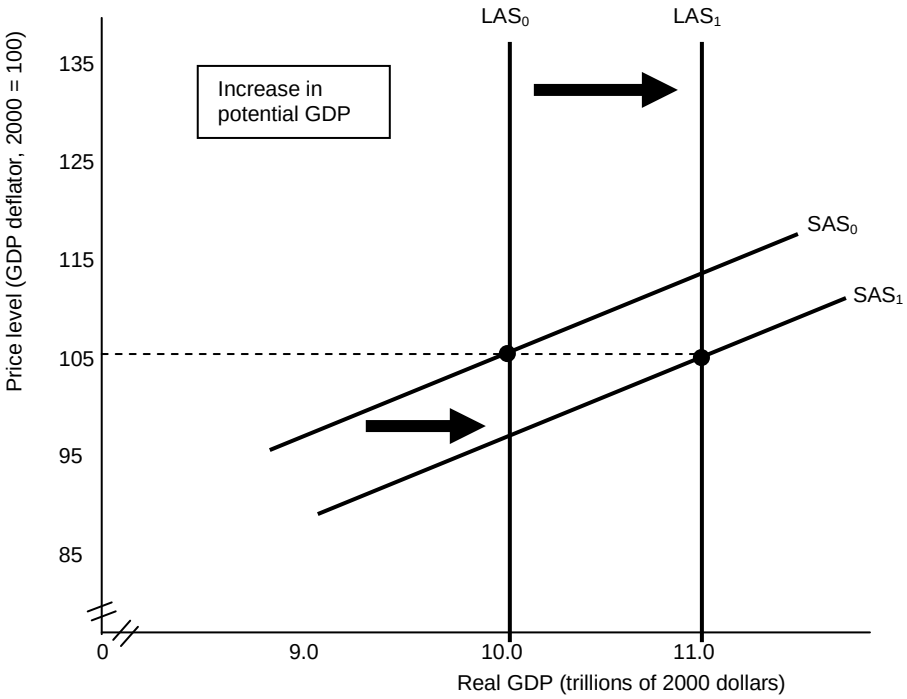
When firms exit the sweater industry, the industry supply curve shifts leftward, from S_2 to S_0. The equilibrium price rises from \$17 to \$20, and the quantity produced decreases from 9,000 to 8,000 sweaters."

Many concepts in Economics will require the students to **read information from a graph**. It will be essential for the student to understand the theory in order to use the graph effectively. Simply having an understanding of graphs will not be enough. The theory is the core part in every section of Economics.

Students will need to know how a **change in one of the variables will affect the graph**, as is shown in the following two examples from Parkin (2005:525,660):

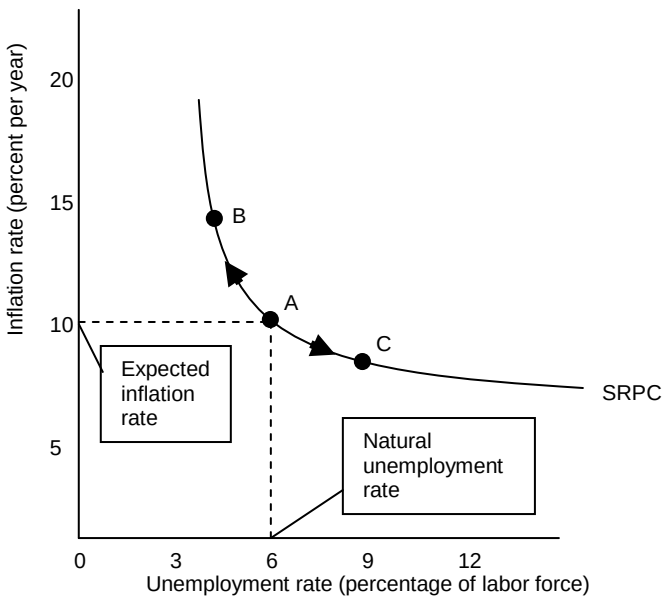
"An increase in potential GDP increases both long-run aggregate supply (LAS) and short-run aggregate supply (SAS) and shifts both aggregate supply curves rightward from LAS_0 to LAS_1 and from SAS_0 to SAS_1."

FIGURE 23.4 A Change in Potential GDP



"The short-run Phillips curve (SRPC) shows the relationship between inflation and unemployment at a given expected inflation rate and a given natural unemployment rate. With an expected inflation rate of 10% per year and a natural unemployment rate of 6 percent, the short-run Phillips curve passes through point A. An unanticipated increase in aggregate demand lowers unemployment and increases inflation – a movement up the short-run Phillips curve. An unanticipated decrease in aggregate demand increases unemployment and lowers inflation – a movement down the short-run Phillips curve."

FIGURE 28.8 A Short-Run Phillips Curve

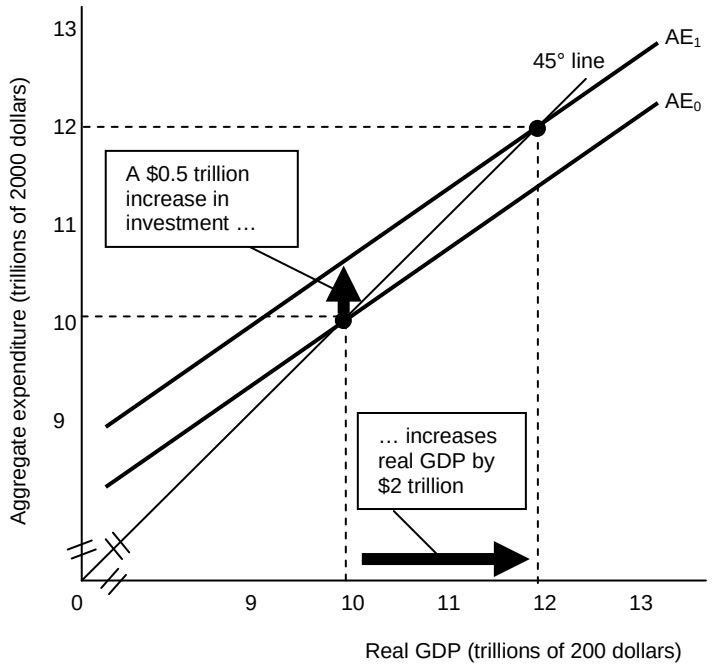


Students must be able to distinguish between when **a change in a variable causes a curve to shift**, and when the **point on the curve moves up or down the curve**.

There are cases where students will have to make **calculations from a graph** where variables change by a specific number. The student will need to know which values are needed for the calculation. With a good understanding of the theory, the student should be able to **evaluate whether the answer they get is realistic**. An example of this is found in Parkin (2005:689):

> "A \$0.5 trillion increase in autonomous expenditure shifts the AE curve upward by \$0.5 trillion from AE_0 to AE_1.
>
> Equilibrium expenditure increases by \$2 trillion from \$10 trillion to \$12 trillion. The increase in equilibrium expenditure is 4 times the increase in autonomous expenditure, so the multiplier is 4.

FIGURE 29.7 The Multiplier



$$\text{Multiplier} = \frac{\text{Change in equilibrium expenditure}}{\text{Change in autonomous expenditure}}$$

$$= \frac{\$2 \text{ trillion}}{\$0.5 \text{ trillion}} = 4\text{"}$$

In this example the students will need to firstly **know the formula** for the multiplier, and then to understand where on the graph to find the values for

equilibrium expenditure and autonomous expenditure. This implies that it is essential for the students to know the theory.

These examples show that the first-year BCom student must be able to:

- sketch graphs from a set of points,
- read information from a graph,
- understand how to shift graphs and the effect of change on the variables,
- know the difference between moving on a graph and shifting a graph.

The topic “Graphs of Non-Linear functions” is included in the list.,Now there is a clearer understanding of which aspects of non-linear graphs and their interpretations are important in Accounting, Business Management and Economics.

5.3.20 Absolute values and graphs of absolute value functions

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.21 Sequences and series

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.22 Areas and volume

No evidence was found that this topic is used in any of the first-year Business Management, Accounting and Economics courses.

5.3.23 Additional topic: Tables

The topic of tables was not included in the original questionnaire, but during the analysis of the textbooks and study guides it became obvious that tables are often used. Tables on their own are not a mathematical *topic*, but using and designing tables is a very important problem solving *tool*. From as early as Grade 3 at school level, students start reading and interpreting data

presented in simple tables, (Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics 2002:31), and from Grade 4 students need to organise and record data by using tables (Revised National Curriculum Statement Grades R – 9 (Schools) Policy, Mathematics 2002:56). The ability to understand information given in tables needs to be developed, and cannot be taught in a single session. The ability to construct a table is an ability that students need to develop practically by regularly constructing tables.

There are a large number of tables included in the textbook for EB101. Students need the ability to read information from a table**, and to also** understand the meaning of the information relevant to the table**. However, the student is not required to deduce any interpretations from the table (see Table 5.2 below).**

Table 5.2 Total population, population distribution and gross domestic product (GDP) (Marx et al. 1998:65)

Province	1994 Total population ('000)	Population groups (1994) ('000)				Distribution of GDP (%)	1990 GDP per capita according to 1990 prices (rand)
		Black	Asians	Coloured	White		
Gauteng	7 915,0	5 175,8	164,0	304,2	2 271,0	39,0	R13 800
Nothern Cape	776,7	242,9	1,9	402,9	129,0	2,7	8 500
Western Cape	3 771,9	627,2	31,6	2 184,2	928,9	12,2	8 300
Mpumalanga	3 334,1	3 003,4	11,4	17,1	312,2	9,5	8 200
Free State	2 903,4	2 435,9	1,0	76,1	390,4	5,8	5 000
KwaZulu-Natal	8 881,5	7 187,9	886,9	124,4	682,3	14,6	4 200
North-West	3 646,4	3 287,4	9,0	43,1	306,9	5,4	3 700
Eastern Cape	7 057,5	6 196,6	15,8	449,8	395,3	7,4	2 700
Northern Province	4 823,6	4 622,2	5,8	8,2	187,4	3,4	1 600
Total	43 120,1	32 779,3	1 127,4	3 610,0	5 603,4	-	R5 990

Students should be able to look at the table and gather data from it. For example, a student may need to be able to find the total population in 1994 in the Eastern Cape. For a student to give the answer as 7057.5 it would mean that the student does not have the ability to understand the table. This would mean that the student did not carefully consider the headings of the table. The student

should be able to see that the total population in 1994 in the Eastern Cape is 7 057 500.

The ability to read figures from the table is not enough. It would be beneficial for students to see for example that the Eastern Cape has a small percentage share of the GDP in comparison to the share in the population. This is not directly required of students to do, but without the ability to see this from the table, it would only be a table with numbers on it to the students, and have no relevance to the topic being studied.

From the document analysis, it is evident that during the EB101 course students must be able to read information from a table. Yet it would be important for students' understanding of the course if they were able to understand the significance of the values in the tables, and be able to draw conclusions from the presented data. It would be ideal if these students were able to see the **relationships between tables, formulas and graphs**. These three different forms of representing data are all linked. For students to be able to identify these links and to be able to move from one form to another would give them a greater understanding of the content with which they are working.

A fourth factor linked to the three mentioned above is the ability to describe the situation in words. For students to have a firm grasp of the relationship between these factors would be very beneficial. The ability to verbalise the problem is therefore very advantageous.

It is easier at times to do **calculations in the form of a table**. This is done when the same calculation needs to be made for different values of the variables. The following example from Parkin (2005:199) illustrates how to calculate economic efficiency:

> "Recall that economic efficiency occurs when a firm produces a given output at the least cost. Suppose that labour costs $75 per person-day and the capital costs $250 per machine-day. Table 9.3 (a) calculates the cost of using the different methods. By inspecting the table, you can see that method B has the lowest cost. Although method A uses less labour, it uses too

much expensive capital. And although method D uses less capital, it uses too much expensive labour.

TABLE 9.3 The Costs of Different Ways of Making 10 TV Sets a Day

(a) Four ways of making TV's

Method	Labour cost ($75 per day)		Capital Cost ($250 per day)		Total Cost	Cost per TV set
A	$75	+	$250,000	=	$250,075	$25,007.50
B	**750**	**+**	**2,500**	**=**	**3,250**	**325.00**
C	7,500	+	2,500	=	10,000	1,000.00
D	75,000	+	250	=	75,250	7,525.00”

In order to do the calculations, students will need to know what economic efficiency is, and once all the calculations are done, the students will then need to decide on the option that is most economical.

These examples show that the first-year BCom student must be able to:

- read and interpret information given in a table,
- do calculations in the form of a table.

5.4 REFINING THE LIST

Many of the algebraic topics are linked together in examples (*refer to sections 5.3.5, 5.3.6, 5.3.14 and 5.3.18*). For example using exponents and being able to manipulate algebraic expressions. For this reason it was decided to group all the algebraic topics together in the list of Mathematical topics. Thus, if the information from the questionnaire, interviews and analysis of content is combined, the list of mathematical requirements can then be refined to the main categories listed in Table 5.3.

Table 5.3 Main mathematical categories

Mathematical Topic
Basic Numeracy
Ratio and Proportion
Percentages and Averages
Basic Algebra
Formulas
Tables and Graphs

These main categories can then be expanded into the following subcategories:

Basic numeracy:

This includes the ability to recognise different types of real numbers and the properties of real numbers.

Rounding a number off correctly (with the focus on two decimal numbers).

Estimating the value of a mathematical calculation effectively.

Performing basic operations (addition, subtraction, multiplication and division) on whole numbers and fractions.

Performing selected operations on decimal numbers.

- Convert from a decimal number to a fraction and vice versa.
- Adding and subtracting decimal numbers.
- Multiplying and dividing decimal numbers by powers of 10.

Doing calculations involving percentages.

- Converting from a percentage to a decimal number and a fraction.
- Finding the percentage of a given number.
- Increasing and decreasing a number by a percentage.
- Finding the percentage by which a number has been increased or decreased.
- Finding the original number after a number has been increased or decreased by a given percentage.

Comparing and arranging numbers according to size.

Ratio and proportion:

Dividing a number into a given ratio.

Solving elementary direct and indirect proportion questions.

Converting units.

Percentages:

Calculating profit and loss, selling price, cost price, mark up and mark up percentage with the selected information given.

Calculating discounted amounts, selling price after discount and marked price before discount.
Calculating commission amounts.
Calculating input and output VAT.
Calculating depreciation using both the straight line and reducing balance methods.

Basic algebra:
Knowing definitions and rules relating to exponents.
Adding, subtracting, multiplying, dividing and factorising algebraic expressions.
Solving linear equations.

Formulas:
Substituting into formulas.
Changing the subject of a formula using algebraic manipulations.
Describing the relationship between variables in a given formula.
Performing calculations in specific relevant contexts, such as simple and compound interest calculations.

Tables and graphs:
Reading information from tables and answering questions relating to given tables.
Sketching a graph when a set of points is given.
Sketching straight-line graphs, finding the equations of straight line graphs and understanding what a gradient and an intercept are.
Reading information from a graph in order to make conclusions or do calculations.
Understanding how to shift graphs and the effect of change on the variables.
Knowing what the difference is between moving on a graph and shifting a graph.

5.5 ANALYSIS OF STUDY GUIDES AND TEXTBOOKS

The analysis of the study guides and textbooks gave insight not only into which mathematical topics are used in first-year Accounting, Business Management and Economics, but also to the extent to which the topics are used. This analysis confirmed that the information gathered from the initial questionnaire was not totally accurate, and that the interviews with the lecturers gave a clearer understanding of the topics needed. Examples have now been identified that illustrate how these topics are used in context.

Many of the topics discussed in the examples have been done at school level up to Grade 7. It has to be questioned whether learners had actually mastered these topics. One cannot simply assume that if a topic was covered, that it was necessarily mastered by the learners.

As was the case with the interviews, it was evident that students will need to be able to evaluate their answers in order to be successful in their courses. This is not a mathematical topic, but a skill that students need to develop. In addition it became clear that students doing accounting should not be intimidated by a large number of calculations. Students will need to be comfortable with doing numerous calculations in order to be successful.

5.6 CONCLUSION

The list of mathematical topics with their subcategories seemed at this stage to be a complete list of the research. However, an analysis of the test and examination papers for Accounting, Business Management and Economics will verify whether saturation level had been reached. Such an analysis should verify whether there were any additional mathematical topics that should be included in the first-year of Business Management, Accounting and Economics.

The following chapter covers the analysis, interpretation and findings. These will either confirm the grounded theory generated so far, or make it possible to refine it yet further.

CHAPTER SIX: TESTS AND EXAMINATIONS

6.1 INTRODUCTION

At this stage of the research, a list of topics was identified to be the mathematical requirements for first-year BCom students (*refer to section 5.4*). In order to verify that this list was complete, an analysis of the tests and examinations for Business Management, Accounting and Economics was made.

Because the research had been conducted from the year 2001, the analysis of the test and examination papers was also done over a number of years. Even though tests and examinations from different years were considered, these did not have any negative impact on the validity of the analysis since the curriculum remained constant over the period of time and the standards of the assessment did not change significantly.

6.2 AN ANALYSIS OF TESTS AND EXAMINATION PAPERS

6.2.1 An analysis of Accounting tests and examination papers

An analysis was made of all the first-year Accounting tests and examinations for 2003 in order to determine what mathematical calculations were included in the assessment.

Other than that part of the first semester test which tested theory, all tests and exams consisted of practical exercises that required the students to do calculations, adjustments, the correction of errors, drawing up of statements and accounts and generally practising accounting.

The mathematical requirements in the first three semester tests (14 March 2003, 25 April 2003 and 16 May 2003) were **basic addition and subtraction**, **percentage calculations** and **interest calculations**. Students were also required to work with a **date line** in order to calculate the data for a required number of months. A large number of **depreciation calculations** were included.

The June 2003 examination required students to do **basic addition and subtraction**, **depreciation** and **interest calculations** and a large number of **percentage calculations**.

The next three semester tests (15 August 2003, 12 September 2003 and 17 October 2003) required the students to do **addition and subtraction**, **sharing amounts in ratios** and **percentage calculations**.

During the November 2003 examination, students were required to do **addition and subtraction**, **depreciation** and **interest calculations** and a large variety of **percentage calculations**. Students also needed to **share profits in a ratio**, **calculate some ratios** and **calculate loan repayments**.

Students would have been able to do these calculations comfortably if they had been practising them repeatedly.

In summary, the essential aspects of mathematics in this section were:

- basic numeracy (including using a number line),
- percentage calculations (including depreciation),
- using formulas (interest calculations),
- calculating ratios,
- proportioning an amount in a ratio.

6.2.2 An analysis of Economics examination papers

In the case of economics, only examination papers were considered since the test papers were not made available to the researcher. It was stated that there was a big similarity between the tests and examinations in economics, so the fact that tests were not available did not hamper this study.

