A PRELIMINARY INVESTIGATION INTO THE USE OF COMPUTERS IN THE TEACHING OF MATHEMATICS

by

GILLES ERNST WILLEM VAN HILLE

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ABSTRACT

Like many South African high school mathematics teachers I have followed the development of computers with interest and I have tried wherever possible to gain some experience on them. Thus when microcomputers became more readily available the mathematics department at our school, Graeme College in Grahamstown, motivated for the school to acquire this powerful new tool.

The eventual outcome was that the Old Boys' Association donated to the school 3 BBC B microcomputers with monitors, a disc drive, a printer and two tape recorders. These have now been in the school for three years.

The acquisition prompted this research project which takes the following form:-

1) An investigation into some of the uses of microcomputers in schools and, in particular, in the mathematics classroom.

2) A statement on the present position adopted by the Cape Education Department on the use of computers in schools.

3) A study of what the experience has been in other countries, particularly in Britain and the United States of America.

4) A description of an investigation which was undertaken at our school using the method of Action Research and Triangulation. Its aim was to investigate the feasibility of using a microcomputer to aid in the teaching of mathematics and the reaction of the pupils to this innovation.

Three different approaches were implemented.

a) The algorithmic approach: In this investigation a class of standard eight pupils were required, with the help of the
teacher, to write, enter and test a short computer program which would solve any pair of simultaneous linear equations of the form, $ax + by = c$. Their reaction to this form of instruction was noted by myself and a non-participant observer. The pupils themselves were also asked to express their reactions, both verbally and by filling in a prepared questionnaire. Examples of worksheets, exam questions and analysed questionnaires are given in the appendix. Short programs which examine various other mathematical concepts are also listed and discussed.

b) The audio-visual approach: In this case use was made of a graphs software package in which the computer would draw either a straight line, circle, parabola or hyperbola when the appropriate variables were entered. This package also includes a graph game facility where participants are required to find the equation of the graph which will pass through three given points. Points are awarded if the correct type of graph is chosen and the variables are entered within a certain time interval. The pupils involved in this investigation were standard eight higher grade mathematics pupils and their reaction to this form of instruction was again noted using the methods described in (a) above.

c) Computer Aided Instruction: Here I was most fortunate to be able to make use of the Rhodes University PLATO Centre. This allowed me to take a class of eighteen standard eight
higher grade mathematics pupils to the Centre. Here during four sessions, each of just over an hour, the pupils interacted with the software on the computer terminal. The software used was a set of five lessons written by Barbara Lederman of the Community College Maths Group, of the University of Illinois in 1976. The lessons give instruction and require the pupils to transform, plot and draw the graphs of linear equations of the form, $ax + by + c = 0$, $x = c$ and $y = b$. They are also taught and required to find the equations of given straight lines. Their reactions to this form of instruction are discussed after each session.

5) In conclusion some thoughts are given on how computers can best be utilised in the school situation, with particular reference to the teaching of mathematics.
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CHAPTER ONE

THE BACKGROUND TO THE INVESTIGATION

1.1 The Advent of Computer Education at School Level

It is generally accepted that the developed nations have entered the computer age. Computers are being used in many forms in commerce, industry, research and more recently in education. This is particularly true with the advent of the more reliable and less expensive microcomputer.

Reactions of individuals to computers and how they should be utilised are varied and tend to be extreme. D H Ahl gives these reasons why computers are not used to a great extent in educational practice, when it has been known since the sixties that they possess great benefits for the educational process:

Why then, you may ask, doesn't every child in every school in the world have a computer available for learning? Teachers will say it's lack of money, administrators will say the teachers aren't interested, the teacher unions will say there is no time for additional responsibilities, researchers will say its a lack of curriculum materials and software, government program administrators will say that proper objectives have not been established, and on and on. As in some multiple choice questions, the real answer is "all of the above".

(Ahl, D.H. 1980, p. VIII)

There are others who ask how much longer a computer illiterate will be considered educated?

In the commercial and educational fields there are at present a large number of microcomputers being operated by persons who have insufficient background and knowledge to realise the true capabilities of the machine they are operating. They can often also not depend on
reliable assistance from superiors and suppliers, and as a result systems and machines may lie idle or be replaced by more expensive equipment. In some cases manual and computer systems run concurrently because of a real or imagined lack of trust in the system. This situation results in unproductive capital and wasted manhours. Computers are here to stay and are going to be utilised to an ever greater extent and there is therefore a definite need for a computer literate society.

Within the educational field many governments are at present doing in-depth studies to investigate how computers can be implemented. There is, however, at this stage very little consensus and overall policy. South African education departments are also at the experimental stage in computer education. Although some project schools have been set up, generally computers are not yet freely available to schools. However, because of various pressures, specified below, most of the established government and private schools have purchased or been donated some sort of computer equipment.

Some of the reasons for acquiring the equipment are:
1) Large schools have acquired computers to assist with administrative functions.
2) There is a social pressure by parents, old boys and pupils for the school to get involved in this modern technology. It is felt that if the pupils are not introduced to computers at this stage they will be inadequately prepared for the work situation. Many adults already feel this computer threat. Computers are thus donated or purchased and consequently add greatly to the prestige of the school, which in turn places greater pressure on other schools.
3) An enthusiastic teacher or parent thus has very little difficulty in persuading a school committee to purchase relatively expensive equipment.

4) Institutions sometimes donate obsolete equipment to schools.

5) Quite a number of pupils already have their own home computers and may want to do school projects on them. These projects then need to be demonstrated and assessed at school.

### 1.2 The Involvement of the Cape Education Department in Computer Education

The present and future development of computers in education in South Africa is given in detail in the booklets published by the Human Science Research Council (Ref. HSRC) and in the article by I Gregory in the book edited by J Megarry, 1983.

In the Cape Province the Cape Education Department has set up 11 project schools where computer studies is offered as a school subject. These schools were equipped, at departmental expense, with a computer laboratory and a suitably qualified teacher. Interested pupils from the surrounding schools can opt to take computer studies as a seventh subject after normal school hours. This subject is examined externally as a matriculation subject. In 1986 it is hoped to increase the number of project schools to 16. The system of project schools has been fairly successful but some problems have been experienced. The major problem at present is that of finding and keeping suitably qualified staff. These teachers, who are paid at the same rate as other teachers, are particularly vulnerable to offers from the business sector. The inconvenience of having to teach after normal school hours and the frustration of irregular attendance and
fairly high drop-out rate by the pupils, for usually legitimate reasons, adds to the negative aspects of the post. Finance for the expanded project is also problematic in the present depressed financial climate.

A large percentage of the White schools have now acquired one or more of their own microcomputers. These are used for administration or for various educational needs. As these are privately funded the Department cannot specify how they are to be utilised. However, it has created an education planning department for computers, the staff of which are willing to use their experience to offer advice as to the type of machines to purchase and how these can best be utilised.

Their advice at present can be summarised as follows:

1) It is important to show the computer as a tool. All useful applications are therefore encouraged. This can include prepared programs in any of the normal school subjects, using the word processor or spreadsheet facility of a computer for project work, to aid with administration of school or sporting events and using small programs to calculate results or illustrate algorithms.

2) That as wide a range of pupils should be given computer experience as possible.

3) That the computer can be introduced at any level but preferably around about standards three or four. The preferred language at this level is LOGO. (The more sophisticated versions of LOGO are now suitable at any level.) How this introduction takes place will depend on the circumstances of the school, the number of computers available and the teachers' interest and knowledge of the subject. Computer education can take place during any school period where appropriate, for example during mathematics, science or youth preparedness.
4) That computer clubs should be formed wherein the other computer languages can be taught.

The Cape Education's Audio Visual Department is willing to advise schools and supply the software it houses free of charge.

The Teacher Centres are also most helpful and have arranged seminars and discussions so as to increase the awareness of computers in the schools in their district.

1.3 Computer Education in Eastern Cape Schools

As the Eastern Cape is part of the Cape Province all the remarks of the previous section are applicable. It has one Project School and there is a possibility of another one being introduced in 1986.

Two of the private schools have their own privately funded computer laboratories. One of their uses is to offer short courses in computer awareness. This is part of the school curriculum and takes place during normal school hours. Each class in standards six and seven has about three computer awareness lessons a week for a full term. Pupils may increase their knowledge after this in their own time. A teacher is usually on duty at all times to assist these pupils. The laboratories are also used for computer aided instruction (CAI) in a number of subjects: mathematics, xhosa, geography and wherever else it is appropriate.

Most of the other well established White high schools have their own microcomputers. The degree to which they are utilised, and in which direction, depends very much on the interests of the teachers concerned. Normally it is one or two teachers who have an interest and can then do very worthy work. The majority of the other teachers
feel it can best be left in the hands of these so-called experts. The unfortunate result of these specialised interests of the few involved teachers is that the direction of computer development lacks permanence.

The most usual applications of computers in the schools are:
The introduction of junior secondary pupils to LOGO;
The formation of computer clubs where pupils follow their own interests;
Specialised work in a specific subject, e.g. mathematics, physics, etc.;
The writing of administrative programs by teachers or pupils.

Teachers have opportunities to exchange ideas through, e.g., seminars organised by the Teachers' Centre or one of the universities; the formation of a Schools' Computer Centre by one of the commercial companies where a software library is to be formed; the newly formed 'Computing News', a journal serving the district.

1.4 General Uses of Microcomputers in Schools
1.4.1 Introduction

It is important that the school has a clear idea of how it is to use computing equipment once it is introduced. The schools are restricted at this stage mainly by the amount of money that is available, and the expertise and willingness of their staffs to get involved in this project. Careful consideration must also be given as to whether the money could not be more profitably spent in some other sphere of education.

A V. Kelly is highly critical of some of the papers aimed at aiding
teachers to use computers effectively.

They have concentrated their attention on what teachers might or should be doing with microcomputers and how they should do these things, but have too often failed to face up to the question of why they should do these things in these ways. They have tended to adopt a simplistic and naive model of education and of curriculum planning.

Thus the message of much of the literature is that we should use this technology because it is there and is available, that we should learn how to use it, that we should teach children how to use it, that - in the context of the Infant School - no child is too young to be taught how to use it, that - in the context of Special Education - no child is too handicapped to use it. But little attention is given to the related questions of what we are all to use this technology for, how it is to be used to further our educational purposes or how it relates to the other kinds of activity or experience we continue to offer. (Kelly, A.V. 1984, p.4)

He goes on to stress that the first criterion of good practice is that it should be based on a clearly worked-out philosophy and secondly that the work in this area should be a genuine extension and development of what is being done elsewhere.

This is rarely being done in the average government school, partially because of the prescriptive approach adopted by the education departments and secondly because there are usually only a few teachers interested or confident in this area.

How computers can be utilised in schools will be divided into three main areas. One or all these areas could be exploited depending on the situation within the school. This discussion is included as it is usually the mathematics teachers who assist in the development of computer applications in schools.
1.4.2. To Assist with the Administration Processes of the School

In a large school a case could be made to justify an expenditure of about R8000 for hardware and about R2000 for software to implement a fully computerised system which will print various lists, analyse marks and print reports. This would mean that the system would eventually be exclusively used by administrative staff and thus it is not an educational question. It could be argued that one way of keeping closer contact with the pupils and how they are progressing as a class would be to do this analysis manually.

In a smaller school where a less sophisticated system is used it would be possible for the computer to be available for other purposes as well. It would, however, probably result in clashes of responsibility and time.

Some interested and capable teachers have written their own administrative programs for schools. This is, however, extremely time-consuming and not within the scope of the average teacher. Where an administrative package is introduced by an enthusiastic teacher there is also the danger that he has great difficulty in getting the administrative staff or other teachers to do their part in feeding the information in. This can result in an unfair distribution of work.

1.4.3 To Assist in the Teaching Process

This area, usually designated as Computer Aided Instruction (CAI), can cover a wide range of activities, from simple drill and practice programs done on a single microcomputer to whole courses being handled by a computer network system. In exemplifying the breadth of computer application, Taylor discusses the computer as a tutor, as a tool, and as a tutee.
To function as a tutor in some subjects, the computer must be programmed by "experts" in programming and in that subject... The computer presents some subject material, the student responds, the computer evaluates the responses, and, from the results of the evaluation, determines what to present next. At its least, the computer keeps complete records of each student being tutored...

To function as a tool, the classroom computer need only have some useful capability programmed into it such as statistical analysis, super calculation, or word processing. Students can then use it to help them in a variety of subjects...

To use the computer as a tutee is to tutor the computer; for that, the student or teacher doing the tutoring must learn to program...

(Taylor, R. 1980, pp. 3-4)

The effectiveness of CAI depends on what one is trying to achieve, the quality of the software, the reliability of the hardware and the management skills of the teacher. At this stage in time although the proponents of CAI have been very enthusiastic about the capabilities of CAI since the sixties the industry has not made as much progress as one would expect. The software available is limited and often of poor educational quality and thus inappropriate to a particular program of instruction. This is mainly due to the fact that it has had a cottage industry type of existence. Programs are often written by teachers with limited programming skills or else by programmers with limited knowledge of educational needs. This situation is, however, improving as more co-ordinated studies are being made. When one considers the animation, diversity and interest that can be built into computer games one wonders why the educational software has lagged behind. Hofmeister offers this caution:
To advocate or criticise all computer applications in education on the basis of the present state of the art of CAI would be most unwise. Advocates of CAI must realise that we will do a disservice to school pupils and CAI by suggesting that CAI is the best and only approach to individualised instruction. Critics must be sensitive to the fact that CAI is still in its infancy, and to condemn it on the basis of its poorer products may restrict the development of the field and our chance to learn what contributions are possible. (Hofmeister, A. 1984, p.3)

The comparatively slow growth of CAI has also been due to the changes that have taken place in computer hardware. In the seventies all CAI required a mainframe computer which was connected to a number of terminals. This was very expensive and beyond the budget of schools. However, projects such as TICCIT and PLATO (these will be discussed in greater detail at a later stage) were developed and a great deal of good software was written for these projects. Much of this material was released for microcomputers in 1982 but is still not readily available. If CAI is to be implemented in a meaningful manner it would mean that the school should have a computer for each pupil in the class or at a minimum one computer for each two pupils. This would imply a minimum of about fifteen computers. Even a network system of this size would cost in the order of R30 000. This cost without government assistance has been prohibitive to nearly all schools. Schools which have only one or two computers are thus greatly restricted in the amount of use they can make of CAI. Pupils who have learning difficulties or need extra stimulation could perhaps use the computer during or after school, but without suitable control and guidance this is of limited value. Software at an average cost of about R40 for a comparatively limited drill and practice package is also discouraging. Thus schools having limited hardware available
have tended to make little use of CAI. Teachers are, however, keeping a close watch on the progress that is being made.

1.4.4 Computer Literacy and Awareness

It is often for the purpose of introducing computer literacy and awareness courses that schools have invested in one or more computers. Pupils then at least have an opportunity of becoming somewhat aware of the capabilities of the new technology. It can be brought into the classroom and its basic functions explained, small group programming courses can be held extramurally, computer clubs can cater for the enthusiast and pupils can be encouraged to utilise the computer to do useful manipulative or administrative tasks. The problem of availability versus control is difficult because of the heavy demand by pupils to have time on the computer, and, if there are only two or three computers, having a teacher present at all times cannot be justified. A fair proportion of pupils are very keen to write their own programs, and as this is very time-consuming, they often want to use computers over the weekends and during school holidays, which introduces problems regarding the security of the school. It is preferable to expose as wide a range of pupils as possible to the computer. Without proper control, however, too much time can be wasted playing games and expensive machinery can be abused.

The educational value of a fairly informal approach to computer use should not be underestimated for the following reasons.

1) Pupils can become aware of and familiar with computer technology in a friendly and unstressful situation. This knowledge and willingness to become involved with computers could be an advantage in
the job situation. As Kelman says:

...experience in using computers is clearly appropriate for students who will be going straight to work after high school. Many of them will work directly with computers in the office, on the assembly line, or in stores. (Kelman, P. 1983, p.19)

2) Extramural activities other than sport which attract a wide cross-section of the pupils need to be encouraged in most South African schools.

3) Programming and its many related possibilities can be a real challenge to the gifted child who is not sufficiently extended by normal school work. In fact pupils ranging in ability have found a new and stimulating interest in this medium and this can affect their whole outlook towards intellectual activity.

4) Programming and related activities require absolute accuracy, logical thinking and long periods of concentration. The possibility of different and more elegant or economical methods is demonstrated. Manuals and reference books need to be used effectively. Pupils also help each other to a large extent, and thus intellectual as well as social skills are exercised.

