SAMPLING IN THE EVALUATION OF ORE DEPOSITS

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This dissertation was prepared in accordance with specifications laid down by the University and was completed within a period of ten weeks full-time study.
"If geology benefits from sampling, it also contributes to it, for sampling can never be reduced to blind rules of thumb; it must be carried out in conformity with geological principles."

(McKinstry, 1948)
ABSTRACT

Sampling is an error generating process and these errors should be reduced to a minimum if an accurate ore reserve estimation is to be made from the sample values. Error in sampling can arise from the sampling procedure as well as where and how each sample is taken from the deposit. Sampling procedure involves sample collection, sample reduction and analysis, and the error from each of these three stages has an equal influence on the total error of the process. Error due to sampling procedure should be identified and eliminated at an early stage in the evaluation programme. An ore deposit should be subdivided into sampling strata along geological boundaries, and once these boundaries have been established they should be adhered to for the evaluation programme. The sampling of each stratum depends on the small-scale structures in which the grade is distributed, and this distribution in relation to sample size controls sample variance, sample bias and the volume of influence of each sample. Cluster sampling can be used where an impractically large sample is necessary to reduce sample variance or increase the volume of influence of samples. Sample bias can be reduced by composing a large number of small samples. Sampling patterns should be designed with reference to the volumes of influence of samples, and in favourable geology, geostatistical or statistical techniques can be used to predict the precision of an ore reserve estimation in terms of the number of samples taken. Different ore deposits have different sampling characteristics and problems which can be directly related to the geology of the mineralization. If geology is disregarded when sampling an ore deposit, an evaluation programme cannot claim to give an accurate estimate of the ore reserves.
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REFERENCES
1. INTRODUCTION

Sampling in the evaluation of an orebody is the process by which the grades of very small portions of the ore are selected as being typical of the grade of the orebody. It is the process upon which the whole ore reserve estimation is based, and includes techniques such as channel sampling, instrumental detection and visual estimates. All methods of ore reserve estimation combine the sample values by assigning them to blocks of ore larger than the sample, and only differ in the way the sample values are assigned to the blocks, and the shapes and sizes of those blocks. In assigning a sample value to a larger block or ore, it must be assumed that the sample is representative of the ore from which it was taken, and that the ore is representative of the larger block. An invalid assumption generates error, and it is this error that can lead to an inaccurate ore reserve estimation.

Error can be subdivided into random and systematic categories. Systematic error or bias means that the sample values are consistently higher or lower than the values they are supposed to represent, and will cause the overevaluation or underevaluation of an orebody. Random error is not as critical as systematic error because there is a good probability that another sample will have a similar but opposite error which, when the two values are combined, will have a counteracting effect. An increase in the random error of a sample decreases the volume of influence of that sample, and the precision of the ore reserve estimate. More or larger samples would be necessary to bring the precision of the estimate within the limits required for an investment decision. Such alteration of the sampling programme will increase cost and may even be impractical or subeconomic. Consequently, it is desirable to keep the precision of the sampling as high as is practically possible.

The aim of this dissertation is to attempt to identify the nature and sources of error in sampling. Chapter two of this dissertation covers the techniques and errors of sample collection, the cause and effects of error in sample reduction, and effects of error in sample analysis which contribute to the total error of the sampling procedure. Chapter three considers how samples should be taken in relation to the
distribution of mineralization in an ore deposit, and chapter four discusses the sampling of certain types of mineralization and some of the problems that may be encountered. The dissertation is concluded with a chapter on recommendations for the design of a sampling programme. This dissertation has been written with the assumption that the reader is familiar with statistical terminology.
2. SAMPLE COLLECTION, SAMPLE REDUCTION AND ANALYSIS

There are three stages in ore sampling viz. sample collection, sample reduction and analysis, and each has its own characteristic variability which contributes to the total error of the process. The value of a carefully taken sample has always been appreciated by samplers (Rickard, 1907), and the error of an analytical technique is often precisely known because of the relative ease with which it can be measured. The error due to sample reduction is often overlooked or left to experience. The purpose of this chapter is to discuss the types and origins of errors that can be expected with the various stages in the sampling procedure, and how they can be eliminated. A geologist in charge of an evaluation project should be responsible for the sampling procedure from the taking of the sample to the size of the aliquot that is used in the analysis.

The total error of a sample is related to the individual errors of the sampling stages by the following equation:

$$\text{Total error} = \sqrt{\text{sample collection error}^2 + \text{sample reduction error}^2 + \text{analytical error}^2}$$

(1)

The error of each stage is the difference between the true and estimated value of the sample and these differences are squared to eliminate positive and negative signs which would cancel each other. The error generated in sample collection, for example, has an equal chance of being increased or decreased by the error generated in sample reduction. But the total error is the maximum error that can be expected for a given probability and therefore the errors of the individual sampling stages are cumulative. Equation 1 shows that any error in the sampling procedure can have an equal influence on the total error of a sample and could, either individually, or in total, invalidate a carefully planned sampling programme. Sometimes the errors are difficult to quantify, especially those due to sample collection, but they can be expected to occur at almost every step in the sampling procedure. Errors due to sampling procedure must be identified and reduced to acceptable levels so that a sample represents the ore from which it was taken.
2.1. SAMPLE COLLECTION

Techniques of sample collection belong essentially to three groups viz. hand-taken samples, drilling, and determination of grade in situ. For any one method there are many variations that have been designed to suit a particular problem. Each has its own advantages and disadvantages, as well as characteristic sources of error created by applying the technique to a particular type of mineralization. The techniques and their errors will be reviewed here, and again in chapter 3 of the dissertation, where their application to specific types of mineralization will be discussed.

2.1.1. Hand-taken samples

Channel sampling is a time proven method of hand sampling described in detail by Rickard (1907), Hoover (1933), Forrester (1946), McKinstry (1948) and Reedman (1979). It involves cutting channels across the face of exposed ore and collecting chips and dust from each channel to make up a sample. Channel samples can have any cross-sectional area, and areas of up to approximately 80 cm$^2$ are known (Basden, 1970). However, the larger the cross-sectional area, the more time consuming and expensive the sample becomes. Groove samples are a narrow variety of channel samples, and have widths which are normally determined by the width of the chisel head.

Before taking a sample, the face should be cleaned with a wet wire brush (Cornish, 1966) because the fines which settle out after blasting are often sulphides and will contaminate the sample. An oxidized surface will also alter the sample grade, so samples should not be taken on badly oxidized surfaces. The sample should be cut perpendicular to the geological structure, but where this is not possible, the apparent width of ore that the sample represents should be corrected to true width by simple trigonometry (Cornish, 1966; Storrar, 1977). A channel sample should be split into a number of smaller samples where there are significant changes in orientation as the combination of sections with different orientations will create bias. The channel should be cut as evenly as
possible; Fig. 1 illustrates the bias that can be generated by taking an uneven channel. A number of precautions can be taken to ensure an even channel, e.g. any protruding rock within the limits of the proposed sample should be chipped away before the sample is taken, and the channel inspected for irregularities after the sample has been taken, but before it has been accepted. When cutting channel samples in the massive

![Diagram of regular and irregular channel samples.](From: Basden, 1970)

sulphides at Mount Isa mine (Morrow, 1976) it is first necessary to cut a groove around the sample boundary to achieve an even cut. A similar technique of cutting the sample boundaries with a diamond saw, for channel samples across a Witwatersrand reef, did not prove to reduce the sample variability (Sichel and Rowland, 1961). If the channel has been subdivided into smaller samples, according to the grade and hardness of the ore, then an uneven channel will not create a significant bias in the sample grade. Sampling of massive sulphides can produce a positive bias where hard gangue minerals may cause a larger proportion of the sulphides to be taken.

When cutting the sample, care should be taken to ensure that
every chip from the channel is caught and extraneous material is discarded. If chips of a certain 'composition' are lost due to their ability to be propelled beyond the collecting device, a bias will be caused in the sample value. At Kilembe mine the cuttings were caught in a pan and spillage collected on a canvas sheet on the floor. Consequently an expected negative bias arising from loss of the friable chalcopyrite was avoided (Davis, 1962).

Chip sampling involves taking a series of chips in a definite pattern, or randomly across the face (McKinstry, 1948). Patterns include taking chips continuously along a line across the face (Morrow, 1976), on a diamond pattern within a large but demarcated area on a face (Fig. 2), and randomly within a demarcated area on the face. Chip

[Diagram of chip sampling]

Fig. 2. Diamond pattern chip sampling used at Hilton mine. A shows the spacing of the chips and B shows the distribution of the chips on the face.

(From: Morrow, 1976)

sampling is easier than channel sampling but dangerous imprecision is created by combining a variety of different chip sizes, especially where there is a correlation between chip size and grade, e.g. soft sulphide minerals as opposed to hard, barren chert.

Bias can be avoided to a certain extent by demarcating an area
on the face from which samples should be taken, and acceptance of only those chips that are representative of the area. Joseph (1962) found that chip samples taken from gold mines in Kalgoorlie were heavily biased but more representative values were obtained by rejecting large chips in favour of a large number of smaller ones, and by avoiding free gold. Bias is reduced by taking a large number of chips at specified intervals over a face as described by Morrow (1976) but a variety of chip sizes will still cause imprecision.

Davis (1962) describes an application of chip sampling which he showed to be reliable for the highly variable massive sulphide ore at Kilembe. The walls and back of a cross-cut were clearly subdivided with chalk lines into areas of uniform grade. Each area was then separately chip-sampled to produce a sample which was large enough to minimise bias. The volume of ore in the cross-cut represented by each sample was estimated from the geological mapping and the value of each sample weighted by the respective volume of ore. The final result compared closely to the grade of the bulk sample. Davis found that the time taken for marking, chipping and estimating the weighting factors underground, was approximately the same as that taken by an experienced sampler to take half the number of channel samples needed to arrive at an equally comparable result.

2.1.2. Drilling

This is one of the most common methods of sampling an orebody, especially in the exploration and early evaluation stages where it is the only means available for a geologist to obtain fresh samples of the ore. Methods of drilling are described in detail by Cumming and Wicklund (1975) and have been reviewed by Bertram (1980). Borehole sampling has been discussed by McKinstry (1948), Peters (1978) and Reedman (1979). The advantages and limitations of collecting drill samples will be reviewed here with particular reference to the errors that may occur.

Diamond drilling yields a core in which the geological boundaries to the mineralization can be established so that samples can be taken relative to these boundaries. This is particularly valuable where samples have to be taken within strict geological limits. The whole core within the prescribed limits can be sampled but it is
preferable to keep at least half of the core as a geological record. The core should be evenly split lengthways, either by cutting with a diamond saw, or a hammer and suitably shaped chisel, with the core held firmly in a type of vice. Core splitting often generates bias and every precaution should be taken to prevent it. If there is significant anisotropy to the mineralization in the intersection, the core should be split across the long axis of the anisotropy so as to moderate the variability in the grade. Core splitting will also produce fines which are often enriched with soft and friable sulphide minerals, compared with their concentrations in the ore. Great care must be taken to collect all the fines, split them equally and combine one half with the split core sample.