The June 2003 examination paper for Microeconomics (EC101) counted a total of 180 marks. The paper consisted of 60 multiple-choice questions,

counting two marks each (where both marks went for the correct answer), and long questions counting a total of 60 marks.

A third of the paper involved some form of mathematics. Of the multiple choice questions, 24 required the students to either **use a graph**, **a table or do some calculation for which they should know the formula**. Of the long questions, 16 marks required the students to **draw graphs**, **understand how graphs shift**, **use tables and do calculations**.

All the graphs that the students were required to draw or use were graphs that they had encountered during the course. It was important for the students to understand **how changes in the variables would influence the graphs**. This would only have been possible if the students had a good understanding of the theory related to each graph.

In order to have **read data off tables or done calculations with the values included in the tables**, the students would have needed to **know the relevant formulas** and have been able to use them. The calculations that were done were basic **addition, subtraction, multiplication and division**, but the students would have needed to understand the theory, and remember to apply the correct formulas.

The November 2003 examination paper for Macroeconomics (EC102) counted a total of 180 marks. The paper consisted of 60 multiple-choice questions, counting two marks each, and long questions counting a total of 60 marks.

A third of the November 2003 paper consisted of some form of calculation, graph or table. **Understanding the graphs**, being able to **predict how a change in a variable would influence the graph** and being **able to draw graphs** made up 16 marks of the paper. Six marks required students to **read data off tables**, while 38 of the marks required students to **do calculations**. None of these calculations could have been done if the student had not

known the theory associated with the question, or the formulas needed to answer the question.

A large proportion of the examination papers were multiple-choice questions. It is important to note that in most of the cases, there were no complex calculations involved in getting the answer. This implies that students were not penalised by the fact that there were no marks allocated for workings. The students would either get the question right or wrong, depending on their understanding and application of the theory.

In order for students to answer any of the mathematical type questions in these examination papers, it was essential for the students to **know the theory and the formulas**. Simply having a good understanding of graphs was not enough to be able to derive the correct answer.

In summary, the essential aspects of mathematics in this section were:

- basic numeracy,
- doing calculations from a graph,
- doing calculations with data in a table,
- knowing formulas,
- substituting into formulas,
- understanding graphs that are related to the theory,
- drawing relevant graphs,
- transformations of graphs,
- predicting how a change in a variable will influence a graph.

6.2.3 An analysis of Business Management tests and examination papers

The Business Management Department at NMMU, university stream, do not release the EB101 or EB102 test or examination papers. The main reason for this is that most of the papers contain many multiple-choice questions, and these can be used repeatedly. It was possible to view the papers in the presence of the lecturer, so this was done to gather information on the types of calculations the students are required to do in the tests and examinations.

The first semester test of EB101 (19 March 2004) did not require the students to do any calculations. The second semester test (14 May 2004) did contain calculations. All the questions containing calculations were multiple-choice questions, so the students were required to give an answer only. At least 10% of this semester test involved calculations (5 multiple-choice questions).

Students were required to calculate the following:

- rate of return percentage
- VAT amounts
- Taxation amounts

The students needed to be able to **remember a formula**, and **substitute into the formula**. In addition to this the student needed to have an understanding of **percentages**.

The EB101 examination (June 2004) did not contain any calculations.

Only 4% of the first EB102 semester test (20 August 2004) contained calculations. From information given the students were required to calculate **labour productivity per day**, as well as **capital productivity per day**. These questions were in the form of multiple-choice questions.

The student needed to **know the formula and substitute into it**.

The second EB102 semester test (17 September 2004) contained a large proportion of calculations (80%, or 40 marks out of 50). All questions were multiple-choice questions, counting 2 marks each (where both marks were for a correct answer). This implies that the focus is on the answers alone, and the calculation was not valued.

From a break-even graph and given information, the students were required to do various calculations. All involved **substituting into formulas**. Students were also required to **calculate cost price** if selling price and mark up were given, as well as **calculating the mark-up percentage** if the cost price and selling price were given.

The semester test contained a question on outsourcing, in which the student had to calculate the quantity at which the company should outsource its manufacturing. This involved **solving a linear equation**, and also making a decision based on the result. The students had to be able to decide if the company should outsource when the quantity was below or above the critical quantity.

From a given table the student was required to calculate a number of **ratios**, for example Debt ratio and Acid-test ratio. The students therefore needed to know the ratios as well as how to get the correct **information from the table**.

Students needed more than just knowledge of percentages, formulas, ratios and linear equations to be able to excel at the semester test. If students were not confident with calculations, it would be very daunting to have to write a test containing such a large number of calculations. If students had a fear of numbers, then this attitude towards numbers would negatively influence the students' ability to perform the operations with numbers which would normally not be a problem.

Calculations made up 26% of the EB102 examination (November 2004). That constituted 44 of the total of 170 marks. Only 8 out of the total of the 44 were multiple-choice questions.

Students needed to calculate whether to accept or reject an order, as well as whether to purchase or manufacture an item. These calculations involved **solving linear equations**, and were similar to those on outsourcing in the semester test.

A balance sheet and income statement were given, and students needed to **calculate ratios** from the given information. The students needed to know the ratios, and where to find the information required on the balance sheet and income statement.

Calculating Economic order quantity and a re-order level were also required, and for this the student would need to know the formulas and **substitute into the formulas**.

A graph of the purchasing process was given, and student needed to **read values off the graph and interpret their meaning**. An understanding of graphs (especially **intercepts of a graph**) was needed to answer this question.

The examination included multiple-choice questions (4 questions of 2 marks each) on calculating the break-even point in terms of units and the monetary value. The student needed to know the formulas to solve this calculation.

The use of multiple-choice questions to test the students' ability to do a calculation is not a good evaluation tool. The reason for this is that the student does not get any credit for workings. The answer that counts here is either right or wrong. The students were required to use the correct formula, substitute the correct values into the formula, and do the calculation correctly. A multiple-choice question is not the correct evaluation tool to use for calculations of this kind.

In summary, the essential aspects of mathematics covered in the tests and exams were:

- remembering formulas,

- substituting into formulas,
- percentage calculations,
- calculating cost price,
- calculating the mark-up percentage,
- solving linear equations,
- calculating ratios,
- reading information from a table,
- reading values off a graph and interpreting the meaning.

6.3 SUMMARY OF RESULTS FROM TESTS AND EXAMINATIONS

After the analysis of the test and examinations, the data were summarised. This summary is reflected in Table 6.1.

Mathematical Topics	**Economics**	**Accounting**	**Business Management**
Basic Numeracy	✓	✓	
Ratio and Proportion		✓	✓
Percentages and Averages		✓	✓
Basic Algebra			✓
Formulas	✓	✓	✓
Tables and Graphs	✓		✓

Table 6.1 Summary of results from first year Accounting, Economics and Business Management tests and examinations

When comparing Table 6.1 to Table 5.2, the summary of the test and examinations therefore indicate that there are no further topics to add to the list. This verified the fact that the list of topics generated in Chapter 5 was complete. Therefore the researcher came to the conclusion that saturation had been reached in terms of investigating the first research objective of the study.

6.4 CONCLUSION

After having presented an overview and analysis of a representative sample of test and examination papers from Accounting, Economics and Business Management in this chapter, the researcher was satisfied that all possible mathematical topics had been identified. Thus the saturation level reached in Chapter 5 had been confirmed. In the next chapter these topics will be presented in the form of learning outcomes and then compared with the content of Mathematics and Mathematical Literacy in the NSC. As a final validation, the first-year lecturers of Accounting, Economics and Business Management were interviewed to confirm their agreement with the findings.

CHAPTER SEVEN: OUTCOMES, COMPARISON WITH NSC AND LECTURERS' VALIDATION

7.1 INTRODUCTION

This study has focused on the mathematical requirements for students doing a BCom degree programme at the NMMU, but not majoring in Mathematics, Statistics or Computer Science and Information Systems. Table 7.1 gives a synopsis of the relevant BCom degree programmes that were considered and the mathematical related modules that are included in the programmes (Prospectus: Faculty of Business and Economic Sciences 2006).

Programme name:	**Mathematics-related course included in Programme:**
General BCom	Mathematics Special (A) or Statistics (1)
General Accounting	Statistics (1)
Accounting for Chartered Accountants	Statistics (1)
Human Resource Management	Mathematics Special (A) and Statistics (1)
Marketing Management	Statistics (1)
Law	Mathematics Special (A) or Statistics (1)
Sport and Recreation Management	Statistics (1)
Financial Planning	Statistics (1)

Table 7.1 Mathematics-related courses included in different BCom programmes

The overall significance of having made an in-depth study of the syllabi of first-year Business Management, Accounting and Economics is illustrated by the fact that in all these programmes, except Law, students will be required to do first-year Business Management, Accounting and Economics. In the Law programme students can choose just two of the three.

The second column in Table 7.1 states which mathematical course the student will be required to do as part of the particular degree programme. It is either Mathematics Special (A) or Statistics (1) or both. Statistics (1) consists of two semester modules named Financial Mathematics and Business Statistics.

7.2 MATHEMATICAL SUPPORT COURSE: ITS NATURE AND PURPOSE

A course to assist the students with the mathematics they will need in their degree programme will not necessarily replace the current mathematical courses they are doing, these being Mathematics Special (A) or Statistics (1). While studying the content of these courses, students may acquire other skills that are beneficial especially for their second and third-year courses. A course to assist students with the mathematics that is included in the first year of their degree programme would only be for those students that really need support to introduce them to the mathematics included in the first year of their degree programme.

7.3 ORIENTATION REMARKS

The mathematical content that directly supports Business Management, Accounting and Economics needs to form the core of the curriculum of a support course. This may be content that lecturers often assume students know and can transfer to their courses. The reality in South Africa is that in the last decade, the quality of education in the schools has changed and lecturers cannot assume that students come with the same knowledge as they previously did (*refer to section 2.5*).

Additional work will be required for many students as they enter higher education. It may not be necessary for all BCom students to do this course, but the level of mathematics with which the student enters the university should be a guide to which students would need such a course. It would also be possible to develop a test to measure how capable students are at doing the calculations that will be needed in their first year of study. All students would probably benefit from such a course. However, there are some students for which this will not be necessary (*Refer to section 2.8*).

7.4 MAPPING THE CONTOURS OF A SUPPORT COURSE BY MEANS OF LEARNING OUTCOMES

In order to develop a course to support BCom students with the mathematics required in their BCom subjects, some of the essential content needed has already been identified in the previous chapters (*refer to section 5.4*). The first step in designing a support course is to develop learning outcomes. These learning outcomes give an indication of the content that is required. The learning outcomes should not be considered in isolation, but rather as the first step in the development of a support course (*refer to section 2.8.7*).

From the list of topics identified in section 5.4, learning outcomes have been developed. (*Refer also to section 8.5.*) These outcomes are a direct result of this research and form part of the original contribution of this research. The learning outcomes that have been developed are:

Learning outcome 1:

The learner understands fundamental concepts relating to numbers and operations on numbers.

We know this when the learner is able to:

1.1 Recognise different types of real numbers and the properties of real numbers.

1.2 Round a number off correctly (with the focus on two decimal numbers).

1.3 Estimate the value of a mathematical calculation effectively.

1.4 Perform basic operations (addition, subtraction, multiplication and division) on whole numbers and fractions.

1.5 Perform selected operations on decimal numbers:

- Convert from a decimal number to a fraction and vice versa.
- Add and subtract decimal numbers.
- Multiply and divide decimal numbers by powers of 10.

1.6 Do calculations involving percentages.

- Convert from a percentage to a decimal number and a fraction.
- Find a percentage of a given number.
- Increase and decrease a number by a particular percentage.

- Find the percentage by which a number has been increased or decreased.
- Find the original number after a number has been increased or decreased by a given percentage.

1.7 Compare and arrange numbers according to size.

<u>Learning outcome 2:</u>

The learner is able to use ratios in a business context.

We know this when the learner is able to:

2.1 Divide a number into a given ratio.

2.2 Solve elementary direct and indirect proportion questions.

2.3 Convert units.

<u>Learning outcome 3:</u>

The learner is able to use the knowledge of percentages to solve problems in a range of different contexts relating to financial issues.

3.1 Calculate profit and loss, selling price, cost price, mark up and mark up percentage with the selected information given.

3.2 Calculate discounted amounts, selling price after discount and marked price before discount.

3.3 Calculate commission amounts.

3.4 Calculate input and output VAT.

3.5 Calculate depreciation using both the straight line and reducing balance methods.

<u>Learning outcome 4:</u>

The learner is able to apply basic algebraic techniques.

We know this when the learner is able to:

4.1 Know definitions and rules relating to exponents.

4.2 Add, subtract, multiply, divide and factorise algebraic expressions.

4.3 Solve linear equations.

<u>Learning outcome 5:</u>

The learner is able to use and manipulate formulas.

We know this when the learner is able to:

5.1 Substitute into formulas.

5.2 Change the subject of a formula using algebraic manipulations.

5.3 Describe the relationship between variables in a given formula.

5.4 Perform calculations in specific relevant contexts, such as simple and compound interest calculations.

<u>Learning outcome 6:</u>

The learner is able to use tables and graphs to answer questions from financial or business contexts.

We know this when the learner is able to:

6.1 Read information from tables and answer questions relating to given tables.

6.2 Sketch a graph if a set of points is given.

6.3 Sketch straight line graphs, find the equations of straight line graphs and understand what gradients and intercepts are.

6.4 Read information from a graph in order to draw conclusions or do calculations.

6.5 Understand how to shift graphs and the effect of change of the variables.

6.6 Know what the difference is between moving a graph and shifting a graph.

Descriptions and examples of topics mentioned in the learning outcomes followed in Addendum B.

One of the aims of the study was to identify the mathematical content included in first-year Business Management, Accounting and Economics (*refer to section 1.3*).

This study did not intend to develop all the aspects of the support course, but once the content of the support course had been identified, the method of

delivery, assessment and aspects of teaching and learning needs could be designed. These all fall outside the ambit of this study but need to be considered in follow-up research. This course has not yet been offered at NMMU.

7.5 COMPARISON BETWEEN MATHEMATICS AND MATHEMATICAL LITERACY IN THE NSC

All South African learners in government schools follow the same Mathematics curriculum from Grade R to Grade 9. In Grade 10 the learner will choose to do either Mathematical Literacy or Mathematics through to Grade 12. The contents of Mathematical Literacy and Mathematics are both very different and articulation between the two subjects is not possible after Grade 10.

A comparison between the contents of the school Mathematics and Mathematical Literacy curriculum (National Curriculum Statement Grades 10 – 12 (General), Mathematical Literacy 2003; National Curriculum Statement Grades 10 – 12 (General), Mathematics 2003), and the learning outcomes identified in Section 7.2 should provide valuable feedback to assist in determining both entry requirements and which students could benefit from a support course.

This was the first step in reaching the second research objective which was to determine whether Mathematics and Mathematical Literacy in the NSC adequately prepare students for mathematical concepts included in the BCom studies.

7.5.1 Meeting learning outcome 1: The learner understands fundamental concepts relating to numbers and operations on numbers

From early in their school careers, South African students are introduced to numbers and operations with numbers. By Grade 4 learners would have been introduced to different types of real numbers and their properties. Concepts

like estimation, arranging numbers according to size, rounding off and using a calculator should be familiar to the learners by the end of Grade 4.

Basic operations (addition, subtraction, multiplication and division) on real numbers (including fractions and decimals) should have been covered by the end of Grade 6. Learners are also introduced to percentages, finding a percentage of an amount and converting between percentages, fractions and decimals by the end of Grade 6.

During Grades 10 and 11 in the Mathematical Literacy curriculum learners revisit topics like estimation, rounding off and basic operations on numbers. Basic percentage calculations are done. However, these are limited to finding the percentage of a number and percentage increases and decreases.

During Grade 10 in the Mathematics curriculum different types of real numbers, estimation and rounding off is covered briefly. Converting between a decimal number and a fraction is also revisited.

7.5.2 Meeting learning outcome 2: The learner is able to use ratios in a business context

During Grade 7 learners start working with ratios, and in Grade 8 learners convert units. The Mathematical Literacy curriculum for Grades 10 to 12 includes ratios as a topic. Learners work with ratios, including dividing a number into a ratio. Solving direct and indirect proportion questions, as well as converting units is also covered. Selected questions are from a business context. The Mathematics curriculum does not cover any of these topics.

7.5.3 Meeting learning outcome 3: The learner is able to use knowledge of percentages to solve problems in a range of different contexts relating to financial issues

The concept of percentages is introduced to learners in Grade 6, but financial specific percentage calculations are not covered. In Grade 10 of the Mathematical Literacy curriculum learners are introduced to using

percentages for profit and loss calculations, cost price and selling price calculations and calculating discounts.

In Grade 11 of the Mathematics curriculum learners perform depreciation calculations, both using the straight line and reducing balance methods. Learners do annuity calculations, which include working with percentages in a financial context as well as using formulas.

7.5.4 Meeting learning outcome 4: The learner is able to apply basic algebraic techniques

During Grades 8 and 9 learners are introduced to exponents and the properties of exponents. Teaching learners to add, subtract, multiply, divide and factorise algebraic expressions should also be covered by the end of Grade 9.

In the Mathematical Literacy and Mathematics curricula learners cover exponents and the solving of linear equations. Further, in the Mathematics curriculum the ability to add, subtract, multiply, divide and factorise algebraic expressions would normally be visited.

7.5.5 Meeting learning outcome 5: The learner is able to use and manipulate formulas in order to answer questions from a variety of contexts

In the Mathematical Literacy curriculum learners substitute into formulas, including simple and compound interest formulas. There is little emphasis on changing the subject of a formula, even though the student will be required to calculate the values of different variables in interest formulas.

During Grade 10 in the Mathematics curriculum learners are introduced to formulas. Substituting into formulas, changing the subject of formulas and identifying the relationship between variables in a formula are all covered. Learners also do simple and compound interest and annuity calculations.

7.5.6 Meeting learning outcome 6: The learner is able to use tables and graphs to answer questions from financial or business contexts

In Grade 7 learners are already using tables and graphs to find required information, and to answer specific questions relating to the tables and graphs. In the Mathematical Literacy curriculum, learners work with tables and graphs, reading information from tables and graphs and answering questions relating to the tables and graphs. Learners sketch straight line graphs and calculate gradients. Learners are also taught to sketch a graph if a set of points are given.

Learners doing Mathematics will work with tables and graphs, reading information from tables and graphs and answering questions relating to the tables and graphs. Learners will also sketch straight line graphs, calculate gradients and find the equations of straight line graphs. Learners are also taught to sketch a graph if a set of points is given.

There is no evidence that learners doing either Mathematical Literacy or Mathematics spend much time on how to shift graphs, and how changes in the variables will affect the graph.