5) The sense of achievement gained when one eventually gets one's first few programs to work is very real.

6) Pupils tend to write programs that are related to their school subjects, such as history or geography quizzes, scientific or mathematical manipulation and animated games which require mathematical shapes and manipulation. This assists the study of normal school subjects in a vital way.
1.5 Microcomputers and the Mathematics Teacher

The association between computers and mathematics teachers is born out by these remarks by Peter Kelman:

Once again, mathematics teachers are at the forefront of computer implementation in schools. In fact, they have never ceased to be. Mathematics educators were among the first to use computers as an aid to instruction in the traditional curriculum. They pioneered so-called computer assisted instruction (CAI). The first large scale curriculum projects were in mathematics. The first educational computer games were mathematical games. And now, the entire mathematics curriculum appears to be the first of the traditional school curriculum areas to be undergoing substantial transformation because of computers.

(Kelman, P. 1983, p.1)

This association between mathematicians and computers is understandable as the first computers were built to assist with mathematical and scientific problems. Although mathematicians seem to be the innovators, applications in the business and commercial fields now far exceed all other forms of application. Professional computer programmers and analysts are no longer people having exclusively mathematical backgrounds, but in fact come from any of the disciplines. However, where computers have been introduced into schools it has almost always been as the result of motivation by maths and science departments. It is almost taken for granted that this should be the case and other subject teachers have been quite content to let this situation continue. The close association between mathematics and computers has in fact a negative effect on non-mathematicians: "I was never very good at mathematics so please don't involve me with anything as complicated as a computer."

Some of the best and most suitable software is in fact available in subjects such as geography, biology and the languages. It will,
however, most probably depend on the mathematics teachers to introduce other subject teachers to it, to show them how to operate the system and to be on constant standby.

As a result of this involvement many mathematics teachers have in fact been lost to commercial computing firms, which has increased the shortage of teachers in this field. Although many mathematics teachers are actively involved in computing it is debatable whether they can in fact afford the time. One of the findings of the Cockcroft report was that:

It therefore seems inevitable that for the time being, and probably for some time to come, a significant proportion of the teaching of computer studies will be undertaken by mathematics teachers, who will in consequence have less time available to teach mathematics. It is probably also the case that mathematics teachers who have displayed the initiative to develop computer studies courses are likely to be among the more enterprising and effective teachers and so the loss to mathematics teaching is correspondingly greater.
(Cockcroft, W.H. 1982, p.117)

1.6 An Overview of the Computer-Aided Mathematics Teaching Investigated in this Project

This investigation is principally aimed at determining how computers can assist in the teaching of mathematics. The aim is to place it in a typical South African situation with our own particular limitations and restrictions. The research has been done according to the basic principles of action research (this is described more fully in chapter three).

Three ways in which computers can assist in the teaching of mathematics will be investigated.

a) The first method is where one uses a single computer in the
classroom to demonstrate various basic algorithms. For example when
the properties of polygons are studied, a short program can be typed
into the computer which will determine such basic properties as the
size of exterior or interior angles when the required number of sides
is typed in. There are many of these short programs available in
mathematical journals or computing magazines. Pupils or teachers with
programming experience may wish to write their own.

This utilisation of the computer has many advantages:

i) It is a novel way of introducing this topic using modern
technology which makes the subject more real life orientated.

ii) It shows the computer being used as it should be used, i.e. as
a practical tool.

iii) Because the computer allows one to consider a large number of
examples quickly without too much 'hard work', pupils remain
interested and the development of patterns can be seen. Questions
can be raised and the program altered to investigate other factors.

iv) Only one computer is needed. The results could be put onto a
TV screen or even written on the blackboard.

The effect of this method on the class will be investigated by
obtaining the opinions of the pupils themselves, those of a non-
participating teacher who will act as an observer and those of the
teacher in charge of the lesson.

b) In the second method use is made of commercial or prepared
programs. There are a number of these software packages available on
the market. (Many of them make full use of the audio capabilities of
the computer. While these can be quite impressive and to some extent
motivating, they seldom add much to the instructional value of the
program and can frequently be annoying and unproductively time-consum ing if the program is run a number of times.) A particularly suitable area is that of graphs where the special qualities of the computer can be fully utilised. This again only requires a single computer, preferably connected to a TV screen for easier viewing. The shapes and properties of various graphs can then be easily demonstrated. The effectiveness of this method depends largely on the quality of the software and how the teacher handles the lesson and utilises the material available. There is again the advantage that only one computer need be used and that one is using the computer as a tool. Pupils with programming experience may want to alter given programs or write their own programs with this sort of application. Those pupils who do this will gain an extremely good insight into the mathematical concepts involved, because one soon learns that it is impossible to program a computer without understanding exactly what one is trying to achieve.

c) The third method is to utilise the computer to its fullest extent in CAI. Here, ideally, each pupil has the use of a computer and he is instructed, guided and possibly assessed by the computer. The effectiveness of this method is determined largely by the quality of the software and its availability. It also requires a large number of terminals. This is an expense that most schools have not yet undertaken although some schools are considering it.

In this investigation, because many terminals are not available at the school, use was made of the PLATO centre at Rhodes University. While I realise that the change of venue and the special qualities of the more sophisticated system will introduce many other factors, it
is, I feel, still relevant, because this software has been released for use on microcomputers and could thus be used in a very similar form in a school having sufficient microcomputers of the required type.
CHAPTER TWO

A STUDY OF THE LITERATURE ON MICROCOMPUTERS AND MATHEMATICS TEACHING

2.1 Some Research Projects

Since the late 1960's there have been a number of government backed research projects set up to investigate the different aspects of the role of computers in education. The aims of some of these projects will be discussed and the findings of these and other smaller investigations will be examined, under appropriate sub-headings, in the next section. The fields of investigation of these projects are much broader than is applicable to this investigation into microcomputers and mathematics. It is therefore particularly this field that will be reported on.

The two major projects on computer assisted learning (CAL) in America have been the TICCIT and PLATO projects. Although there had been smaller and less successful projects before this, in 1971 the National Science Foundation of America (NSF) decided to invest $10 million over five years on two large-scale CAL projects. The two projects had two quite distinct approaches to CAL. O'Shea remarks:

The directorship of the TICCIT (Time-shared Interactive Computer Controlled Information Television) project was entrusted to the MITRE Corporation which had been developing cable television systems. MITRE was to design and develop hardware and software for a computer assisted delivery system. A related contract to develop course material was given to what became the Institute for Computer Uses in Education at Brigham University. NSF also provided for an evaluation of the TICCIT project by the independent Educational Testing Service.

The aim of the project was to demonstrate that computer assisted learning "can provide better
instruction at less cost than traditional instruction in community colleges" (Mitre Corporation, 1974). TICCIT was not designed to be used as an adjunct to regular classroom teaching, but to be used as the main source of the delivery of instruction.

(O'Shea, T. 1983, p. 86)

The main emphasis of TICCIT was that it was learner controlled. Lesson material was designed and programmed by a team of experts but the hardware consisted of a fairly modest minicomputer linked to a comparatively simple learner controlled keyboard. One of the NSF-funded programs was to develop courses in pre-calculus mathematics (reviewing basic arithmetic and teaching intermediate algebra, logarithms, systems of linear equations, permutations and progressions).

The other major project funded by the NSF was the already well established PLATO (Programmed Logic for Automatic Teaching Operation). PLATO designers believed in very large networks of terminals and the latest technological development. Software as it was produced could be entered onto the main computer and would then be available to all its users. The cost-effectiveness of the project would depend on the number of terminals that were connected to mainframe PLATO computers. It also offered an author language, TUTOR, which would allow instructors who were comparatively inexperienced programmers to write programs. The one-terminal PLATO 1 of 1960 had grown to the NSF-funded PLATO 4 with 950 terminals located at about 140 sites and about 8000 hours of instructional material contributed by over 3000 authors in 1980. (O'Shea & Self, 1983)

Since then the number of PLATO users has increased significantly both in America and internationally. PLATO material is now also available for microcomputers (MICROPLATO) on discs. Although this
type of installation is less expensive, many of the special features, such as management and tracking, are lost. At present in South Africa there are six, mainframe or micro, PLATO installations with 1200 terminals. At the comparatively small installation at Rhodes University, with only 10 terminals, 30 000 contact hours were logged in its first two years of existence. Some of the software available on PLATO is discussed and evaluated in chapter six.

One of the earlier British projects to examine the possibilities of Computer Managed Instruction was the Hertfordshire Computer Managed Mathematics Project (HCMMP). Its origins go back to 1972 and it consisted of modules of computer based instruction, which lasted two to three weeks, and allowed pupils to work at their own rate to improve arithmetic skills. The whole course lasted two years in the middle school. (Tagg, W. 1981. ed. Howe, J. & Ross, P.)

In 1980 the British Government set aside a sum of nine million pounds for a four year project, the Micro-electronics Education Programme (MEP), which was to investigate how micro-electronics could best be utilised in the education system. The aim of the project was to encourage a greater understanding and use of micro-electronics in schools through a programme of information dissemination, in-service training and curriculum development. The curriculum was divided into four main areas: Computer Studies, Business Studies, Microelectronics, and the last domain embraced all the conventional school subjects. It is the work within this fourth domain and particularly in mathematics that will be discussed. This project did not allow for the purchase of large quantities of equipment. A special project, 'Micros in Schools', was initiated by the Department of Industries to help schools acquire computers. It allowed for each
secondary school to purchase one or two microcomputers at half price. One of the computers available on this deal was the fairly advanced 32K BBC microcomputer. As a result most secondary schools in England now have at least one microcomputer. In 1982 this offer was extended to primary schools. The MEP also supports the major centres of educational software production, so a major effort has gone towards the production of 'educational software packages' which will be available at discount rates to the schools. Fothergill reports that:

As a result of this initiative, the programme could plan with some confidence that teachers could practice on and use a computer in their school. With only a short time to make a significant impact, it was important that the initiatives started by the Programme should continue after government funding ended. Responsibility for schooling rests with local education authorities (LEAs), and it is therefore essential to involve them directly in the work of the Programme, both to attract additional resources and also to perpetuate some of the work that the Programme started. At the same time, it was hoped that such an approach would assist in disseminating the materials and ideas into the schools and the minds of the teachers. (Fothergill, R. 1983, p.133, ed. Megarry, J.)

During my recent visit to England in October 1985 I was most fortunate to visit one of the centres where work was being done for the MEP. The centre I visited was the West Sussex Institute of Higher Education (W.S.I.H.E.) in Bognor Regis. Here I met Mr Adrian Oldknow, the director of the MEP Curriculum Development Project in Mathematics, and Mr John White his Project Officer. This gave me the opportunity to find out at first hand some of the work they were doing and their general opinions. This will be given in more detail in the next section.

Other organisations, in Britain and elsewhere, some of which are
supported from central funds, are listed below.

The 'London Institute' Projects - Childrens' Logo Project and Capital Logo Project, directed by Richard Noss and Prof. Celia Hoyles of the London University Institute of Education.

The ITMA Project of the Shell Mathematics Centre at Nottingham University, Directed by Prof. Hugh Bunkhardt and assisted by Richard Phillips and Rosemary Fraser.

The CAMP Project, directed by Prof. David Johnson, at the Shell Centre for Science and Mathematics Education, Chelsea College, London.

In an article by N Rushby, 'Microcomputers in the classroom in Continental Europe', basic differences in approach to microcomputers in the different continental countries are outlined. Much of this depends on the degree of control that central government has over the educational process. Switzerland and the Netherlands have far more fragmented policies than the more centrally co-ordinated France. (ed. Howe, J. & Ross, P. 1981)

J Megarry's book, titled Computers and Education, has articles on three other national case studies:-

The '10,000 microcomputers plan' in France by Jacques Hebenstreit;
Educational computing in the Province of Quebec by Louise Dubuc;
Software and Geography teaching in New Zealand by Pip Forer.


2.2 Evaluation Reports

In attempting to evaluate the position of microcomputers as related to
the teaching of mathematics, from the research literature, I have found that although there has been a tremendous amount written on the subject, very little quantitative research has been done in an attempt to measure its effectiveness. This is because of the very large range of factors that can affect any teaching situation and the fact that microcomputers are still a relatively new innovation and by no means standardised or readily available in the mathematics classroom. The wide variety of uses of computers has also complicated the issue. There have however been a number of councils, project groups, evaluation committees and inspectorates which have investigated the role of microcomputers in the teaching of mathematics and have written reports and discussion papers on it.

Some of these findings will be discussed. Two of the major councils in America are the National Council of Supervisors of Mathematics (NCSM) and the National Council of Teachers of Mathematics (NCTM). A project that advises and collects relevant data on mathematics teaching is the Priorities in School Mathematics (PRISM) Project.

In Britain the three main evaluation reports that have been published on this subject are:

The Cockcroft Report published in 1982, which was a large scale investigation into the then present position of mathematics and the needs and directions it should take in the future; in it is a chapter on the role of calculators and computers.

The discussion paper by T J Fletcher (HMI, 1983, Department of Education and Science (DES)), titled 'Microcomputers and Mathematics in Schools'. In Fletcher's words:
This paper is addressed to those who are concerned with the development of the school curriculum as a whole and to those who have a particular interest in mathematics. The aim is to promote discussion between these two overlapping groups, so that the growing use of computers in mathematics can be coordinated with other areas of the curriculum. It results from a series of visits to schools in many parts of the country and from consultation with a number of HMI with a variety of interests. (Fletcher, T. 1983, Foreword to paper.)

The other report resulted from the DES and MEP jointly sponsored Curriculum Conference held at Pendley Manor in May 1983 to advise on policy and further developments within MEP. It was titled 'Mathematics and Microcomputers: A Pendley Manor Report'. (Boys, G. HMI, et al. 1983)

The conference was chaired by Dr. T. Fletcher HMI and the rest of the group consisted of various project chairmen: Prof. D. Johnson, R. Phillips, J. Warwick, F. Watson, W. Wynne-Willson and A. Oldknow.

It is from these and other research reports that the role of microcomputers in mathematics will now be considered under certain headings.

2.3 The School Curriculum

The ever greater use of computers in nearly all facets of our daily lives has prompted a general call for some form of computer studies in the school curriculum. This is particularly true since the advent of the cheaper microcomputer. As an editorial in Mathematics Teacher (November 1981) noted:

It is no longer a question of whether educators should be involved with computers. With the pervasiveness of computers, it is essential that students learn about them. For some individuals, this may simply be a matter of computer literacy, that is, familiarity with their capabilities and the kinds of functions they can perform. For
The place of computer studies within the curriculum must depend on the aims and objectives that need to be met. In an article M Suydam (1984) reports that as early as 1978 both the NCSM and the NCTM endorsed statements recommending the study of computers. The NCSM listed computer literacy as one of the ten skill areas of mathematics. The results of the PRISM project, designed to assess priorities of both educators and lay people before initiating curriculum change, showed that 95% gave support to secondary pupils having access to a computer. If computer studies is considered as one of the ten mathematical skill areas, does this imply that it should form part of the mathematics curriculum? The types of uses which have been identified are Computer Assisted Instruction (CAI), Computer Managed Instruction (CMI) and Programming and Problem Solving.

The British MEP also tended to divide the role of the computer into different areas. The Fletcher report states that the HMI papers Curriculum 11-16 (DES 1977) argued that compulsory schooling is concerned with introducing pupils to certain 'areas of experience'. These areas are the aesthetic and creative, the ethical, the linguistic, the mathematical, the physical, the scientific, the social and political and the spiritual. However, a study of computers should ideally include statistics, electronics, programming as well as conventional mathematics. Some sort of compromise therefore needs to be reached so that pupils interested in the field of computing are not too restricted in their 'areas of experience' because of the heavy mathematical bias that computer studies demands.
2.4 The Place of Mathematics

To avoid the bias referred to above Fletcher recommends:—

If the curriculum is to be planned around areas of experience then the area of mathematical experience must be more extensive than the area defined by current syllabuses; it must include some aspects of computer use and statistical analysis; and it must be planned not simply as a self-contained discipline but planned in relation to the other areas of experience as well. (Fletcher, T. 1983, para. 16)

The Pendley Manor Report makes the statement that a significant number of secondary schools have displayed commendable initiatives in experimenting with the use of microcomputers in the teaching of mathematics. But that change on a substantial scale will require an increased availability of machines, modest encouragement from examination boards and substantial in-service training. The fact remains that it would be beneficial if most pupils had some contact with computers and as there are many financial problems associated with the large scale introduction of computer studies, it will for the foreseeable future depend on the more traditional subjects to include some form of computer awareness and computer literacy.