Core loss is a source of error that will bias a sample if there is preferential loss of gangue or ore minerals. The nature of the bias depends on the properties of the ore, the gangue and the country rock. For example, core loss in many porphyry deposits is caused by the disintegration of the rock adjacent to the mineralized cracks. This also involves a disproportionate loss of the ore mineral, causing a negative bias in the sample value. Hazen and Berkenkotter (1962) studied the effects of random core loss on samples taken from different core sizes. They found a significant reduction in the variability of poorly recovered samples in NX core (54.8 mm diam.) compared with AX core (29.4 mm diam.). Core loss in most geological situations is not random and will cause some bias, so preference for a larger core size should not be based on this reasoning alone.

When core recovery is not good, assay values will be consistently higher either in the core or in the drilling sludge. An indication of the bias caused by the core loss can be obtained by sampling the sludge as well as the core. A simple comparison of sample values obtained from the sludge and core will show if the core samples have been positively or negatively biased. Various techniques have been proposed to quantify the sludge and core sampling so as to obtain a reliable sample value (Basden, 1970). Koch and Link (1970) have shown that assays of core and
sludge can be combined according to the following formulae:

\[ w = \frac{k_1 w_c + k_2 w_s}{k_1 + k_2} \] ....... (2)

and

\[ s^2 = \left( \frac{k_1}{k_1 + k_2} \right)^2 s_c^2 + \left( \frac{k_2}{k_1 + k_2} \right)^2 s_s^2 + 2 \frac{k_1 k_2}{(k_1 + k_2)^2} \frac{s_c s_s}{n} r_{cs} \] ....... (3)

where \( k_1 \) and \( k_2 \) are the per cent recoveries of the core and sludge, \( r \) is the correlation coefficient and subscripts \( c \) and \( s \) stand for core and sludge respectively. The formulae were tested on a core loss problem at Chuquicamata copper mine, Chile. Sludge samples can only be taken over set depth intervals during the drilling and are difficult to compare with samples taken according to geological boundaries. Furthermore, heavy mineral separation in the hole will bias the value of a sludge sample. The relationship between core and sludge was tested at Kilembe mine, using five different holes with a total length of 280 ft (Davis, 1962). Davis found a wide variation in detail but both averaged 1.4 per cent Cu over the total test footage. This shows that a close comparison between grades of sludge samples and core samples is dubious, and care should be taken in quantifying the data for core loss problems.

Sampling by percussion drilling involves collecting the chips and dust expelled from the hole. Methods of percussion drilling and sampling are reviewed and discussed by Peele and Church (1945), Moehlman (1945), Cheetham and Inett (1954), Harding (1956), Cumming and Wicklund (1975), Clark (1976) and Bertram (1980). There are many sources of error in percussion drilling which will bias samples and increase their variability.

Mineral segregation can occur among the chips and dust in the hole, especially where the expulsion of the material is slow. Up to 30 minutes lag has been measured between the time that rock chips leave the bottom of the hole and the time that they arrive at the collar when a liquid drilling medium is used in a hole 100 m deep (Bertram, 1980). Mineral segregation depends on the energy of the drilling medium, and the time lag is considerably shorter with air flushing. Samples can be
contaminated by the incorporation of wall rock material removed from the sides of the hole by abrasion, but this is largely overcome by using reverse circulation techniques.

Samples can be collected with buckets, trays, troughs or cyclones or by using a mechanical separator. Errors in sample collection arise from contamination, clogging and loss of fines. Contamination can be prevented by keeping the sampling area and containers as clean as possible at all times. Clogging can occur when a small cut of a much larger sample is taken from the particle stream being expelled from the hole. It is often caused by a high moisture content in the chips which arises when drilling beneath the water-table. The clogging causes a disproportionate collection of chips which will bias the sample if it is correlated with the occurrence of mineralization. Sample error can be caused by taking a small cut from a particle stream, if the cut size is incorrect for the type of mineralization and the distribution of particle sizes in the chips. The reasons for this error and the methods by which it can be estimated have been reviewed in section 2.2. Negative sample bias will occur if the valuable mineral is concentrated in the fines and there is a loss of fines, either as a sludge or as dust.

A major disadvantage of percussion drilling is that the samples cannot be taken accurately with respect to geological boundaries. Consequently this method of sampling can best be used where the sample lengths are small with respect to the separation of the geological limits to the mineralization. In broken ground, where significant core losses may be suffered during diamond drilling, better sampling may be achieved with percussion drilling.

2.1.3. Determination of grade in situ.

A number of techniques are available for the determination of ore in situ, and their application avoids the errors caused by sample collection. The techniques are normally cheaper than conventional methods in terms of cost per sample, and a larger quantity of sampling data can be obtained because of the ease with which the techniques can be applied to suitable ore. Direct measurement of grade allows immediate assessment of the mineralization with respect to the geology of the face
being sampled. The techniques include portable X-ray fluorescent analysers, gamma ray scintillometers and visual sampling.

Portable X-ray fluorescent analysers have been applied to the problems of gold sampling on the Witwatersrand (Davies et al., 1979a, b, and Davies et al., personal communication), and to the problems of sampling tin mineralization in Cornwall (Bowie et al., 1965) and in the Cleveland mine, Australia (Cox, 1968a). The instrument works on the principle of irradiating the rock with gamma rays from a source mounted in a hand held probe. The gamma rays have a frequency which excites the element of interest causing it to emit X-rays which are detected by a sensor mounted in the same probe (Bowie et al., 1965). The gamma rays leave the source in a cone shaped pattern which, for the portable gold-analyser, has a collimation angle of $120^\circ$. The depth penetration of the instrument is a function of the energy of the X-ray fluorescence emitted by the element in question. The portable gold analyser used by Davies et al., is able to determine the concentration of gold to within an error of 18 per cent inside a frustrum of rock approximately 8 cm in diameter and $2\frac{1}{2}$ cm deep. Strong concentrations of gold outside this volume, but within the cone of gamma radiation, will still be detected by the sensor, so the instrument has an equalizing effect on patches of high values. The instrument used by Bowie et al. (1965) for the estimation of tin only had a depth penetration of approximately 3 mm.

A sample is taken with a portable X-ray analyser by moving the probe at a slow speed, e.g. 1 metre per minute across the portion of the face to be sampled. Errors are due to counting statistics, geometric effects, and the nature of the mineralization. Errors due to counting statistics are inherent to the instrument and are normally outlined in the instruction manual. Errors due to geometric effects include differential rate of moving the probe across the rock surface, the uneven rock surface and the change in the distance at which the probe is held away from the rock face. Careful handling of the probe and careful rock surface preparation prior to sampling was found to minimize error due to these effects (Cox, 1968a). Errors due to the nature of the mineralization are caused by changes in grain size, matrix composition and heterogeneity of the mineralization. Bowie et al. (1965) found that with two samples having identical tin
content, the sample with the largest grains gives the lowest count rate. Discrepancies due to this effect are most marked in low grade material. Errors due to the matrix effect are caused by varying amounts of associated heavy minerals. Bowie et al. (1965) and Cox (1968a) found that, with samples having identical tin content, those with a greater amount of heavy minerals, e.g. pyrite, pyrrhotite and marcasite, gave the lowest count rate. This can be largely eliminated by using different combinations of energy filters. Water will increase the radiation scatter and decrease count rates proportionally in all energy channels. Cox found that heterogeneous distribution of the mineralization gave spurious results, yet experiments on the Witwatersrand showed the instrument to have an equalizing effect for the same problem. The contrast in error due to heterogeneity is probably related to the different depths of penetration for the X-ray fluorescence.

Experiments on the Witwatersrand (Bowie et al., 1965 a, b) and at Cleveland mine (Cox, 1968a) showed good correlation between results from the X-ray analyser, and groove, channel and chip samples. On the Witwatersrand the range of error in sampling a panel is ± 10 per cent, and Davies et al. (1979b) found the analyser to be 2½ times more precise than chip sampling over a 3 m length of reef that was approximately 8 cm thick. Cox found that the operating time was shorter with the analyser than the time to cut groove samples. He also found that the operating cost of the analyser was less than one half the labour cost in cutting a groove sample.

Gamma ray logging is a common instrumental sampling technique largely employed during the exploration and evaluation of uranium deposits. The method involves lowering a probe down a hole and measuring the level of gamma ray radiation at known depths. The counts obtained in potassium, uranium and thorium energy channels are proportional to the concentrations of the three elements. Haycraft (1976) has tested this technique in holes drilled by three different methods in the Yeelirrie calcrete uranium deposit in Australia, viz. reverse circulation, conventional auger and dry stick auger drilling. He found that the sample values obtained from all three types of drilling were comparable, but that there was a slight negative bias to the grades determined by the gamma ray logging method for
all three holes. Gamma ray logging was found to be affected by contamination of the sides of the holes drilled by the auger techniques, but was more sensitive to the detailed distribution of uranium than the sampling of drill chips at one metre intervals.

Visual sampling is discussed by Jewett (1956), Van Graan (1964) and Mellett (1979) and is a technique that does not receive the recognition that it deserves, yet visual estimates of grade should be made by a geologist. A sample is taken by drawing a straight line of known length over the section to be sampled following the principles of demarcating a channel sample. The portions of the line which lie on ore minerals are measured with calipers and cumulative totals of length determined for each ore mineral. The grade of the sample is then calculated using the formula given below:

\[
\text{per cent metal} = \frac{(a_1 \times b_1 \times c_1) + (a_2 \times b_2 \times c_2)}{(a_1 \times b_1) + (a_2 \times b_2) + (A - a_1 - a_2) B}
\]  \hspace{1cm} (4)

Where
- \(a_1, 2\) = cumulative lengths of valuable minerals 1 and 2;
- \(b_1, 2\) = specific gravities of minerals 1 and 2;
- \(c_1, 2\) = per cent valuable metal in minerals 1 and 2;
- \(A\) = total sample length;
- \(B\) = specific gravity of host rock or gangue.

The disadvantages of the visual sampling method are that it is only a surface sample and it can involve a high degree of personal bias if the location of the sample line is influenced by the position of ore minerals. It can only be applied to coarse grained mineralization and imprecision is caused by careless measurement. The advantages are that the sampling is cheap and easy to take and data on structure, crystal size, distribution of mineralization and waste inclusions can seldom be economically obtained any other way. The method has been used extensively at Messina mine where samples are taken at five foot intervals perpendicular to a base line in a stope or roof of development. The resulting data has to be adjusted by an assay plan factor of 95 per cent indicating that it is a reliable method.

2.1.4 Measurement of porosity and specific gravity

Measurement of the porosity and specific gravity of an orebody is
as important as determining the grade itself because a well conducted sampling programme could be nullified by the bias resulting from use of an incorrect specific gravity factor. For instance, use of a specific gravity factor larger than the true value will lead to overevaluation. Likewise, disregard of significant porosity will lead to overevaluation, especially if the value of a sample is expressed as an accumulation of the concentration of the valuable component and the length of that sample.