7.5.7 Summary of the extent to which NSC Mathematics and Mathematical Literacy meet learning outcomes

It does seem that to some extent both Mathematical Literacy and Mathematics do cover most of the mathematical topics that students will encounter during their first year of Business Management, Accounting and Economics, but the ability to apply the knowledge in context varies from student to student.

A summary of the results can be found in Table 7.2

Table 7.2 Comparison of Mathematical Topics with Mathematical Literacy and Mathematics in the NSC

Mathematical Topics	Grades R - 9	Mathematical Literacy	Mathematics
Numbers and operations on numbers	Grades R - 9	Grades 10 - 11	Grade 10
Ratios	Grades 7 - 8	Grades 10 - 12	Not
Percentages	Grade 6	Grade 10	Grade 11
Basic Algebra	Grade 8 – 9 (minimal)	Not	Grades 10 - 12
Formulas	Minimal	Grades 10 - 12	Grade 10
Tables and graphs	Grade 7	Grades 10 - 12	Grades 10 - 12

7.6 FINDINGS ON MATHEMATICS ENTRY REQUIREMENTS FOR A BCOM DEGREE AT NMMU

Mathematical Literacy should prepare a learner to be comfortable working with numbers. If a learner is able to perform well in Mathematical Literacy, (the meaning of “performed well” would only be able to be quantified exactly once the level of assessment was clear), this learner should not be intimidated by a large volume of calculations. This is an important criterion that surfaced in Chapter 5.

Apart from sketching straight line graphs (Grade 10) and solving linear equations (Grade 11), Mathematical Literacy contains very few other algebraic concepts. It could be argued that even though the student will be confident with numbers, the possibility of encountering algebraic manipulations in their first-year courses could be intimidating to the student.

It has to be emphasised again that the comparison was made between the mathematical content of first-year Business Management, Accounting and Economics, and the school Mathematics and Mathematical Literacy curricula.

A student doing a BCom degree programme at NMMU will also be required to do either first-year Mathematics Special or first-year Statistics (*refer to section 7.2*). A comparison still has to be made between the mathematical content of these two courses and the school Mathematics and Mathematical Literacy curricula.

Intuitively it does seem that with the large amount of statistics included in the Mathematical Literacy course, a student with a strong Mathematical Literacy background could possibly cope with the first-year Statistics course. The first-year Mathematics Special course could be a problem for a Mathematical Literacy student because of the absence of algebra in Mathematical Literacy.

Hence, a student who performed well in Mathematical Literacy in Grades 10 to 12 should be able to cope with the mathematical topics included in first-year Business Management, Accounting and Economics. The main concern is the lack of Algebraic topics in Mathematical Literacy.

It could however be beneficial for these students to do a refresher course in the mathematical topics included in first-year Business Management, Accounting and Economics. Thus, they could become confident in doing the required mathematics in context. Specifically working with formulas and graphs should be emphasised. In this refresher course students could be introduced to any algebraic topics needed.

Mathematics Grades 10 to 12 does not contain as many number operations as Mathematical Literacy, but the statistics component of Mathematics should give the learner confidence to work with a large volume of calculations. There is a considerable emphasis on algebraic manipulations, so a student able to cope with Mathematics up to Grade 12 should not have any problem with the algebra contained in the first-year Business Management, Accounting and Economics courses.

A learner that did not perform well in Mathematics at Grade 12 level might need to be exposed to the mathematical topics in the first-year Business Management, Accounting and Economics courses in context.

To summarise, both Mathematics and Mathematical Literacy in the NSC contain enough of the required mathematical topics that are included in the first year of the BCom degree programme for a student who was able to cope well with Mathematics and Mathematical Literacy in the NSC to be comfortable with the mathematics included in their degree programme. Those areas where students might struggle could be covered by a support course for these students.

7.7 LECTURERS' VALIDATION OF THE RESEARCH FINDINGS

In the research process, three lecturers were interviewed. Lecturers were asked to answer an open question, namely what mathematics they felt the students needed to do their course. This information was used, with other sources of data, to determine the mathematical requirements of first-year BCom students.

In order to validate these results, the researcher returned to the same lecturers at this stage of the research (*refer to section 3.5*). During the interview, the lecturer was given the list of topics that were identified. The researcher briefly discussed the findings and the lecturer had the opportunity to give personal feedback.

All three lecturers were in total agreement with the topics that were identified. Even though all the topics were not relevant to each of the subjects, Accounting, Business Management and Economics, each lecturer could identify the topics on the list that were relevant to their subject.

Once the researcher had discussed the findings concerning Mathematics and Mathematical Literacy, the Economics lecturer expressed concern that first-year economics students would need some form of Algebra. The Accounting lecturer was encouraged by the number of calculations involved in

Mathematical Literacy, while the Business Management lecturer did not have strong feelings about Mathematics or Mathematical Literacy in the NSC.

The last chapter will deal with the summative conclusions to the study.

CHAPTER EIGHT: THE SUMMATIVE CONCLUSION

8.1 INTRODUCTION

Having completed the collection of data required for this study, and having analysed and interpreted them at the various phases of the research process, this chapter will now consider a reflective summary of the total process. Furthermore, final conclusions will be made and relevant topics for further research will be given.

8.2 THE OBJECTIVES OF THE STUDY

The first objective of this study was to identify the mathematical concepts which are essential for BCom students to be able to cope with the mathematical content included in the first year of the core subjects in their BCom degree programme. This was to ensure that no students are excluded from BCom studies, based on their high school mathematics mark (if in fact they should have been accepted), and secondly, to ensure that all students that are accepted for a BCom degree are equipped with sufficient mathematical knowledge to cope with the mathematical content in the first-year core subjects of their BCom degree programme.

Through the grounded theory process followed in the research the mathematical topics included in the first-year core subjects of the BCom degree programme were identified. The topics were categorised under the following headings:

- Basic Numeracy
- Ratio and Proportion
- Percentages
- Basic Algebra
- Formulas
- Tables and Graphs

These topics are expanded on in section 5.4.

The approach was very systematic and many sources of information were consulted.

The researcher is satisfied that the approach followed allowed for saturated categories to be generated. The verification thereof was further enhanced by the opinions of lecturers teaching the first-year core subjects of the BCom degree programme (*refer to section 7.6*).

Examples of how the identified mathematical content are used in context were highlighted (*refer to section 5.3*). Examples were selected from texts used in the first-year core subjects of the BCom degree programme. The examples given covered most of the mathematical topics identified in the study, but many examples showing similar uses of the mathematical concepts were omitted, since this would have led to unnecessary repetition, serving no purpose.

The identified content was then formulated into learning outcomes that could be used as the first step towards developing a support module for the BCom students (*refer to section 7.3*).

The second objective of this study was to determine whether Mathematics and Mathematical Literacy in the NSC would adequately prepare students for mathematical concepts included in the BCom studies. It was found that there was no definite yes or no answer to this question (*refer to section 7.5*).

A learner who passes Grade 12 Mathematics should be able to cope with the mathematical content, but may need assistance in applying that content in context. Nothing in the content material should be new to the learner.

A learner who passes Mathematical Literacy in Grade 12 well should be able to cope with most of the mathematical content in the first year of the BCom degree. The definition of “performed well” will be determined once the assessment of Mathematical Literacy becomes more clear.

The first cohort of Grade 12 learners to write the Mathematical Literacy examination will do so in November 2008, and thereafter the standard will be assessed.

Initial speculation would place “perform well” at about 70%.

It needs to be noted that the fact that there are no algebraic topics included in Mathematical Literacy is a point of concern. These topics should be included in a support module. The benefit of Mathematical Literacy is that a large number of calculations are included in the curriculum. This will prepare students well for a subject like Accounting where students cannot afford to be intimidated by large volumes of calculations.

8.3 THE LITERATURE REVIEW

As described in Chapter 3, grounded theory is not rooted in the existing literature, but rather grounded in the data themselves. The literature review in Chapter 2 was therefore not carried out with the purpose of dictating methods for conducting the research. The literature provided in Chapter 2 placed the study in context by describing the current state of higher education worldwide and more specifically in South Africa.

From the literature (*refer to section 2.5*), there is evidence that more and more students are struggling at higher education institutions. There is worldwide concern that students are not as prepared for higher education as they used to be previously. Universities are taking the responsibility to put various programmes in place in order to accommodate students that are struggling.

The final step in grounded theory research is to compare the emerging theory with the existing literature, and look for similarities and differences (*refer to section 3.3.3*).

Very little evidence was found of documented studies similar to the current study. Similar studies should be conducted, yet formal documentation was not easily found. Two similar studies are described in section 2.7. The current

research used a combination of the methods used in the studies discussed in section 2.7. Rather than consult only the lecturers, or mainly using the texts involved, the researcher decided to use a combination of these methods to ensure that theoretical saturation was reached.

8.4 THE RESEARCH METHODOLOGY

This research followed a grounded theory approach, where a grounded theory is one that is inductively derived from the study (*refer to Chapter 3*). It was a very effective approach to follow since the list of topics was developing as the research progressed. At every stage of the research the researcher was able to evaluate where there were still gaps and how to progress to the next phase of the research in an effort to reach saturation level.

What has developed during this research, which was not necessarily one of the aims, was that a framework has emerged for finding mathematical (or other) topics that should be included in specific studies. If any researcher repeats this study at his own institution, In order to determine whether the present cohort of learners passing Mathematics and Mathematical Literacy in Grade 12 will be able to cope with the mathematical content in their learning programmes, it is necessary for reliable, verifiable results to be obtained by further research.

8.5 TOWARDS CONCEPTUALISING A FRAMEWORK FOR A SUPPORT COURSE FOR BCOM STUDENTS

In the development of a support course to assist students with the mathematical content included in the first year of their BCom programme, attention needs to be given to more than just the learning content. The learning outcomes identified in this study could form the framework for the support course.

As seen in Chapter 2, topics like mathematics study skills would be an essential addition to such a course. The course would also have to aim at addressing students' fear of mathematics. (*Refer also to section 7.4.*)

The type of offering will depend on the resources available. The support course could be offered as a series of workshops before lectures commence. Alternatively, it could take on the form of a course being offered in concurrence with the first year of study. Similarly, deciding between self- study courses or offering lectures, will depend on the resources available.

Attention also needs to be given to the way the course will be assessed. Assessment forms an intricate part of any course and should be developed together with all the other aspects of the course. The course will also need to be evaluated each time it is offered. This is necessary in order to measure the effectiveness and to any identify areas needing improvement.

8.6 FURTHER STUDIES

8.6.1 Tracking Students

Follow-up research has already been planned to track the performance of the students entering first-year BCom from the NSC. The performance of these students in their BCom subjects will be compared with their results in Grade 12 Mathematics or Mathematical Literacy. Students will also be asked to fill in a questionnaire to determine their impression of the school curriculum that prepared them for university.

Once the performance of the first group of learners from the NSC has been determined, systems could be put into place to assist these students, if they need help. As this cohort will start higher education studies from 2009, this thesis has ideally laid the foundation to proceed with such research.

8.6.2 Beyond the conceptualision phase of a support course

A pilot course needs to be designed and implemented.

8.6.3 Comparing the pilot course with first-year responses

Once the pilot course has been implemented, the effectiveness of the course needs to be evaluated by considering the first-year students' responses.

8.6.4 Second-year BCom degree programmes

The main focus of this study was the content of first-year Business Management, Accounting and Economics at NMMU. These three subjects form the core of a general BCom degree. It would be of benefit to consider not only the first year, but also the further years of study in these subjects. This will give an indication of mathematical topics that are essential to support further BCom studies.

The reason the initial focus was on the first year, was to determine specific mathematical topics that BCom students are expected to know when they enter university.

With the change in the school syllabus to the NSC, universities will be compelled to develop new entry requirements for all degrees. In order to make informed decisions, attention needs to be given to the content of the degrees together with the content of the subjects in the NSC.

As part of the BCom degree, students still do some mathematical or statistical subjects (*refer to section 7.2*). Hence, the content of these subjects could be influenced if the content of the further years of Business Management, Accounting and Economics were to be analysed.

8.6.5 NMMU entry requirements

Current NMMU entry requirements state that students with Mathematical Literacy at Grade 12 will be accepted for a BCom degree programme if they get a mark of at least 70%. It is however stated that students may be required to do some form of mathematics intervention. The type of intervention and the nature of the intervention are not yet clear.

Using the support module that will be developed, students can be streamed to do either the complete course, or sections of the course based on their abilities. These abilities could possibly be determined by an entry test.

8.6.6 Different universities in South Africa

An interesting comparison would be between the first-year Business Management, Accounting and Economics contents of the different universities. This would show the different mathematical expectations at different universities, if any such differences existed.

An umbrella research could be considered where various researchers research their own institutions. Then they could then combine their findings and come up with a typical core course or auxiliary programmes for students who are at risk. Pooling the best expertise of each institution could create a support tool that should work very effectively.

In addition, to determine what intervention strategies are done at different universities might lead to areas of cooperation.

8.7 CLOSURE

Since 1994 there have been many changes in South Africa. We are attempting to rebuild a nation and to make effective changes to systems. The change in the school curriculum was intended to assist South African learners to become citizens able to meet the economical challenges of our country, especially those that need higher education graduates in commercial sciences.

The effectiveness of such changes will only be measured in time, but as a university we need to anticipate any problems that changes in curricula might bring. For this reason this research is critical for the time in which it has been done.

ADDENDUM A: QUESTIONNAIRE
Mathematical Requirements for Economic Sciences students.

Dear Lecturer

We request 10 minutes of your time to complete the following questionnaire.

The aim of this questionnaire is to determine which mathematical topics students in the Accounting, Economics and Business Management Departments need to be successful in their studies. Remember, we are not looking for the ideal student, but for the minimum requirements to pass.

For any queries please contact Marguerite Walton at X 2663.

Which of the following topics from the school Mathematics syllabus do you regard as ESSENTIAL, as RECOMMENDED, or as NOT NEEDED AT ALL as pre-knowledge for your course: (Mark the appropriate block)

Topic	**Essential**	**Recommended**	**Not needed**	**Unsure**
Basic Trigonometry				
Applications of Trigonometry (e.g. Sine and Cosine Rules)				
Graphs of Trigonometric Functions				
Trigonometric Equations and Identities				
Geometry				
Factorisation and Products of Algebraic Expressions				
Solutions of Linear and Quadratic Equations				
Remainder Theorem and Solutions of 3rd degree Polynomial Equations				
Systems of Equations (Solving simultaneous equations)				
Solving Linear Inequalities				
Linear Programming				

Topic	Essential	Recommended	Not needed	Unsure
Solving Quadratic Inequalities				
Exponents - Basic Properties and Applications				
Logarithms - Basic Properties and Applications				
Graphs of Exponential and Logarithmic Functions				
Differentiation - Basic Properties and Rules				
Differentiation - Advanced Applications				
Graphs of Linear and Quadratic Equations				
Absolute Values				
Graphs of Absolute Value Functions				
Interpreting of Graphs				
Formulas - substitution, manipulations and interpretation				
Percentages and Averages				
Ratio and Proportion				
Sequences and Series				
Areas and Volume				

Do you have any other comments, or other Mathematical topics that you expect your students to know?

_____

_____

_____

_____

_____

Name: _____ **Department:** _____

ADDENDUM B: CONTENT OF A SUPPORT MATHEMATICS MODULE FOR BCOM STUDENTS

With the aim to further expand on the learning outcomes stated in Chapter 7, clear descriptions and examples of the intended learning outcomes are provided here. This can form the basis of a support course, not a fully developed course as the examples only cover selected cases. To fully develop the support module, these descriptions of the learning outcomes need to be expanded further to include additional examples and explanations to assist the students in their understanding. Some of the content has been adapted from content of the Commercial Mathematics (Walton 2002), Foundation Mathematics (Snyders & Walton 2005) and Numerical Skills for Humanities (Hendricks 2004) offered in the Foundation Programme at NMMU. The authors, M Walton, MA Hendricks and AJM Snyders have given their permission for the use of selected sections from the courses.

Fundamental number operations

Real Numbers (Learning outcome 1.1)

Types of Real Numbers

Natural Numbers are the numbers 1; 2; 3; 4; … Denoted by □

Natural Numbers, including zero are the numbers

0; 1; 2; 3; … Denoted by □ $_0$

Integers are the numbers …; −3; −2; −1; 0; 1; 2; 3; … Denoted by □

Rational numbers: Are all numbers that can be expressed as a fraction of the form: $\frac{a}{b}$; where a and b are integers. Denoted by □

Irrational numbers: Numbers with an infinite non-recurring decimal form.

Real numbers are all rational and irrational numbers. Denoted by □

Any number that can be constructed from other numbers through addition, subtraction, multiplication, division, root extraction or raising to a power is a real number, but note:

- Division by 0 is undefined.
- An even root of a negative number, e.g. $\sqrt{-4}$, $\sqrt[4]{-12}$, is not a real number.
  (These are called <u>non-real</u> (imaginary) <u>numbers</u>.)

<u>Properties of real numbers</u> (where a, b, c and d are real numbers):

is <u>Commutative</u> under Addition:	a + b = b + a
is <u>Commutative</u> under Multiplication:	ab = ba
is <u>Associative</u> under Addition:	(a + b) + c = a + (b + c)
is <u>Associative</u> under Multiplication:	(ab)c = a(bc)

Multiplication in is <u>left Distributive</u> over addition or subtraction:

$a(b \pm c) = ab \pm ac$

Multiplication in is <u>right Distributive</u> over addition or subtraction:

$(a \pm b)c = ac \pm bc$

Zero is the <u>additive identity</u>: a + 0 = a and 0 + a = a

Every real number has an <u>additive inverse</u> or negative:

a + (-a) = 0 and (-a) + a = 0

Note: 0 is its own additive inverse.

Subtraction is the same as adding the additive inverse:

$a - b = a + (-b)$

The number 1 is the <u>multiplicative identity</u>:

$a \times 1 = a$ and $1 \times a = a$

Note: 1 is its own multiplicative inverse.

Every non-zero real number has a <u>multiplicative inverse</u>:

$a \times \frac{1}{a} = 1$ and $\frac{1}{a} \times a = 1$

Division is the same as multiplying with the multiplicative inverse:

$$a \div b = a \times \frac{1}{b}$$

<u>Rounding off</u> (Learning outcome 1.2)

In business, many calculations involve money. For this purpose it is important to be able to round off to two decimal places, in order to have an answer in terms of rands and cents.

If asked to round a number off to **two decimals**, follow the following steps:

1. Find the number in the third decimal place, (ignoring any numbers in later decimal places).
2. Decide whether the number is less than 5 or greater than or equal to 5.
3. a) If the third decimal is less than 5, simply rewrite the first two decimals and rounding off is complete.