M Suydam makes this distinction: computer awareness means becoming aware of the extent to which computers influence our lives and computer literacy means being able to take an idea and express it in a way that the computer can carry out your intent. Using these definitions it would seem that computer awareness could become part of the social studies region of the curriculum and that computer literacy falls into the scientific or mathematical region. R L Petosa claims there is evidence that computer programming can benefit the development of algorithmic thinking and thus lead to greater exploration of mathematical phenomena. G H Elgarten also points out
that much of programming is a mathematical exercise. However, it is important that while programming is being taught one does not lose sight of mathematics and that the mathematical concepts are stressed.

2.5 What is Currently to be Seen in Schools

Much has been said about the need to introduce computers into the classroom. The Cockcroft Report, however, sounds these warnings:

The fact that a school possesses one, or several, microcomputers will not of itself improve the teaching of mathematics or any other subject. It does no more than make available an aid to teaching which, if it is to be properly exploited, requires the teachers who have the necessary knowledge and skill and who have been supplied with, or have the time to prepare, suitable teaching programs. It does, however, also provide a valuable source of which individual pupils can make use and from which some are likely to derive considerable benefit. (para. 404)

Furthermore it has been pointed out to us that much of the limited amount of software which is available is of poor quality, with programs that are badly written and documented, sometimes inaccurate and sometimes merely 'gimmicks'. There can be no point in producing software to teach a topic which can be taught more effectively in some other way. (Cockcroft, W.H. 1982, para. 407)

In America where for the past twenty years it has been realised that computers can be used effectively in mathematics instruction, PRISM data shows that perhaps for the same reasons as above, computers have in fact made comparatively small inroads into class teaching. In fact, only 14 and 12 percent of the 13 and 17 year-olds, respectively, indicated they had used computers when studying mathematics.

However, many successful applications have been observed and reported. G Kulm describes some short programs that can be used effectively at the Junior High School level. W Stannard illustrates how short programs can be written as a 'Guess and Check' method to
solve word problems. MEP project workers, A Oldknow (1983), D Smith & N Bufton (1983), and J White (1984) have written a number of booklets to encourage this use of small programs and the algorithmic approach. Teachers in numerous countries have written their own software to illustrate mathematical concepts. The larger American projects TICCIT and PLATO in conjunction with Control Data have produced many hours of CAI. A Maddison in his book Microcomputers in the Classroom, (1982), summarises some of the various uses of microcomputers in the different subjects.

As a result of many visits to schools in Britain by HMI, Fletcher reported that considerable use was made of microcomputers in mathematics classrooms, but that the manner of use varied greatly and that it was not easy to give a tidy classification, and even less easy to qualify the various types of usages that occur.

Some of the interesting observations that were made by Fletcher were that where there was only one machine in the classroom in the primary school it would most likely be in the hands of children but that in the secondary school it was more likely to be in the hands of the teacher. It was also found that far more boys were computer enthusiasts than girls and that some special measures needed to be taken to restore the balance.

Drill and practice programs are not usually looked upon favourably, however mathematics at the primary level requires a certain amount of this. Major difficulties associated with excessive drill and practice are boredom and the feeling of inadequacy, anxiety or even panic associated with direct questions that the child is unable to answer. Unhappy memories of such experience colour the attitude toward mathematics for a number of adults. The advantage of computer-managed
drill and practice programs is that this is largely absent.

At a recent DES Regional Course A Oldknow summed up the problems associated with pupils in a lesson where computers are introduced in this way:-

Kids - There are real problems here, too. Girls can and do get pushed out of the way. Some homes have computers and others don't. Some kids are pests, speak hex and make the machine do silly things. Some kids come from primary schools where they're used to being able to use a computer. Much of the problem is that we're prone to focus explicit attention on the computer (they're rare, new, expensive and our rooms and teaching strategies are not geared up to making them feel like part of the natural environment of the maths classroom). Perhaps we can evolve subtler strategies which prompt the need for the mathematical use of the micro and which don't lend themselves to exhibitionist displays? Perhaps life will be easier when the novelty has worn off and kids are more blase? Is it already happening? (Oldknow, A. 1985, para. 9)

The role which computer games should have is discussed at some length in the Fletcher Paper. These are some of the observations and comments he makes:-

1) That much of the initial excitement and interest in computers is motivated from games. It is this aspect of computers that is most well known. Even serious programming is considered 'playing with the computer' by the uninformed. This resulting informality is one of the major factors which makes computers appealing to children.

2) That although many feel games should be restricted to after school activities others have claimed that some restricted game playing in the classroom has resulted in goodwill which has paid ample dividend in the form of improved pupil-teacher relationships and an increased amount
of work being done in the remainder of the lesson.

3) The genuinely creative work that has gone into creating games can lead to a great deal of friendly discussion and the urge to create one's own games or alter existing ones. This type of activity will usually result in the need to resolve mathematical situations.

4) Some mathematical software is presented in the form of a game and this fun characteristic of mathematics should be exploited. Other software can be turned into a game by allowing one half of the class to compete with the other.

So we see that many of the computer applications which have been seen in schools are not immediately mathematical, but they quickly involve mathematics if they are followed through. The use of computers is not necessarily mathematical; the reflective use frequently is. If we analyse the curriculum of the pupil in terms of areas of experience we need to consider mathematics not only as a course defined by a syllabus but also as an activity in many areas of the curriculum; and it is part of the professional expertise of the mathematics staff in a school to relate to this activity. (Fletcher, T. 1983, para. 96)

Much of the work involving microcomputers at schools is with the use of the programming language LOGO. S Papert in his book Mindstorms illustrates the many advantages and possibilities that this computer form has to offer to the learning of mathematics. Because of its initial simplicity and its appealing graphical qualities LOGO is generally used as the language to which children are first introduced. As a result a tremendous amount of research and research literature is on this topic. I am, however, not going to discuss it here as it represents a whole new field of experiences and I have personally not used LOGO to a great extent.

The question of advantages and disadvantages of different
programming languages is settled in this manner by A Oldknow who advocates the use of short programs:

A lot of hot air is talked about programming languages, mainly by those with an allegiance to Computer Science. I am not in the business of teaching people 'bad habits', but with the length and complexity of the majority of programs I worked with over recent years it doesn't make a scrap of difference what language the computer uses - from a mathematical viewpoint it is the ALGORITHM that matters. Through programming learners can develop algorithms for themselves and then challenge their robustness, efficiency and generality.

(Oldknow, A. DES Regional Course, 1985)

2.6 Implications for Resources, Staffing and In-Service Training

The minimum hardware requirements needed for the introduction of any form of computer use in a school are a computer, monitor and TV screen if it is to be used in the classroom, a disc drive and preferably a printer. (Tape recorders are sometimes recommended instead of disc drives, to reduce costs, but I have found these completely unreliable when used by a large number of people in the school situation.) This initial outlay is approximately R 4000. Extra terminals with disc drives and monitors will cost in the region of R 2000 per set. These are substantial amounts, and the equipment is bulky and difficult to move about and so creates safety and security problems. Careful planning is therefore required to determine who is to use it, where and for what purpose. A single computer will most probably be needed in different locations so safe mobility is important. If a large number of computers is to be purchased the language laboratory set-up is usually used. This however creates problems if different classes wish to use the facility or if a single computer is required in the classroom. In the computer laboratory set-up it has been found that
this facility tends to be monopolised by the computer studies/science groups. The Pendley Manor Report states that they consider it as a major objective to employ more machines on a regular basis in the mathematics classroom and that it is hoped that this will encourage industry to develop still lower cost equipment. The machines pupils use in mathematics classrooms need not have large storage capacities or sophisticated peripherals but good graphic facilities are important. The wide range of machines that are available and their incompatibility in the use of software is a factor that needs to be rationalised.

If expensive equipment of this nature is to be acquired or supplied it is extremely important that it is correctly utilised. Teachers thus need to be trained. A Penny in his report to the HSRC states that it has been the British experience that some teachers have made use of microcomputers when other activities are more suitable. He also refers to the studies by MacIntosh which state that computer activities have been introduced in an ad-hoc manner in many secondary schools. There is thus a very real need to offer some form of in-service training. A Penny reports fully on the content and experiences related to six teacher training establishments that offered in-service courses. There were a number of external pressures and problems encountered by the teacher training establishments. One of the courses offered is General Computer Awareness and is of about twenty hours duration. Courses such as this can be held in the evenings or on a day-release basis. A Penny has listed some of the aims of these courses as:-

(i) To introduce some educational applications of microcomputers for learning and teaching;
(ii) To provide an opportunity for exploring the
uses of micros in schools;
(iii) To ensure that students have a grasp of the educational issues raised by the new technology;
(iv) To familiarise students with some of the social implications of information technology.
(Penny, A. 1985, p.10)

Although these courses were generally over subscribed one cannot be sure what pressures were put on the teachers to attend them, or what the response will be once the more enthusiastic teachers have been catered for.

Some of the reasons for the misuse and nonuse of innovations advocated by in-service training courses, and some of the concerns expressed by the teachers attending these courses have been summarised by Barbara Signer (1983), from available research literature. Some of these reasons are given here:-

1) Teachers are not involved in the planning stages of in-service programs.
2) Extra time is not provided for in-service programs. Instead teachers are asked to use weekends or take night courses.
3) Teachers are not helped to understand how to interact with students using the new material.
4) Administrators need to encourage and reward teachers for participating in in-service training programs.
5) In-service programs are not flexible enough. Teachers are forced into group information sessions.
6) In-service programs are not sufficiently relevant to actual in-class problems of teachers.
7) The innovations were not markedly superior to the familiar methods.
8) The innovations were incorporated on the directive of higher authority, not by the expressed need of the teachers.
3.1 The Quantitative and Qualitative Traditions

The assessment and collection of data of any research project is biased towards one of the two traditions, namely a quantitative or qualitative approach.

A quantitative approach emphasises measurement and prediction and the testing of hypotheses by means of changing selected features of the experimental situation while keeping others constant. This form of investigation has been called "experimental", "scientific", "objective" or "the agricultural botanical model" (Parlett, 1977, p. 14). For this form of research to be considered really worthwhile it should be on a relatively large scale and adhere strictly to the various statistical rules and restrictions.

In education, human behaviour forms a major influence on any results that are obtained, and the difficulty in measuring such a wide and interwoven set of factors statistically bends this type of research more towards the qualitative approach. This tradition is associated with such terms as "the hermeneutic", "illuminative", "participant observer" or "social anthropological" model. This type of evaluation depends on the interpretation of human actions and behaviour by the investigator from a social and anthropological viewpoint (Bynner and Stribley, 1979).

The main shortcoming of the quantitative approach is the difficulty in evaluating complex human behaviour statistically, while the qualitative approach has the problem of not giving unbiased and
3.2 Action Research: The Method Selected for the Investigation

An investigation of the nature I wished to undertake posed definite problems of assessment.

The main problems were:

a) The wide variety and comparative innovativeness of the experiences that were being examined.

b) The smallness of the groups that could be exposed to the teaching experiences.

c) The large number of diverse factors that could influence any form of outcome.

d) The uniqueness of any particular occasion and the difficulty in arranging repetitive testing experiences.

e) The very small scale of the investigation.

f) The very limited ways in which data could be collected, due to the fact that the investigator would need to be actively involved during the sessions which were being investigated.

The only form that this investigation could take is "action research". As Cohen and Manion put it:

"action research is small-scale intervention in the functioning of the real world and a close examination of the effects of such intervention." (Cohen, L. and Manion, L. 1985, p.208)

The main features of action research are:

a) It is situational. It is concerned with diagnosing a problem in a specific context and attempting to solve it in that specific context.

b) It is participatory. Team members themselves take part directly or indirectly in implementing the research.
c) It is self-evaluative. Modifications are continually evaluated in the ongoing situation.

This form of research thus would seem ideally suited for the type of investigation which I had in mind. In fact Cohen and Manion specifically state as one of the categories where action research could be used in the classroom:

it is a means of injecting additional or innovatory approaches to teaching and learning into an ongoing system which normally inhibits innovation and change.
(Cohen, L. and Manion, L. 1985, p.211)

The first stage is the identification, evaluation and formulation of the problem perceived as critical in an everyday teaching situation. Here it is suggested that the word 'problem' should be interpreted loosely so that it could refer to the need to introduce innovation.

The "problem" to which this investigation was to address itself was that a number of schools had recently, usually by private means, purchased one or more microcomputers. The question was now, how could this versatile machine best be utilised in the mathematics classroom? The experience has generally been that it is the science and mathematics teachers that are most involved in the initial motivation for and purchase of the machine. It therefore falls upon these teachers to ensure that it is used to the fullest extent. One important aspect would be as a teaching aid. The aim of this first exploratory study was thus to investigate the variety of ways that the microcomputer can be used in the mathematics classroom, and to try and assess the reaction of the pupils and the teacher himself to this innovation. The actual effectiveness as a teaching aid to improve mathematical performance is not something that can be established with
any degree of reliability in such an initial study.

Another stage involves preliminary discussion among interested parties. Here I was most fortunate in that the other mathematics teacher at the school was experienced and was also very interested in this utilisation of the computer. The interest and expertise of the Rhodes University Education and Computer Science Departments was also most helpful. The interest and facilities of the Rhodes University PLATO CENTRE allowed me to extend the investigation to include possible applications if a computer laboratory, with more terminals, were to be established in a school. Two private schools in Grahamstown now have this facility and their experience has been most useful.

3.3 Data Collection

The data collection stage is concerned with selection of research procedures, sampling, choice of materials and methods of teaching.

It needs to be stressed that the aim of the investigation was not an attempt to assess the relative effectiveness of some sort of computer assisted teaching against that of more conventional methods. It was rather trying to establish the attitudes of the teacher and pupils to this innovation and the practicality of it, regarding e.g. portability, effectiveness and availability of hardware, reliability and relevance of software.

The question of practicality of the innovation is in fact a major portion of the investigation and is discussed at various stages. In trying to assess the pupils' attitudes I found during pretest sessions that to merely discuss the experience with the pupils as a group or
singly did not give sufficiently detailed responses in the time available. Similarly a request to form undirected written comments seldom produced more than a one line favourable comment, especially with the standard six and seven pupils.

It was therefore decided to involve the standard eight pupils in this investigation. The classes were also smaller at this level which was more practical from a logistical point of view, although it did result in a decrease in the size of the sample. To assess the attitudes of the pupils it was decided to use short questionnaires at the end of each session. These would involve a number of directed questions with two or more possible responses. An opportunity for a general comment was also given. Their general behaviour and attitude during the whole lesson were closely observed and noted by both myself and the non-participant observer whilst the pupils were completing their questionnaires.

Although the frequency for each response on the questionnaires has been given, no attempt has been made to try to analyse these any further by statistical means. Further statistical analysis would have no value because the size of the sample is too small and there are too many other factors which could influence the investigation and the pupils' responses.

3.4 Triangulation

The process of data collection was done with the principles of triangulation in mind. As Cohen and Manion explain it:

Triangulation may be defined as the use of two or more methods of data collection in the study of some aspect of human behavior.

(Cohen, L. and Manion, L. 1985, p.254)
They go on to say that the use of triangular techniques helps to overcome the problem of what is termed 'method-boundedness'. It is for this reason that as many forms of data collection were employed as were felt to be feasible. The impressions of a non-participant observer are therefore also included. The non-participant observer teaches the other standard eight mathematics class. The fact that he is very familiar with the pupils and the work being done was a great advantage as it was felt his comments would be more relevant. It could be argued that it may have been advantageous to include another observer who was less involved in the system and could thus give an alternative impression. It was, however, felt that besides the extra problem of finding a suitable and available person, we would be introducing a stranger who could become another factor in influencing the behaviour of the pupils. For the same reason I decided against a tape recording or taking a video-tape of the lessons. Although informal discussions with pupils took place it was felt these did not elaborate on the opinions already expressed in the questionnaires, so they have not been documented here.
AN INVESTIGATION OF THE ALGORITHMIC APPROACH IN MATHEMATICS TEACHING

4.1 Introduction

Although the present SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS HIGHER GRADE does not mention the word "algorithms" and the study thereof, it is I feel, implied in the general aims which are:

To develop a love for, an interest in and a positive attitude towards Mathematics, by presenting the subject meaningfully.

To enable pupils to gain mathematical knowledge and proficiency.

To develop clarity of thought and the ability to make logical deductions.

To develop accuracy and mathematical insight.

To instil in pupils the habit of estimating answers where applicable and, where possible, of verifying answers.

To develop the ability of pupils to use mathematical knowledge and methods in other subjects and in their daily life.