Specific gravity and porosity are related by the following equation:

\[
\text{per cent porosity} = 100 \left(1 - \frac{\text{Bulk S.G.}}{\text{True S.G.}}\right) \quad (5)
\]

where Bulk S.G. is the specific gravity of the ore with pores and the True S.G. is the specific gravity of the ore without the pores. Specific gravity can be measured by determining the weight of the sample in air, \(w_1\), and the weight of the same sample in water, \(w_2\), and combining them according to the formula:

\[
\text{S.G.} = \frac{\text{weight of sample}}{\text{weight of displaced water}} = \frac{w_1}{w_1 - w_2} \quad (6)
\]

Other methods of determining specific gravity follow the same principle but make use of the Jolly balance, beam balance and pycnometer (Dana, 1949). The pycnometer method is particularly suited to the determination of the specific gravity of powders. It is filled with distilled water and weighed, taking care that all extraneous water is removed. The selected sample is also weighed. The sample is then placed inside the pycnometer which is then refilled with water. The difference between the sum of the sample and pycnometer weights, and the weight of the pycnometer, is equal to the water displaced by the sample. This is divided into the sample weight to give the specific gravity. To determine the specific gravity of a porous ore, the sample can first be sprayed with a waterproof varnish (Martin and Allchurch, 1976) or coated with a paraffin wax (Box and Reid, 1976) before submerging in water. The true specific gravity of a porous ore can be determined from a sample of the pulverized ore using the pycnometer method.

Values of specific gravity in an orebody are as variable as the
grade and their estimations are equally prone to error. A sample selected for the determination of specific gravity should be typical of the ore it is supposed to represent. Consequently, unless one is attempting to measure bulk specific gravity, all specific gravity determinations for inconveniently large samples should be carried out on properly reduced sub-samples. Often there is very good correlation between specific gravity and grade, and correlation graphs can be constructed. The graphs can then be used to determine the specific gravity for a given grade, with only a small increase in imprecision than if the specific gravity of the sample were determined directly. This has been used successfully by Box and Reid (1976) who found significant contrasts in the porosity of the Cockatoo Island iron orebody due to silica leaching and iron enrichment. Once a correlation between grade and specific gravity had been determined, they were able to weight small increments of ore reserves with the appropriate specific gravity.

2.1.5. Detection of error in sample collection

Error in sample collection is very difficult to detect, and can only be confidently estimated by comparing the sample results with a reliable reference value obtained from a bulk sample or the mill returns from a specific block of ore. Such comparisons will reveal even small biases in the various sampling techniques, e.g. positive bias in split core samples from Kilembe mine (Davis, 1962). Often reliable reference samples are unavailable, especially at the beginning of an exploration programme where fresh ore is seldom exposed. In this case, sampling error should be continuously guarded against and small experiments conducted to gauge the error that can be expected from poor sampling technique. For example, Sichel and Rowland (1961) found that the error in underground channel sampling on the Witwatersrand could be controlled by resampling five per cent of the channels and comparing the two results. The variance between the values had to lie within acceptable limits. Other experiments have been described in previous subsections, e.g. sludge sampling to gauge the degree of core loss.

2.2. Sample reduction

Sample reduction is the process by which samples collected in the field, underground or with drills, and which are too large to be
submitted directly to the analytical laboratory, can be reduced to a smaller size or subsample. Davis (1963) defines the aim of sample reduction as, "... to reduce the original coarse bulky sample to a relatively small quantity of finely divided and homogeneous powder, from which any small portion chosen for assay represents the metal concentrations present in the whole of the original sample".

There are numerous different methods by which a sample can be reduced or split and these include:

1. coning and quartering;
2. Jones riffler or segmented drum that is rotated below the outflow of a bin;
3. automatic sampler in particle feed which periodically diverts a cross-section of the stream of material which is retained as the sample;
4. spot and grab samples selected by rule;
5. spot and grab samples selected at random.

The methods listed above are self explanatory and it is assumed that the reader is familiar with them. Descriptions can be found in McKinstry (1948), Milner (1962), Cornish (1966), Basden (1970), Levin (1974) and Gy (1979). Each method has its own advantages and disadvantages and will be particularly suited to different types of ores. The splitting methods remove between \( \frac{1}{2} \) to \( \frac{1}{10} \) of the original sample. Significant errors can be generated in subsampling due to the heterogeneity in the original sample and the inhomogeneity in the crushed sample.

The original sample is normally heterogeneous because the valuable components in the geological unit being sampled are segregated into minerals and the minerals concentrated into laminae or other geological structures. Homogenization of the sample can be achieved by crushing and pulverizing because a reduction in particle size of the sample will destroy the heterogeneity preserved in each particle. Homogeneity of a crushed sample therefore depends on the proportion of liberated to composite grains in the material.
Once a sample has been crushed to below a specific particle size and mixed, it can be split into a subsample. The subsample must be large enough so that a chance omission or addition of grains of the valuable mineral does not change the grade by more than a permissible error. The distribution of results from the subsamples will then be approximately normal or Gaussian and the mean is representative of the true value of the original sample. This is shown in Fig. 3 where the

Fig. 3. Histograms illustrating the reproducibility of mine assay values when the identical sample is split after (a) coarse crushing, (b) fine crushing, and (c) pulverizing.

(From: Koch and Link, 1970)
values of the subsamples taken from the pulverized sample are virtually normally distributed. However, if the size of the subsample is such that it will only contain a small number of grains of the valuable component, then the subsample is not representative of the sample and the Gaussian approximation breaks down. The variance between subsamples from the same sample increases and the mean of the subsamples becomes biased with respect to the true value of that sample. Although this bias is found at low concentrations of the valuable component, it depends on how the valuable component occurs. For example, if the valuable component is 100 ppm Ag that is diffused in galena, of which there are a large number of grains in the subsample, then the results will be unbiased and the variance low. If the valuable component is, for example, 100 ppm Zr occurring in zircons, then 1,0g of pulverized rock passed through 300 mesh will contain approximately 200 grains of valuable mineral. Lister (1980) found that emission spectrography analysis for Zr at the above specifications gave results that ranged from 30 - 300 ppm, and that reproducibility could not be achieved without taking larger subsamples.

The distribution of values in subsamples where the values depend on a small number of grains, can be compared to Poissonian statistics, where the variance of the distribution is equal to its mean (David, 1977). For example the coefficient of variation for three grains (assuming all grains to have equal size) is:

\[ \sqrt{3} / 3 = 58\% \]

for ten grains;

\[ \sqrt{10} / 10 = 32\% \]

and for twenty grains;

\[ \sqrt{20} / 20 = 22\% \]

The reduction in variance due to smaller particle size, and therefore a larger number of grains of the valuable component, is illustrated in Fig. 3. The diagram shows how the spread of subsample values is reduced with decreasing particle size.

A subsample value will also become negatively biased if the value
depends on a small number of particles of the valuable component, corresponding to a positive skewness in the histogram. This is clearly shown in Fig. 3. A positive bias or negative skewness will occur where a subsample is composed almost entirely of the valuable mineral, and its concentration depends on the presence of a few particles of a foreign mineral. This is common in the sampling of pegmatites, for example, where the sample consists almost entirely of one mineral, e.g. lepidolite or amblygonite, and the concentration of Li depends on the presence of a few grains of impurities. If such bias occurs in the subsampling procedure, it will give an artificial distribution to the estimated grade of the ore and seriously affect the ore reserve estimation.

The Poisson distribution predicts this bias and shows theoretically that significant bias will occur in a subsample value if that value depends on six particles or less in the subsample (David, 1977). However, the Poissonian distribution is too simple to apply to the problem of subsampling because the assumption that all particles have the same size is unjustified when dealing with a crushed ore. Furthermore, it does not account for the proportion of grains that have not been liberated from their matrix, and therefore carry with them some heterogeneity inherent in the original sample. It is important to find a relationship between particle size, mineral grain size, size of the subsample and error, because subsampling is an added expense to the sampling programme and should be carried out efficiently and with a minimal amount of error.

Attempts to resolve the problem of sample reduction were made by Brunton (1896) and Richards (1903) who published tables of subsample size versus grain size for a variety of different ore types. The tables were based on both mathematical and experimental work. Later improvements were made by Taggart (in Cornish, 1966) who based his work on Richards, Demond, Halferdahl and Argall. Davis (1963) has shown this work to be conservative when applied to the sampling of the Kilembe mine (see Fig. 6).

Clifton et al. (1969) studied the problem of sample size specifically for gold ores. Their approach required that a sample must contain a minimum of 20 gold grains and their reasoning is based on the binomial and Poisson distributions. This gives estimates of minimum
sample weight that are high when compared with estimates using the same
data in Gy's equation. Becker and Hazen (1961) have studied the
application of the binomial, multinomial, compound binomial and compound
multinomial distribution functions to broken ore and subsampling condi-
tions. They show how, by screening the ore mixture into particle sizes
and estimating the mineral contents for each size of particles, the
sampling moments and therefore the standard deviation and skewness may be
determined for a range of particle sizes. However, their approach has
been largely superseded by the concepts of two independent workers,
Ingamells and Gy. Ingamells developed his ideas to suit a geochemical
laboratory and Gy's work has been designed for metallurgical problems,
although essentially the two approaches are the same.

2.2.1. Ingamells' approach

Ingamells' work is given in detail in Ingamells (1974a, b)
although the reader is also referred to Engels and Ingamells (1970),
Ingamells and Switzer (1973) and a brief summary in David (1977).
Ingamells considers a two-mineral mixture that can be compared to an ore
with approximately cubic shaped grains in a disseminated background. He
relates all the variables to a sampling constant $K_s$ by the following
equation:

$$K_s = \frac{10^4 (K-L) (H-L) u^3 d}{K^2} \quad \ldots \ldots (7)$$

which is valid when $H \gg K$, i.e. when the valuable component is concen-
trated into ore minerals. The sampling constant is the minimum weight of
sample (in grams) needed for a one per cent relative standard deviation
in the analysis at 68 per cent confidence, assuming that the analytical
error is negligible. $K$ is the true concentration in per cent of element
$(X)$ of interest; it is estimated from weighted mean of all analytical
results. $H$ is the $X$-content of the minor mineral component, i.e. the ore
mineral and rich in $X$, and $d$ is the density of the mineral; both $d$ and $H$
can be estimated after the ore mineral has been identified. The grain
size, $u$, is estimated by inspection of unscreened material, or from the
mesh size of the sieve through which screened material has passed.
$L$ is the $X$-content of the major component of a two-mineral mixture. It is
normally the gangue and may account for a low background value for $X$. $L$
is difficult to determine, but two methods are described in Appendix I.
The sampling constant $K_s$ can be used to determine the weight, $W$, of sample needed for a specified degree of precision. For a given grain size, the variance of a determination (assuming negligible analytical error) is inversely proportional to the mass of a sample, and is given by the equation:

$$
\sigma^2 = \frac{A}{W}
$$

where $A$ is a constant. Because $K_s$ is that mass which will yield a one per cent precision, one can write:

$$
\sigma = \sqrt{\frac{K_s}{W}} \quad \text{or} \quad W = \frac{K_s}{\sigma^2}
$$

Sampling and subsampling diagrams (Ingamells, 1974a) provide a means of measuring and controlling subsampling error through visual presentation (Fig. 4).