   Example 1: 3.87296 rounded of to two decimals is 3.87

   b) If the third decimal is equal to 5 or greater than 5, then increase the second decimal by one.

   Example 2: 19.2378102 rounded off to two decimals is 19.24

A similar procedure can be followed for rounding off to any number of decimals places.

If asked to round a decimal number of to the nearest **whole number**, follow the following steps:

1. Find the number in the first decimal place.
2. Decide whether the number is less than 5 or greater than or equal to 5.
3. a) If the first decimal is less than 5, simply rewrite the whole number and rounding off is complete.

   Example 3: 45.162 rounded of to the nearest whole number is 45

   b) If the first decimal is equal to 5 or greater than 5, then increase the whole number by one.

   Example 4: 12.533 rounded off to the nearest whole number is 13

If asked to round a number of to the nearest **10**, follow the following steps:

1. Find the number in the “units” position.
2. Decide whether the number is less than 5 or greater than or equal to 5.
3. a) If the number is less than 5, simply replace the “units” number with a zero and rounding off is complete.

Example 5: 562 rounded of to the nearest 10 is 560

b) If the number is equal to 5 or greater than 5, then increase the value in the “tens” position by one, and replace the “units” number by zero.

Example 6: 128 rounded off to the nearest 10 is 130

A similar procedure can be followed for rounding off to hundreds, thousands, etc.

The symbol ≈ is used to show that two things are approximately equal. The symbol = implies that the one side of the equal sign is exactly equal to the other side of the equal sign.

It will be true to say that : 1.5899 **≈** 1.6,
but when asked to round 1.5899 off to one decimal place then:
1.5899 rounded off to one decimal place **=** 1.6.

Use these symbols correctly!

<u>Approximation</u> (Learning outcome 1.3)

Approximation involves either estimating the result of a mathematical calculation, or making an educated guess about the result of a mathematical calculation. Rounding off can be used to estimate answers for simple calculations, for example:

Example 1: Estimate the value of 23.4 + 19.8 + 0.3
23.2 + 19.8 + 0.3 ≈ 23 + 20 + 0 = 43

Example 2: Estimate the value of 96 x 31
96 x 31 ≈ 100 x 30 = 3000

Approximation techniques can be used to evaluate answers that have been obtained by using a calculator in a mathematical calculation.

Example 3: Which of the following could be possible answers (rounded off to a whole number) for 2.4 x 5.9:

a) 9 b) 18 c) 14

9 could not be possible since it is less than 2 x 5
18 is not possible since it is equal to 3 x 6
14 is a possibility since it is close to 2.5 x 6 (which is 15)

It is essential when doing calculations with a calculator that the answer obtained is evaluated. It is easy to make errors when typing into the calculator; hence it is an essential part of any calculation to evaluate the answer obtained.

Example 4: Here are some results obtained for calculations by using a calculator. Which of them are obviously incorrect?

a) $1\frac{1}{2}+3\frac{5}{6}+2\frac{2}{3}=5$ b) 120 x 45 = 1800 c) 25.4 + 18.7 – 12.4 = 41.7

d) $1\frac{1}{2}-3\frac{5}{6}+2\frac{1}{3}=0$:

a) $1\frac{1}{2}+3\frac{5}{6}+2\frac{2}{3}=5$ is not possible.
Consider the whole number parts, 1 + 3 + 2 which gives 6, so the answer needs to be larger than 6.

b) 120 x 45 = 1800 is not possible.

120 x 10 gives 1200, so 120 x 45 needs to give an answer that is more than four times 1200, and 1800 is not close.

c) 25.4 + 18.7 – 12.4 = 41.7 is not possible.

Consider again the whole number parts, 25 + 18 – 12 = 25 + 6 = 31, and the decimals would not be able to add another 10 to the answer.

d) $1\frac{1}{2}-3\frac{5}{6}+2\frac{1}{3}=0$ is possible.

<u>Fractions</u> (Learning outcome 1.4)

A **common fraction** is a real number written in the form: $\frac{a}{b}$;

where a is called the **numerator** and b is called the **denominator**.

2 ÷ 3 is written in common fraction form as $\frac{2}{3}$ $\frac{\leftarrow \text{ numerator}}{\leftarrow \text{ denominator}}$

A **proper fraction** has a value less than 1 (one), thus it has the numerator smaller than the denominator**.**

.

Examples: $\frac{1}{2}$; $\frac{3}{5}$; $\frac{23}{30}$

An **improper fraction** has a value greater than 1 (one), thus it has the numerator larger than the denominator**.**

Examples: $\frac{3}{2}$; $\frac{5}{3}$; $\frac{30}{23}$

A **mixed number** consists of a whole number and a fraction written together with the understanding that they are added together.

Examples: $3\frac{1}{4}$; $2\frac{5}{7}$; $12\frac{3}{5}$

<u>Converting improper fractions and mixed numbers</u>

Example 1: Convert $\frac{17}{5}$ to a mixed number.

Solution $\frac{17}{5}$ means **17 ÷ 5**

5 divides into **17** , **3** times and leaves a remainder of **2**

$$\therefore \frac{17}{5} = 3\frac{2}{5}$$

Example 2: Convert $2\frac{7}{8}$ to an improper fraction

Solution $2\frac{7}{8} = \frac{2 \times 8}{8} + \frac{7}{8} = \frac{16+7}{8} = \frac{23}{8}$

Simplification of fractions

To simplify a fraction to its simplest form, the numerator and denominator must be divided by the largest whole number which will divide both exactly.

Example 1: Reduce (Simplify) $\frac{12}{30}$

Solution $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$

It is not necessary to show the above working. Divide the numerator and denominator by the same number mentally as follows : $\frac{\overset{2}{\cancel{12}}}{\underset{5}{\cancel{30}}} = \frac{2}{5}$

A fraction can also be simplified by repeated steps if you do not at once see the largest number which can be divided exactly into both the numerator and denominator.

Example 2: Reduce (Simplify) $\frac{128}{288}$

Solution $\frac{128}{288} = \frac{128 \div 4}{288 \div 4} = \frac{32 \div 8}{72 \div 8} = \frac{4}{9}$

Addition and subtraction

If the denominators are the same, simply add the numerators together.

Example 1: $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{4}{5}$ **and** $\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$

The denominators are different with both numerators equal to 1.

Example 2:

$$\frac{1}{3}+\frac{1}{5}$$
$$=\frac{1\times 5}{3\times 5}+\frac{1\times 3}{3\times 5}$$
$$=\frac{5+3}{3\times 5}$$
$$=\frac{8}{15}$$

and

$$\frac{1}{5}-\frac{1}{3}$$
$$=\frac{1\times 3}{5\times 3}-\frac{1\times 5}{5\times 3}$$
$$=\frac{-2}{5\times 3}$$
$$=\frac{-2}{15}$$

If the denominators are different and both numerators not equal to 1, first make the denominators the same.

Example 3:

$$\frac{3}{5}+\frac{2}{3}=\frac{3\times 3+5\times 2}{5\times 3}=\frac{19}{15}$$

and

$$\frac{4}{5}-\frac{2}{3}=\frac{4\times 3-5\times 2}{5\times 3}=\frac{2}{15}$$

A mixed number first needs to be changed to an improper fraction.

Example 4:

$$2\frac{1}{4}+1\frac{2}{3}$$
$$=\frac{9}{4}+\frac{5}{3}$$
$$=\frac{9\times 3+4\times 5}{4\times 3}$$
$$=\frac{47}{12}$$

and

$$2\frac{1}{4}-1\frac{2}{3}$$
$$=\frac{9}{4}-\frac{5}{3}$$
$$=\frac{9\times 3-4\times 5}{4\times 3}$$
$$=\frac{7}{12}$$

Multiplication

The product of two fractions is found by multiplying the numerators and then multiplying the denominators. The product is, where possible, simplified.

Example 1: Simplify $\frac{2}{3}\times\frac{4}{5}$

Solution:

$$\frac{2}{3}\times\frac{4}{5}=\frac{2\times 4}{3\times 5}=\frac{8}{15}$$

Example 2: Simplify $\frac{2}{3} \times \frac{6}{7}$

Solution:

$$\frac{2}{3} \times \frac{6}{7}$$

$$= \frac{2 \times 6}{3 \times 7}$$

$$= \frac{12}{21} = \frac{4}{7}$$

or $\frac{2}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{7} = \frac{2 \times 2}{1 \times 7} = \frac{4}{7}$

To multiply mixed numbers first convert them to improper fractions.

Example 3 : Simplify $2\frac{1}{2} \times 1\frac{2}{5}$

Solution:

$$2\frac{1}{2} \times 1\frac{2}{5} = \frac{5}{2} \times \frac{7}{5} = \frac{7}{2}$$

Division

As explained in the section on Real Numbers, division is the same as multiplying with the multiplicative inverse.

Example: Simplify: $\frac{2}{3} \div \frac{1}{2}$

Solution $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$

<u>Decimal Numbers</u> (Learning outcome 1.5)

Decimal numbers and common fractions are simply different ways of expressing the same number (different notations).

When written as decimal numbers,

$$\frac{1}{10}\ ;\ \frac{2}{100}\ ;\ \frac{3}{1000}\ ;\ \frac{4}{1000}\ ;\ \frac{5}{10000}\ ;\ \text{etc}$$

become: 0.1 ; 0.02 ; 0.003 ; 0.000 4 ; 0.000 05 ; etc.

Note: These decimals are also known as **terminating decimals** as the number of digits is finite (terminating).

Decimal numbers where the digits are repeated are called **recurring decimals.** (Excluding the case where 0 is repeated, e.g. 5.2300000…)

Examples: 0. 1111 …… ; 1. 3333…. ; 0. 5 23 23 23 ……

Recurring decimals are often written with a dot or dash over the digit or digits that are repeated.

Examples: $0.\dot{1} = 0.111.....$ **;** $0.\overline{12} = 0.12121212...$ **;** $0.3\dot{2} = 0.32222....$

There are decimals that are neither terminating nor recurring (irrational numbers):

Examples:

0.01001000100001…. and 1.234567891011121314……

These two decimals follow a pattern, but they are not repeating patterns.

Conversions of decimal and common fractions

To write a decimal number as a common fraction, write it as a common fraction with a power of ten as a denominator, and then simplify it:

Example 1:

a) $0.12 = \frac{12}{100} = \frac{3}{25}$

b) $0.025 = \frac{25}{1000} = \frac{1}{40}$

To write a common fraction as a decimal number, first write it as a common fraction with a power of ten as a denominator:

Example 2:

a) $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$

b) $\frac{6}{25} = \frac{6 \times 4}{25 \times 4} = \frac{24}{100} = 0.24$

Another way to write a common fraction as a decimal number is to divide the numerator by the denominator:

Example 3:

a) $\frac{3}{4} = 3 \div 4 = 0.75$

b) $\frac{9}{8} = 9 \div 8 = 1.125$

Note: Writing a recurring decimal as a common fraction will not be considered at this stage.

Addition and Subtraction

Addition and subtraction of decimals is very similar to the addition and subtraction of whole numbers.

Remembering that digits of the same place value must be written in the same column. To get this right keep the decimal points one below the other.

Example 1:

a) 8.271 + 95.35 =

```
   8 . 2 7 1
+ 9 5 . 3 5 0
 103 . 6 2 1
```

b) 8.42 – 3.243 =

```
  8 . 4 2 0
- 3 . 2 4 3
  5 . 1 7 7
```

Multiplication and division with powers of 10

When multiplying a decimal number with a power of 10, the resulting number should be larger than the original number by that factor of 10, hence the decimal point move to the right, as many times as there are factors of 10.

Example 1:

a) 23.36 x 10 = 233. 6

b) 13.037 x 100 = 1303 .7

c) 1.3406 x 1000 = 1340.6

d) 23.52 x 1000 = 23 520

When dividing a decimal number with a power of 10, the resulting number should be smaller than the original number by that factor of 10, hence the decimal point move to the left, as many times as there are factors of 10.

Example 2:

a) 23.36 ÷ 10 = 2.336

b) 13.037 ÷ 100 = 0.13037

c) 2345.39 ÷ 1000 = 2.34539

<u>Percentages</u> (Learning outcome 1.6)

We have already seen that the same (rational) number can be expressed as a fraction or a decimal number. In this section a third way of writing numbers, namely **percentages** will be discussed.

A **percentage** is a common fraction with a denominator of 100**,**

Example: $\frac{23}{100}$ is written as **23% ,** and this is read as **"23 percent".**

<u>Conversions</u>

To convert a decimal number or a fraction to a percentage, multiply by 1 (the multiplicative identity) in the form $\frac{100}{100}$ = 100% (Where the percentage symbol, %, means $\frac{1}{100}$)

Example 1:	Convert $\frac{4}{5}$ to a percentage	
Solution:	$\frac{4}{5} \times 100\% = 80\%$	(multiply by 100%)
Example 2:	Convert $\frac{31}{25}$ to a percentage	
Solution:	$\frac{31}{25} \times 100\% = 124\%$	(multiply by 100%)
Example 3:	Convert 1.38 to a percentage	
Solution:	$1.38 \times 100\% = 138\%$	(multiply by 100%)

To convert a percentage to a fraction, write the percentage symbol, %, as "$\frac{1}{100}$":

Example 4: Convert 15% to a common fraction
Solution: $15\% = \frac{15}{100} = \frac{3}{20}$

To convert a percentage to a decimal number, use the percentage symbol to mean “divided by 100”:

Example 5: Convert 57% and 13.7**%** to a decimal number

Solution: $57\% = \frac{57}{100} = 0.57$ and

$13.7\% = \frac{13.7}{100} = 0.137$

<u>Finding a percentage of a given amount</u>

To find the percentage of an amount, multiply the amount with the percentage in the form of a fraction with denominator 100 or a decimal number.

Example 6: Find 10% of 70

Method 1: $\frac{10}{100} \times 70 = 7$

Method 2: 0.1 x 70 = 7

Example 7: Find 8.5% of 120

Method 1: $\frac{8.5}{100} \times 120 = 10.2$

Method 2: 0.085 x 120 = 10.2

Some percentages should be easy to calculate without a calculator, for example 10%, 25%, $33\frac{1}{3}$%, 50%, 75% and 100%.

Hints:

To find 10% of a number; divide the number by 10

To find 25% of a number; divide the number by 4

To find $33\frac{1}{3}$% of a number; divide the number by 3

To find 50% of a number; divide the number by 2

To find 75% of a number; divide the number by 4 and multiply by 3 (Find three-quarters of the number)

100% of a number is simply the number itself.

Increasing and decreasing quantities

To increase a quantity by a percentage, add the percentage of the quantity to the quantity itself. This implies adding a percentage of a quantity to the whole quantity (100%).

Example 1: Increase R80 by 10%

Method 1:
10% of R80 = R80 x 0.1 = R8

therefore R80 + R8 = R88

Method 2: (Preferred)
To increase R80 by 10%, you are trying to find 110% (100% + 10%) of R80

$$80 \times \frac{110}{100} = \text{R } 88$$

To decrease a quantity by a percentage, subtract the percentage of the quantity from the quantity itself. This implies subtracting a percentage of a quantity from the whole quantity (100%).

Example 2: Decrease R1100 by 12%:

Method 1:
12% of R1100 = R1100 x 0.12 = R132

therefore R1100 - R132 = R968

Method 2: (Preferred)
To decrease R 1100 by 12%, you are trying to find 88% (100% - 12%) of R1100

R1100 x 0.88 = R968

Finding the percentage increase or decrease

The formula that is generally used to find the percentage change is:

Percentage Increase (decrease) =

$$\frac{\text{amount increased (decreased)}}{\text{orginal amount}} \times 100\%,$$

Note that there are special cases where percentage change is calculated differently, but then it will be stated as such.

Example 1: The price of an article increased from R20 to R25. Find the percentage increase.
Solution: Amount Increase = R25 - R20 = R5

$$\text{Percentage increase} = \frac{5}{20} \times 100\% = 25\%$$

Example 2: The price of an article decreased from R200 to R180. Find the percentage decrease.
Solution: Amount Decrease = R200 – R180 = R20

$$\text{Percentage decrease} = \frac{20}{200} \times 100\% = 10\%$$

Finding the original amount

In the previous section, when increasing an amount by for example 20%, view the original number as 100%; hence the new number is then 120% of the original number.

In the case where you are given the number after a percentage change and asked to find the original number, you are then trying to find the 100%. Consider the following examples:

Example 1: 20% of a number is 60, what is the number?

View the original number as 100%
Reduction (60) is 20%
Hence 60 is the 20% and we are looking for the 100%:

$$60 \div 20 \times 100 = 300$$

$$\text{or } 60 \times \frac{100}{20} = 300$$

Test: 20% of 300 is: $300 \times \frac{20}{100} = 60$

Example 2: Following a pay increase of 10%, an accountant earns R19 800. What was the accountants' salary before the increase?

Original salary: 100%
New salary: 110% = R19 800

$$\text{Hence original salary} = \text{R}19800 \times \frac{100}{110} = \text{R}18000$$

Comparing numbers (Learning outcome 1.7)

We have discussed different types of Real numbers, and seen how to convert between fractions, decimals and percentages. It is important to be able to order numbers according to size.

Example 1:
Arrange the following numbers 9 ; – 4 ; 5 ; – 7 ; – 1 ; 3 in ascending order (smallest to largest)

Solution:
– 7; – 4; – 1; 3; 5; 9

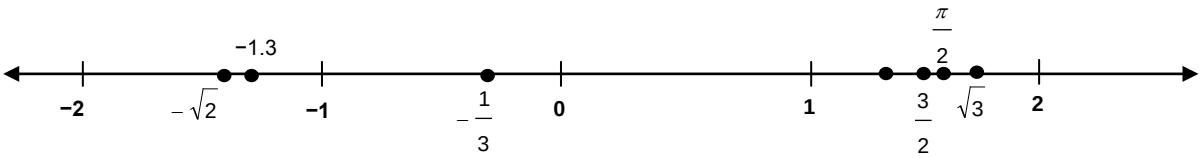
When arranging numbers, it is helpful if all the numbers are converted to the same format (often decimals are the easiest to use).

Example 2: Place the following numbers on a number line:

$\frac{1}{2}$; -1.3; $-\sqrt{2}$; $-\frac{1}{3}$; $\frac{\pi}{2}$; $\sqrt{3}$; $\frac{3}{2}$

Solution: Firstly, rewrite the numbers in decimal form (rounded off to two decimal places where needed):

$\frac{1}{2} = 0.5$; -1.3; $-\sqrt{2} = -1.41$; $-\frac{1}{3} = -0.33$; $\frac{\pi}{2} = 1.57$; $\sqrt{3} = 1.73$; $\frac{3}{2} = 1.5$



It is important to be comfortable with the different notations used for large numbers. Firstly, you need to know the meaning of the terms one, ten, hundred, thousand, ten thousand, hundred thousand, million and billion.