To provide basic training for future study and careers. (19 July 1984)

These aims are very encouraging to the teacher who wishes to introduce computers into the classroom and adopt a more algorithmic approach to the teaching of mathematics. Although at present there has not been an indepth study to examine where within the syllabus computers and an algorithmic approach would be feasible, it will with the greater availability of computers and the pooling of teachers' ideas find many applications. There is anyway much to be said for introducing some small topic to stimulate interest, even if it is not
specifically in the syllabus. According to the Pledley Manor Report:

Research with children in which algorithms and computing were an integral part of their mathematical experience has shown that:
- embedding (selected) mathematical ideas and concepts in an algorithm (or computer program) results in better understanding
- investigating mathematical ideas and solving problems through computer programming (algorithmic design) contributes significantly to the development of problem-solving ability (WITH transfer)
- exploring mathematical ideas through an algorithmic approach
  (a) enables children to develop a dynamic view of important concepts and relationships (for example: 'primeness' is a means of testing whether or not a number is a prime number rather than a static definition)
  (b) provides a context in which a child is involved in assessing the result (output) of a particular decision (input) and making modifications. This type of environment is very much one of LEARNING (see Papert's MINDSTORMS) (MATHEMATICS AND MICROCOMPUTERS; A PLEDLEY MANOR REPORT, 1983)

My own experience in this area has been taking standard four to standard seven pupils for an introductory course in computer awareness during the youth preparedness or mathematics periods. The classes at this level have between 25 and 30 pupils and their computer experience varies greatly. To some this was their first contact with a computer while others have their own home computer. I would end the course of about six lessons with a problem which required the programming of the computer to achieve some mathematical result or investigation. Listings of some of these programs are given.

I have not used LOGO to any great extent mainly because at the time the version of LOGO available for the BBC was fairly simplistic. I also experienced problems loading it from tape.

Some of the problems which I experienced during these sessions and which I feel would also apply if one occasionally adopted an
algorithmic (programming) approach in the mathematics classroom were:
The group is too large for the whole class to have a clear view of the
TV screen.
The pupils with programming experience try to dominate the lesson and
make use of more complicated procedures than are ideal for such a
mixed group.
If one tries to allow inexperienced pupils to type in input it is
painfully slow. However, to only allow the experienced pupils to do
this would be to defeat the object. One usually resorts to doing it
oneself or choosing one pupil.
Interest varies widely within the class, those who are not interested
can claim they can't see the screen clearly, and anyway the material
taught is not going to be examined.
For it to be really meaningful each pupil should, in his own time,
have an opportunity to type in and correct his own version of the
program. There were, I found, about 25% who would make the effort to
do this. Those who did, found it a most worthwhile experience.
The other times I have used this type of program were during extra-
mural programming courses. Here the group consists of about six
pupils who work in pairs at a terminal. In this case they are
expected to do the programming themselves rather than in the larger
group situation where an experienced programmer is always present to
sort out the minor programming difficulties. In this situation it is
difficult to differentiate between mathematical and programming
problems.

4.2 Some Examples of Short Programs
Some of the short programs that I have used with standard four - seven
classes or with smaller programming courses will be listed and discussed.

At this stage the pupils have been shown the meaning of the following BASIC commands, PRINT, INPUT, GOTO, FOR-NEXT loops and the mathematical operations. Statements such as \( X = X + 1 \) are, after an explanation, accepted and understood quite readily.

The first example program which merely prints out a copy of a times table is very popular with younger pupils, as learning of tables is still a very real everyday issue.

Although there is very little mathematics in this program the following factors are important and are appreciated by the pupils:

that a computer is only a tool and that it only does what it is told to do - merely to obtain a printout that is neatly spaced requires a certain amount of thought and correction;

some important programming skills are required such as determining which statements must be in the loop and which must not, and the need to define variables;

that when the program eventually does work it is a very simple matter to change variables to get all sorts of different tables, and that it is necessary to check a few of these mentally to make sure that they are correct - even decimals, very large and very small positive or negative numbers can be tried;

that the whole exercise can be rewarding and fun.

The program and some results are listed below:

```
10 REM ** PRINTING OF TIMES TABLES **
20 INPUT "WHAT TIMES TABLE DO YOU WANT TO PRINT? " X
30 PRINT " THE ";X;" TIMES TABLE"
40 PRINT
50 FOR A = 1 TO 12
60 B = A*X
70 PRINT A;" ";X;" = ";B
```
This next program which is similar to the times table program but does involve a more complicated formula and more layout problems, and thus is more suitable for older pupils, does again illustrate a real life situation. The changing of the variables can also give some startling results.

10 PRINT" Calculation of inflation or capital growth "
15 PRINT
20 INPUT "AMOUNT? R"AMT
30 INPUT "FIRST YEAR? "YR
40 INPUT "RATE? %"RTE
50 INPUT "NUMBER OF YEARS? "NYRS
60 PRINT
70 PRINT " YEAR",TAB(17)"AMT. IN RANDS"
80 PRINT
90 FOR X = 1 TO NYRS
100 PRINT YR,TAB(18)AMT
110 YR = YR + 1
115 REM CALCULATION OF ANNUAL CAPITAL
120 AMT = AMT*(RTE+100)
125 REM ROUNDING OFF
130 AMT = INT(AMT)/100
140 NEXT X
150 END

RUN
Calculation of inflation or capital growth

AMOUNT? R567.89
FIRST YEAR? 1985
RATE? %16
NUMBER OF YEARS? 12

<table>
<thead>
<tr>
<th>YEAR</th>
<th>AMT. IN RANDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>567.89</td>
</tr>
<tr>
<td>1986</td>
<td>657.72</td>
</tr>
<tr>
<td>1987</td>
<td>762.95</td>
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<td>1988</td>
<td>885.02</td>
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<tr>
<td>1989</td>
<td>1026.62</td>
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<td>1990</td>
<td>1190.87</td>
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<td>1991</td>
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<td>1995</td>
<td>2501.19</td>
</tr>
<tr>
<td>1996</td>
<td>2901.38</td>
</tr>
</tbody>
</table>

The conversion program below, in this case km/hr to m/s, illustrates some of the problems that have to be overcome to get easily readable results. In this case 'How does one get rid of all the unwanted decimal places?' Different methods can be tested. The applications for conversion programs are numerous and they can be expanded very easily to make comments on the value of the result obtained. For example limits can be included and comments such as 'A safe driving speed' or 'Faster than sound' can accompany the result.
10 PRINT "Conversion of km/h to m/s"
15 PRINT
20 INPUT "Value in km/h? "K
30 M = K*100000/3600
35 M=INT(M)/100
40 PRINT K," km/h = ";M;" m/s"
50 END

RUN
Conversion of km/h to m/s
Value in km/h? 78.67
78.67 km/h = 21.85 m/s

This type of program has the advantage of being within the range of the average pupil as well as being useful. The need to check results and to estimate others is brought home very clearly.

From practical experience I have found various problems. That although this type of problem is most successful with the brighter pupils and the computer enthusiasts, the average pupil is disappointed that so many difficulties can occur. Their idea of a computer is that it can do anything very quickly and without too much effort on their part. It is therefore difficult to make them appreciate the important mathematical concepts among the many trivial programming problems that also occur. In a class situation it is particularly difficult to suppress the enthusiasts and to slow them down so that the majority don't feel lost and lose interest.

The program below to find the mean of a set of numbers is quite successful in illustrating the mathematics involved and is also a good program from which to move on to DATA statements.

10 PRINT" To find the mean of a set of numbers "
15 PRINT
20 A = 0
30 T = 0
40 PRINT "TYPE IN NUMBERS. PRESS RETURN AFTER EACH NUMBER."
45 PRINT
50 PRINT "TYPE -1 AFTER TYPING IN YOUR LAST NUMBER."
55 PRINT
60 INPUT N
70 IF N = -1 GOTO 120
80 A = A + 1
90 T = T + N
100 GOTO 60'
120 PRINT "MEAN AVERAGE IS ";T/A
130 END

RUN
To find the mean of a set of numbers

TYPE IN NUMBERS. PRESS RETURN AFTER EACH NUMBER

TYPE -1 AFTER TYPING IN YOUR LAST NUMBER.

?7
?24
?11
?13
?15
?-1
MEAN AVERAGE IS 14

The next two programs written by G Kulm, although very simplistic, illustrate that this approach can be utilised in junior standards.

The point that needs to be understood as early as possible is that a computer, unlike the machine shown in the cartoons, cannot be asked any random question and give an answer.

10 REM ** From G Kulm - Mathematics Teacher Nov. 1984 **
20 REM ** Converting fractions to decimals **
30 INPUT " FRACTION NUMERATOR? "N
40 INPUT " FRACTION DENOMINATOR? "D
50 A = N/D
60 PRINT " DECIMAL EQUIVALENT: ";A
70 END

RUN
FRACTION NUMERATOR? 17
FRACTION DENOMINATOR? 13
DECIMAL EQUIVALENT; 1.30769231
The next two programs also by G. Kulm are both very relevant to class work and can be incorporated when teaching the section on triangles. They can be used to mark a set of examples done for homework. One is thus then re-inforcing the algorithm as well as showing the computer being used as a tool.

10 REM ** From G Kulm - Mathematics Teacher Nov. 1984 **
20 REM ** Converting coins to Rands **
30 PRINT "HOW MANY"
40 INPUT "Cents? "C
50 INPUT "Two Cents? "T
60 INPUT "Five Cents? "F
70 INPUT "Ten Cents? "TE
80 INPUT "Twenty Cents? "TW
90 INPUT "Fifty Cents? "FI
100 A = .01*C + .02*T + .05*F + .1*TE + .2*TW + .5*FI
110 PRINT "Total is R ";A
120 END

The next two programs also by G. Kulm are both very relevant to class work and can be incorporated when teaching the section on triangles. They can be used to mark a set of examples done for homework. One is thus then re-inforcing the algorithm as well as showing the computer being used as a tool.

10 REM ** From G Kulm - Mathematics Teacher Nov. 1984 **
20 REM ** Classifying triangles by their lengths **
30 PRINT "Give the lengths of the sides of the triangle, smallest to largest"
40 PRINT "Type a comma between each pair of lengths"
50 INPUT "What are the lengths? "A,B,C
60 IF A+B = C THEN PRINT "Not a triangle":GOTO 200
70 IF A=B AND B=C THEN PRINT "EQUILATERAL TRIANGLE":GOTO 200
80 IF A=B OR B=C THEN PRINT "ISOSCELES TRIANGLE":GOTO 200
85 IF A^2+B^2=C^2 THEN PRINT "RIGHT ANGLED TRIANGLE":GOTO 200
90 PRINT "SCALENE TRIANGLE"
200 END.
RUN
GIVE THE LENGTHS OF THE TRIANGLE, SMALLEST TO LARGEST
TYPE A COMMA BETWEEN EACH PAIR OF LENGTHS
WHAT ARE THE LENGTHS? 5,12,13
RIGHT ANGLED TRIANGLE

RUN
GIVE THE LENGTHS OF THE TRIANGLE, SMALLEST TO LARGEST
TYPE A COMMA BETWEEN EACH PAIR OF LENGTHS
WHAT ARE THE LENGTHS? 9,9,9
EQUILATERAL TRIANGLE

10 REM ** From G Kulm - Mathematics Teacher Nov. 1984 **
20 REM ** To find area, base or height of a triangle **
30 PRINT "ENTER ANY TWO (AREA, BASE OR HEIGHT)"
40 PRINT "ENTER ZERO FOR THE UNKNOWN QUANTITY"
50 INPUT "BASE "B
60 INPUT "HEIGHT "H
70 INPUT "AREA "A
80 IF B=0 THEN LET B=2*A/H
90 IF H=0 THEN LET H=2*A/B
100 A=B*H/2
105 PRINT
110 PRINT "BASE = ";B
120 PRINT "HEIGHT = ";H
130 PRINT "AREA = ";A
140 END

RUN
ENTER ANY TWO (AREA, BASE OR HEIGHT)
Enter zero for the unknown quantity
BASE 5.67
HEIGHT 0
AREA 23.09

BASE = 5.67
HEIGHT = 8.14
AREA = 23.09

The following set of programs by D Smith illustrates how the characteristics of prime numbers can be investigated. It also shows how a program can be developed to be more effective. It does unfortunately use the MOD statement but this could easily be done another way as shown by the program of G Kulm. The booklet by D Smith 'Computer Help In Mathematical Problem Solving' (CHIMPS) has many other examples of number investigation.
REM **From CHIMPS - WSIHE**
100 REM **PRIME-1**
110 INPUT "TEST WHAT NUMBER?" N
120 D = 2
130 IF N MOD D = 0 THEN PRINT "Not Prime."; END
140 D = D + 1
150 IF D > N THEN GOTO 130
160 PRINT "Prime"

RUN TEST WHAT NUMBER? 18
Not Prime.

RUN TEST WHAT NUMBER? 29
Prime

REM ** From CHIMPS - WSIHE **
100 REM **PRIME-2**
105 REM **slow listing of prime numbers below a certain given number**
110 INPUT "WHAT LAST NUMBER?" LAST
120 PRINT 2
130 FOR N = 3 TO LAST
140 D = 2
150 IF N MOD D = 0 THEN GOTO 190
160 D = D + 1
170 IF D > N THEN GOTO 150
180 PRINT N;
190 NEXT N

RUN WHAT LAST NUMBER? 130
  2
  3  5  7  11 13 17 19 23
 29 31 37 41 43 47 53 59
 61 67 71 73 79 83 89 97
 101 103 107 109 113 127

5 REM **From G Kulm - Mathematics teacher - Nov. 1984 **
10 REM ** Prime vs composite number **
20 INPUT "WHAT NUMBER?" N
30 FOR D = 2 TO N/2
40 IF N/D = INT(N/D) THEN 70
50 NEXT D
60 PRINT N," IS PRIME": GOTO 80
70 PRINT N," IS COMPOSITE"
80 END

RUN WHAT NUMBER? 67
67 IS PRIME

RUN WHAT NUMBER? 465
465 IS COMPOSITE
The program given below is an example of what is sometimes termed a 'Black Box' program in that the user does not need to understand the procedure but merely changes the variables. According to White:

Fletcher believes that a more urgent need now is the development of "modest programs which clarify simple pieces of quite ordinary mathematics". Nor is it essential that children should be prevented from listing or editing the program, since this may remove them a stage further from the mathematics at the core of the program. To illustrate: 'CLOWN' can be written as an eight line procedure to draw an arbitrary ellipse in colour, and a number of calls to this procedure to form a clown's face.

(White, J. 1984, p.6)

Variables H & V are the values of the X & Y co-ordinates, RX & RY indicate the X & Y axes of the ellipse and C gives the different colours.

Although the pupils cannot understand the procedure which draws the ellipse a great deal of fun is had seeing the effect of changing the variables, and they are at the same time learning about variables, positions on a co-ordinate plane and the effect of changing axes. My eleven year old son and his little brother were absorbed with this program for over an hour. J. White reports similar interest from a group of underachievers.

2 REM ** From J White - WSIHE **
5 REM ** Draw the face of a clown using different sizes and positions of ellipses **
10 MODE 2
20 PROCellipse(640,512,500,500,7)
30 PROCellipse(400,660,100,20,0)
36 PROCellipse(820,660,100,20,0)
37 PROCellipse(640,200,150,30,0)
38 PROCellipse(640,512,20,20,4)
40 PROCellipse(100,500,45,200,6)
41 PROCellipse(1180,500,45,200,6)
42 PROCellipse(640,980,300,70,5)
43 PROCellipse(300,900,70,100,5)
44 PROCellipse(980,900,70,100,5)
4.3 The Investigation

For the experience I wished to monitor I used a standard eight higher grade mathematics class of twelve pupils. This is somewhat smaller than normal but the experiment should work as well for up to about twenty pupils. Seven of the twelve pupils had had some programming experience.

The section to be covered was that on solving a system of two linear equations. This had been covered in the conventional manner and thus this would act as a revision lesson just before the end of year examinations. The aim of the lesson was not to concentrate merely on method, and obtaining a correct result, but rather to take the algorithmic approach, and try to understand the effect of how and why one applied the rules, and to examine special cases. Although this is always an aim in teaching mathematics, one finds that pupils need to be able to achieve the results themselves before they are able to fully investigate how and why they got them. As a result both the teacher and pupil are often satisfied once the method (by rote) can be successfully completed. From personal programming experience I have found that even when one fully understands the method one is programming the computer to follow, one is seldom successful the first time and one invariably learns something.

"The best way to learn something is to teach it, and a computer is a
very patient but unintelligent pupil."