Fig. 4.  
(From: Ingamells, 1974a)

Ideally a sampling diagram is prepared by repeatedly measuring the X-content in samples of different weights, and plotting averages ($\bar{x}$), standard deviations ($\sigma$) and ranges (error bars) against sample weight.
In practice, sampling and subsampling diagrams are constructed from the following equations:

\[
\frac{(K - L)^2}{S^2} = \frac{1}{3}, \quad \frac{Y - L}{K - Y} = \frac{K - L}{H - L} \cdot \frac{W}{u \cdot d}
\]  

\[(10)\]

By estimating \( K, L, H, u, \) and \( d \) for specific sample weights \( W \), then \( S \) (the standard deviation at 68% confidence) and \( Y \) (the most probable result of the determination) can be calculated. The values are then plotted on either side of \( K \) which is represented by a line parallel to the ordinate. Fig. 4 shows how, as the sample size is reduced, the variance increases and the "most probable result" (corresponding to the peak of a frequency curve) becomes negatively biased. Note how the distribution of values from the 100 g samples has the same skewness as the histogram in Fig. 3, but that the bias is eliminated by taking a larger sample.

2.2.2. Gy's equation

Gy's equation is summarized by Ottley (1966). Derivations can be found in Gy (1976) and Gy (1979). Briefly summarized, the equation is:

\[ M = \frac{Cd^3}{S^2} \]  

\[(11)\]

where \( M \) is the sample weight in grams, \( d \) is the top particle size of the crushed sample and \( S \) is the standard deviation of the values that can be expected from a sample of mass \( M \). \( C \) is the sampling constant which remains constant for a given ore, but will be altered by the largest particle size. Details of the practical application of this equation can be found in Appendix II.

The equation is simple and versatile, and by rearrangement it can be used to calculate the weight of subsample required for a given error limit and top particle size; the subsample error knowing the subsample size and particle size; and the top particle size for a given subsample size and error limit. The equation is applicable to well mixed ores where there is no bias from sampling technique or device. The user may find it necessary to make slight adjustments to the constants to suit a particular ore. The Pierre Gy Sampling Slide-Rule has been based on this formula.
Sample reduction steps can be planned from Gy's equation. If the acceptable error in the final sample is $S_f$, then $S_f^2$ will be the sum of the variances ($S^2$) produced by each sampling step. The calculated error can then be judged against the practical and economic implications of each step. Fig. 5 A and B shows how, by plotting the logs of sample weight versus sample size, a graph can be subdivided into two by an oblique straight line corresponding to Gy's equation for a specific limit of error. Sample reduction steps to the left of the line are sound (Fig. 5A) but to the right of the line are unsound (Fig. 5B) for that error. Davis (1963) experimentally established the curve at an error of 3 per cent for the Kilembe copper ore. It was found to correspond to the 2 per cent error line as calculated by Gy's equation (see Fig. 6) for the same ore.

2.2.3. Other errors in sample reduction

Every care should be taken to ensure that all machinery, e.g. crushers, pulverizers, and screens that come into contact with the sample

Fig. 5. Diagrams showing safe (A) and unsafe (B) sample reduction.

(From: Gy, 1979)
are clean. Samples should be large enough so that inadvertent contamination will be negligible. Samples containing marginal grades should not be handled immediately after samples from rich ores, without thorough cleaning.

When passing a sample through a screen, surreptitious discard of screen oversize must be avoided as this will bias the result. Davis (1963) found that the copper values from the Kilembe ore were increased

![Diagram](Fig. 6. From: Davis, 1963)
by 3 per cent for every 5 per cent of 100 mesh sample oversize that was
discarded. This positive bias was probably caused by the soft
chalcopyrite concentrating in the fines. In contrast, Davis found a
decrease in cobalt values for the same oversize discarded because the
metal occurred in pyrrhotite and linnaeite which have a mixed hardness.
Had this practice of discarding the screen oversize been allowed to
continue it would have resulted in a 3 per cent overevaluation for copper
and up to a 3 per cent underevaluation for cobalt in the Kilembe orebody
(Davis, 1963).

Grains of heavy minerals in a pulverized sample tend to concen­
trate during mixing, especially if the sample is rolled (Davis, 1963).
Jewett (1956) suggested that large gold particles in a sample should be
screened out and weighed before assay and recombined with the assay value
on a pro rata basis. This would avoid the bias generated by coarse
gold erratics.

Most errors associated with the splitting are produced by the
fines as it is this particle size fraction of a crushed ore which normally
contains the highest values. Loss of fines as dust should be avoided
and any concentration of fines in one part of the sample (e.g. the apex
of the cone in coning and quartering) should be avoided. Pantony (1980)
concludes "No method will provide a truly representative sample without
application of honesty and almost obsessional care at both planning and
execution stages."

2.2.4. Detection of errors in sample reduction

Although the errors involved in sample reduction can be predicted
mathematically, they should be confirmed by experiment. This can be done
by repeatedly splitting a crushed sample of fixed particle size into
smaller fractions, and determining the concentration of the valuable con­
stituent in each fraction. The mean and standard deviation of the results
obtained from equivalent fractions taken from the same sample can be
calculated. A curve of approximate minimum sample weight for various
maximum particle sizes can be constructed from this data, as shown in Figs
5 and 6 (Davis, 1963). This method can also be used to compare the
errors caused by different steps taken in the subsampling procedure.
2.3. ANALYSIS OF SAMPLES

Analytical error has the same influence on the total error of a sampling procedure as the error in sample collection and sample reduction. Therefore, analytical error should be kept within acceptable limits. The causes of analytical error and how they can be prevented will not be discussed here but the interested reader is referred to Hill (1940) and Nicholls (1974) who discuss the errors generated in assaying for precious metals. Craven (1954) describes a method for the statistical estimation of the accuracy of an analytical technique. Lister (1977) presents details of an interlaboratory survey and includes information concerning the precision and accuracy in the determination of metals in ores.

2.4. CONCLUSIONS

Sampling errors generated by sample collection and sample reduction are numerous and must be controlled if a sampling programme is to be efficient and reliable. Errors in sample collection are often suspected but they are difficult to measure unless they can be compared to a reference value such as the grade of the bulk sample. They can be reduced to acceptable levels by meticulous attention to detail and by application of experience gained from other orebodies of similar mineralization. Errors due to sample reduction are as important as those from sample collection and analysis, and can occur up to and including the selection of the aliquot size taken from the reduced sample for analysis. Sample reduction errors are more easily measured and predicted than errors in sample collection, and are controlled by the interrelationship between ore type, particle size and subsample size. Analytical errors should be kept within acceptable limits. For the remainder of this dissertation, errors due to sampling procedure will be assumed to be negligible.
Having identified the source of errors in sample collection, sample reduction and analysis, and shown how the errors may be reduced or avoided, the next problem that arises is to establish how can samples be taken most profitably from an orebody. The samples must have every chance of being representative of the ore from which they are taken, they must have the lowest practical variance between them, and they must have the largest possible area of influence, so that the least number of samples are needed. In practice it is almost impossible to establish the necessary shape, size and number of samples in any great detail. However, by identifying the characteristics of a deposit which will influence the dimensions and number of samples needed to accurately and efficiently determine the grade of the orebody, the most can be achieved from a sampling programme.

Procedures of sampling mineral deposits belong essentially to three categories (Hazen, 1967 a);

a) Random sampling;
b) Systematic sampling;
c) Stratified sampling.

Complete mathematical discussions of the statistics of these methods can be found in Cochran (1963). Random sampling requires that each element or item to be sampled be given an equal chance of being selected. This means that every possible sampling unit that may exist in a mineral deposit has an equal chance of being selected, each time a sample is withdrawn from the population. Random sampling is seldom achieved in the geological context. Griffiths (1974) presents an algorithm (Fig. 7) designed to achieve random samples in the evaluation of structure, and shows how it can be applied to the problem of determining structure in a fluvio-glacial gravel terrace. The algorithm randomises the data and applies statistical checks to detect bias caused by the interaction of the sampling arrangement with the element arrangement. Systematic sampling is basically a repeated or successive sequence at some definite selected interval, e.g. drilling on a regular grid or sampling at a regular interval on a face or along drill core. This may or may not
yield random data depending on the presence and extent of small-scale structures which control the distribution of the mineralization, compared with the sampling interval. Stratified sampling is the process of dividing a large population into two or more subpopulations, and is the method best employed for the evaluation of mineralization. Stratified sampling can be either random or systematic.

3.1 THE SAMPLING STRATUM

The valuable component of an ore is not evenly distributed throughout the orebody but is contained within minerals, the grouping of which is determined by the physical and chemical conditions which prevailed during deposition. For example, the mineral may be concentrated in
laminae along the bedding planes of a sediment, or deposited in irregular fractures and veinlets as found in a porphyry deposit. This is the inherent variability of an ore. An orebody can be subdivided into geological units each of which contain mineralization with the same grade and variability, and which formed under essentially similar conditions.

A geological unit in an orebody may exist on more than one scale and the various scales of units will be "nested" inside each other in order of decreasing size. For example, consider a volcanogenic massive sulphide deposit. The deposit as a whole may be lenticular, corresponding to a depression in the sea floor. Within it, stratiform sulphide bodies may be distributed according to successive episodes of brine exhalations. Within the bodies, the sulphide minerals may be distributed in bands, and within the bands the minerals may be grouped into aggregates. Each of these scales corresponds to a geological unit and one will be more suitable for sampling than the others.

Otto (1938) has studied the value of the sedimentation unit in sampling, and although his observations have been confined to the strata in aeolian sand, they are equally applicable to the general problems of identifying the relevant geological boundaries in orebodies. He observed that the growth of a sand dune in prevailing fair weather conditions is principally related to the magnitude and direction of the wind velocity. It is assumed that for short periods of time, growth factors such as sand supply, moisture conditions, availability of sand, and dune size and shape are virtually constant. The wind variations include local erratic fluctuations as well as more extensive regular fluctuations. The erratic fluctuations in the form of eddies and gusts cause a complex pattern in the laminae, with individual units traceable for only a few centimetres, e.g. as found in ripple marks which migrate and change their dimensions in response to the many changes in magnitude and direction of the wind. The regular fluctuations are caused by diurnal winds and the regular weather pattern, and their effects are recorded in the sediment as gradual changes in composition which can be traced over large areas or significant thicknesses of strata, e.g. a bed characterized by a particular grain size or ripple mark.
Influences on the growth of the sand dune can be categorized as local or extensive, depending on their area or volume of influence. Local influences due to erratic wind fluctuations produce inconsistent laminae that can be traced for only a few centimetres. However, the effects of these individual laminae can be averaged by considering a number of adjacent laminae as one unit. The unit chosen should have boundaries between which the local influences have remained essentially constant, and includes material deposited under extensive influences such as diurnal wind fluctuations and the regular weather pattern. This is the sedimentation unit for which Otto (1938) outlined a number of steps in field recognition. These steps, with some modification of the concepts, can be used as follows to outline any geological unit of value to the problem of sampling an orebody:

a) Define the ore to be sampled in terms of the agencies of transportation and causes of deposition. If two agencies of deposition act alternately and portions of the deposits formed by each are left unchanged, the alternate layers cannot be one geological unit.

b) List the environmental factors that determine the statistical properties of a single mineral aggregate and groups of aggregates. The factors would include the composition of the ore-bearing fluids, fluid supply, direction of flow, rate of deposition, size of depository and host rock porosity, and they control the mineralogy, grade and distribution of grade within the geological unit.

c) Determine which of these factors belong to the category of extensive influence. For example, the size and shape of a depository and direction of fluid flow can be extensive influences, while host rock porosity and rate of deposition may only be local influences.

d) Determine whether the remaining factors are local influences. This may reveal unsuspected extensive influences.

e) Trace the effects of the extensive influences on the nature and composition of the mineral aggregates, by considering
what happens if the influence undergoes its expected range or cycle of variation. The purpose of this step is to identify those conditions which introduce a sharp change to the pattern of ore deposition, as these mark the boundaries of the geological unit. The magnitude of the effect must be large enough to remain unaffected by the smaller fluctuations in the local influences.

f) As a check to the inductive reasoning, re-examine the mineral aggregates for discontinuous and rapid changes not indicated by the inductive analysis. This could reveal an unsuspected extensive influence.