One	1
Ten	10
Hundred	100
Thousand	1 000
Ten thousand	10 000
Hundred thousand	100 000
Million	1 000 000
Billion	1 000 000 000

Example 3: Consider the following listed vehicles prices:

Make	Price (R'000)
Type A	140
Type B	125
Type C	85
Type D	79,5

The unit for price is not given as rand, but as R'000. What this means is "thousands of rands". Hence a Type A vehicle actually costs R140 000, while a Type D vehicle costs R79 500.

Ratio, Proportion and Unit Conversions

Ratio (Learning outcome 2.1)

Ratio is a way in which the sizes of two or more numbers are compared, e.g. a ratio of 2 : 1 between two numbers indicates that the first number is double the second number while a ratio of 2 : 3 : 6 indicates that for every 2 units of the first number, the second number has 3 units, and that the third number is double the second number. The ratio between two numbers can also be written as a fraction, e.g. 2 : 3 is the same as $\frac{2}{3}$.

A ratio in its simplest form satisfies the following conditions.

- All the numbers in the ratio are integers.
- The numbers in the ratio are relatively prime (do not have any common factor).

Example 1: The ratio between 5 and 20 is 5 : 20 or $\frac{5}{5} : \frac{20}{5}$, i.e. 1 : 4 in its simplest form.

Example 2: To write $\frac{3}{8} : \frac{9}{12}$ in its simplest form, first multiply by 24 to change the numbers to integers, i.e.

$$\frac{3}{8} \times \frac{24}{1} : \frac{9}{12} \times \frac{24}{1} = 9 : 18 = 1 : 2.$$

Dividing a value in a ratio

To divide a value in a ratio, say 1:2, it implies for every ‘1’ that the first person gets, the second person gets ‘2’. This means that the dividing will happen in groups of ‘3’ (1 + 2). This in turn implies that of the total value, the first person will receive ‘1 out of the 3’, or $\frac{1}{3}$, while the second person will get ‘2 out of the 3’, or $\frac{2}{3}$.

These fractions will then be used to calculate the separate values. Note that the ratio between the separate values will then be the same as the original ratio.

Example 1: To divide an amount, say R36 in a ratio of 4 : 5 between two people means that for every R4 the first one gets, the second one gets R5. The money can hence be divided in groups of R9 at a time, where person A get 4 out of the 9 while person B gets 5 out of the 9.

Hence A gets: $\frac{4}{9} \times \frac{36}{1}$ = R16 and B gets: $\frac{5}{9} \times \frac{36}{1}$ = R20.

Test: R16 + R20 = R36 and 16 : 20 = 4 : 5

<u>Proportion</u> (Learning outcome 2.2)

A <u>proportion</u> is formed if two ratios are equal to each other,

e.g. 2 : 3 = 4 : 6 or $\frac{2}{3} = \frac{4}{6}$.

Two quantities are <u>directly proportional</u> if the ratio between their values stays the same, irrespective of the values. If one of the quantities is multiplied by a factor, then the other one is increased by the same factor.

Example 1: Apples are sold at 25c each.
The number of apples and the total cost are directly proportional. If the number of apples are doubled, so is the total cost.

To solve direct proportion questions, simply set the two ratios equal to each other and solve for the variable.

Example 2: If 25 articles cost R450, what would the cost of 17 articles be?
Write this information in the form of a ratio,
25:450 = 17 : x,

Then $\frac{25}{450} = \frac{17}{x} \Rightarrow x = \text{R306}$

Two quantities are <u>indirectly proportional</u> if the ratio of the one to the inverse of the other is a constant. If one quantity is increased by a factor, the other quantity is decreassed by the same factor. This implies, that if the one

quantity is multiplied by a constant, the other quantity is multiplied by the inverse or reciprocal of the constant.

Example 3: It takes 2 people 10 days to complete a task.
4 people working at the same tempo will complete the task in 5 days. The number of people and the time it takes to complete the task are indirectly proportional.

To solve indirect proportion questions set the information in the form of two ratios equal to each other, where the first value is in ratio with the inverse of the second value.

Example 4: From a piece of timber 12 pieces each 180 mm long can be cut. How many pieces each of length 240 mm could be cut from the same length of timber?

$$12 : \frac{1}{180} = x : \frac{1}{240}$$

Then x = 9 pieces

There are many different methods to solve direct and indirect proportion problems, these are only examples of one method.

<u>Unit Conversions</u> (Learning outcome 2.3)

Direct proportion can be used to solve unit conversion problems. There are times when easier methods are available. When converting between Rand and cent, the easiest would be to simply multiply or divide by 100. This is because R1 = 100c.

Example 1: Convert R0.32 to cents: R0.32 x 100 = 32c

Convert 567c to Rands: 567 ÷ 100 = R5.67

There are many conversions that could be encountered, but most of them are simply multiplication or division calculations. When converting from one currency to another it might be easier to use a formal direct proportion method.

Example 2: On 8 August 2006, the following information was found on www.news24.com:

R/$ = 6.8

R/£ = 12.96

R/€ = 8.74

Convert R14 to Dollar: R/$ = 6.8 $\Rightarrow$ ratio Rand: Dollar = 6.8 : 1 = 14 : x

$$\text{Hence } \frac{6.8}{1} = \frac{14}{x} \Rightarrow x = \$2.06$$

Convert €100 to Rand: R/€ = 8.74 $\Rightarrow$ ratio Rand to Euro = 8.74 : 1 = x : 100

$$\text{Hence } \frac{8.74}{1} : \frac{x}{100} \Rightarrow x = R874$$

It is important when converting currency to evaluate the answer to see if it makes sense. It is easy to make a mistake with the calculation, hence checking the validity of the answer is essential.

Application of Percentages

In the previous section percentages were introduced. The following topics were explained:

- What is a percentage
- Converting between percentages, decimals and fractions
- Finding a percentage of a given amount
- Increasing or decreasing an amount by a percentage
- Finding the percentage increase or decrease
- Finding the original amount

These techniques will now be applied to situations that you may encounter in business.

Profit and Loss (Learning outcome 3.1)

The difference between what an article costs to purchase or to make and what it is sold for is the profit or loss. That is, if the cost price (CP) is less than the

selling price (SP) a profit (P) is made and if the cost price (CP) is more than the selling price (SP) then a loss (L) is made.

$$P = SP - CP \text{ or } L = CP - SP$$

Example 1: The cost of 15 articles is R180. The articles are sold for R25 each. Calculate the total profit.

Total CP = R180
Total SP = R25 x 15 = R375
Hence Total Profit = R375 – R180 = R195

Example 2: The cost of 15 articles is R180. The articles are sold for R25 each. Calculate the profit per item.

Total CP = R180
CP per item = R180 ÷ 15 = R12
SP per item = R25
Hence Profit per item = R25 – R12 = R13

Profits and losses are usually expressed in terms of a percentage of the cost price. This is the only case considered at the moment. Profit percentage is also refered to as **Mark Up** percentage.

Example 3: The profit on an article that cost R110 is R20. What is the profit percentage?

$$\text{Profit percentage} = \frac{\text{profit}}{\text{cost price}} \times 100\% = \frac{20}{110} \times 100\% = 18.18\%$$

Example 4: What is the profit percentage if an article's cost price is R215 and the selling price is R250?

Profit = SP − CP = 250 – 215 = R35

$$\text{Profit percentage} = \frac{P}{CP} \times 100\% = \frac{35}{215} \times 100\% = 16.28\%$$

Example 5: What is the loss percentage if an article that cost R28.50 is sold for R25?

Loss = CP − SP = 28.50 – 25 = R3.50

$$\text{Loss percentage} = \frac{3.50}{28.50} \times 100\% = 12.28\ \%$$

In the case where the profit (or mark-up) percentage is given, you may be required to calculate the Cost Price (CP), Selling Price (SP) or Profit (P). A similar procedure will be followed to that used to find the original amount if a percentage increase or decrease is given.

Example 6: An article sold for R33 and yields a profit of 10%. What was the cost price?

CP = 100%
P = 10%
SP = 110% and R33

We have the 110% and we are looking for the 100%, so

$$CP = R33 \times \frac{100}{110} = R30$$

Example 7: An article is marked up by 25%. It is sold for R350. What profit is made on the article?

CP = 100%
P = 25%
SP = 125% and R350

We have the 125% and we are looking for the 25%, so

$$P = R350 \times \frac{25}{125} = R70$$

Transactions between Manufacturer, Wholesaler, Retailer and Consumer

Consider the following trade chain:

Manufacturer sells to wholesaler, wholesaler sells to retailer, retailer sells to consumer. Everyone in this chain wants to make a profit!

Remember: The Manufacturer's SP is the Wholesalers CP,
the Wholesalers SP is the Retailers CP.
the consumer buys it from the Retailer at their SP.

Example 1: A manufacturer makes an article at a cost of R50. He sells it to the wholesaler at a profit of 10%, who sells it to the retailer for a 20% profit, who in turn sells it to you for a 15% profit. What did you pay for the article?

Manufacturer: CP = 100% and R50
P = 10%
SP = 110%

$$\text{Hence SP} = R50 \times \frac{110}{100} = R55$$

Wholesaler: CP = 100% and R55
P = 20%
SP = 120%

$$\text{Hence SP} = R55 \times \frac{120}{100} = R66$$

Retailer: CP = 100% and R66
P = 15%
SP = 115%

$$\text{Hence SP} = R66 \times \frac{115}{100} = R75.90$$

You pay R75.90

Note that a given percentage is always a percentage of a specific value; this is why the different profit percentages cannot simply be added together!

Example 2: A manufacturer sells to the wholesaler at a profit of 10%, who sells to the retailer for a 20% profit, who in turn sells to you for a 15% profit.. You paid R45.54 for an article. What did it cost the manufacturer to make?

Retailer: CP = 100%
P = 15%
SP = 115% and R45.54

Hence CP = $R45.54 \times \frac{100}{115} = R39.60$

Wholesaler: CP = 100%
P = 20%
SP = 120% and R39.60

Hence CP = $R39.60 \times \frac{100}{120} = R33$

Manufacturer: CP = 100%
P = 10%
SP = 110% and R 33

Hence CP = $R33 \times \frac{100}{110} = R30$

The article cost the manufacturer R30 to make.

<u>**Discount**</u> (Learning outcome 3.2)

Discount is an allowance given to the customer to pay less than what is actually owed on purchases or services. The different kinds of discounts, cash-, trade-, and bulk discounts, all reduce the cost of purchases.

Example 1: After receiving a discount of 10% on an article you pay R207. What was the marked price?

Marked price: MP = 100%
Discount : D = 10%
Selling price : SP = 90% and R207

So to find MP: $R207 \times \frac{100}{90} = R230$

Test: 10% discount on R230 $\Rightarrow R\,230 \times \frac{90}{100} = R207$

The abbreviations used are:

Cost Price: CP

Profit: P

Marked Price: MP

Discount: D

Selling Price: SP

Note: MP - D = SP and SP + D = MP

The retailer purchases an article (CP) with the intention to sell it to a customer, in order to make a profit (P). The article is marked at a specific price (MP). The retailer then gives the customer a discount (D) on the purchase, and the amount the customer pays is the actual selling price (SP).

Example 2: At what price must an article costing R100 be marked at if after allowing a discount of $16\frac{2}{3}$ % on the marked price, a profit of 50% on cost is made?

CP = 100% and R100		MP = 100%
P = 50%		D = $16\frac{2}{3}$ %
SP = 150%	and also	SP = $83\frac{1}{3}$ %

Please note that SP on both sides have the same value, but in different situations different percentages are attached to them. First calculate the SP on the left side (since this is where an amount is given, the CP) then use that SP amount in the right side.

So SP = R100 $\times \frac{150}{100}$ = R150

Now

MP = R150 x $\frac{100}{83\frac{1}{3}}$ = R180

Example 3: An article marked at 15% above cost is sold for R133,40 after allowing a discount of 20%. What is the article's cost price?

CP = 100%	MP = 100 %
Mark-up = 15%	D = 20%
MP = 115%	SP = 80% and R133.40

So MP = $133.40 \times \frac{100}{80}$ = R166.75

Now

CP = $166.75 \times \frac{100}{115}$ = R145

In examples 2 and 3, the retailer wanted to make a specific profit on the Cost Price (CP), hence there is a relationship between CP, P and SP.

In example 4, the retailer marks an article at a specific percentage above cost, so the profit made after discount will be less than the mark-up percentage.

Example 4: At what price must an article costing R80 be marked at if after allowing a 25% discount off marked price, a profit of 75% on cost is required?

CP = 100% and R80	MP = 100%
P = 75%	D = 25%
SP = 175%	SP = 75%

So SP = R80 $\times \frac{175}{100}$ = R140

Now

MP = R150 $\times \frac{100}{75}$ = R186.67

<u>**Commission**</u> (Learning outcome 3.3)

Some people work to sell other people's goods, and so earn a part of the selling price as renumeration. This earning is called commission.

Example 1: A car salesman sells a car for R75 950 and earns a commission of $3\frac{1}{2}$%. What was the commission?

75 950 x $\frac{3\frac{1}{2}}{100}$ = R2 658.25

<u>Value Added Tax (VAT)</u> (Learning outcome 3.4)

Value Added Tax is an indirect tax charged on the spending by people or businesses. If a business is registered for VAT, it may claim back the VAT that it has paid to suppliers.

The VAT that a business pays to suppliers is called Input VAT, while the VAT that a business charges its customers is called Output VAT. (Output VAT is usually more in a business than Input VAT). The difference between the Output VAT and the Input VAT, is the amount that is payable to the South African Revenue Service (SARS).

The VAT percentage is currently 14%, and is included in the selling price. If the sales amount is given, and the VAT amount needs to be calculated, the 14% VAT is included in the sales amount so, sales is seen as 114%, of which 14% is VAT.

Example 1: A Manufacturer purchased raw materials from someone who is not registered for VAT for R50. The wholesaler bought a finished product from the manufacturer for R300. The wholesaler sold the product to the retailer for R500, who in turn sold it to the consumer for R750.

What amounts must the manufacturer, wholesaler and retailer pay to SARS?

Manufacturer:
CP: R50
Input VAT: 0
SP: R300
Output VAT: $R300 \times \frac{14}{114} = R36.84$

Amount payable to SARS: Output VAT – Input VAT = R36.84 – 0 = R36.84

Wholesaler:
CP: R300
Input VAT: R36.84
SP: R500
Output VAT: $R500 \times \frac{14}{114} = R61.40$

Amount payable to SARS: Output VAT – Input VAT = R61.40 − R36.84 = R24.56

Retailer:
CP: R500
Input VAT: R61.40
SP: R750
Output VAT: $R750 \times \frac{14}{114} = R92.11$

Amount payable to SARS: Output VAT – Input VAT = R92.11 – R61.40 = R30.71

Depreciation (Learning outcome 3.5)

Straight Line Method:

In the straight line method of depreciation, a fixed amount or percentage of the assets' cost price is written off at the end of the accounting period each year for the life of the asset. The amount written off each year, is called depreciation or annual depreciation. The total amount that the asset has been depreciated at any time is called the accumulated depreciation. The book value of the asset is the cost price minus the accumulated depreciation.

The first example of the straight line method is when an equal (fixed) amount is allocated to each accounting period over the life of the asset.

Example 1:
Your company buys a car on 1/1/2006 for R50 000, and you expect it to have a trade-in value of R27 500 on 31/12/2010. On the straight line method, what will the annual depreciation be?

Amount to be written of to depreciation

= R50 000 – R27 500
= R22 500

Number of years = 5

Annual depreciation = $\frac{22500}{5}$ = R4 500

Depreciation table:

Date	Cost Price	Annual Depreciation	Accumulated Depreciation	Book value
1/1/2006	50 000	-	-	50 000
31/12/2006	50 000	4 500	4 500	45 500
31/12/2007	50 000	4 500	9 000	41 000
31/12/2008	50 000	4 500	13 500	36 500
31/12/2009	50 000	4 500	18 000	32 000
31/12/2010	50 000	4 500	22 500	27 500

The next example of the straight line method is when a fixed percentage of the Cost Price is allocated to each accounting period over the life of the asset.

Example 2: Suppose a company has an accounting period from 1 January to 31 December. The company buys a truck on 1/1/99 for R25 000. Depreciation is provided for at 20% per year on cost.

On 31/12/99 depreciation will be:

25 000 x 20% = 25 000 x 0.2 = R5 000 Thus the book value at 1/1/00 is

The depreciation table will look as follows:

Date	Cost Price	Annual Depreciation	Accumulated Depreciation	Book value
1/1/99	25000	-	-	25000
31/12/99	25000	5000	5000	20000
31/12/2000	25000	5000	10000	15000
31/12/2001	25000	5000	15000	10000
31/12/2002	25000	5000	20000	5000
31/12/2003	25000	5000	25000	0

The last example of the straight line method is when an asset is bought during the accounting period. When an asset is purchased during the accounting period, the depreciation for the first year must be adjusted for that fraction of the year that the asset is owned.

Example 3: Suppose a company has an accounting period from 1 January to 31 December. The company buys a piece of machinery on 1 July 2005 for R60 000. Depreciation is provided for at 25% on cost price (per year). What would the accumulated depreciation be by 31/12/2007?

For the year ending 31/12/2005, the asset cannot be depreciation over the full year, only the fraction of the year that the asset was in possession.

From 1 July to 31 December is 6 months, so the asset was owned for 6 months out of 12.

That is $\frac{6}{12}$ or $\frac{1}{2}$ of the year.

Now, depreciation for the year ending 31/12/2005:

60 000 x $\frac{1}{2}$ x 25% = 60 000 x $\frac{1}{2}$ x 0.25 = R7 500.

On 31/12/2005 accumulated depreciation totals R7 500.

Depreciation for the years ending 31/12/2006 and 31/12/2007 are as follows:

60 000 x 0.25 = R15 000 per year.

Accumulated depreciation on 31/12/2006 is 7 500 + 15 000 = R22 500.

Accumulated depreciation on 31/12/2007 is 22 500 + 15 000 = R37 500.

Book value at 1/1/2008 is R60 000 – R37 500 = R22 500.

Reducing Balance Method:

With the reducing balance method of depreciation, a fixed percentage of the assets' book value is written of at the end of the accounting period.

The first example is for an asset that was purchased at the beginning of the accounting period.

Example 1: Suppose your company buys a motor vehicle for R80 000 on 1/1/2003. The accounting period is from 1 January to 31 December. Depreciation is 20% per year on the reducing balance method.