The class were given the worksheet (see appendix p.102) to work through as far as they could in about twenty minutes without interruption. I then, with discussion, worked through it with them. Occasionally I would give them the opportunity to try to complete a section by themselves. After about an hour a rough working version of the program had been achieved (see below). Ideally the pupils should then be given a week to type in their own programs, document and test them, after which a listing could be handed in. Unfortunately because of the closeness of the end of year examinations this was not feasible.

After this the pupils were given the questionnaires (see appendix p.103) to complete and the non-participant observer was asked to make his comments.

A listing of the program and results for the first two examples of the worksheet are given below.

```
5 REM ** From Problems in Mathematics Teacher Nov. 1983 **
10 REM ** Program to solve a system of equations **
20 PRINT
30 PRINT "TO SOLVE A SYSTEM OF THE FORM"
40 PRINT
50 PRINT TAB(9); "ax + by = c"
60 PRINT TAB(9); "dx + ey = f"
70 PRINT
80 INPUT "ENTER a,b,c "A,B,C
90 INPUT "ENTER d,e,f "D,E,F
100 LET P = A*E-B*D
110 IF P = 0 THEN GOTO 170
120 X = (C*E - B*F)/P
130 Y = (C - A*X)/B
140 PRINT
150 PRINT "THE SOLUTION IS X = ";X;" Y = ";Y
155 GOTO 190
160 PRINT
170 PRINT "THERE IS NO SOLUTION"
180 PRINT
190 PRINT "DO YOU WISH TO SOLVE ANOTHER SYSTEM? "
```
200 PRINT
210 PRINT "ENTER 1 FOR YES AND 0 FOR NO."
220 INPUT W
230 IF W = 1 THEN GOTO 80
240 ENQ

RUN

TO SOLVE A SYSTEM OF THE FORM

\[ ax + by = c \]
\[ dx + ey = f \]

ENTER a,b,c, 2,5,8
ENTER d,e,f 3,-1,-5

THE SOLUTION IS \( x = -1, y = 2 \)
DO YOU WISH TO SOLVE ANOTHER SYSTEM?

ENTER 1 FOR YES AND 0 FOR NO.
?1

ENTER a,b,c 2,3,4
ENTER d,e,f 1,-5,1

THE SOLUTION IS \( x = 2.53846154, y = 0.307692308 \)
DO YOU WISH TO SOLVE ANOTHER SYSTEM?

ENTER 1 FOR YES AND 0 FOR NO.
?0

4.4 Comments after the Lesson

Pupils' comments:

Results of pupils' comments from the questionnaire can be seen from the analysis of the options in appendix p.103.

The overall experience seems to have been a positive one, especially the results of questions seven, eight, nine and ten, with probably the same two dissenters in each case.

I feel, however, that they exaggerated the degree of understanding they gained from the lesson.
My comments:

All but about three boys were quite confident and capable when algebraically solving the equations. (worksheet appendix p.102)

When asked the meaning of the values they had calculated and the results obtained in examples (c) and (d) they all seemed very vague and tended to mumble incoherent answers under their breath. The large majority did not really understand fully the implications of their results. They had learned a method and could obtain results but that was all.

The solving of the general case was a new experience for them and needed a lot of prompting, but the method was, I feel, understood by the majority.

The writing of the program also needed to be initiated mainly by the teacher but again they appeared to follow the steps involved.

During the solving of the general equation and the writing of the program the class seemed very vague and unwilling to speak out, but this was because it was a completely new experience for them. Even the solving of general equations is not something that is normally considered at the standard eight level.

It is most probably too much to expect pupils to program lines 120 and 130 by themselves, but one would like them to understand what is being done. The special case, line 110, should be suggested by them to remedy the division by zero when such an example is entered.

If this approach were to be adopted more regularly, even in the lower standards, as shown with some of the simpler programs of the previous section, then a more positive response could be expected.

The pace at which the general result is handled and the programming is done, should, I feel, be fairly brisk, otherwise the whole thing
becomes laboured, time-consuming and boring. The interested pupils will in their own time attempt to figure out any problems and try to tidy up the program.

Non-participant observer's comments:

A disappointing lack of response from the pupils. Is it ignorance or are they merely unfamiliar with this approach and thus unsure of what is expected from them?

Working out a program would probably emphasise the mathematical process involved - as long as it wasn't merely putting a formula into program form, without the process of deriving the formula. Then it becomes a mere lesson in programming which is a different subject.

Calls for an examination of the maths syllabus to see what topics lend themselves to this sort of treatment.

Indicates the importance of a thorough knowledge of algebraic manipulation to get matters to such a stage that a computer can be used.

Can you program for factorisation? Emphasis again seems to be on process.

When would you introduce the computer? When they are familiar with all the processes or as they master processes required for a specific application? Can it cope with every problem that may arise - conceptual difficulties peculiar to an individual (but then are all teachers perfectly sensitive to the problems being experienced by every pupil in their class).

I don't think that at this stage it is flexible enough to assist teaching effectively.
4.5 Conclusions

Although the pupils were somewhat subdued in the class they seem to have enjoyed the experience and would like more of this form of interaction in the mathematics class.

Their subdued attitude could be due to any one of a number of factors, e.g. the particular class, the fact that it was the first period of the day or the closeness of the end of year examinations.

Their responses on the questionnaire could be motivated by the type of response they feel I would like to have, or are they merely asking for some variety in the maths teaching?

For this type of approach to be effective, a fairly large number of applications must be found so that some form of continuum is established. If it becomes an integral part of the teaching practice should there not be some form of assessment, and how does one accommodate the wide range of programming abilities?

I personally feel that there is great merit in this approach and that a careful study of the syllabus will reveal other situations where it will be applicable. There also seems to be a definite request from the pupils to vary the teaching approaches that are used. The computer with its modern high-tech connotations is always popular.

It is most important that the computer should be seen in its correct role, i.e. that of a useful machine. Care must therefore be taken that its applications are not too trivial and thus negate its useful role.
5.1 Introduction

Using a single computer in a classroom to demonstrate mathematical concepts and to interact with the class as a whole has tremendous potential. However, before this form of teaching can be explored, three main prerequisites must be met:

a) Easy access to a reliable microcomputer, disc drive, monitor and TV screen. Although many South African schools now have one or a few
micros on the premises these are often not easily moved from classroom to classroom. It was for this reason that I made the trolleys seen in the photographs. The hardware in fact needs to be screwed or strapped onto the trolley as the negotiating of stairs is still quite hazardous. Departmental OHP trolleys can be used but they are a bit small and not quite suitable.

b) A teacher who has some experience of working with micros and is willing to exploit this in the mathematics classroom.

c) Suitable software that is flexible enough to cater for the different needs of teachers, yet simple to operate. It should also be relevant to the section of work being covered, and utilise fully the special features that a microcomputer has to offer. Suitable software of this sort is at present limited, but a tremendous amount is being written, both professionally and by the teachers themselves. The quality of this software varies greatly and very little has been officially evaluated and classroom tested. My experience has been that much of the commercial software is of the drill and practice variety and although it may have quite impressive sound and graphics motivational techniques, these can be overdone. It is often difficult to move backwards or forwards through the program without having to endure all sorts of noises. Amateur programs, although not as sophisticated, are sometimes more useful. They also have the advantage of being fairly simply written and a teacher with some programming experience can therefore alter them to better suit his individual needs.
5.2 The Investigation

Although I have used this method of teaching on a number of occasions with varying degrees of success, I will describe more fully the activities of three lessons which took place just before the December examinations in a standard eight HG mathematics class. The section to be revised was that of graphs. During the year they had covered the section on graphs, with particular reference to the general equations, plotting, symmetry and finding the equations of given graphs of the form:

a) the straight line, \( ax + by + c = 0 \) or \( y = mx + c \)

b) the circle, \( x^2 + y^2 = r^2 \) or \( y = \pm \sqrt{r^2 - x^2} \)

c) the hyperbola \( xy = k \) or \( y = k/x \ (x \neq 0) \)

\( \text{d) the parabola } \ y = ax^2 + c \)

This class of 12 pupils had been taught in the traditional manner (without the use of computers) by my colleague who for these experimental three lessons acted as the non-participant observer. I would normally teach another standard eight class but was at this time on long leave. The class consisted of about four boys who could be considered as above average HG mathematics pupils, four average, and four slightly below average. Generally they appeared well motivated. The normal teaching time for this section would be between four to five weeks. My aim during these three revision lessons was to review this section by considering all four types of graphs simultaneously.
and highlighting their recognition, particular properties and the different methods that are needed to find the equations of the graphs and draw them. The intention was to use a balance of teaching methods, namely teacher-pupil-blackboard, pupil-worksheets and computer-pupil-teacher interaction. The first session took the form of a double period lasting in all one hour and ten minutes. The lesson was started with the handing out of the worksheets (see appendix p.104) and the pupils were asked to complete as much of the worksheet as possible without referring to notebooks. About half the group found it necessary to consult their notebooks to make any progress at all. Their teacher was in fact rather disappointed in their apparent lack of knowledge as he had done some work on graphs as recently as a week previously. I allowed them to continue working on the worksheet for about fifteen minutes after which I interrupted them, suggesting that we go through it as a class. Some of the pupils had at this stage nearly finished the worksheet while others had not progressed very far at all. Question one gave the most problems; this was perhaps because they were not used to working with all four types of graphs simultaneously and my approach differed somewhat from my colleague's. The method of drawing each type of graph was also discussed at this stage. A four column table was drawn on the board so that the different methods and properties could be easily compared. The recognition of the type of graph by observing the powers of the dependent and independent variables was noted, as well as the number of independent constants in each equation. This was discussed with particular reference to question four. This form of interaction took about 30 minutes. The pupils were then asked to gather around the computer so that these aspects could be observed using the special
facilities that the computer can offer. There were about six chairs available in front of the normal rows of desks, so it was possible with this comparatively small group for each pupil to have a fairly clear view of either the monitor or the computer screen while sitting on a chair or in one of the front desks. Using this type of arrangement I think it would be possible to give up to about eighteen pupils a good view of a screen without the pupils having to sit on the floor, which can easily result in discomfort and disorder.
For larger classes more TV screens would be necessary. This would require special connections from the computer to the TV screens, but should not be too difficult. The software package we were using was written by Mr P Collett, a computer studies and mathematics teacher at a local school. The package has four main components:

a) A graph drawing facility, which allows one to draw the graphs under discussion by typing in the values for the constants. These can be drawn, one type at a time or all types, on the same set of axes or new axes.

b) A similar facility but with trigonometrical graphs.

c) A graph game, where three points are shown on the screen, and the aim is to choose a type of graph, and then type in the relevant constants so that a minimum number of lines or curves pass through the given points. The points are not random as in many cases only one graph is necessary to pass through the three given points.

d) The same game but with trigonometrical graphs.

This graph drawing facility can, I feel, be most useful for it allows the class to examine first the properties of each type of graph separately and at a later stage together. This session of about twenty minutes allowed us to draw about thirty graphs and interest and discussion were at a high level. There was the disadvantage that the program draws the set of axes with a full graph paper grid. This tended to distract from the clarity of the picture and it is also different from the sketch graphs pupils are expected to draw.

The following day during a single period the revision exercise was continued. I started the lesson off by writing a few questions on the board which the pupils were to complete in their classwork books. These consisted of questions relating to the general equations of the
graphs and how one would draw them, and questions which required the actual drawing of graphs. They were given ten minutes to complete this and it was followed by a ten minute discussion and correction session. Their ability to answer the questions and enter into discussion was of a much higher standard than the previous day. They then once again gathered around the screen to try the 'graph game'. It was found that the reading of the co-ordinates of the points caused some difficulty, and initially I found it necessary to draw a copy of the points on the board. However, after some discussion, the aim and method of the game was grasped by nearly all the pupils. Further attempts showed another problem in that because the author's aim was to write a game, if answers were not typed in quickly enough then the player got no points and the next example was put on the screen. The class tended to be a bit slow because I had told them to work out or at least write down their equations. From previous experience I have found this advisable if one wants to prevent the weaker pupils from sitting back and allowing the others to do the thinking. The pupils were then asked to complete the questionnaires (see appendix p.105) and my colleague and I wrote down our immediate impressions.

5.3 Comments after these Lessons

Distributions of the pupils' responses to the options given on the questionnaire are given (see appendix p.105).

The distributions on questions six and seven would seem to indicate favourably the use of computers in this form. The pupils' written comments endorse this view.

"Computers could be useful in teaching, they stimulate interest and
concentration."

"The graphs were explained much better and were easier to understand. I feel that perhaps a clearer picture could be obtained if you eliminate all the blocks and just have an X and Y axis. Also if you just type your formula in and the graph is drawn, not go through choices of what graph and what constant, because this is time-consuming."

"This method should be used more often."

"It would help if each boy could use the program for an afternoon to get a thorough, complete and sure understanding."

"This lesson helped me to understand more about graphs and now I have got a better idea of plotting graphs. It should be used more often."

"It was interesting and different. It would be nice if we could have more of these lessons."

"I enjoyed it but it would have been better if the TV screen was more clear."

"This should be done on a regular basis. It was enjoyable."

Non-participant observer's comments

If the teacher and the computer are going to 'interact' in a lesson, one needs a very flexible program. A teacher's approach and method of teaching are very personal and can the program be flexible enough to allow for this?

It seems to require quite a complicated package to demonstrate fairly simple ideas.

Too much revision of the work was required before the pupils remembered enough for them to cope with the program. This resulted in the graph game to be a bit rushed.
When they were gathered around the screen they certainly seemed to get involved. Even with its limitations a bit of technology focuses their interest.

Some of the limitations: visibility - rigid program - breakdowns.

Pupils tend to show more frustration if the TV set goes on the blink than if the teacher dropped dead!

My comments

The pupils seemed to enjoy the lesson and were very alert as a result of the fact that a different approach was being used. Modern technology seemed to impress.

Applicability very dependent on the standard of the software.

TV screen and monitor should be sufficient for about 18 pupils if they are allowed to come forward, and with some of the pupils sitting on chairs rather than in their desks.

The correct balance between normal teaching (this includes interaction between pupil and teacher and written work by pupils) and computer-teacher-pupil interaction is very important.

Some pupils seemed to be mesmerised by the action on the computer screen rather than trying to understand why.

Technical malfunctions even after thorough preparation and testing are always a possibility, and can have a disastrous effect on the whole lesson. How long should one fiddle with malfunctioning equipment?

5.4 Conclusions

a) For the computer to be used widely and effectively as an audio-visual aid to the teaching of mathematics:

(1) the computer must be permanently housed in the mathematics
department or be easily moveable;
(2) the mathematics teacher should be familiar with the equipment and have some knowledge of programming so that he can alter programs to suit his needs;
(3) the software must be very flexible or it must be a well documented simple program so that the teacher can himself alter it to suit his needs;
(4) screen visibility vs cost will always be a factor but where visibility is satisfactory the computer is a welcome and effective aid;
(5) the final effectiveness will depend on the suitability of the program, but even comparatively simple programs can be effective;
(6) it is very important that this form of interaction with the pupil forms an integrated part of the whole lesson - individual written work by the pupil must not be neglected;
(7) if technical malfunctions occur which cannot be remedied almost immediately it is better to abandon the computer's use until the problems are corrected.

b) It was found that class time is often too short for pupils to take full advantage of the program. This was in fact the case in this investigation. However, on the boys' own suggestion they were given an opportunity to work on the program by themselves. They tended to do this in groups of about four or five and would spend about an hour systematically working through the program. Their comment was that they found it too rushed in class but that they had now gained real understanding. Watching a group working on their own and noting their confidence and competence was most pleasing.
c) Drill and practice programs can be handled in a similar fashion. They can be introduced in the classroom as a novelty or as a team game, but the real benefit is gained if the pupil is then so motivated that he will request to try more examples in his own time. It should always be one's aim, however, to introduce a wide range of capabilities of the computer to the pupils.

d) Teachers who are successfully using micros to assist in their teaching should make an effort to communicate their experiences to other mathematics teachers. There is also some very good software in the other subjects and it often falls upon the mathematics teacher to investigate and demonstrate these programs to other fellow members of staff. It is often the case that all mail on computers and available software is routed to the "teacher in charge of computers". Demonstrations and lectures are also only attended by these persons. If this is the case then they have the very large and difficult responsibility of trying to pass on this information and motivating the right people.

e) Although some programming knowledge is always an advantage it should not be a prerequisite for using computers in the classroom.

f) One must not expect to see an immediate improvement in exam results for a particular section covered by this method. In this investigation, where this fairly comprehensive exercise was done on graphs just before the examinations, it was disappointing not to be able to claim a marked improvement. The examination question on graphs (see appendix, p.106) tested the section covered using the
software in a very similar fashion, however no improvement could be noted in the marks obtained for this section when compared to the whole paper, or when compared to the other class who were not exposed to the software. It should be noted that these are merely impressions as the group was too small and there was insufficient data to do a valid statistical analysis.
AN INVESTIGATION OF CAI IN MATHEMATICS

6.1 Introduction
This chapter describes the make-up of the experiment and the events that took place when computers were used for computer aided instruction. Ideally each pupil should have his own terminal and be instructed by and required to interact with the computer. As there were insufficient computers available at the school I was extremely fortunate to be able to utilise the facilities offered by the PLATO Centre of Rhodes University. This made available to me nine terminals and all the software on this most sophisticated system.