Choice of a geological unit for sampling depends on the proportion of the ore deposit represented by that unit, and the size, shape, and number of samples that must be taken from that unit so as to obtain an unbiased estimator of the grade of the unit, within an acceptable level of confidence. The necessary size, shape and number of samples must be weighed against the practicalities and economics of obtaining them, and the level of confidence at which they will estimate the grade of the unit. Circumstances may dictate the choice of a geological unit which eases the problem and cost of obtaining the samples, but which will yield an estimate at an inferior level of precision. Whatever geological unit is chosen for sampling, it will be referred to here as the sampling stratum.

Once the sampling strata of an orebody have been recognized, the problem of sampling them can be related to the size, shape and orientation of the samples relative to the size of the strata, and the size of the small-scale structures within them. All combinations of relative sizes of the sampling strata and small-scale structures can be found. For example in stratiform ores, the sampling strata will be beds and the small-scale structures will be laminae of ore minerals within the beds. The laminae are often very extensive, allowing correlation between samples over large distances. In porphyry deposits the smallest sampling stratum larger than the small-scale structures may be very large with respect to the size of samples. This stratum corresponds to the dome shaped
orebodies surrounding the intrusive plug. The small scale structures are fine fractures and veinlets, each of very small extent compared with the stratum size, and often within the dimensions of the sample.

3.2. MEASUREMENT OF THE DISTRIBUTION OF SAMPLE VARIANCE IN A SAMPLING STRATUM

The inherent variability of grade in a sampling stratum is due to the distribution of mineralized structures within the stratum. Sample variance is caused by the selection of small portions of the variable ore and is therefore a function of the size of the structures and the contrast of grade from one part of the stratum to the next. For example, the variance between samples will be greater in a sampling stratum that contains patches of high grade mineralization interspaced by barren country rock, compared with a sampling stratum containing stratiform lead and zinc sulphides. A maximum variance between samples will be reached at a closer sample spacing in the stratum containing patchy mineralization than in the stratum containing the stratiform sulphides. Sample variance can be reduced to a certain extent by increasing the sample size and/or decreasing the spacing, but eventually points are reached beyond which an increase in sample size or decrease in spacing will not have a significant effect. It is therefore desirable to be able to measure the distribution of sample variance within an orebody so that the effects of sample size, shape and spacing in reducing the sample variance can be predicted from it.

One of the best and most popular mathematical representations of the distribution of sample variance in a sampling stratum is the semi-variogram, which is expressed by the following equation:

\[ \gamma(h) = \frac{1}{2N_h} \sum_{i=1}^{N_h} (g(i) - g(i + h))^2 \]

where \( \gamma(h) \) is the variance between the grades \( g(i) \) and \( g(i+h) \) of samples separated by distance \( h \). \( N \) is the number of samples that are separated by distance \( h \). The semivariogram can be determined in any direction by taking a number of closely spaced samples and determining the average variance for all sample pairs separated by a fixed distance. This is
repeated for increasing distances of separation which are multiples of the original sample spacing. Details of the construction of a semivariogram can be found in David (1977), Rendu (1978), Journel and Huijbregts (1978) and Clark (1979 a, b).

Once an experimental semivariogram has been established, it can be fitted to a model, of which there are two basic types, viz. models with sills, and models without sills. Models without sills include:

Linear \[ \gamma(h) = \rho h \]  \hspace{1cm} (13)

Generalized linear \[ \gamma(h) = \rho h^{\lambda} \text{ where } 0 < \lambda < 2 \]  \hspace{1cm} (14)

and \( \rho = \) slope.

De Wijsian \[ \gamma(h) = 3 \alpha \ln h \]  \hspace{1cm} (15)

where \( 3 \alpha = \) slope and \( h \) is the separation between samples. Fig. 8 shows the linear and generalized linear semivariograms and Fig. 9 illustrates a De Wijsian semivariogram from a Witwatersrand gold mine.

Fig. 8. Linear and generalized linear semivariograms.

(From: Clark, 1979a)
Fig. 9. De Wijsian semivariogram for gold values in the Leader Reef in the President Steyn gold mine.

(From: Krige, 1978)

Fig. 10. Varieties of semivariograms with a sill.

(From: Rendu, 1978)
Models with sills include:

- **Exponential**
  \[ \gamma(h) = C \left( 1 - e^{-(h/a)} \right) \]  
  \[ (16) \]

- **Spherical**
  \[ \gamma(h) = C \left( \frac{3h}{2a} - \frac{h^3}{2a^3} \right) \]  
  \[ (17) \]

\[ \text{if } h \leq a \]

\[ \gamma(h) = C \]  
\[ (18) \]

where \( h \) is the separation between samples and \( a \) = range of the semivariogram.

The two models are illustrated in Fig. 10. For small values of \( h \) they are essentially linear, but their slopes are different. The Gaussian semivariogram is a third model with a sill but it is not often encountered in nature.

The theory of the semivariogram requires that the distribution of grade in the sampling stratum obeys the intrinsic hypothesis. The conditions of the intrinsic hypothesis are:

a) The expected value of the difference \( g(i) - g(i+h) \) is independent of the point \( i \) for any distance \( h \);

b) The semivariogram is independent of the point, \( i \), for all possible distances, \( h \) (Rendu, 1978).

These conditions mean that the distribution of the difference in grade between two samples is the same over the entire deposit and that it depends only on the distance between, and the orientation of, the points. In other words, differences in grades must be consistent, not constant, over the deposit (David, 1977; Clark, 1979a). The conditions are less restrictive than those of second order stationarity which must be satisfied to justify the use of correlograms or covariograms, alternate methods of measuring the inherent variability of grade in a sampling stratum. The conditions of the intrinsic hypothesis are satisfied whenever the second-order stationarity conditions are satisfied, which makes the semivariogram a more versatile model than the correlogram or covariogram.

Nature is seldom stationary but many experiments have shown that semivariograms are hardly affected by strong departures from the hypothetical conditions. The effect of non-stationarity or the existence of a trend is to add a parabola to the semivariogram. If the parabola appears
beyond the range of the semivariogram, the trend will not necessarily interfere with the statistical value of the semivariogram. If, however, the parabola is apparent at a distance smaller than the range of the semivariogram, it indicates that there is a very strong trend in the sampling stratum and a semivariogram model is not justified (David, 1977).

The most commonly used semivariogram model is the spherical model as it approximates many natural distributions of sample variance.

![Spherical semivariogram showing the range of influence and proportions of variance due to spatial and random causes.](From: Rendu, 1978)

Fig. 11 shows that the maximum variance that can be achieved between samples is represented by the sill that the curve forms beyond the range of influence. This variance is the sum of random and spatial components. The random variance is due to mineralization controlled by and contained within geological structures that are smaller than the samples in which they occur. This variance accounts for the fact that two samples taken adjacent to each other will not have the same values and is commonly known as the nugget effect. The spatial variance is related to the distance that separates the samples. As the distance between the two samples increases, so the quantity of small-scale structures and mineralization common to both samples decreases, corresponding to an increase in the variance between them. The variance reaches a maximum or sill at the range of influence of the sample. For example, in a deposit where the mineralization occurs in laminae, the range would correspond to the average length of the laminae along the direction in which the semivariogram was determined. If the samples are separated by a distance greater than the extent of the small-scale structures, then the semivariogram will
have an apparent pure nugget effect, as shown in Fig. 12. The De Wijsian model is useful where the structures controlling the mineralization extend further than the maximum sample spacing used to determine the semivariogram, or the range of a spherical semivariogram is larger than a convenient sized ore unit, e.g. mining block. In the latter case, the slope of the spherical semivariogram within its range can be approximated to the De Wijsian model. The De Wijsian model is popular because of the comparatively simple mathematics associated with it. The Gaussian model is encountered in the determination of variance in the thickness of some bedded deposits. The model differs from other semivariograms with a sill because it is parabolic near its origin. This corresponds to the high degree of continuity in thickness between closely spaced measurements.

The use of experimentally determined semivariograms is limited by the fact that they average the distribution of sample variance, and are matched to restricted mathematical models. Where the distribution of sample variance is even, as can be expected in a stratiform deposit, the averaging is more justifiable than in very variable mineralization. However, in certain instances this averaging can be advantageous (Sandefur and Grant, 1976). Representation of the distribution of sample variance by a single mathematical equation has a further averaging effect. Normally the distribution will be related to a number of different structures each with their own size, configuration and mineralization. If mineralization is related to two different scales of structures, each superimposed on the other, then the experimental semivariogram will be the sum of the two semivariograms corresponding to the two different structures as if
they occurred separately. This is shown in Fig. 13. For many ores this, too, is an oversimplification and experimental semivariograms will not necessarily show two different slopes separated by an inflexion point. In many cases the effects of large structures will be lost in the sill of the semivariogram. Fig. 13 does show that the range of influence of a sample will be a summation of the two structures unless the upper slope of the semivariogram is nearly flat. In this case the significant range of influence will be that of the smaller structures. But for all the criticisms, the semivariogram clearly illustrates the random and spatial components in sample variance. Some of the uses and successes of semivariograms will be discussed, and the effects of sample size, shape, and patterns will be related to it.

Fig. 13. Resultant semivariogram for sample variance due to two different structures.

(From: Rendu, 1978)

In some cases it will not be possible or practical to determine the semivariogram for a deposit. Consequently the sample variance cannot be quantified with respect to distance of separation. Correlation between samples must then rely on geological interpretation.