1/1/2003 Cost Price	
31/12/2003 Depreciation = 80 000 x 0.2 :	16 000
1/1/2004 Book Value	
31/12/2004 Depreciation = 64 000 x 0.2 :	12 800
1/1/2005 Book Value	
31/12/2005 Depreciation = 51 200 x0.2:	10 240
1/1/2006 Book Value	
31/12/2006 Depreciation = 40 960 x 0.2 :	8 192

After 4 years the book value of the vehicle is R32 768 and the accumulated depreciation is R47 232.

When an asset is purchased during the accounting period, the depreciation for the first year must be adjusted for that fraction of the year that the asset is owned.

Example 2: A company has an accounting period from 1 January to 31 December. You purchase a piece of machinery on 1 April 2004 for R10 000. Depreciation is provided for at 25% per year on book value.

Note: "on book value" implies the reducing balance method.

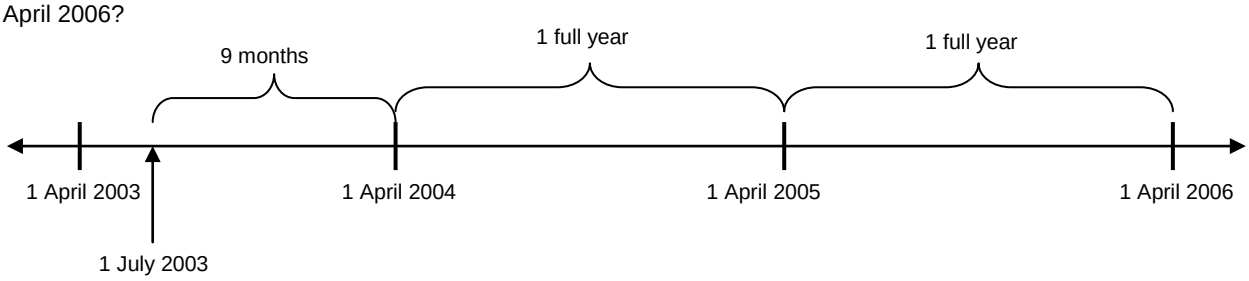
What is the accumulated depreciation on 31/12/2006?

1/4/2004	Cost Price		
31/12/2004	Depreciation:	8 125	10 000 x 0.25 x $\frac{9}{1}$
31/12/2005	Depreciation:	6 093.75	8 125 x 0.2
31/12/2006	Depreciation:	4 570.31	6093.75 x 0.2

Book value on 1/1/07 is R4570.31 and accumulated depreciation is R5429.69.

In all the examples so far, the accounting period was from 1 January to 31 December. This is not always the case for companies. In the cases where the accounting period starts during a year, and not in January, special attention needs to be given when calculating the number of months for which an asset has been owned. A number line might assist in the calculation.

Example 3: A company has an accounting period from 1 April to 31 March. On 1 July 2003 a vehicle was purchased for R85 000. Depreciation is provided for at 20% of book value. What was the vehicle's book value on 1 April 2006?



The first depreciation calculation will be on 31 March 2004, and calculated as follows:

$$R85\ 000 \times 0.2 \times \frac{9}{12} = R12\ 750$$

Thereafter the depreciation will be calculated as 20% on book value for the next two full years.

The depreciation table will then look as follows:

Date	**Cost Price**	**Annual Depreciation**	**Accumulated Depreciation**	**Book value**
1/7/2003	85000	-	-	85000
31/3/2004	85000	12750	12750	72250
31/3/2005	85000	14450	27200	57800
31/3/2006	85000	11560	38760	46240

Hence the vehicle's book value on 1 April 2006 is R46 240.

Elementary Algebra

A variable is a letter of the alphabet normally (x,y etc) that is used as a place holder for some number. This is done:

(i) If the actual value of the number is unknown or

(ii) To indicate a property that is valid for any number value of the variable.

Exponents (Learning outcome 4.1)

If a number is multiplied by itself a number of times, use exponents to write it in an easier way.

Example 1: $2 \times 2 \times 2 \times 2 = 2^4$

The following definitions form the basis for exponents:

Definitions

- $a^n = a.a.a\ldots\ldots$ (n factors) $n \in \mathbb{N}$, $a \in \mathbb{R}$.
- $a^0 = 1$ if $a \neq 0$, $a \in \mathbb{R}$.
- $a^{-n} = \dfrac{1}{a^n}$ if $n \in \mathbb{N}$, $a \neq 0$, $a \in \mathbb{R}$.
- $\sqrt{a} = b \Leftrightarrow a = b^2$ for $a \geq 0$.
- $\sqrt[n]{a} = b \Leftrightarrow a = b^n$ for $n \in \mathbb{N}$ and $a \geq 0$ if n is even.
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ if $n \in \mathbb{N}$, $a \geq 0$ if n is even.
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ if $n \in \mathbb{N}$, $a \geq 0$ if n is even.

Note that the symbol "$\in$" means "element of"

From these definitions rules can be deduced, (we will not be considering the proofs), some of these rules are:

Rules

For a, b $\in$ □ ; a, b ≠ 0; m, n $\in$ □ ,

- $a^m \cdot a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$
- $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$
- $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
- $\sqrt[n]{a^n} = a$ if n is odd.
- $\sqrt[n]{a^n} = |a|$ if n is even.

These definitions and rules are used to simplify expressions that contain exponents.

Example 1:

a) $x^4 \times x^3 = x^7$

b) $\dfrac{x^5}{x^2} = x^3$

c) $(x^4)^5 = x^{20}$

d) $(2x)^3 = 2^3x^3 = 8x$

e) $\dfrac{\sqrt{45}}{\sqrt{5}} = \sqrt{\dfrac{45}{5}} = \sqrt{9} = 3$

<u>Algebraic Expressions</u> (Learning outcome 4.2)

A <u>term</u> is a combination of numerals and variables involving multiplication and divisions. Different terms are separated by addition or subtraction.

Examples of terms:

8y; 3; $4x^3$; $5x^2y^3$; 2(x + y); $\dfrac{4}{x^3}$.

An <u>algebraic expression</u> is a collection of terms combined by addition, subtraction or both. (An algebraic expression is an expression in which variables are used to represent numbers.)

An algebraic expression containing only one term is called a <u>monomial</u>, one with two terms a <u>binomial</u> and one with three terms a <u>trinomial</u>.

Examples of monomials:	**Examples of binomials:**	**Examples of trinomials:**
4	$2x^3 - 4x$	$3x^5 - 2x^4 + 8x$
$\frac{1}{2}x^7$	$x - 4$	$2x^4 - 5x - 1$

A <u>polynomial</u> is an expression of the form
$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$ when $a_0, a_1, a_2, \ldots, a_n$ are any constants, $a_n \neq 0$ and $n \in \mathbb{N}_0$.

The <u>degree of a polynomial</u> is the highest power of the variable in the polynomial.

Examples of polynomials:

$4x^5 + 3x^4 - 2x^3 + x^2 - 3x + 5$,	has degree 5
$\frac{1}{3}x^2 + x - 1$,	has degree 2
x^7,	has degree 7
2,	has degree 0
$\sqrt{3} \cdot x + 1$,	has degree 1

<u>Addition and subtraction of Polynomials</u>

To add or subtract polynomials all the like terms (exactly the same variables with the same exponents) are added or subtracted.

Example 1:

a) $(x^3 + 4x^2 – 3x + 1) + (4x^2 – 2x + 3)$
$= x^3 + (4x^2 + 4x^2) + (-3x – 2x) + (1 + 3)$
$=x^3 + 8x^2 – 5x + 4$

b) $(x^3 + 4x^2 – 3x + 1) – (4x^2 – 2x + 3)$
$= x^3 + (4x^2 – 4x^2) + (-3x –(-2x)) + (1 – 3)$
$= x^3 – x – 2$

Multiplication of Polynomials

To multiply two polynomials (removing the brackets) each term of the first polynomial is multiplied with each term of the second polynomial after which all the like terms are added.

Example 1:

a) $4x(2x + 3y) = 8x^2 +12xy$

b) $(x + 4)(3x – 5) = 3x^2 - 5x + 12x - 20 = 3x^2 + 7x – 20$

c) $(x^3 + 4x^2 – 3x + 1)(4x^2 – 2x + 3)$
$= 4x^5 – 2x^4 + 3x^3 + 16x^4 – 8x^3 + 12x^2 – 12x^3 + 6x^2 – 9x + 4x^2 – 2x + 3$
$= 4x^5 + 14x^4 – 17x^3 + 22x^2 – 11x + 3$

d) $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$

e) $(2x – 4)(2x + 4) = 4x^2 – 8x + 8x - 16 = 4x^2 - 16$

Take note of the following Special Products:

$$(A + B)^2 = A^2 + 2AB + B^2$$

Example 2: a) $(x + 5)^2 = x^2 + 10x +25$
b) $(3x – 4y)^2 = 9x^2 - 24xy + 16y^2$

$$(A – B)(A + B) = A^2 – B^2$$

Example 3: a) $(x – 3)(x + 3) = x^2 – 9$
b) $(2x - 7y)(2x + 7y) = 4x^2 – 49y^2$

Division of Polynomials

To divide a polynomial by a monomial, divide the monomial into each term of the polynomial.

Example 1:
a) $(6x^3 + 9x^2 - 3) \div 3x$

$$= \frac{6x^3 + 9x^2 - 3}{3x} = \frac{6x^3}{3x} + \frac{9x^2}{3x} - \frac{3}{3x} = 2x^2 + 3x - \frac{1}{x}$$

Factorisation of Polynomials

Factorisation of a polynomial is the process of writing the polynomial as the product of polynomials, (each of which has a degree lower than the original polynomial.)

- Factorisation of polynomials can be compared to factorisation of numbers, e.g. 20 is factorised as 5×4 or $5 \times 2 \times 2$.
- Factorisation is the opposite of finding the product of polynomials.
- The result of factorisation is one term.
- Factorisation can be tested by finding the product of the factors. This should give the original polynomial.
- It can be proved that any polynomial of degree higher than two can be factorised.
- Not all quadratic expressions (2nd degree polynomials) can be factorised rationally. A non-factorisable quadratic expression is called irreducible.

The Methods of Factorising that will be considered are common factor, difference between squares and quadratic polynomials.

a) Common Factor:

When taking out a common factor, find the number that is a factor of all the terms, as well as the combination of variables that are present in each term.

Example 1:

a) $4x - 6x^3 = 2x(2 - 3x^2)$

b) $15w^3 - 45w^2 + 30w^6 = 15w^2 (w - 3 + 2w^4)$

c) $a^2 b^3 c^2 - ab^2 c^2 + 2a^3 b^2 c^4 = ab^2 c^2 (ab - 1 + 2a^2 c^2)$

d) $(x - 3)(2x + 1) - (x - 3) = (x - 3)(2x + 1 - 1) = (x - 3)(2x)$

b) Difference between squares:

For any A, B,

$A^2 - B^2 = (A - B)(A + B)$

Example 1:

a) $y^2 - 49 = (y - 7)(y + 7)$

b) $9x^2 - z^4 = (3x - z^2)(3x + z^2)$

c) $25u^2 - 49v^2 p^4 = (5u - 7vp^2)(5u + 7vp^2)$

d) $x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1)$

Note: The sum of two squares is irreducible (rationally)

c) Quadratic trinomials:

An expression of the form $ax^2 + bxy + cy^2$ is called a quadratic trinomial and some of these can be factorised rationally as the product of two linear expressions.

Example 1: The following trinomials can be factorised,

a) $x^2 + 7x + 10 = (x + 2)(x + 5)$

b) $x^2 - 7x + 10 = (x - 2)(x - 5)$

c) $x^2 + 3x - 10 = (x - 2)(x + 5)$

d) $x^2 - 3x - 10 = (x + 2)(x - 5)$

e) $15p^2 - 23p + 6 = (3p - 1)(5p - 6)$

Note: Since the last term in the trinomial is positive the signs in the brackets are the same (negative because the middle term of the trinomial is negative)

f) $3a^2 - 16a - 12 = (3a + 2)(a - 6)$

Note: Since the last term in the trinomial is negative the signs in the brackets differ.

<u>**Linear Equations with one variable**</u> (Learning outcome 4.3)

An <u>equation</u> is formed if two algebraic expressions are joined by an equal sign.

For any real numbers A, B and C.

(a) $A = B \Leftrightarrow A + C = B + C$

(b) $A = B \Leftrightarrow A - C = B - C$

(c) $A = B \Leftrightarrow AC = BC$ if $C \neq 0$.

(d) $A = B \Leftrightarrow \frac{A}{C} = \frac{B}{C}$ if $C \neq 0$.

Note that the symbol "$\Leftrightarrow$" means "if and only if".

A <u>linear equation</u> is an equation in which the highest power of any term is one.

Examples of linear equations:
$2x + 3 = 4$ and $2x + 4y = 5$

NB $2x + 4xy = 6$ is <u>not</u> a linear equation.

The properties (a) – (d) above can be used to solve linear equations by adding, subtracting, multiplying or dividing with the same number on both sides of the equation.

NB! Whatever is done on one side of the equation must also be done on the otherside. [This is called balancing an equation.]

A <u>solution (root)</u> of an equation is the value(s) for the variable(s) that will satisfy the equation.

A linear equation can have either 1 solution, no solutions or all real numbers can be solutions. (As seen in the examples that follow).

Example 1:

a)
$$8x + 2 = 6$$
$$8x + 2 - 2 = 6 - 2$$
$$8x = 4 \quad \Rightarrow \quad x = \frac{1}{2}$$

b)
$$\frac{1}{3} - \frac{4}{x} = \frac{3}{2x} \qquad \text{Conditions: } x \neq 0$$

$\times$ (6x): $1(2x) - 4(6) = 3(3)$
$$2x - 24 = 9$$
$$2x = 33$$
$$x = \frac{33}{2}$$

c)
$$\frac{5}{x-4} = \frac{6}{x-3} \qquad \text{Conditions: } x \neq 3;\ 4$$

$\times$ (x – 4)(x – 3): $5(x - 3) = 6(x - 4)$
$$5x - 15 = 6x - 24$$
$$-15 + 24 = 6x - 5x$$
$$9 = x$$

d)
$$2x - \frac{5x}{3} = \frac{x}{3} + 2$$
$\times$ 3: $6x - 5x = x + 6$
$$x = x + 6$$
$$0 = 6$$
$\therefore$ No solution for x

e)
$$4(x + 2) = 2(2x + 1) + 6$$
$$4x + 8 = 4x + 8$$
$$0 = 0$$
$\therefore x \in \square$

Another example for solving a linear equation is from Parkin (2005:77):

Example 2: Calculating market equilibrium, assuming demand and supply curves are straight lines:
The demand for ice-cream cones is $P = 800 – 2Q_D$.
The supply of ice-cream cones is $P = 200 + 1Q_S$.
The price of a cone is expressed in cents, and the quantities are expressed in cones per day.

To find the equilibrium price (P^*) and equilibrium quantity (Q^*), substitute Q^* for Q_D and Q_S and P^* for P. That is,

$P^* = 800 – 2Q^*$
$P^* = 200 + 1Q^*$
Now solve for Q^*:
$800 – 2Q^* = 200 + 1Q^*$
$600 = 3Q^*$
$Q^* = 200.$

And
$P^* = 800 – 2(200)$
$= 400.$

The equilibrium price is 400 cents a cone and the equilibrium quantity is 200 cones per day.

Formulas

A formula is an equation containing letters of the alphabet instead of numbers. Each letter represents some physical quantity.

The letter representing the quantity that can be calculated directly appears on its own on the left hand side of the equal sign and is called the SUBJECT of the formula.

Substitution into formulas (Learning outcome 5.1)

If the values of all the quantities, except the subject, are known, the formula can be used to find the corresponding value of the subject.

Example 1: Consider the formula for compound interest: $F_V = P_V (1 + r)^n$

a) Find the future value, F_V, if present value, P_V = R1000, interest rate, r = 8% and time, n = 3 years.
Then: $F_V = P_V (1 + r)^n = 1000(1+0.08)^3$ = R1259.71

b) Find r if: $F_v = 1000$, $P_V = 800$, n = 2
Then: $F_V = P_V (1 + r)^n$
So, $1000 = 800(1 + r)^2$

$$\frac{1000}{800} = (1+r)^2$$

$$\sqrt{1.25} = \sqrt{(1+r)^2}$$

$$1.11 = 1 + r$$

$$\Rightarrow r = 0.11 \text{ or } 11\%$$

<u>Changing the subject of formulas</u> (Learning outcome 5.2)

If the values of the subject and all but one of the other quantities are known, the unknown quantity can be calculated by rewriting the formula to make the unknown quantity the subject of the formula. (Which could be an easier method than example 1b) above if the same calculation needs to be made for different sets of values.)

The following procedure can be used to change the subject of a formula:

- Identify the quantity that must become the new subject of the formula.
- Remove all fractions in the formula by multiplying by the LCM of the denominators.
- Rewrite the formula with all the terms containing the new subject on one side of the equation.
- Take out the new subject as a common factor (if necessary).

“Transpose” all other quantities on the same side of the equation as the subject, starting with the quantity that is mathematically “furthest away” from the new subject by using opposite operations:

a) If it is added, subtract on both sides
b) If it is subtracted, add on both sides
c) If it is multiplied, divide on both sides
d) If it is divided, multiply on both sides
e) If it is raised to a power, find a root on both sides
f) If there is a root, raise both sides to the same power

Example 1: Consider the formula for compound interest: $F_V = P_V (1 + r)^n$

Make r the subject of the formula:
$F_V = P_V (1 + r)^n$

$$\frac{F_V}{P_V} = (1+r)^n$$

$$\sqrt[n]{\frac{F_V}{P_V}} = 1 + r$$

$$\Rightarrow \sqrt[n]{\frac{F_V}{P_V}} - 1 = r$$

$$\text{or } r = \sqrt[n]{\frac{F_V}{P_V}} - 1$$

<u>Relationships between variables in a formula</u> (Learning outcome 5.3)

All formulas indicate some relationship between variables. The formulas that will be considered all contain an equal (=) sign. This implies that the equation is balanced. If changes are made to any of the values of the variables in the equation, and the equation must still remain true, then there needs to be another change in the formula to account for the first change.

Example 1: From Parkin (2005:480), the following variables and equation are given:

Consumption expenditure (C)	Investments (I)	Government purchases (G)
Net taxes (T)	Exports (X)	Imports (M)
Net exports (X – M)	Aggregate income (Y)	Private savings (S)

Then: Y = C + I + G + X – M.

What would happen to Aggregate income if Government purchases decrease?
If G decreases then Y must decrease.

What would happen to Aggregate income if Imports increase?
If M increases then Y would decrease.

Example 2: From Parkin (2005:690), the multiplier is given as:

Multiplier = $\frac{1}{1-\text{MPC}}$; where MPC is the Marginal propensity to consume. (Note that $0 \leq \text{MPC} \leq 1$)

What would happen to the size of the multiplier if MPC increases?