6.2 The Software Used
PLATO offers many thousands of hours of CAI in a wide variety of subjects. These are catalogued according to subject or author as well as giving detailed instructions on how to operate the system. I thus thought that I would have no problem finding suitable software to which I could introduce my pupils. This, however, turned out not to be the case as there is in fact very little written for high school mathematics. Most of the material is American based and therefore not quite suitable for South African schools.

Some of the programs which used the special qualities of animation of a computer well and took the form of mathematical games were:
1) darts: a program where the player must estimate fractions to pop a balloon using darts.
2) pogo: a game requiring the estimation of decimal distances to make
a ball roll down various layers avoiding obstacles.

3) pinwheel: a mathematical game requiring skills in addition and multiplication.

4) skywriting: allows one to make patterns by travelling a certain distance and then turning a certain angle; this combined with a repeat command allows one to make a large number of varied and interesting patterns.

Some other similar programs were available but as with the above they were aimed at junior school pupils.

There is also available on PLATO the material produced by Control Data under the title of SOUTH AFRICAN SECONDARY SCHOOLS CURRICULA (SASSC) - a computer-based education system. The following excerpt from the SASSC Information Booklet outlines the project:

SASSC Development Programme

Because of the need for high-quality instructional materials at the secondary level and the potential benefits of the PLATO Computer-based Education system, the Courseware Development Department of Control Data (Pty) Limited decided in 1980 to commence development of SASSC - the South African Secondary Schools Curricula programme.

Control Data's SASSC development programme is a large scale effort to develop computer-based curricula that are intended to assist students to successfully complete the National Senior Certificate (NSC), Joint Matriculation Board (JMB), or other Senior Certificate examinations.

By providing high-quality computer-based curricula in selected matriculation subjects, the SASSC programme will assist individuals to qualify for higher education and more rewarding employment. It will also assist commercial organizations in their efforts to improve their employees' job skills. Furthermore, by providing supplementary teaching, diagnostic testing, and record-keeping, it will be of great assistance to teachers in educational and training institutions. (SASSC Information Booklet, p.2)
The syllabuses for subjects such as mathematics, physical science and biology have been covered in the form of CAI. Although I have not studied all the material in detail I did spend some considerable time examining the material available for standards eight, nine and ten mathematics, both higher and standard grade. My general impression of this material was that it was merely a computerised version of a rather poor programmed learning book. Most of the material consisted of static multiple choice questions. The degree of difficulty was also not always in keeping with the expected standard and the marking system often faulty. I found it lacked planning, imagination and did not use the capabilities of the computer at all. It was thus completely unsuitable for what I had in mind. I would go so far as to say that in its present state it would be inadvisable for anyone to use it. I eventually found some software on the PLATO system which was ideally suited to my investigation. It is a number of programs written by Barbara Lederman of the Community College Maths Group, of the University of Illinois in 1976. The programs cover the various concepts and properties relating to straight line graphs. The subject matter and the approach was very similar to that which I used in the standard eight mathematics course. I had already covered this section during June of 1984 and therefore felt it would be an ideal way to revise this just before the September examinations. A detailed description of the work covered by the programs follows.
<table>
<thead>
<tr>
<th>Program name</th>
<th>Est. time</th>
<th>Description</th>
</tr>
</thead>
</table>
| Oline1       | 30 min    | 1) Getting the table  
2) What's my line? Drawing graph from table  
3) What's my line? Drawing graph from eqn.  
By using the cursor keys points can be plotted on the graph. |
| Oline2       | 10-15 min | Intercept \( y = mx + B \) (find \( B \))  
1) Give \( Y \) intercept - comp. draws graph  
2) Comp. draws graph - name \( Y \) intercept |
| Oline2a      | 30-60 min | Gradient \( y = Mx + b \) (find \( M \))  
1) What is slope given equation?  
2) What is the eqn. given the slope?  
3) Reading the slope from a graph.  
4) Finding the slope given two points. |
| Oline3       | 30 min    | The line \( y = mx + b \)  
1) Given equation identify \( m \) and \( b \)  
2) Given a graph - find equation  
3) Given equation - plot graph |
| Oline4       | 10-15 min | 1) The line \( y = b \)  
2) The line \( x = c \) |
| Oline5       | 30-60 min | The line \( ax + by + c = 0 \)  
1) Identify \( a \), \( b \) and \( c \)  
2) Change to \( y = mx + b \)  
3) What is slope - given equation  
4) What is intercept - given equation  
5) Graph given line |

All through the programs good use is made of the capabilities of the computer. Animation is used to plot graphs using the cursor keys: if the pupil is plotting graphs incorrectly it makes him go back through the previous stage: 'help' in the form of an explanation is available at a number of stages and an option to do more examples is always given if reinforcement is necessary. If work is done correctly the pupil is told 'good' or 'that is correct'. Praise is thus not overdone.

I have evaluated the straight line graph software according to the format suggested by A Hofmeister (see appendix, p.107).
6.3 Aims of the Investigation

The aims of this investigation were deliberately open-ended. A situation was set up and I was keen to try and gauge how the pupils adapted to it. Some points which I was particularly interested in were:

1) Did the pupils adapt to this form of instruction easily? Did they have much trouble using the computer keyboard and following its instructions?

2) Did the pupils prefer to work in pairs or by themselves? What happens if the pair have different abilities?

3) Did they prefer this form of instruction and was it successful?

4) Do the pupils work at widely different rates? A case for individualised instruction?

5) Does the constant interaction demand a higher degree of concentration and perseverance?

6) Can the pupils work by themselves or does the teacher have to play a very active part?

7) Are there peculiar discipline problems involved with this form of instruction?

8) How does this form of instruction affect examination results?

I intended getting the answers to some of these questions by using the method of triangulation.

The pupils could express their opinions by means of questionnaires which were completed after each session, and by their behaviour and conversations with myself or each other.

There was also an observer in the form of another maths. teacher, who was intended to merely observe the proceedings at each session and then to note them down.
My own impressions as the master in charge were also recorded.

6.4 The Make-up of the Group and Logistics

At the school the standard eight mathematics class is divided into three groups:

1) The top 12 mathematicians in the higher grade class;
2) A group of 16 pupils who also take higher grade but are very much weaker;
3) A standard grade group of 12 pupils.

I taught the second group and I intended to use them for this study. As a higher grade class they were very weak and unmotivated. Some of the boys had in fact not even passed mathematics in standard seven. The policy of the school was not to interfere with their subject choice too much at this early stage. The removal of the top pupils has in retrospect had a bad influence on the class as it tends to lower the expectancy of the pupils themselves and perhaps also the demands of the teacher.

As there were nine terminals available at the PLATO centre and one boy was absent, three boys from the other class were included. Two of these were also weak but one was a capable mathematician.

It was arranged that four sessions of one hour would be held on the computers. The school was allowed to use all the terminals during these sessions. Three of these were held during normal double mathematics periods and the other on a Wednesday evening from 6.45 to 8.00 p.m. The pupils were taken to and from the centre by school bus, accompanied by the two teachers. The room housing the terminals is comparatively small and with two pupils at each terminal rather crowded. It resulted in making it rather difficult to walk between
the terminals when trying to observe the pupils' progress and offer assistance. This was particularly true of about four terminals. At the end of each session the pupils were given about ten minutes to fill in questionnaires and to express their opinions both verbally and on paper.

6.5 Session 1

Monday 3rd September 1984. 12.00 - 1.10 p.m.

We arrived promptly at 12.00 noon and the boys filed in fairly quietly though obviously quite excited at the prospect of this new experience. Nobody seemed in the least apprehensive. The students who were using the machines left promptly and one of the staff of the PLATO centre assisted us to sign on. This was fairly easy as I had arranged a general sign-on which then immediately displayed a menu, listing the six programs. Once they all had this on their displays I told them to start with the first program and work through the others in the order given and that if they had any problems they could ask me. They had been told previously that the programs would cover the section on straight line graphs but I had not done any recent revision on it. It must, however, be stressed that this was a revision exercise as the section had been covered in the traditional manner in the second term.

In the first section they are presented with an equation e.g. \( y = x + 4 \), a set of co-ordinate axes and a table on which they must fill in \( x \) and \( y \) values which satisfy the given equation. If correct values are typed in by the pupil they appear on the table. If the \( y \) value is incorrect the equation appears on the screen in large print and they are taken through the steps of substitution until correct values are
attained. This is repeated until five sets of values are listed on the table. Correct values also appear immediately as points on the co-ordinate plane. When five points have been correctly plotted the straight line through the points is drawn, the pupil is congratulated, "well done", and the next equation appears on the screen and the process is repeated. The gradient and the y-intercept may now be a negative or a fraction. After four successful executions the pupil is offered the option of trying another one or proceeding to the next section. In the next section the points do not appear automatically on the screen but have to be plotted by means of a small arrow which can be moved across the plane by the arrow keys. When the desired position is reached the letter P is depressed and the point appears on the screen. If it is correct it appears as a dot and if it is incorrect it appears as a dot with a cross through it, and the pupil is told to try again. If the HELP key is depressed at this stage the pupil will again be taken through the substitution process. After three successful attempts the pupil may proceed to the last section of the first program where no table is drawn but points are plotted directly onto the co-ordinate plane. Again if mistakes are made the pupil is taken through the substitution process.

In the first session the majority of the pupils did not complete all three sections. Only the pair that had the competent mathematician speeded ahead, mainly because he was able to substitute values quickly and correctly.

The program attempts to highlight the following concepts:
1) That a point on a plane has an X and a Y co-ordinate.
2) That for a point to lie on the line it must satisfy the given equation.
3) That incorrect values do not satisfy the equation.
4) That a number of points are required before the line can be drawn.
5) That the independent variable can be randomly chosen. If too large a value is chosen the pupils are told it does not fit on the axes given.
6) It gives practice in substitution.

Ten minutes before the end of the session pupils were told to sign off and the questionnaire (see appendix, p.108) was handed out, and they were told to complete and return it before leaving.

Opinions of the pupils as expressed on the questionnaire:
1) Did you have problems using the computer and following its instruction?
   No - 8 pupils; Yes - 0 pupils; Sometimes - 5 pupils; In the beginning - 5
2) How do you find this way of learning mathematics as compared to classroom teaching?
   "It was much more exciting and I learnt a lot more than I usually learn in class."
   "Much-Much Better - You never get bored doing problems on the computer."
   "It is easier when it comes through your ear but it explains enough."
   "It is better, you are finding out more about the problem by yourself."
   "I find it more absorbing, because you have something interesting to do instead of being preached at all day."
   "With a teacher, it would be very good."
Etc. etc.
3) Would you prefer to work by yourself or in pairs? Give your reasons:
In pairs - 13 pupils; Alone - 5 pupils.
In favour of pairs - "If you are stuck then maybe your friend can help."
Against pairs - "My partner is pigheaded and wants to do all the work."
4) Do you think you have a better understanding of the section covered and could it have been covered more successfully in the classroom?
Better understanding - 15 pupils; maybe - 1; Not better - 2.
5) List some of the advantages and disadvantages of this form of instruction.
Advantages:
"No writing. Personal help; it is like having a teacher for each pupil explaining at the same time."
"Makes you do the problem over if you are wrong. In the classroom you waste time doing a problem which is wrong."
"Interesting, easy to comprehend, patient."
"No time limit."
"Its a new method of doing these sums."
"Better because you can erase mistakes."
"Don't have to write on paper. Less messy. Tells you your mistakes and makes you correct them."
Disadvantages:
"Expensive."
"If you don't understand it does not explain how to do it."
Comments from observer straight after session 1.

A number did appear to have problems getting started. They wanted to hear someone telling them what to do. Sat looking at instructions and not doing...this reaction seemed to disappear as they became more familiar. As they progressed they became more confident - however, some of the arithmetic seemed to cause problems. Those whose arithmetic was better developed galloped ahead.

Do they connect this with classroom work?

My comments after session 1.

The attitude during the whole session was very positive and the pupils were very actively involved, far more so than in the ordinary classroom situation. Although the pupils made light of the problems involving the keyboard and following instructions, about six pupils took at least half an hour to really understand what was expected of them. They were entering any values for x and y, not realising that they had to satisfy the equation. Equations of the form $y = (2/3)x + 1/2$ caused tremendous problems and pupils would resort to giving random values instead of trying to carefully work it out on the scrap paper provided. They would also try to get rid of the problem by signing off and then on again. Generally I felt the session went very well and that the pupils had been active and had learned a great deal. I felt their comments on the questionnaires very valid but I was interested to see how long this positive attitude would be maintained. I, however, had my doubts as to how well this newly attained knowledge could be transferred to the paper and pencil situation of the classroom.
6.6 Session 2

Wednesday 5th September 1984. 6.45 - 8.00 p.m.

This session had to be held in the evening as we only have two double periods during the week and I felt there should not be too great a time lapse between sessions. The pupils were told that the session was going to be held at this time but that attendance was not compulsory. They were also allowed to come casually dressed. All pupils except one turned up and seemed keen to continue. The work in this session, programs 2 and 2a, involved the drawing of the line by the gradient-intercept method, and also finding the equation of a given line using the same method. The pupils were told that they could start from where they left off if they felt confident, or if they preferred they could quickly repeat the work covered in session 1. The concept of gradient was soon giving a number of pupils problems. A line would be drawn on the set of axes and the pupil was expected to find its equation by first stating its gradient and then the y-intercept and then writing the equation. The gradient could be found by using the arrow keys to pace out a horizontal and vertical component. Whether through lack of understanding or through laziness many would just enter random values for the gradient. The computer would respond by drawing the straight line with this gradient to show how close they were to being correct. This, however, would not deter them and they would enter an equally absurd value without really thinking about where they went wrong. When the correct answer was eventually obtained this would be greeted with great jubilation and then they would continue in the same random way. Both the observer and I were now in constant demand to assist with difficulties. In many cases it was purely a reluctance to read the instructions. The
pair with the good mathematician tried to complete the whole program as quickly as possible, trying to get away with only doing one example if possible. When they had finished they asked to be excused to go and see an exhibition that was being held at the Great Hall. When their terminal became vacant another pair quickly split up so that they could each have their own terminal. Two boys who were perhaps getting frustrated with their problems and with their partners, who insisted on doing everything, started walking round and making a nuisance of themselves with other pairs. This had to be quickly discouraged.

This part of the program attempts to highlight the concepts:

1) Gradient as being vertical component/horizontal component, and as being the co-efficient of the $x$ term when the equation is in the $Y$-form.

2) The $Y$ intercept as being the $Y$ value where the straight line cuts the $Y$-axis, and the constant term in the equation.

3) That the straight line can be drawn knowing these facts from the equation or vice-versa.

I did not ask them to fill in a questionnaire after this session as I felt it might be inhibiting. Instead they were given a blank piece of paper and were asked to make any comments that they thought to be appropriate.

Some of the pupils comments after session 2.

"It was very interesting. It should be done more often."

"It was exciting and time passed quickly. I remembered what we did last time and understood it a lot quicker."

"It is very good fun, it is also a very good learning program. Very
easy to follow. Also an interesting way to learn maths."
"I understood it better this time as I knew what to do. I think I understand the section of work we have done but there are still some things I do not understand. This is very interesting and exciting."
"It was interesting. Working by myself was more interesting. I got stuck a few times but soon found out. I remembered the work from Monday's lesson."

Observer's comments after session 2.
Beginning to wonder how efficiently some kids could function on their own - they seem to need cues or prompts at virtually every step. Them or the way they have been spoonfed up to now?
Also when getting the sum wrong - does the program provide sufficient encouragement/motivation/incentive to keep them trying?
System certainly caters for different rates of progress more successfully than the normal classroom.

My comments after session 2.
Generally still favourably impressed. Even though this session was taking place during their own free time nearly all pupils were positive and tried their best. Their lack of arithmetic ability and their reluctance to read and follow instructions and do some thinking on their own is disappointing. Perhaps this comment is exaggerated as one tends to be helping or observing pupils when they are having problems. It was encouraging to see some pupils doing more than the required examples. At the end of the session many of the boys were trying to break passwords so as to get into other programs.
6.7 Session 3
Friday 7th September 1984. 11.30 a.m. - 12.45 p.m.