3.3. ANISOTROPY AND SAMPLING WITH RESPECT TO THE SAMPLING STRATUM

The inherent variability of grade in most orebodies is anisotropic, in other words it is not distributed evenly in all directions. Anisotropy is the direct result of the prevailing physical and chemical
conditions during formation of the deposit. For example, alluvial deposits have a greater semivariogram range parallel to the current directions; semivariograms from deformed massive sulphide deposits have greater ranges along the fold axis directions than across them. The anisotropy of an orebody or sampling stratum can be estimated by determining the semivariogram in two or three directions perpendicular to each other, and orthogonal to the geological structures that control the distribution of the mineralization (Fig. 14). The two or three axes of anisotropy can then be represented by an ellipse or ellipsoid, with the lengths of their axes corresponding to the ranges of the semivariograms.

![Fig. 14. Anisotropy in sample variance.](From: Journel and Huijbregts, 1978)

Some small-scale structures will extend beyond the boundaries of the ellipsoid while others will fall short of it, but by establishing the ellipsoid from the semivariograms, an average length for each axis of anisotropy will be found. The anisotropy modulus is the ratio of the two ellipsoid axes in any plane. For anisotropic De Wijsian semivariograms, the modulus is determined by the ratio of the ranges for a given variance (Blais and Carlier, 1968).

A sample should be taken across the small-scale structures so that it will average the maximum variability in grade caused by them (Griffiths, 1974). Fig. 15 illustrates the various types of structure that can be expected in a sampling stratum. A 'traverse' as referred to in the diagram, can be considered as the long axis of a single sample or a succession of samples through the sampling stratum. Samples that are taken right across the sampling stratum parallel to the shortest axis of anisotropy, will show a minimum variance between their values.
<table>
<thead>
<tr>
<th>Structure</th>
<th>Massive</th>
<th>Regular Bedded</th>
<th>Irregular Patchy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traverses</td>
<td>Zero</td>
<td>Large</td>
<td>Large</td>
</tr>
<tr>
<td>Traverses</td>
<td>Zero</td>
<td>Large</td>
<td>Large</td>
</tr>
<tr>
<td>Within</td>
<td>Large</td>
<td>Zero</td>
<td>Large</td>
</tr>
<tr>
<td>Total variation</td>
<td>Equal to all cases.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 15. Sample variances for different structural configurations.

(From: Griffiths, 1974)

Fig. 16. Vertical semivariograms computed for increasing sample lengths in an Australian uranium deposit. The semivariograms for the shorter sample lengths show a marked "hole effect".

(From: David, 1977)
Values of samples that have dimensions smaller than the thickness of the stratum, parallel to the shortest anisotropy axis, will have a variance larger than the minimum variance. This is because the samples will be independently influenced by different small-scale structures (see Fig. 15). In this case the samples should be limited to the sub-boundaries within the sampling unit so as to maximise the correlation between them. Fig. 16 shows the variance between samples taken through a uranium orebody approximately 10 m thick and composed of alternating rich and poor bands (David, 1976). Each semivariogram represents samples of different lengths. Samples that have lengths shorter than the thickness of the orebody show a high variance between their values and produce a "hole effect" in the semivariogram. As the length of the samples increases, the variance between their values decreases until the sample length approaches the thickness of the orebody, i.e. 9.6 m. At this length, the "hole effect" is eliminated and the values show a minimum variance. This would indicate that the "hole effect" is caused by the sampling as opposed to the variable mineralization as suggested by David (1977).

Fig. 16 also shows that once a sampling stratum has been isolated from the rest of the deposit, it does not really matter how the samples are taken with respect to the alternating rich and poor bands, as long as the individual samples are combined, with appropriate weightings, and related back to the sampling stratum as a whole, e.g. 9.6 m length samples. The decision on how to sample the alternating bands would rest on the potential biases that may occur through uneven sample taking, as discussed in subsection 2.1.1 on hand taken samples. If, in the course of sampling, it is evident that there is a significant boundary separating the alternating bands into two groups, then it is possible that the incorrect boundaries have been chosen for the sampling stratum. Recconsideration should be made of the steps to identify the boundaries and the stratum reassessed.

Where the boundaries to the sampling stratum are either faults or unconformities, and part of the original stratum has been removed, it will not be possible to take samples so as to achieve a minimum variance corresponding to the original distribution of the mineralization. In this
situation either the sub-stratum of ore affected by the transgressive boundary can be isolated and sampled separately, or a larger variance for the whole stratum will have to be accepted.

If geological units are disregarded in sampling, the sample variance will be a function of the ratio between the size of the sample and the size of the geological unit so that the variance will not be independent of the sample size. This type of relationship can produce grossly heterogeneous variance (Ehrlich, 1964). Sichel (1947) shows that error is introduced into samples taken from a Witwatersrand reef if the sampling limit is extended beyond the geological boundary. Therefore an orebody should never be sampled to a minimum stope width, unless the stope width is smaller than that of the geological unit enclosing the ore. If the wall rock is thought to be mineralized it should be sampled separately from the body carrying the majority of the mineralization. Samples taken according to geological boundaries can be recombined mathematically to any fixed lengths, but not vice versa, because mathematics cannot remove heterogeneous variance from sample values.

3.4. SAMPLE VARIANCE AND BIAS AS CONTROLLED BY SAMPLE SHAPE AND SIZE

Both random and systematic errors can occur when samples are taken from a sampling stratum. They are caused by the inability of the samples to even out the inherent variability of the mineralization. If the samples are small with respect to the small-scale structures in a sampling stratum, their values will be negatively biased and will have a high variance. Fig. 17 illustrates this type of variance in values of copper obtained from borehole samples compared with the grades of the corresponding bulk samples. It shows that if the sample size is increased with respect to the small-scale structures, the variance and bias is reduced. The errors are therefore a function of the size and shape of the samples relative to the size, shape and configuration of small-scale structures that control the distribution of mineralization. A number of attempts have been made to find the relationship between these factors, and some will be reviewed here.
Fig. 17. Histogram of the values of 10 ft samples compared to the histogram of the grades of 50 ft blocks in a porphyry copper deposit. (From: David, 1977)

3.4.1. Sample variance

The distribution of grade in an orebody is not random but controlled by the many factors which influenced its formation. However, if the size of the structures which control the distribution of ore are small with respect to the size of the samples, then the structures can be considered as being random relative to the samples. Kendall, quoted by Griffiths (1974) states "I contend that there is no such thing as absolute randomness .... Randomness is relative". Randomness, as defined by Hazen (1967a)"implies either a lack of order or of conformity of measurements when the measurements are listed in the order required, and it inherently implies inclusion of the measurable attributes in direct proportion to their relative importance. A random sample should show independence between successive sample units". For a discussion on randomness in geology, the reader is referred to Mann (1970), Simpson (1970) and Smalley (1970).

Edelman (1962) has shown how the binomial theorem can be used to relate sample weight to sample variance. The theory requires that the distribution of the source (mineral) of the component must be random.

Where samples can be considered random, Becker and Hazen (1961)
have shown that there is a simple relationship between sample volume and sample variance. This is given by:

$$S_1^2 \lambda_1 = S_2^2 \lambda_2$$

where $S_{1,2}^2$ are the variances of the samples and $\lambda_{1,2}$ are the volumes of the samples. This relationship has been used by Hazen (1961, 1963) to test for randomness in sample data.

Where sample data are not random, they should not be randomised by taking samples at different intervals especially if it involves taking a sample across a sampling stratum. Instead a more fitting relationship between the volume and variance of a sample should be sought. In a general sampling theory, Visman (1969) has proposed that for segregated materials, the sampling error for any weight $W$ may be expressed in terms of a variance made up of the sum of variance due to random non-uniformity, and variance due to segregation. This relationship is given as:

$$S^2 = \frac{A}{W} + \frac{B}{N}$$

where $S^2$ is the overall variance expected in the average of $N$ results on a total weight of samples, $W$, or:

$$W = \sum_{i=1}^{N} w_i$$

where $w_i$ are the individual weights of samples taken from the segregated material. $A$ is a homogeneity constant related to $K$ (Ingamells sampling constant - see subsection 2.2.1.) by the expression:

$$A = K^2 K_S \times 10^{-4}$$

where $K$ is the true content of the valuable component in the ore. $B$ is the segregation constant defined as the contribution of the segregation to the variance when a single sample is taken. Where there is no segregation, then $B = 0$. $A$ and $B$ are known as the Visman constants.

Ingamells (1974a, b) used Visman's general sampling theory to
determine minimum and optimum sampling weights and draw sampling diagrams for orebodies (Fig. 18). The diagrams are similar to those established for homogeneous mixtures (see section 2.2.1., Fig. 4), but it is also necessary to estimate the segregation constant \( B \) as well as a homogeneity constant \( A \). The method of estimating the constants is given in Appendix III. Minimum and optimum weights are then estimated by:

\[
\begin{align*}
    w_{\text{min}} &= \frac{A}{(K - L)^2} \quad \ldots \ldots \quad (23) \\
    w_{\text{opt}} &= \frac{A}{B} \quad \ldots \ldots \quad (24)
\end{align*}
\]

where \( L \) = background concentration for the valuable component of the ore (see subsection 2.2.1). The minimum weight, \( w_{\text{min}} \), is that weight at which
Z (defined in Appendix III) is one grain, on average, per sample. The optimum precise weight, \( w_{opt} \), is that weight of sample which will yield a most precise estimate of the X-content of an orebody for any given total weight of field samples (Ingamells, 1974b).

The sampling diagram can then be constructed by substituting the various values of \( w \) into the following equations (see Fig. 18):

\[
S = \frac{A}{\sqrt{w}} \quad \text{where } B = 0 \quad \ldots \ldots \quad (25)
\]

\[
S_v = \frac{2A}{\sqrt{w}} \quad \text{where } w = w_{opt} \times N \quad \ldots \ldots \quad (26)
\]

\[
S'_v = \frac{A + B}{\sqrt{w}} \quad \text{for } N = 1 \quad \ldots \ldots \quad (27)
\]

where

- \( S \) = deviation expected from unsegregated material or when the number of samples taken is very large.
- \( S_v \) = overall deviation in the average of \( N \) results using samples of optimum weight, \( w_{opt} \). Although values for \( S_v \) generate a continuous curve on the sampling diagram, they are only meaningful at weights which are multiples of \( w_{opt} \).
- \( S'_v \) = deviation of a single result using weight \( w \).

If the weights of samples used are such that \( Z < 6 \), i.e. less than \( 6 \times w_{min} \) (see Appendix III) then the error estimates \( S, S_v \) and \( S'_v \) are no longer reasonable because of the increasing asymmetry in the distribution of results. It is at this point that Gaussian statistics lose their validity.

Fig. 18 is an example of a sampling diagram for a molybdenum orebody that has a grade of 0.3 per cent Mo, of which 0.060 per cent Mo is uniformly distributed. The constants \( A = 69 \) and \( B = 0.006 \) were calculated from 3 kg and 15 kg samples. The diagram shows that the variance of the sample values can be improved by increasing the sample weight to that of the optimum weight (11.5 kg), but will not be significantly improved by taking samples in excess of this weight. The diagram
predicts that samples having weights equal to or larger than the optimum sample weight will have grades of between 0.2 - 0.4 per cent Mo, 68 per cent of the time. This was confirmed in 17 bulk samples, each weighing more than 10 tonnes, and extracted by underground mining; 10 of the 17 grades fell within the predicted range.