If MPC increases then (1 – MPC) will decrease.
If (1 – MPC) decreases then $\frac{1}{1-\text{MPC}}$ will increase.
Hence if MPC increases, the multiplier will increase.

<u>**Interest Calculations**</u> (Learning outcome 5.4)

Interest calculations involve using percentages as well as formulas to do calculations.

Notation:

P_V: Present Value

I: Interest (in Rand)

F_V: Future Value

r: Interest Rate (%)

n: number of years

<u>*Simple Interest*</u>

Simple interest is calculated on the initial amount borrowed/lent/invested. Calculating simple interest is a normal percentage calculation.

$I = P_V \cdot r \cdot n$

$F_V = P_V + P_V\, r \cdot n$ or $F_V = P_V\,(1 + r \cdot n)$

Example 1: Invest R10 000 for 3 year at 15% interest per year.
Calculate the total interest received and the future value of the investment after the 3 years.

I = R10 000 x 15% x 3
= R10 000 x 0.15 x 3
= R4 500

F_V = R10 000 + R4 500 = R14 500

Example 2: Invest R10 000 for only 9 months at 15% simple interest. Find the value of the investment after the 9 months.

$F_V = P_V\,(1 + r\,n)$

$$= R\,10000\left(1 + 0.15 \times \frac{9}{12}\right)$$

= R11 125

Interest could be calculated over a number of days, but this needs to be stated specifically in the question. If months are given, do not use days.

Example 3:

a) Invest R15 000 at the bank for 200 days at 15% interest. Calculate the total interest.

$$I = R15\,000 \times 15\% \times \frac{200}{365} = R1232.88$$

b) Loan R15 000 from the bank from 7 April to 28 September at 20% simple interest. Calculate the total amount owed at the end of the period.

Days in April: (30 – 7)	23
Days in May	31
Days in June	30
Days in July	31
Days in August	31
Days in September	28
	174

$$I = 15\,000 \times \frac{20}{100} \times \frac{174}{365} = R1\,430.34$$

$F_V = 15\,000 + 1\,430.34$
$= R16\,430.34$

Note: The first day (7 April) is not added in, but the last day (28 September) is.
In a leap year February has 29 days and then there are 366 days in the year.

Compound Interest

After an interest period, interest is added to the principal amount (P_V) so that interest for the new period is calculated on the new principal amount.

The formula for compound interest is:

$F_V = P_V (1 + r)^n$ after year n

Notation: For year 1, let interest be I_1, and future value be F_{V_1}. Similarly for years 2 and 3, let interest be I_2 and I_3, and future value be F_{V_2} and F_{V_3}.

Example 1:

Invest R5 000 for 3 years at a compound interest rate of 15% per year.

$F_V = 5\,000 (1+0.15)^3 = R7\,604.38$

Calculating P_V, r or n

The following needs to be noted:

- If the interest is not stated as simple or compound, compound will always be assumed.
- The formula cannot be used for the full time period if the interest rate changes in that time.
- If you are required to calculate the value of P_V, r or n, either change the subject of the formulas, so that you have formulas to find P_V, r and n, or substitute into the original formula and calculate the unknown.

Simple interest:

$$P_V = \frac{I}{r \cdot n} \quad \text{or} \quad P_V = \frac{F_V}{1 + r \cdot n}$$

$$r = \frac{I}{P_V n} \quad \text{or} \quad r = \left(\frac{F_V}{P_V} - 1\right) \div n \quad \text{or} \quad r = \frac{F_V - P_V}{P_V n}$$

$$n = \frac{I}{P_V n} \quad \text{or} \quad n = \left(\frac{F_V}{P_V} - 1\right) \div r \quad \text{or} \quad n = \frac{F_V - P_V}{P_V r}$$

Compound interest:

$$P_V = \frac{F_V}{(1 + r)^n}$$

$$r = \sqrt[n]{\frac{F_V}{P_V}} - 1$$

The case for finding n in compound interest will not be considered as for that knowledge of logarithms is needed.

Example 1:

a) Invest money at the bank for 2 years at a simple interest rate of 20%. Receive R2 000 interest, how much money was invested?

n = 2 r = 20% I = R2 000, Find P_V

$$P_V = \frac{I}{rn} = \frac{2000}{0.2 \times 2} = \text{R5 000}$$

b) Loan R1 500 from the bank at a simple interest rate of 15%. After a certain time period the value of the loan is R2 175. What was the time period?

P_V = R1500 r = 15% F_V = R2175, Find n

$$n = \frac{F_v - P_v}{P_v r} = \frac{2175 - 1500}{1500 \times 0.15} = 3 \text{ years}$$

c) Invest R28 000 at a compound interest rate. After 3 years the investment is worth R40 500, what is the interest rate?

$$r = \sqrt[n]{\frac{F_V}{P_V}} - 1 = \sqrt[3]{\frac{40500}{28000}} - 1 = 0.13 \quad \text{or} \quad 13\%$$

Changes in interest rate or amount invested

In cases where the interest rate or the amount borrowed/lent/invested changes during the time period, adjustments need to be made. Calculations can only be done over a time period where those values do not change.

Example 1: Loan R10 000 for 3 years at 15% interest a year. After the second year the interest rate increases by 1%. What is the total amount you have to pay back after the 3 years?

The interest rate remains constant for the first two years, so

$$F_{V_2} = \text{R}10000(1 + 0.15)^2 = \text{R}13225$$

For the last year, the interest rate increases to 16%, note that the investment remains on 16% for 1 year only (the last year of the investment).

$$F_{V_3} = \text{R}13225(1 + 0.16)^1 = \text{R}15341$$

Interest calculated Semi-Annually, Quarterly and Monthly

In all the examples up to now the assumption has been made that interest is calculated only once a year (annually), but in reality this is very rarely the case. Now, consider cases where interest is compounded Semi-Annually, Quarterly and Monthly.

Normally an annual compound interest rate is given, so the interest rate and the time period need to be adjusted.
Assume an annual interest rate of 14% is given. If compounding occurs semi-annually, an interest rate of 14% per year, would become a rate of 7% for half a year.

Assume a 3 year investment is made. If compounding occurs semi-annually, interest would be calculated twice a year, so interest would be calculated six times in the three years, n = 6.

Example 1:

Find the future value of R10 000 invested for 3 year at 14 % compound interest if compounding occurs:

a) annually
b) semi-annually
c) monthly

a) r = 0.14 (14 %) and n = 3, so
$F_V = P_V (1 + r)^n = 10\,000\,(1.14)^3 = R14\,815.44$

b) Interest is 14% per year, so for half a year it will be: $r = \frac{0.14}{2} = 0.07$ (7%)
and interest gets calculated semi-annually, which means twice a year, hence $n = 2 \times 3 = 6$

$F_V = P_V (1 + r)^n = 10\,000\,(1.07)^6 = R\,15\,007.30$

c) Interest is 14% per year, so per month it would be: $r = \frac{0.14}{12} = 0.0117$
Interest gets calculated monthly, which means 12 times per year, hence $n = 12 \times 3 = 36$
$F_v = 10\,000\,(1.0117)^{36} = R15\,200.68$

Tables and Graphs

Using tables (Learning outcome 6.1)

Tables are used to summarise data in an orderly and structured manner. It is important to be able to read relevant information from a table and to interpret information given in the form of a table.

Consider the following table:

Province	**1994 Total population ('000)**	**Population groups (1994) ('000)**				**Distribution of GDP (%)**	**1990 GDP per capita according to 1990 prices (rand)**
		Black	**Asians**	**Coloured**	**White**		
Gauteng	7 915,0	5 175,8	164,0	304,2	2 271,0	39,0	R13 800
Northern Cape	776,7	242,9	1,9	402,9	129,0	2,7	8 500
Western Cape	3 771,9	627,2	31,6	2 184,2	928,9	12,2	8 300
Mpumalanga	3 334,1	3 003,4	11,4	17,1	312,2	9,5	8 200
Free State	2 903,4	2 435,9	1,0	76,1	390,4	5,8	5 000
KwaZulu-Natal	8 881,5	7 187,9	886,9	124,4	682,3	14,6	4 200
North-West	3 646,4	3 287,4	9,0	43,1	306,9	5,4	3 700
Eastern Cape	7 057,5	6 196,6	15,8	449,8	395,3	7,4	2 700
Northern Province	4 823,6	4 622,2	5,8	8,2	187,4	3,4	1 600
Total	43 120,1	32 779,3	1 127,4	3 610,0	5 603,4	-	R5 990

Table 5.1 Total population, population distribution and gross domestic product (GDP) (Marx et al. 1998:65)

Example 1: From the table given above, answer the following questions

a) What was the total population of the Eastern Cape in 1994?
b) Which province made the lowest contribution to the GDP?
c) Which province had the higest GDP per capita in 1990?

Solutions:

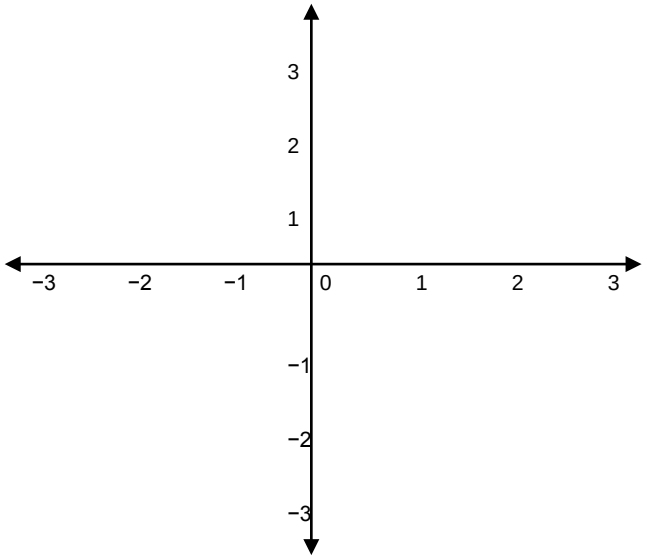
a) 7 057 500, just over 7 million.
(Note the answer is not 7 057,5, since the unit of population is ('000), which means thousands.)

b) Northern Cape contributed 2.7% of the total GDP.

c) Gauteng, with R13 800 per capita.

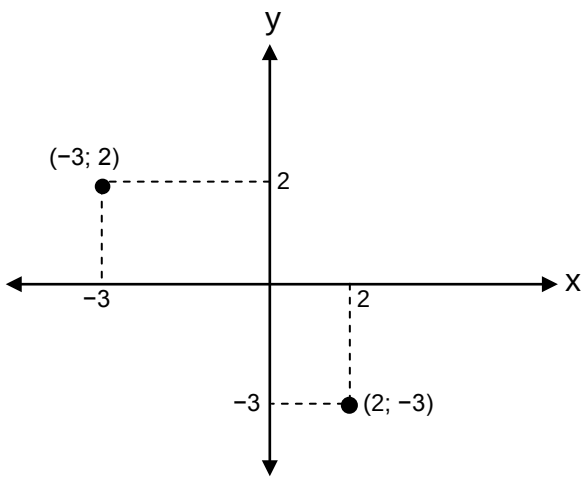
Sketching a graph from given points (Learning outcome 6.2)

The Cartesian Plane is a system of two perpendicular number lines intersecting at the 0-points of each.



The point of intersection of the number lines is called the origin.

An ordered pair is a set of two numbers written in a specific order. The ordered pair (2; −3) has 2 as first co-ordinate (x – value) and −3 as second co-ordinate (y − value) and is not the same as the ordered pair (−3; 2). Both ordered pairs are represented on the Cartesian Plane below:



A graph is defined as the set of all ordered pairs that satisfy a given condition. There are times when some of the points rather than the equation are given. To sketch the graph, simply plot and connect the points.

Example 1: Parkin (2005:61) gives the following table:

Original demand schedule CD burner $300		New demand schedule CD burner $100	
Price	Quantity (millions of	Price	Quantity (millions of

The following observation could be made from the graph and the table:

- The graph illustrates an increase in demand. When demand increases, the demand curve shifts rightward and the quantity demanded is greater at each and every price (Parkin 2005:60).

At the original price of the CD burner, an increase in price leads to a decrease in quantity demand, this is a move on the original demand curve.

Straight Line Graphs (Learning outcome 6.3)

The graph of any function with an equation of the form ax + by = c where a, b and c are any constants is a straight line.

Example 1: The graphs of all the following are straight lines

$3x + 2y = 6$	$y = -2x + 5$
$y = 4$	$2x + y = 0$
$y = 4x$	

Generally if the variables x and y are used, x is the independent variable, while y is the dependant variable. This means that the value of y depends on the value of x, or y is a function of x.

To sketch a straight line, the independent variable (x) will be represented on the horizontal axis, while the dependant variable (y) will be on the vertical axis. To draw the graph of a straight line at least two points on the line are needed. Since the x-coordinate of any point on the y-axis is 0 and the y-coordinate of any point on the x-axis is 0, the easiest points to find on the graph of a straight line are the intercepts on the axes:

For x-intercept: let y = 0 and find x.

For y-intercept: let x = 0 and find y.

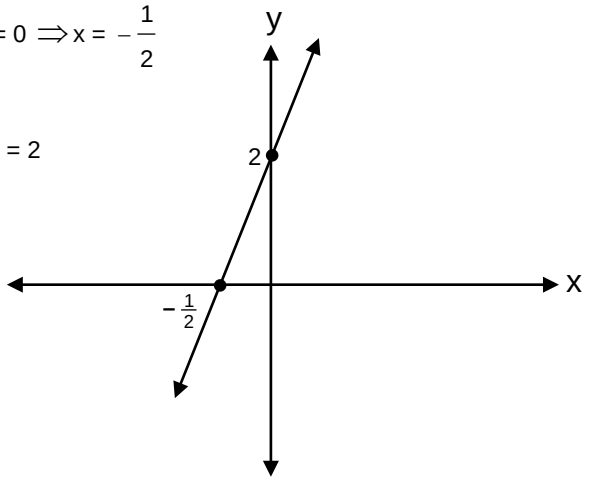
Example 2: Sketch the graph of y = 4x + 2

x-intercept: let y = 0 $\Rightarrow$ 4x + 2 = 0 $\Rightarrow$ $x = -\frac{1}{2}$

$\left(-\frac{1}{2};0\right)$

y-intercept: let x = 0 $\Rightarrow$ y = 0 + 2 = 2

(0; 2)



In the cases where the graph intersects with the origin, both the x- and y-intercepts are 0, another point is needed to draw the graph that satisfies the given equation. A point on the line can be found by choosing an arbitrary x-value and calculating the corresponding y-value by substituting into the equation or by choosing any y-value and calculating the corresponding x-value.

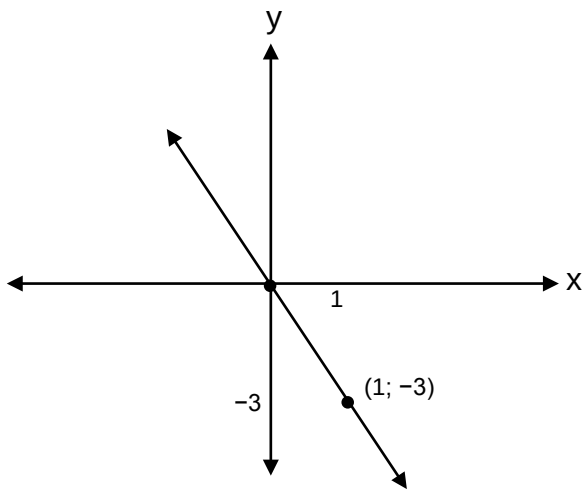
Example 3: Sketch the graph of $2y = -6x$

x-intercept: let $y = 0 \Rightarrow x = 0$

y-intercept: let $x = 0 \Rightarrow y = 0$,

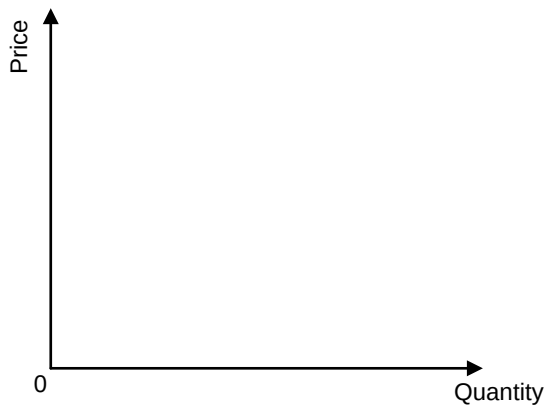
hence the graph goes through the origin (0; 0).

Another point: choose any value for x, say $x = 1 \Rightarrow 2y = -6 \Rightarrow y = -3$. So the point (1; −3) is on the graph.



Often in business variables other than x and y are used. Although the variables are different, similar rules would apply for sketching the graphs. It must also be noted that often these varialbes can only take on positive values, so the graph would only be represented in the area where both variables are positive.

In Economics, Price is always on the vertical axis, while Quantity is always on the horizontal axis. This was decided since there are different schools of thought on which is the independent variable. Both of these variables can only be positive, so the Cartesian Plane used would look as follows:

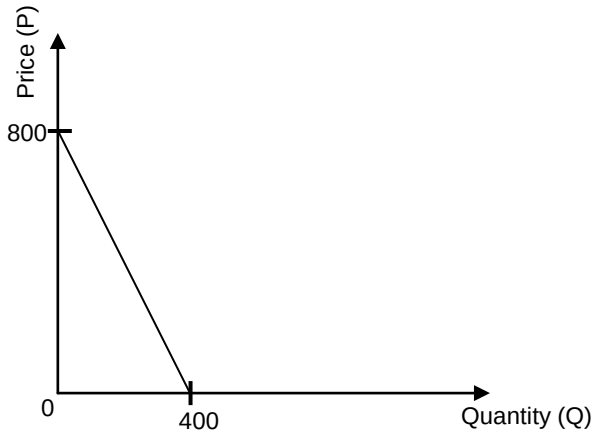


Example 4: Parkin (2005:77) gives an example where the demand for ice-cream cones is $P = 800 - 2Q_D$.

To sketch this demand function:

Q-intercept: let $P = 0 \Rightarrow Q = 400$

P-intercept: let $Q = 0 \Rightarrow P = 800$,

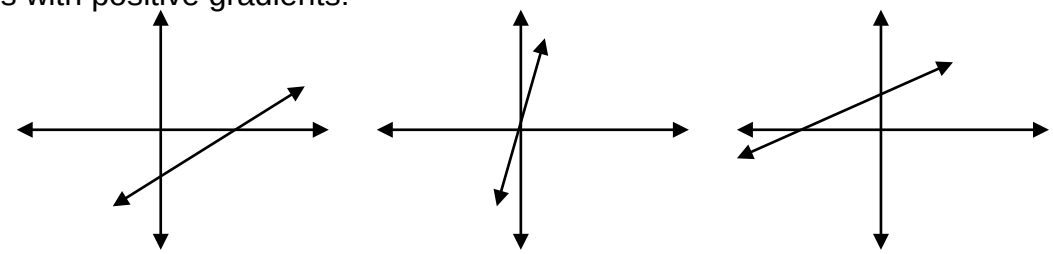


For any two points $(x_1; y_1)$ and $(x_2; y_2)$ on a given straight line, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is a constant. This constant is called the <u>gradient</u> or <u>slope</u> of the line.