The boys were still keen to go - get out of school? On arrival I told them that they were free to start anywhere. The starting points thus varied widely depending on their ability, confidence, curiosity and thoroughness. Program 4 illustrates the lines $y = b$ and $x = c$ and program 5 requires the equation of the line to be drawn from the form $ax + by + c = 0$. The examples in this final section are easy at first but get progressively more difficult as fractions are introduced as co-efficients.

The boys worked more quietly and tended to ask less questions than on the Wednesday evening session. Some boys seemed to prefer to battle on in vain by themselves rather than ask for assistance.

An example of the questionnaire and an analysis of their opinions are given (see appendix p.109).

My comments after session 3.
Although much of the novelty of this form of instruction was beginning to wear off most of the pupils were still applying themselves diligently. The fact that they had to work in pairs was causing a degree of frustration in quite a few groups. One member of the pair wants to dominate all the action even when his partner knows he is doing it wrong. Nearly all the pupils were reluctant to use pencil and paper to do substitutions or converting to $y$-form; they would rather guess randomly. At the end of the session most pupils were on the last section but few had completed it. From the results of the questionnaire it would seem that most pupils were confident that they were beginning to master the work. The observer and I were, however,
a bit more doubtful than they were prepared to say. The pupils generally tended to want to express themselves favourably towards this form of instruction, more because it was something different rather than because it was so effective.

6.8 Session 4
Monday 10th September 1984. 12.00 - 1.10 p.m.

All the pupils were still keen to attend this last session. Most of them had started on the last or second last section. The fact that they had to work in pairs was becoming more of a problem. The inactive partner would sometimes stretch over to hit a random key merely to be disruptive. The observer and I experienced moments of complete frustration at their lack of understanding. The whole exercise seemed a waste of time because quite a few still had no idea of the concept of gradient! The boys also seemed overconfident in that they were quite satisfied that they had mastered the section when they could do the first simple example. When told to do the next one it could again cause all sorts of errors to appear. One or two pupils were beginning to show their apathy and boredom. This, however, is a constant problem with these pupils. I would estimate that about 80% still used their time constructively. Both the observer and I were kept constantly busy answering questions and helping pupils.

Pupils' comments after session 4.
These are adequately summarised by the way they filled in their questionnaires (see appendix p.110 & 111). The assessment of these results is more difficult because one must wonder about their motives.
1) Are their responses to the questions completely honest?
2) Do they give answers they think I want to have?
3) Do they enjoy it merely because it seems not quite related to school work?

Observer's comments after session 4.

Does CAI alone provide enough re-inforcing to get the subject matter into the learner's mind? Can it operate effectively without written work and note taking? Some appear to forget with great rapidity even the most basic concepts. Once they start interacting with the machine, it does seem to hold their attention, though some appear to be getting flippant in their approach.

My comments after session 4.

It was during this session that I began to have doubts about the effectiveness of this method of instruction. So many of the pupils were still having problems with such basic concepts as a point on a plane, gradient and the y-intercept. When at this stage one sees a boy plotting the point (4;3) as two points one on each axis, one is ready to despair. I often felt the urge to want to stop everyone and do some teaching. I would have liked to use a blackboard and explain again what is meant by gradient, etc. The difference between a positive and a negative gradient also still a problem.

6.9 Analysis of the Experience

It needs to be emphasised again that although this is a higher grade group they are particularly weak mathematically; seven pupils in fact changed to standard grade after the September examination. They are
also not very motivated, perhaps because they are weak and because the
top mathematicians are in another class. In the September
examinations which followed soon after this exercise they were given
the graphs question shown in appendix p.112. The first part of the
question, where they are required to find the equations of the given
lines is very similar to the work done during the CAI sessions. Their
marks for this question were disappointing as no improvement could be
noted. The exercise may have been more successful if it could have
been alternated with normal teaching and classwork.

Some of the facts that emerged:
1) The pupils enjoyed the experience because: It was different; it
took them out of school; it gave them a chance to work on computers.
2) The pupils were more actively involved than they would be in a
classroom situation.
3) This particular program had these advantages for the pupil:
   a) It allowed them to work at their own pace.
   b) It had animation - it drew graphs and axes quickly and neatly.
      Many more examples could thus be done because of the time saved.
   c) There was immediate feedback if mistakes were made.
   d) Some help and explanation was given by the computer.
   e) Any number of graded examples was available.
4) That pupils had an opportunity to interact with a computer in a
   meaningful manner was a worthwhile experience.
5) Working in pairs was not an ideal situation. This, however, means
   a doubling of the cost of the hardware for a prospective installation.
6) It would seem that it was possible for pupils with the best
   intentions to complete the course without having gained much insight.
A possible solution would be to alternate CAI sessions with classroom teaching to reinforce the concepts that are being put forward.

7) During CAI sessions the role of the teacher was a vital and busy one.

8) It has not been established how well knowledge gained from the computer can be transferred to a paper and pencil situation.

9) There is only a very limited amount of good high school mathematical software available at present.

10) Pupils are very appreciative of alternate forms of instruction, and if suitable facilities and software were available within the school, CAI could be used to good effect.
7.1 The Present Situation

The present situation regarding the use of computers in South African White Schools will be summarised briefly in point form.

1) There is an overall pressure from pupils, teachers, school committees, parents, old pupils and the business sector for schools to give pupils an opportunity to work with computers. Their reasons for wanting this are varied and range from being very sound educational ones to those which are based mainly on competition with other schools and prestige.

2) Most schools as a result of this pressure have now acquired one or more micros. These have been acquired through private funds and the project has been motivated by one or two teachers, usually from the mathematics or science departments, who have an interest in computers. These teachers are then normally made responsible for the implementation of the project. These teachers have seldom had more than a few weeks of formal training on computers. The extensiveness to which the computers are then used at schools will vary with the interests and convictions of the teacher in charge, but will usually include some administration work, computer clubs for the interested pupils, a class introduction at the standard six or seven level, usually using LOGO, some CAI and word processing. The pupils will be most aware of the presence of the computer by the fact that they can play games on it, and if allowed to do this there will always be a rush to book time on a computer.
3) Because these micros are privately funded there can be no definite directive from the education departments as to which make of microcomputer should be bought and how they should be used. This situation is however improving as both Teachers' Centres and the education departments have liaised to give standard advice.

4) A small number of departmentally funded project schools have been set up where computer studies can be taken as a matric subject, but their influence is at present still very small.

5) In most schools the use of the computers is still reduced to the few highly interested pupils and one or two members of staff. This group have in many cases produced some highly sophisticated programs, but this is extremely time-consuming. There are now a number of competitions or festivals where these can be exhibited. The enthusiast is therefore fairly well catered for but as a result of this he tends to monopolise computer time and intimidate the rank beginner. Because of this individualised nature of the use of the computers, continuity often suffers when pupils and teachers leave.

6) The software available, other than games, is limited and often of poor educational quality, trivial, or of the drill and practice variety.

7) Although to a large extent it has been mathematics teachers who have been responsible for the introduction of microcomputers into the school, they have tended to concentrate on more general applications rather than finding specific uses in the mathematics classroom.

8) Mathematics is generally considered as one of the vital career subjects in the school curriculum, the standard however is often lower than in other subjects. It is generally considered the most difficult because of the abstractness and lack of simple real world
applications. Interest, understanding and marks tend to decrease at the standard five to seven level. Class averages below fifty percent at the standard six and seven level are not uncommon. However, because of the career importance of the subject many of the weaker pupils feel forced to continue with the subject and thus tend to lower the standard of the senior classes and encourage drill and practice methods of instruction.

9) The syllabus for school mathematics is at present prescribed in full at all standards. This together with an inspectorate system and the external matriculation examination has resulted in most teachers adopting a rigid text-book approach which is inappropriate for innovation.

7.2 Some Recommendations
In this section I would like to express a few recommendations that I feel would benefit the role of computers in education. I am using the broad term education, even though this particular investigation is principally concerned with computers in mathematics teaching, because as has been stated, programming can be considered a mathematical activity. Any form of interaction with a computer will directly or indirectly promote an interest in other computer activities and hence illustrate a need for mathematics. This is because although a computer is an extremely versatile machine it is basically mathematical in construction, and thus its successful use will sooner or later involve some practical mathematics. School mathematics, particularly at the junior secondary stage, is considered by the more able pupil to be easy and by the less able as being too abstract.
Boys, in particular, are at this age interested in modern technology and here is therefore an ideal opportunity to exploit this interest in the mathematics classroom. Using a computer to illustrate and investigate mathematical concepts, one will promote the image of mathematics to a more exciting and practical one. This should be done even if it means changing the present syllabus slightly to include sections which are more suited to this form of investigation. These could include such activities as planning effective layouts using a spreadsheet or the word processor, using BASIC graphics to make geometrical shapes or pictures, using LOGO, investigating number patterns or using the computer to assist with calculations encountered in other subjects. Pupils should ideally find these projects so interesting that they are prepared to investigate them more fully in their own time. The recommendations will be directed to the following:

1) To the Teacher Training Institutions.

Because of the wide range of teaching opportunities that the computer can offer and because of the irreversible fact that computers are being used increasingly in all facets of modern life, it has become almost essential that children have some contact with this form of education. Even if it is only to realise its limitations. However, once a teacher is in the school situation, he is soon involved in a large number of activities both during school and extra-murally. This does not mean that he has no time or interest to learn new applications, but because of the wide range of worthwhile activities that he can involve himself in, it will always only be those who have this particular interest who get involved. The most effective time to
introduce teachers to this medium and to highlight its capabilities and wide range of useful applications is during their teacher training year(s). This exposure should not be restricted only to mathematics teachers but be applicable to all students. There are now useful applications in all subjects. Spreadsheets and in particular the comparatively cheap word processing facility are applications that any teacher can appreciate. More and more schools are computerising their whole administrative procedure and prospective teachers therefore need to know some of the peculiarities associated with this computerisation as well as the meaning of the statistical data that now becomes automatically available to them.

Prospective mathematics and science teachers should be given a short course in computer studies, and suitable classroom applications should be found and tested.

In-service courses should be offered to mathematics teachers to illustrate how computer applications can be used to motivate pupils into taking a greater interest in mathematics. This will however only be effective if the applications really are easy and relevant. The project therefore needs to be well prepared and researched.

2) To the Education Departments.
Although one can appreciate the difficulties, in the present political and economic situation, for the Education Departments to make money available for schools to purchase computer equipment, it should be possible, together with the business community and the computer dealers, to set up a system similar to the British 'Micros in Schools Project'. This project allowed schools to purchase up to two micros
at half price if they could supply sufficient motivation. This would also reduce the number of different models available to schools. How these computers are to be used should be left to the discretion of the schools as applications can vary greatly and depend on the interests of teachers and pupils. There is in the present dispensation sufficient leeway in the internal examination system of the lower standards for teachers to include innovative projects as part of the examination assessment and so make it more meaningful. A more effective software library could be built up which would make schools aware of what was available and indicate each item's degree of applicability. Mathematics text book writers should be encouraged to include possible computer applications as part of the text.

3) To the Teachers' Centres.
Courses or meetings arranged at the Teachers' Centres should become more specialised. They should look at the applications within a particular subject rather than being of a general nature. There are for example quite a few teachers who are making use of computer applications in the mathematics classroom but there is at present insufficient communication between mathematics teachers and thus useful experiences are not being shared. Listings of suitable programs as well as software and articles could be housed at the Centre and teachers should be notified of what is available in their subjects and they should be encouraged to contribute their own material.

4) To Headmasters.
The degree to which the computer is successfully utilised in school or
within a particular subject is dependent on the amount of time this activity can be supervised and controlled. This supervision is not always stimulating and should therefore be divided amongst a few staff members. Regular teaching and activity sessions should be arranged and these should be recognised as bona fide extra-mural duties. Headmasters should make it their duty to be aware of the possible applications and the limitations microcomputers have in a school.

5) To the Teacher in Charge of the Computers.
The success of computer applications in schools is determined by the number of different persons who use the machine and the number of different ways it is used. To assist this it is therefore important to ensure that the system is well maintained and that hardware and software are not abused. These contradicting factors make the task a difficult one. As with all audio visual hardware and software its usefulness depends on reliability and the ease with which it can be brought into the classroom. A system that takes ten minutes to set up is not going to be used. This is particularly true in the case of the mathematics classroom where the computer is being used as a mathematical tool and its reliability and effectiveness thus is a reflection on the subject. The more experienced teachers should therefore ensure that the computer is used effectively in the mathematics classroom as otherwise its use could be counter productive. It is impossible, especially if there are only a few micros, for a teacher to be present at all times. It is therefore important that a sensible trust be built up between the teacher and users, even if it means harsh treatment for those who abuse this privilege. It is also necessary to maintain a pleasant balance
between the more serious and the fun aspects of computers. The first contact with computers is usually through games, so this aspect should not be repressed to the extent that new pupils are no longer attracted. Another problem that occurs is that the enthusiastic computer pupils want to monopolise all the available time with very 'serious' programming and get annoyed with the beginners whose pursuits are more trivial.

6) To the Mathematics Teacher.
Some applications of computers in the mathematics classroom have now been discussed; it is important that these and others do get used. Mathematics teachers experience a considerable pressure from parents, pupils and headmasters for their pupils to do well in this 'important' subject. On the other hand one usually feels one is dealing, by and large, with incompetent and unmotivated pupils. One cannot therefore afford the time to deviate from the syllabus or use anything but traditional methods. It is perhaps this type of thinking that is the main cause of the lack of success of so many pupils. There is a great need, particularly in the junior secondary stage, to make mathematics more exciting, applicable to real life situations and a subject at which one can succeed. Class averages at this level should be in the sixties rather than at the forty percent level. There are far too many pupils at all levels who are not achieving or even making any attempt to achieve because they feel the subject has become too abstract and they 'got lost' two weeks or a year ago. This negative attitude towards mathematics can also affect other pupils and thus compound the problem. However, because of the career importance of the subject, parents are correctly reluctant to allow the child to
give up the subject. There is thus a very real responsibility on the mathematics teacher to ensure that pupils achieve their full potential. The problem is in many cases a lack of motivation rather than a lack of ability. The two best motivating factors are success and interest in the subject. The first can best be achieved by ensuring that the pupils know precisely what is expected of them. This can be done through regular testing, identifying problem areas and remedial work. However, if this method is adopted too rigorously brighter pupils get bored and weaker pupils get discouraged. The interest factor is therefore most important, and it is the capabilities of the computer which should be exploited because it has the advantage of being considered both 'high tech' and games orientated by the pupils.

One of the factors that emerged very strongly from the investigations which have been described is the positive attitude the pupils expressed towards any approach involving computers in the mathematics class. Their reasons are perhaps not as clear, and the effectiveness of any of the methods described has not been verified. However, the pupils did seem to welcome a change from the normal class routine and were more attentive during these sessions. This alone indicates that further investigation should be carried out. The fact that one is exposing the pupils to this new technology is in itself important. Teachers must however be well prepared, because if too much time is taken setting up the system or if leads are faulty or if pupils cannot see the screen(s) clearly then the whole exercise can become counter productive. The teacher must also be clear in his own mind what his objectives are.
7.3 Some Final Comments on the Methods Investigated

The Algorithmic Approach:

This method is perhaps the most beneficial as it best illustrates the capabilities of the computer and it is the most versatile method of investigating and testing mathematical concepts. It does however have the disadvantage in that it expects pupils to have some programming knowledge, but this many would argue is already essential. What is required is that suitable and meaningful applications need to be found, from within the syllabus or otherwise, and these together with a suggested approach should be included in text books or teachers' guides, so that all teachers become aware of these possibilities. Many of these investigations are unfortunately time consuming so ideally pupils should complete an investigation in their own time.

The Audio Visual Approach:

What is most surprising here is that more suitable software is not yet available. Mathematics teachers and professional programmers need to get together to fully investigate the possibilities this approach has to offer. The graphical and simulation capabilities of the computer are not at this stage being fully exploited. There is also much that can still be done in the form of remedial lessons that pupils can work through in a more informal way, singly or in small groups, in their own time. The quality and reliability of the software needs to be of a high standard as there is no advantage in using a computer if the same result can be achieved as successfully using more conventional means.
Mathematics teachers should not underestimate the role they have to play when computers are introduced. This is particularly true in the case where the pupils are themselves interacting with the computer as in CAI. This is because the pupil needs to be in a position to interact confidently with the computer before he can concentrate fully on the mathematics. This confidence and understanding of the capabilities of the computer takes time and should ideally be gained through practice in the pupil's own time. Computers are also very limited in the explanations they can offer and the corrections they can make to faulty reasoning. The teacher must therefore keep a constant vigil on the progress of the pupils. Pupils also have a greater opportunity to work at their own pace which implies that a more individualised approach is required. CAI is therefore not the solution for poorly qualified or inexperienced teachers.
APPENDIX

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**Worksheet for Algorithm Investigation.**

Simultaneous linear equations worksheet.