The tests show that the theory is applicable to the type of orebody on which the experiments were conducted. However, it depends on the two sizes of samples, viz. 3 kg and 15 kg sets, which must be larger than the small-scale structures controlling the mineralization, if the diagram is to be applicable to larger portions of the body. If a significant small-scale structure exists and there is correlation between samples, then one can expect spurious results. Consequently its use should be restricted to certain types of porphyry mineralization.

Where the small-scale structures in a sampling unit are larger than the samples, then both the random and spatial components of the inherent variation of grade will affect the sample variance. The extent to which it will influence the sample variance can be estimated from the slope, range and nugget effect of the semivariogram.

Because the nugget effect is the random component in the variability of grade, the volume-variance relationship of a sample holds (see equation 19). Consequently, if a sample has a nugget effect of 3, then another sample with a volume six times larger will have a nugget effect of 3/6 = 0.5 (David, 1976). It is very important to establish that the nugget effect is due to the geology of the sampling unit and not due to the sample preparation. If it is due to sample preparation, then it can be reduced by improving the technique and not the sample size.

Where the range of the small-scale structure is larger than the sample size, the degree by which the sample variance is reduced by an increase in sample volume can be estimated from the slope of the semivariogram. This can be done by considering some geostatistical theory which concerns the variance of point samples within samples and samples within blocks, and which has been confirmed by experiment (David, 1977).
The variance of sample values taken from a sampling stratum is equal to the variance of point samples within the samples minus the variance of point samples within the sampling unit. This can be written:

\[
\sigma^2(V) = \sigma^2(V) - \sigma^2(v) = F(V) - F(v) \quad \ldots \quad (28)
\]

where  

- \( V \) = volume of sampling unit;
- \( v \) = volume of sample;
- \( 0 \) = point sample within a volume.

\( F(V) \) and \( F(v) \) are known as the F-functions and can be determined from charts that have been drawn for spherical and exponential semivariograms (in David, 1977; Rendu, 1978; Journel and Huijbregts, 1978).

Examples of these charts are given in Figs. 19 and 20. In Fig. 19 the function that is charted is:

\[
1/C \left( F \left( \frac{h}{a}, \frac{h}{a}, \frac{1}{a} \right) \right)
\]
Fig. 20. Variance \( \sigma^2 (0/S) \) of a point within a rectangle (h, l) for the spherical model, function \( (1/C) F (h/a; 1/a) \).

(From: David, 1977)

where

\[ F = \text{F-function}; \]
\[ C = \text{sill of the semivariogram}; \]
\[ h = \text{dimensions of sample and ore block}; \]
\[ l = \text{dimensions of sample and ore block}; \]
\[ a = \text{range of the semivariogram in each direction, perpendicular to one another.} \]

For a two dimensional problem, the equation can be altered to:

\[ \sigma^2(s/S) = F(S) - F(s) \]

where \( s \) is the area of the sample and \( S \) is the area to be investigated.

Suppose a variable has a semivariogram with ranges of influence in two directions of 10 m x 20 m (spherical model) within an area of 50 m x 80 m, and is investigated by samples with dimensions of 0,1 m x 0,5 m (assuming the third dimension to be negligible). Then \( F(S) \), from Fig. 20, is equal to:

\[ F \left( \frac{50}{10}, \frac{80}{20} \right) = 0,972C \]

By the same diagram, \( F(s) \) is very small and will not alter \( F(S) \).
significantly. If the dimensions of the sample were changed to 0.2 m x 2.0 m, \( F(s) \) would still remain very small, consequently the larger samples are not worth the effort. If, however, the semivariogram ranges in the two directions were 1 m x 2 m, then \( F(s) = 0.26C \) for the smaller samples and \( F(s) = 0.475C \) for the larger samples. This would significantly reduce the value of \( \sigma^2(s/S) \). In this case it would be practical to enlarge the sample dimensions and achieve a desired variance. Similarly the question of core size can be answered by approximating the rectangle to a circle and \( h = l = \text{circle diameter} \). \( F(s) \) for NX core (54.8 mm diameter, assuming a perpendicular intersection) is 0.32C while \( F(s) \) for AX core is 0.18C. Oblique intersections will increase \( F(s) \) but will become less accurate in the perpendicular component through the orebody.

If the distribution of variance in a sampling unit fits the De Wijsian model, a more simple approach can be taken. The slope of a De Wijsian semivariogram on a logarithmic scale is called the intrinsic dispersion coefficient and written \( 3\alpha \), so that the variance of a sample value in a sampling unit is given by:

\[
\sigma^2 \left( \frac{v}{V} \right) = 3\alpha \ln \left( \frac{v}{V} \right) \quad \text{......... (30)}
\]

so long as \( v \) (sample volume) and \( V \) (ore block volume) have similar shapes. The volumes \( v \) and \( V \) can be replaced by their linear equivalents \( l \) and \( L \) (David, 1977) so that:

\[
\sigma^2 \left( \frac{v}{V} \right) = 3\alpha \ln \left( \frac{l}{L} \right) \quad \text{......... (31)}
\]

The linear equivalents for a three dimensional block that has measurements of \( a, b, c \) where \( a \geq b \geq c \) is approximately equal to \( a + b + 0.5c \). This means that ideally the variance of samples taken from a De Wijsian model can be reduced by increasing one or two of the three dimensions of the sample. This has been shown experimentally (David, 1977) for samples taken in a gold mine, and is illustrated in Fig. 21.

3.4.2. Sample bias

Negative sample bias or underestimation will be a function of sample size when the mineralization has a greater chance of being excluded
Fig. 21. Variance - size relationship for samples of different length taken from a gold deposit for which the sample variance fits the De Wijsian model.

(From: David, 1977)

Fig. 22. Regression curve of values from bulk samples versus values from samples taken from coaxial boreholes in the Cerro Colorado deposit, Rio Tinto, Spain. Note how low values in the holes tend to underestimate the grade of the bulk samples, while high values in the holes tend to overestimate the grade of the bulk samples.

(From: Pryor et al., 1974)
in smaller samples than in larger samples. This will arise when the mineralization is distributed in high grade patches separated by low grade patches such that the distance of separation is larger than the sample size, and there are more low grade patches than high grade patches. Small samples may then differ radically in the essential characteristics of the ore from which they are taken, and exhibit characteristics of their own. Consequently the sample distribution curve will become positively skewed.

Fig. 22 illustrates this type of bias in values of copper obtained from borehole samples compared with the grades of the corresponding bulk sample. It shows how the majority of values obtained from borehole samples undervalue the ore with respect to the coaxial bulk samples. Hallbauer and Joughin (1974) point out that the sampling practices on some Witwatersrand gold reefs yield samples of which 90 per cent of the values lie below the average value. This means that the sampling practices are very biased against further mining and exploration. Sample values from the Merensky Reef do not show this negative bias and their distribution is approximately normal (Barry, 1979). This is a result of the fine grain size and even distribution of the platinum group minerals in the reef.

One solution to the problem of sample bias is to counteract the skewness mathematically. Sichel (1947) proposed the use of a lognormal model to represent the positively skewed distributions of data obtained from the Witwatersrand reefs. Fig. 23A and B show that the approximation will hold for a large number of samples taken over a large area.

Fig. 23A. Histogram and frequency curve for 28 334 gold values from the Blyvooruitzicht mine.

(From: Krige, 1978)
Fig. 23B. Normal histogram and frequency curve on a log-scale for the same values as for Fig. 23A.  

(From: Krige, 1978)  

If, in converting the values to their logarithms, the distribution becomes negatively skewed it can be normalized by adding a constant to the raw data (Krige, 1960). This is the three parameter lognormal distribution. The advantage of approximating the skewed distribution to a lognormal model is that the mean is then greater than the mode and a supposedly better estimator of the grade of the reef (as shown in Fig. 17A).

The justification for correcting an unrepresentative sample solely by mathematics is questionable because if the sample is small, the reason for its being unrepresentative of the ore is not altogether mathematical. Hallbauer and Joughin (1974) have shown that the probable reason for the lognormal distribution to the samples is that they were taken without any regard to the positions of the places of enrichment. The way to make a sample more representative of the small-scale structures is to increase the size so that a larger variety of small-scale structures will be incorporated within it. Fig. 18 shows how the most probable value (Y) of a sample taken from a sampling unit approaches the true value of that orebody as the sample size is increased. The size of the sample necessary to be an unbiased representation of the ore depends on the size
of the small-scale structures and the distribution of mineralization within them. In some cases very large samples are necessary, for example Fig. 17 compares the very skewed values from 10 ft samples with the less skewed values from 50 ft blocks taken from a porphyry copper deposit. But not all samples have to be big to be unbiased; at Kilembe mine, Davis (1962) found that the mean of two channel samples, each having a weight per unit length of less than 3 lb per foot, gave results with a very low bias. There are indications too that the split AX core samples at Kilembe would have given unbiased results if it were not for the systematic error generated by the sample preparation and assay.

3.4.3. Composite samples

Although a very theoretical approach to the problem of sample size has been presented, there is no better substitute in determining a suitable sample size than experiment. Experimentation with this aim in mind should be encouraged in an exploration programme once there are indications of economic mineralization. Where the conclusions to the experiments point to an impractically large sample, methods of composing a large number of samples should be investigated. Composite sampling appears to have been neglected in sampling practices and involves collecting samples from a block of ore according to the configuration of the small-scale structures within the block. The values are then combined by weighting them according to the relative volumes of the structures in the block. If the geology of the ore within the block is well understood, then the weightings will be reliable and the final result will be equivalent to a bulk sample. The composite samples can be made up by detailed underground sampling in conjunction with geological mapping or from a series of closely spaced drill holes, e.g. by deflections in a small volume. Underground composite samples have been successfully applied at Kilembe mine (Davis, 1962) and described in subsection 2.1.1; the pattern chip sampling of Morrow (1976) is a comparable method and also described in subsection 2.1.1; a proposal has been made in section 4.3 for this approach to be used on the Witwatersrand. Composite sampling as defined here should not be confused with cluster sampling (see subsection 3.5.1.) which is designed to reduce the nugget effect in sample variance. Cluster sampling will not necessarily reduce the sample bias because the sample values are not weighted by the volumes.
of the small-scale structures to which the mineralization is related.

3.5. THE DISTRIBUTION AND NUMBER OF SAMPLES IN A SAMPLING STRATUM

The question of how many samples are needed to confidently estimate the grade and tonnage of an orebody is often asked because a sampling programme is time consuming and expensive, especially if it involves drilling. Overdrilling or oversampling is a waste of time and money although it is the lesser of two evils when compared with undersampling and the inherent danger of incorrect assessments of the ore reserves.

There are five important factors which determine the distribution and number of samples taken in the evaluation of an orebody:

a) Geology of the orebody;

b) Required sampling method;

c) Accessibility of the orebody;

d) Method of ore reserve estimation;

e) Mining method.

This section will concentrate on the influence of the first two factors. Accessibility will be disregarded but in normal circumstances inaccessibility will be detrimental to a sampling programme. The method of ore reserve estimation should be chosen to suit the samples that are best for the orebody, but once the method has been established, it can dictate the location of infill sampling should this be necessary. A proposed mining method may require some infill sampling, e.g. where the inclusion of subgrade ground is unavoidable.