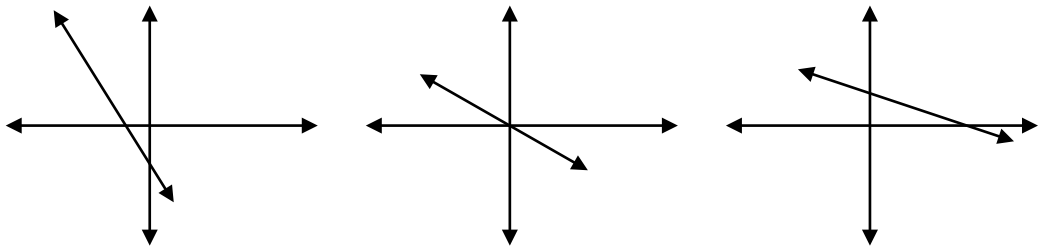
- The gradient of a line is an indication of the "steepness" of the line. The larger the value of the gradient, the "steeper" the line.

- The gradient of a straight line gives an indication of the direction of movement of the line.

Lines with positive gradients:



Lines with negative gradients:



- Two lines are parallel if and only if they have the same gradient and two lines are perpendicular if the product of the two gradients equals – 1.

- The gradient of a straight line is also interpreted as the rate of change of the y -value relative to a change in the x-value.

- If the equation of a straight line is written in the form y = mx + c then the value of m is the same as the gradient of the line and the value of c the same as the y-intercept of the graph.

Example 5: Consider again the example from Parkin (2005:77) where the demand for ice-cream cones is $P = 800 – 2Q_D$.
The gradient of the line is −2, which implies that the rate of change of price with regard to quantity is −2, that is, for every increase of one unit of quantity, the price will decrease by 2 units.

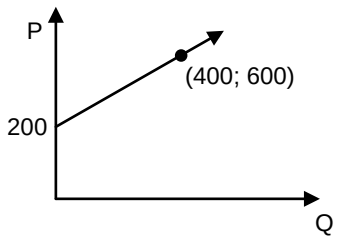
Suppose the gradient m of a line and the co-ordinates of one point $(x_1; y_1)$ on the line are known. For any other point (x, y) on the line $\frac{y - y_1}{x - x_1}$ must be equal

to the gradient of the line, i.e. $\frac{y - y_1}{x - x_1} = m$ or $y - y_1 = m(x - x_1)$. This is called the **point-gradient form** of the equation of a line.

Example 6:
a) Find the equation of a straight line with gradient 4 and passing through the point (1; −2):

$y - y_1 = m(x - x_1)$
$\Rightarrow y - (-2) = 4(x - 1)$
$y = 4x - 6$

b) Find the equation of the following straight line:



Point (0; 200) and (400; 600) are on the line, so

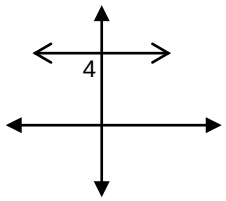
Gradient = $\frac{600 - 200}{400 - 0} = 1$

$P - 200 = 1(Q - 0)$
$P = Q + 200$

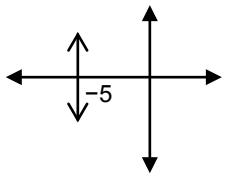
The equation of any line parallel to the x-axis has the form y = c and the equation any line parallel to the y-axis has the form x = c.

Example 7: Sketch the graphs of
a) y = 4
b) x = −5
c) TFC = 50 (Total fixed cost)

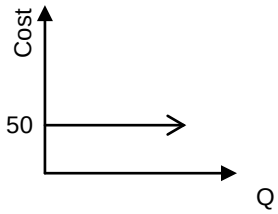
a)



b)



c)



Knowledge on straight lines and gradients can be applied to a business context. Consider the following break-even graph:

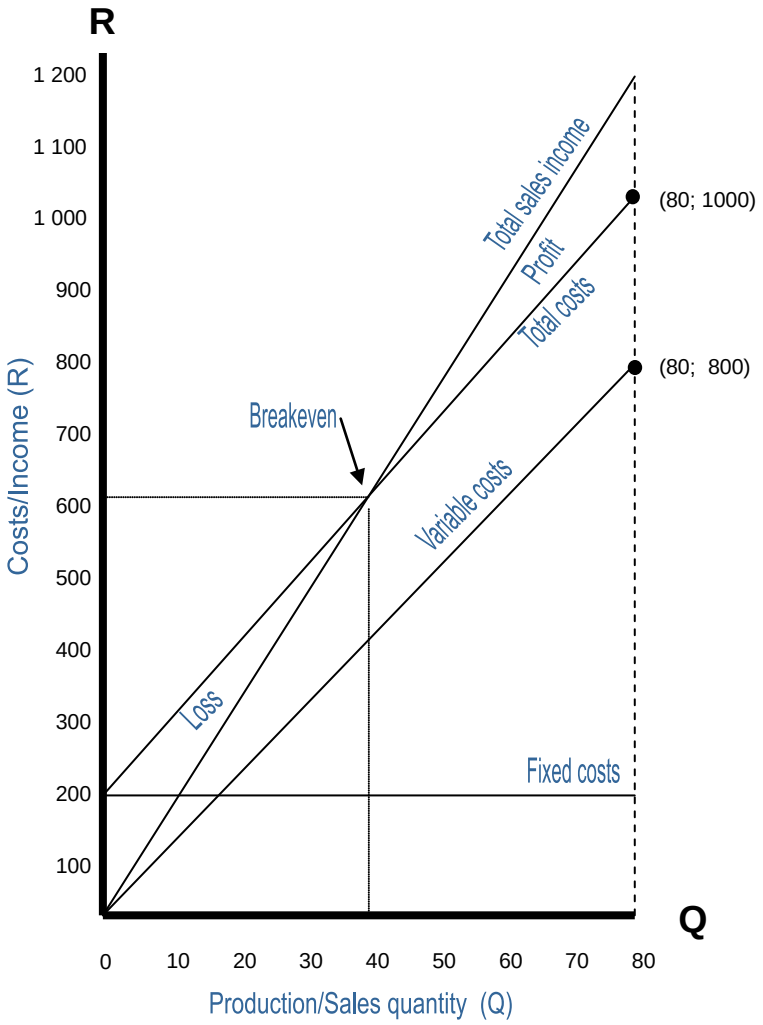


Figure 5.1 Breakeven Graph (Marx et al. 1998:203)

Example 8: From figure 4.1, answer the following questions:

a) i) Find the gradient of the Variable Costs graph.
ii) Interpret this gradient in terms of costs and quantity.

b) Find the equation of the Total Costs graph.

c) Find the equation of the Fixed costs graph.

d) i) Give the R intercept of the Total Costs graph.
ii) Interpret this in terms of costs and quantity.

<u>Answers:</u>

a) i) Gradient = $\frac{\Delta R}{\Delta Q} = \frac{1000 - 0}{80 - 0} = 12.5$

ii) If Quantity increases by 1 unit, cost increases by R12.50

b) Total Cost: R = 12.5Q + 200

c) Fixed Cost: R = 200

d) i) 200

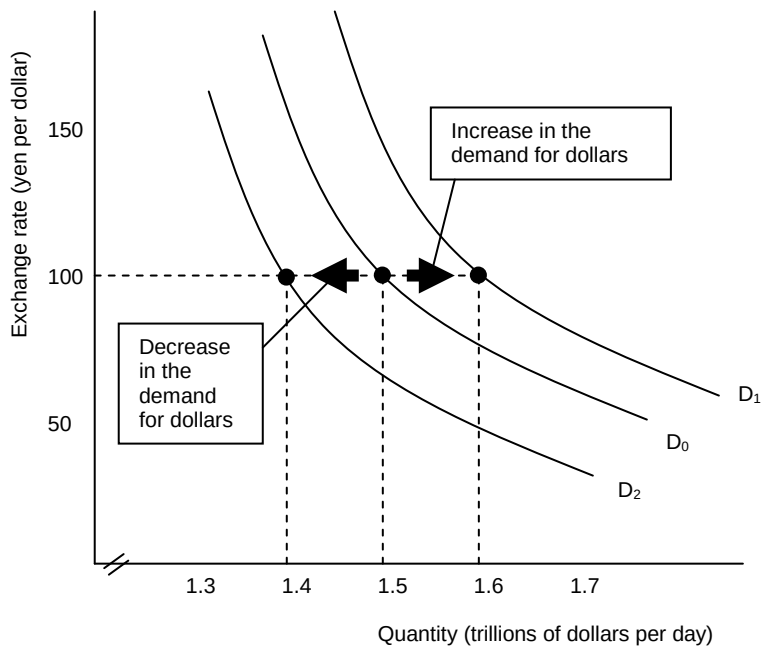
ii) When 0 units are produced, the cost is R200.

<u>Reading information from a graph</u> (Learning outcome 6.4)

Valuable information is often given in the form of a graph. It is important to be able to use a graph to obtain essential information.

Example 1:

Changes in the Demand for Dollars



Consider the graph from Parkin (2005:817) that describes the changes in demand for Dollars and answer the questions that follow:

a) A rise in the future expected interest rate will shift the demand curve rightward from D_0 to D_1. What will happen to the demand for Dollars?

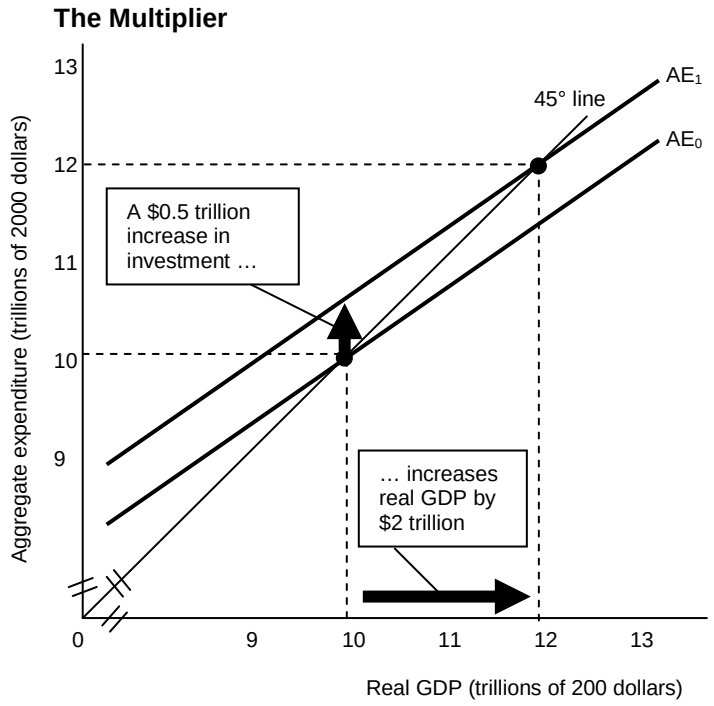
The demand for dollars will increase from 1.5 to 1.6 trillions of dollars per day.

b) A rise in the exchange rate will have what effect on the amount of Dollars demanded?

The amount of Dollars demanded will decrease.

Graphs are also used to obtain information that will be used in calculations. An example of this is found in Parkin (2005:689):

Example 2: This graph shows firstly what happens to the AE curve if Autonomous expenditure increased with \$0.5 trillion. The AE curve shifts upward by \$0.5 trillion from AE_0 to AE_1.



Calculate the value of the multiplier in the example mentioned in Parkin (2005:689):

$$\text{Multiplier} = \frac{\text{Change in equilibrium expenditure}}{\text{Change in autonomous expenditure}}$$

and equilibrium expenditure increases by \$2 trillion from \$10 trillion to \$12 trillion with the increase in autonomous expenditure of \$0.5 trillion, hence:

$$\text{Multiplier} = \frac{\$2 \text{ trillion}}{\$0.5 \text{ trillion}} = 4$$

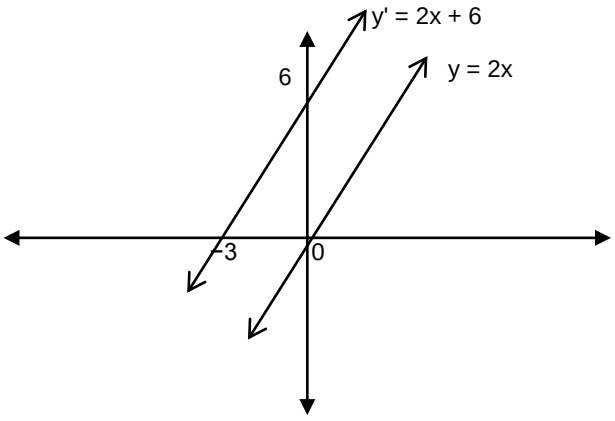
<u>Shifting graphs and the relationship between variables</u> (Learning outcome 6.5 and 6.6)

Graphs show a relationship between two variables, if a relationship exists. A number of changes can cause a graph to shift; consider some of these:

Adding a constant to the equation of the dependent variable will shift the graph either up or down, depending on the value of the constant.

Example 1: Let y = 2x and y' = 2x + 6

Then the graphs will be:

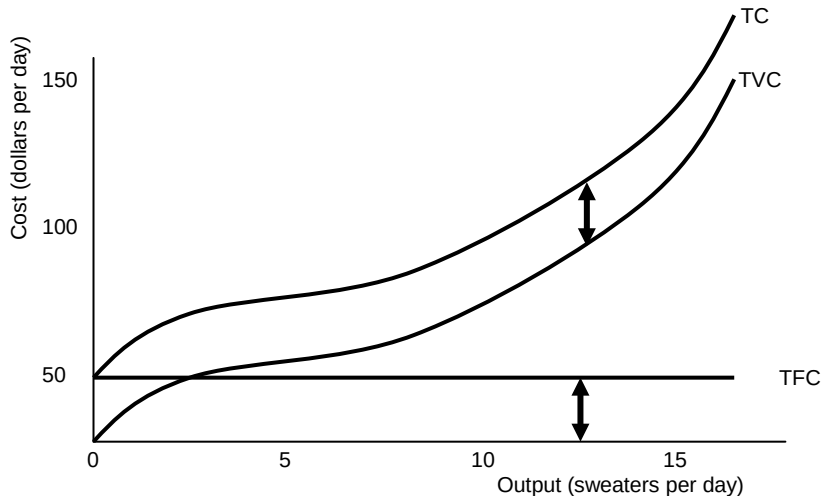


The graphs are parallel, (since they have the same gradient), but at every point, the graph of y' is six units above the graph of y.

This principle can be applied to cost curves where Parkin (2005:223) states a firm's total cost (TC) is the cost of all the factors of production it uses. Divide total cost into total fixed cost and total variable cost. Total fixed cost (TFC) is the cost of the firm's fixed inputs. Total variable cost (TVC) is the cost of the firm's variable inputs. Total cost is the sum of total fixed cost and total variable cost. That is, TC = TFC + TVC.

Example 2: When sketching these graphs, we get:

Total Cost Curves

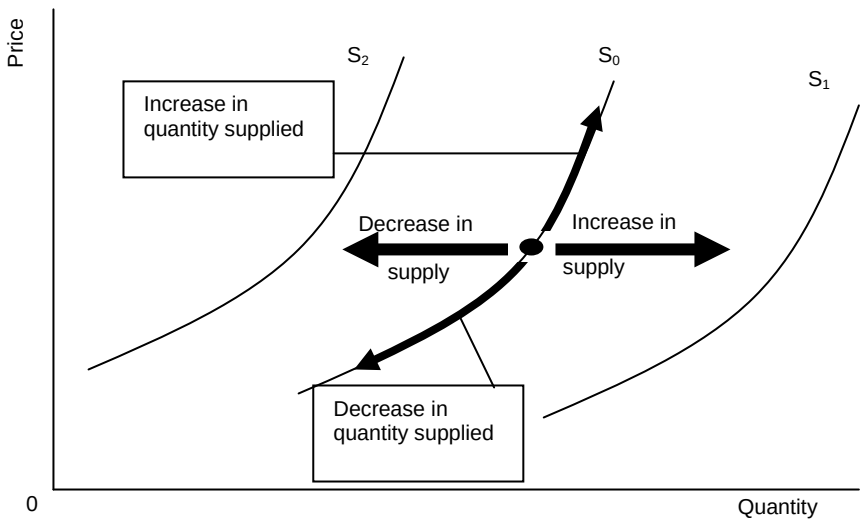


Total fixed cost (TFC) is constant – it graphs as a horizontal line – and total variable cost (TVC) increases as output increases. Total cost (TC) increases as output increases. The vertical distance between the total cost curve and the total variable cost curve is total fixed cost, as illustrated by the two arrows, Parkin (2005:223).

Various factors influence the moving of demand and supply curves.

Example 3: Parkin (2005:67) states that when there is a change in the price, there is movement along the demand and supply curves, but when there is an increase in supply, the supply curve shifts to the right (from S_0 to S_1), and when there is a decrease in supply, the supply curve shifts to the left (from S_0 to S_2). This relationship is represented in the following figure:

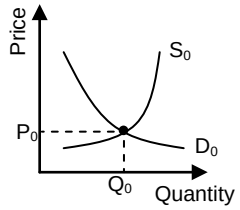
A Change in the Quantity Supplied Versus a Change in Supply



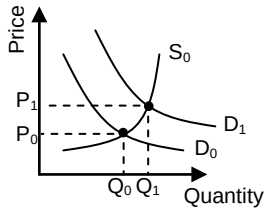
It is important to be able to read information from a graph, especially with respect to the relationship between the variables in the graph if there are changes in the graphs.

Example 4: The point of equilibrium price and quantity is where the demand and supply curves cross. What would happen to the point of equilibrium if the are changes in demand or supply?

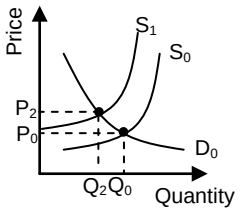
Consider the following graph of demand and supply curves:



Equilibrium point (Q_0; P_0)



An increase in demand will shift the demand curve to the right (from D_0 to D_1), and the equilibium quantity and price will increase (P_1; Q_1).



A decrease in supply, on the other hand, will shift the supply curve to the left (from S_0 to S_1)and the equilibrium quantity will decrease while the equilibrium price will increase (Q_2; P_2)

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