Solve the following equations simultaneously:

- \(a\) \(-2x + 3y = 7\)
  \(3x + 2y = -4\)

- \(b\) \(2x + 3y = 6\)
  \(x - 5y = 1\)

"What do these values of \(x\) and \(y\) mean?"

By what other method could these values for \(x\) and \(y\) be found and what would they stand for?

\(c\) \(4x - 12y = 1\)
\(-3x + 9y = 2\)

\(d\) \(4x + 12y = 16\)
\(x + 3y = 4\)

What is the meaning of the results obtained in:

\(c\) \(\) ________________
\(d\) \(\) ________________

\(e\) Solve the general form of two simultaneous linear equations using the method you used in \(a\):

\(ax + by = c\)
\(dx + ey = f\)

Write a computer program to solve any pair of linear equations.

How will the equation have to be modified to allow for situations encountered in \(c\) and \(d\)?

Does the program work equally well with coefficients that are decimals and fractions?

In your own time try to document and tidy up this program so that it could be easily used by anyone.
Questionnaire for Algorithm Investigation.
************************************************************

The numbers under the option offered indicate the frequency.

<table>
<thead>
<tr>
<th>GRAEME COLLEGE</th>
<th>COMPUTER PROJECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Were you able to do the examples and the questions on the first page?</td>
<td></td>
</tr>
<tr>
<td>all of it</td>
<td>examples only</td>
</tr>
<tr>
<td>(2) (7)</td>
<td>(9) (3)</td>
</tr>
<tr>
<td>2) Were you able to work out the general solution for two linear equations?</td>
<td></td>
</tr>
<tr>
<td>completely</td>
<td>some of it</td>
</tr>
<tr>
<td>(2) (6)</td>
<td>(4)</td>
</tr>
<tr>
<td>3) Have you had any programming experience?</td>
<td></td>
</tr>
<tr>
<td>quite a lot</td>
<td>a little</td>
</tr>
<tr>
<td>(4) (4)</td>
<td></td>
</tr>
<tr>
<td>4) Were you able to write a program to solve two linear equations?</td>
<td></td>
</tr>
<tr>
<td>most of it</td>
<td>a little</td>
</tr>
<tr>
<td>(2) (2)</td>
<td>(4)</td>
</tr>
<tr>
<td>5) Once the method of programming had been explained did you follow the steps that were being taken?</td>
<td></td>
</tr>
<tr>
<td>completely</td>
<td>most of it</td>
</tr>
<tr>
<td>(7) (3)</td>
<td>(2)</td>
</tr>
<tr>
<td>6) Which section gave you the most problems?</td>
<td></td>
</tr>
<tr>
<td>the programming</td>
<td>the algebra</td>
</tr>
<tr>
<td>(2) (7)</td>
<td>(3)</td>
</tr>
<tr>
<td>7) Would you like the computer to be used more often in the mathematics classroom?</td>
<td></td>
</tr>
<tr>
<td>as often as possible</td>
<td>only when applicable</td>
</tr>
<tr>
<td>(10)</td>
<td>(2)</td>
</tr>
<tr>
<td>8) Programming is not part of the syllabus, do you feel this type of application helps your understanding of the mathematics?</td>
<td></td>
</tr>
<tr>
<td>adds interest but does not help your maths</td>
<td>wastes time</td>
</tr>
<tr>
<td>(10)</td>
<td>(2)</td>
</tr>
<tr>
<td>9) As the computer can now find the solution for you. Do you feel it is a waste of time doing more examples algebraically on paper?</td>
<td></td>
</tr>
<tr>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>(6)</td>
<td>(4)</td>
</tr>
<tr>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>10) Do you feel you would like to learn more about computer programming?</td>
<td></td>
</tr>
<tr>
<td>yes even after school</td>
<td>only during school</td>
</tr>
<tr>
<td>(10)</td>
<td>(2)</td>
</tr>
<tr>
<td>11) Can you suggest any other areas in your mathematics or school work generally where the computer could be usefully used in the classroom?</td>
<td></td>
</tr>
<tr>
<td>Graphs, geometrical problems, accounting, science and all aspects of mathematics, project work.</td>
<td></td>
</tr>
</tbody>
</table>

General Comments:

a) need more computers.
b) knowledge of both computer and manual calculation will be useful in later life.
c) the computer helped to understand the work more clearly.
d) it helps in a certain extent to understand more easily, when solving and programming the equation.
e) overall very interesting.
f) computers are an everyday part of our lives.
Worksheet for Visual Aid Investigation.

STD 8 GRAPHS WORKSHEET

1) Write down the general formula for the following graphs.
   a) The circle: ____________________________
   b) The parabola: ____________________________
   c) The straight line: ____________________________
   d) The hyperbola: ____________________________

2) What type of graph do the following equations represent?
   a) $y = x^2$ ____________________________
   b) $y = x$ ____________________________
   c) $xy = 5$ ____________________________
   d) $2x - 3y = 0$ ____________________________
   e) $x + y = 2$ ____________________________
   f) $y = -3/x$ ____________________________
   g) $x + y = 9$ ____________________________
   h) $x + y = 9$ ____________________________
   i) $4xy = 16$ ____________________________
   j) $x = -3$ ____________________________

3) About which lines are the following graphs symmetric?
   a) The hyperbola? ____________________________
   b) The circle? ____________________________

4) What is the minimum number of points lying on the graph that you must know to find the equations of the following graphs?
   a) the circle ____________________________
   b) the straight line ____________________________
   c) the hyperbola ____________________________
   d) the parabola ____________________________

Can you formulate a rule for the above and what is a general method for finding the equation of graphs given a certain number of points lying on the graph. ____________________________
Questionnaire for Visual Aid Investigation.

The numbers under the options offered indicate the frequency.

<table>
<thead>
<tr>
<th>GRAEML</th>
<th>COLLEGE</th>
<th>COMPUTER</th>
<th>PROJECT</th>
</tr>
</thead>
</table>

1) Did you have any difficulty seeing the screen? Yes A little No (2) (3)

2) Which media do you feel is more effective for the following?

<table>
<thead>
<tr>
<th>(A) Meanness</th>
<th>(B) Ease of understanding</th>
<th>(C) Variety of examples</th>
<th>(D) Working at your pace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (1)</td>
<td>9</td>
<td>4 (2)</td>
<td>8 (3)</td>
</tr>
</tbody>
</table>

3) Who do you feel should operate the computer?

<table>
<thead>
<tr>
<th>The teacher only</th>
<th>The teacher and some boys</th>
<th>Each pupil in turn</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

4) Did you understand what was expected of you in the game? Yes No (9) (1)

5) Where are you able to give the correct answer in the game?

<table>
<thead>
<tr>
<th>Hardly ever</th>
<th>Sometimes</th>
<th>Most of the time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(3)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

6) Did you feel you got more understanding from this lesson than normal?

<table>
<thead>
<tr>
<th>Yes</th>
<th>About the same</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)</td>
<td>(1)</td>
<td>(1)</td>
</tr>
</tbody>
</table>

7) If you answered yes to the previous question was it because?

   a) A computer was used? (1)
   b) The lesson was a bit different? (7)
   c) You concentrated more because the exams were soon? (1)
Examination Question on Graphs following Visual Aid

Investigation.

QUESTION FIVE

(a) Calculate the equations of the following graphs:

(b) Draw rough sketches to illustrate the graphs of:
   (i) \( y = mx + c \) if \( m < 0 \) and \( c > 0 \)
   (ii) \( y = ax^2 + c \) if \( a > 0 \) and \( c > 0 \)
   (iii) \( y = ax^4 + c \) if \( a < 0 \) and \( c > 0 \)
   (iv) \( xy = k \) if \( k > 0 \)

(c) On the same set of axes sketch the graphs of \( y = -x^2 + 4 \)
   and \( y = 2x - 1 = 0 \). Use a scale of 1 cm = 1 unit on both axes.
   (a) From your graphs find:
      (i) for which values of \( x \) is \( -x^2 + 4 > 2x + 1 \)
      (ii) for which values of \( x \) is \( -x^2 + 4 = 2x + 1 \).

(d) (i) On a different set of axes sketch the graph of \( xy = 4 \).
   Use a scale of 1 cm = 1 unit on both axes.
   (ii) Draw the graph of \( y = x \) and let it cut the hyperbola
   at \( P \) and \( Q \). Mark the coordinates of \( P \) and \( Q \) clearly
   on your graph.
   (iii) Draw the circle which has the origin as centre and
   which passes through \( P \) and \( Q \). What is the radius of
   this circle?

(e) What equation is one getting when one reads off the
   \( x \)-values of the points of intersection of the circle and
   the hyperbola? Give your answer in the form:
   \[ ax + c = 0. \]

Total: 180 marks
**COURSEWARE EVALUATION FORM**

**Program Name:** Graphs

**Publisher:** B. Lederman

**Price:**

**Content:**

- Straight line of the form $y = mx + b$
- $y = b$
- $x = c$
- $ax + by + c = 0$

See also page 74

**Instructional Design:**

- Objectives are fully and clearly defined.
- Target audience is clearly defined.
- Outside activities are appropriate and effective.
- Inappropriate skills are clearly defined.
- Content is presented clearly and logically.
- Content is transferable and generalizable.
- Content is consistent with objectives.
- Vocabulary level is appropriate for subject area and learner level.

**Ease of Use:**

- Content on screen is clear and easily readable.
- Program can be used independently.
- Learner interacts only with appropriate segments.
- Program allows a variety of display and response modes.
- Program allows a variety of display and response modes.
- Program can be adjusted by user for level.
- Appropriate use of graphics is used.
- Feedback is useful and appropriate.
- Instructions in active rather than passive.
- Learner expectations are met.
- Program has consistent display rate.
- Displays are clear, understandable and effective.

**Package Contents**

- Straight Line Graphs

**Hardware Requirements**

- PLATO

---

**Record Keeping & Management**

The only keeps a record of what has been completed.

Program keeps an ongoing student record.

Program includes diagnostic/evaluative testing.

Program generates student assignments.

Program includes a variety of student programs.

Program provides statistical information on student progress.

Program allows screen and print display of student records.

**Instructional Design**

Good use is made of computer's graphic abilities.

- Arrow keys used to plot points.

**Ease of Use**

- Suitable for scales 0-10 pupils.

- Program contains provided are comprehensive and effective.

- Program is reliable in normal use.

- HELP procedures are available.

- Program can be used by student or automatically when appropriate.

---

**Program Strengths and Weaknesses**

**Validation**

- PROGRAM TESTED:
- EVALUATION SITE:
- DATE:
- EVALUATED BY:
- CONTACT PERSON:
- ADDITIONAL INFORMATION:
Questionnaire after Session One of CAI.

1) Did you have problems using the computer and following its instructions? Discuss briefly:


2) How do you find this way of learning mathematics as compared to classroom teaching?


3) Would you prefer to work by yourself or in pairs? Give your reasons:


4) Do you think you have a better understanding of the section covered and could it have been covered more effectively in the classroom?


5) List some of the advantages and disadvantages of this form of instruction.


Questionnaire after Session Three of CAI.

The numbers after the options offered indicate the frequency.

GRAEME CAI PROJECT

11 Do you ever use the "HELP" key?
- never - 0
- sometimes - 15
- never - 3

Does it provide the help you need?
- yes - 9
- sometimes - 0
- usually not - 1

21 Do you feel a teacher's presence and help is necessary while doing this work?
- definitely - 6
- only sometimes - 11
- not at all - 1

31 Do you think you could master this section on straight line graphs without first having covered this work in class last term?
- definitely not - 1
- perhaps - 15
- yes - 2

41 Are you frustrated in that you do not know what you are doing wrong and the computer does not let you carry on?
- often - 3
- sometimes - 6
- never - 7

Any particular section? State which. Pullis seemed unwilling to commit themselves to any answer here.

51 Do you think you have mastered the sections you have covered so far?
- not really - 0
- nearly - 15
- yes - 3

6 Does it worry you that other groups may be well ahead of your group?
- not at all - 9
- a little - 9
- yes - 0

71 Are there any sections you would like to repeat to make sure you understand them fully?
- most sections - 0
- one or two - 16
- none - 2

81 Any other comments? no meaningful comment here besides good, exciting, enjoyable, better, interesting etc.
Questionnaire after Session Four of CAI.

The following is the questionnaire completed after session 4. The option 5 4 3 2 1 has been replaced by the frequency that option was chosen.

GRAEME COLLEGE CAI PROJECT

Section A

Rate on a scale of 5 the importance of the following aspects of "computer aided learning" which added to your enjoyment of this form of instruction.

5 - very important, 4 - fairly important, 3 - of some importance
2 - only slightly important, 1 - not at all important.

1) The fact that the computer does not call or think you are stupid if you give the wrong answers. 7 4 3 2 2
2) The fact that if you are stuck you can push the "help" button and get some immediate assistance. 11 3 2 0 1
3) The fact that you can work at your own speed. 14 2 1 0 0
4) The fact that you are kept busy all the time. 11 5 2 0 0
5) The fact that there is a teacher present to help you if you are having difficulties. 9 6 2 1 0
6) The fact that you don't have to write anything down. 3 6 4 2 1
7) The fact that your work is always neat. 11 2 4 1 0
8) The fact that you could repeat a section if you were still unsure. 17 0 1 0 0
9) The fact that the computer could draw the lines so quickly. ie. Immediate results to what you are thinking. 10 4 2 1 1
10) The fact that it is new and different. 12 1 4 1 0

OTHER FACTORS? _______ if you can't do an equation it gives you the answer and explains what is happening.
The number after the option indicates frequency. One questionnaire not filled in.

Section B

You have now spent 8 school periods being taught with the aid of a computer system.

1) Do you feel the work could be covered as effectively, if perhaps not as excitingly, in the classroom?
   
   completely - 0  
   fairly well - 17  
   same as before - 0

2) Do you feel the time you spent on this section of work was:
   
   too long - 0  
   about right - 11  
   too short - 6

3) Do you feel you now understand this section of work?
   
   completely - 0  
   fairly well - 17  
   same as before - 0

4) Would you have liked the computer to give you a test as part of this last session?
   
   yes - 11  
   not yet ready for a test - 6  
   rather not - 0

5) Do you think you will do well on this section in the examinations next week?
   
   definitely - 5  
   still not confident - 12  
   probably not - 0

6) If this facility were available at school, how often would you like it to be used to teach mathematics?
   
   only sometimes - 0  
   about 50% of the time - 5  
   most of the time - 12

7) If there were two terminals available at the school do you think you would use them voluntarily in your own time if you were having difficulty with a particular section?
   
   definitely - 12  
   maybe sometimes - 5  
   not at all - 0

8) Do you feel that if a terminal were available at school, that the brighter pupils who finished an exercise first should be allowed to use the terminal to do more complicated work?
   
   yes - 5  
   only sometimes - 5  
   no - 7

9) Do you think if you used this method of learning regularly it would eventually become boring?
   
   yes - 1  
   slightly boring - 2  
   no - 14

OTHER COMMENTS ____________________________________________________________
Examination Question on Graphs after CAI Investigation.

QUESTION FIVE

1 (a) Give the coordinates of two points on (i) line a
   (ii) line b
   (b) What is the gradient of
      (i) line a
      (ii) line b
   (c) Give the equation of
      (i) line a
      (ii) line b

2 (a) Write the equation $2x + 5y - (x + y) - 6 = y$ in the form $y = mx + c$ and hence give the gradient and the $y$-intercept of the line.
   (b) What is the equation of the line perpendicular to the given line passing through the origin?

3. On the same system of axes draw the graphs of:
   $4x + 3y - 9 = 0$
   and $3y - x - 6 = 0$ and find the coordinates of the point of intersection of the two lines.

4. On the same set of axes draw the graphs of
   $y = -x$;
   $y = 3$;
   $y = 2x - 4$

Shade the area given by:

$$\{(x,y) : y \geq -x\} \cap \{(x,y) : y \leq 2x-4\} \cap \{(x,y) : y \leq 3\}$$
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<td>Maddison, A.</td>
<td>Microcomputers in the Classroom.</td>
<td>1982</td>
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