There are both geological and statistical approaches to the problem of where and how many samples need to be taken from an orebody to confidently estimate its grade and tonnage. The final sampling pattern that is chosen is normally a compromise between what is best for the geology of the orebody, and what can be most effectively achieved with respect to terrains, underground access and methods, e.g. type of drill rig, which are available. It is not intended to discuss what a final sampling pattern
might look like, as this will depend on the peculiarities of each situation. However, there are some basic relationships between sample size and spacing applicable to all types of mineralization. The relationships can be quantified statistically.

The geological approach to sampling relies on interpretation and geophysics to judge when adequate data has been obtained. Correlation between samples is based on the geologists' understanding of the mineralization combined with models and experience gathered from other similar types of deposits. Strategically taken samples to maximise correlation between them can be advantageous to a sampling programme (Bancroft, in Richard, 1907) but there is always the human tendency to collect oddities (Koch and Link, 1970). This can be counteracted by sampling and drilling on regularly spaced sections through an orebody, with the spacings judged so that the probability of a significant element of a deposit being overlooked is acceptably low (Jewett, 1956). Ore deposits for which a geological approach is best applied will be discussed in the fourth chapter of this dissertation.

The statistical approach depends on a statistically determined volume of influence to the samples, which is an average of the volumes of influence of a number of similarly shaped samples taken from the sampling stratum. Samples are correlated with each other by their variance which is at a maximum when they are separated by distances greater than their volumes of influence, and which drops towards a minimum (the nugget effect variance) if they lie within their volumes of influence. Samples separated by distances greater than their volumes of influence are said to be random and methods of random statistics can be applied to them. Samples which lie within their volumes of influence show a degree of correlation for which the theory geostatistics has been evolved.

3.5.1. The volume or area of influence of a sample, sample size and sample spacing

Samples can be taken from a sampling stratum so that their values are either dependent or independent of each other. Dependency or independency is achieved by altering the sample spacing relative to the
volumes of influence of those samples. Every sample has a volume of influence that is related to the distances which the mineralized small-scale structures extend beyond the sample limits. If two samples contain mineralized structures that are common to both of them, then the samples lie within each others volumes of influence. A different volume of influence can be assigned to every sample taken from a deposit, but this requires a detailed knowledge of the shape, extent, configuration and grades of each structure. These can be predicted with good geological knowledge of the deposit, but structures become increasingly variable away from the sample, so the confidence that can be placed on interpretive correlation diminishes with distance. Attempts to define the area of influence of samples have been made by Koch and Link (1970) using correlation coefficients and Hazen (1968) using the mean square successive difference test. However, these approaches are based on the statistics for random samples and their application to regionalized variables is not altogether justified. A better approach is to estimate volumes of influence by determining the semivariograms in three directions perpendicular to each other, and orthogonal to the geological structure. Use of semivariograms to define volumes of influence of samples is not justified when these volumes for successive samples may be highly variable. In this case the volume of influence of each sample will have to be determined separately by geological interpretation. Where the sample values are reduced to a two dimensional distribution of accumulations of grade times thickness, then the volume of influence becomes an area of influence.

The volume of influence of a sample is unaffected by an increase in sample size, unless the sample dimensions exceed the range of the small-scale structures. Consider for example the hypothetical deposit represented by the semivariogram in Fig. 13. If the sample size was increased but remained less than \( \alpha \), the volume of influence of the sample would be the range of \( \alpha_1 \), assuming the slope between \( \alpha_1 \) and \( \alpha_2 \) to be nearly flat. If the dimensions of the sample equalled or exceeded \( \alpha_1 \) then the small-scale structures to which \( \alpha_1 \) is related would cause a nugget effect of \( C_1 \), and the volume of influence for the sample would be the range of \( \alpha_2 \). The slope of the semivariogram between \( \alpha_1 \) and \( \alpha_2 \) would then become significant with respect to the larger samples. Fig. 16 illustrates the relationship between range and sample volume as revealed by actual samples. The range
of the semivariogram for samples less than 10 m in length is 10 m because this is the average thickness of the deposit. Once the samples exceed 10 m in length, the range of the semivariogram increases to 20 m corresponding to a larger, unknown structure.

Cluster sampling can be used to evaluate mineralization distributed by two scales of structure where both scales of structure have a significant influence on the volumes of influence of the samples. Fig. 24 illustrates typical sampling clusters. The samples in each cluster are separated by a distance greater than the smaller range of the small-scale structures \(a_1\) and \(a_2\) so that the volume of influence due to these structures will not be inherited by the samples. Consequently the sample variance due to the small-scale structures becomes the nugget effect for the larger structures by averaging the sample values in each cluster. Cluster sampling has been successfully applied to a Minette iron ore deposit in Lorraine (David, 1976) and described in subsection 4.1.1. Davis (1962) used cluster sampling in the evaluation of Kilembe mine by combining the values of samples taken on both walls of the cross-cuts. Cluster sampling should not be confused with composite sampling (see subsection 3.4.3) for although it will reduce sample variance, it will not necessarily reduce the sample bias.

Correlation between samples can therefore be achieved by the
interrelationship of sample size, volume of influence, and sample spacing. If, for example, the volume of influence of a sample is small and circumstances (e.g. cost) mitigate against the number of samples required, then the spacing must be increased. Consequently, for these wider spaced samples to be correlated, the sample size must be increased so that its volume of influence depends on a suitably larger-sized structure. If this sample size is impractical it can be improvised by composite sampling. The only other solution is to revert to the smaller, more practical, but independent samples at a sample spacing larger than their volumes of influence, and a consequent decrease in the precision of the grade estimate.

3.5.2. The geostatistical approach to optimum sampling patterns

As the number of samples taken from an orebody increases, the random sampling errors begin to balance each other out. Fig. 25 shows that a point is approached beyond which further sampling maintains that balance at a certain precision. The precision has been shown in

![Fig. 25. Relative precision (one standard deviation) for the estimation of the copper and nickel quantities in the Expo Ungava orebody as a function of the number of holes drilled. (From: David, 1977)](image)

subsection 3.4.1 to be a function of the sample size and the inherent variability of grade in the sampling stratum. It is, therefore, possible to predict with the aid of a semivariogram, how, for a given sample size, the precision of the grade estimate will improve with an increasing number of samples, and what minimum level of precision will be reached.
It is then possible to estimate how many samples will have to be taken for a given precision, and whether or not further sampling will be cost effective in improving the precision.

A sampling pattern is chosen so that the volumes or areas of influence of the samples overlap, and are greater than the volumes or areas to which the values of the samples are assigned. There are three possible ways of choosing the volumes or areas to which the sample values are assigned:

a) by sampling on a regular grid so that each square or rectangular grid cell is evaluated with a sample in the centre;

b) by random stratified sampling over square or rectangular grid cells so that one sample lies somewhere in each cell;

c) by random stratified or irregular grid sampling and weighting each sample by a polygon of influence.

Blais and Carlier (1968) show how the sample spacings of a regular grid can be chosen according to the isotropy or anisotropy of the distribution of variance in the sampling stratum. If it is isotropic, i.e. has identical semivariograms for two perpendicular directions, it is preferable to use a square drilling grid, unless it is necessary to probe geological structures that affect the continuity of the mineralization. If the variance is anisotropic, the sampling pattern should be modified to restore anisotropy, by proportioning the drill spacing by the anisotropy modulus. The drill spacing should not exceed the range of the semivariogram in either case.

Matheron (1971) and summarized in David (1976) has derived the relationship of the standard error of the mean, $\sigma_m$, in terms of the semivariogram for the mineralization, the length of the side of a square block, $L$, and the number of samples $N$. The relationship is given by:

$$\sigma_m = \sqrt{\frac{0.25 La + C_0}{N}}$$

(32)

where $a$ is the slope of the semivariogram and $C_0$ is the nugget effect.
Where the distribution of variance is anisotropic, a rectangular grid cell can be converted to the square cell by the anisotropy modulus.

Where there is a constant sampling density to which a regular cell pattern can be fitted, but the sample stations do not fit into a regular grid, the standard error of the mean is given by:

\[ \sigma_m = \sqrt{(0.52 L + C_o)/N} \]  

for a square cell of length L. If the grid is not square, an approximate answer can be obtained by considering the diagonal L of a rectangular grid cell, and the standard error of the mean is given by:

\[ \sigma_m = \sqrt{(L/3 + C_o)/N} \]  

These formulae replace that for the standard error of the mean applicable to random data (see subsection 3.5.3.). For example, consider 25 holes drilled on a random stratified pattern into a stratiform copper deposit (David, 1976; 1977). The average semivariogram calculated for these holes has a range that is linear to 400 ft and a nugget effect of 0.06. The density of drilling is such that each hole can be assigned to a square cell of length 400 ft. The standard error of the mean is 0.07 according to the above equation but 0.10 using the equation for random data, which is a 40 per cent overestimation. Similar overestimations have been demonstrated by Brooker (1975, 1976).

Where the sampling is random stratified or based on an irregular grid, the sample values are assigned to a polygon of influence and weighted by the area or volume of this polygon in determining the average grade of the deposit. The error of the grade estimate \( \sigma^2_e \) is given by the following expression (Parker, 1977 a, b):

\[ \sigma^2_e = \sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j \sigma^2_{ij} \]

where \( c_i \) and \( c_j \) are weighting coefficients and \( \sigma^2_{ij} \) are the covariances between samples i and j. Calculation of \( \sigma^2_e \) is long and monotonous and normally done by computer. Details are provided in Appendix II. A
computer programme for this calculation and some examples are given in David (1977).

The estimate has been tested on a roll-front uranium deposit in Wyoming by Sandefur and Grant (1976) using data of grade-thickness accumulations from boreholes spaced less than 100 ft apart. Each hole was assigned a polygonal area of influence that was within the area of influence of the sample defined by the semivariogram in Fig. 26.

![Fig. 26. Semivariogram for the Shirley Basin uranium mineralization.](From: Sandefur and Grant, 1976)

The testing involved the recalculation of the ore reserves for four different grids at five different borehole spacings taken from the high density drilling, and comparing the calculated reserves to a reference value. The data is given in Table 1 which compares the experimentally determined error with the geostatistically predicted error at the 68 per cent confidence limits. It can be seen from Table 1 that the precision of the grade estimate from approximately 175 boreholes, with spacings of approximately 200 ft, is not significantly improved by a further 500 boreholes at spacings of 100 ft. Sandefur and Grant concluded that had the semivariograms for the deposit been determined early in the exploration programme, the overdrilling could have been avoided.

3.5.3. The statistical approach to the optimum number of samples

Statistics designed for random samples (Hazen, 1967) can be used to determine the relationship between the precision of the sample...
### Table 1. Experimental error compared with geostatistically predicted error for various drill spacings: combined errors in total ore reserve estimate.

(From: Sandefur and Grant, 1976)

<table>
<thead>
<tr>
<th>MINIMUM SPACING (feet)</th>
<th>CASE*</th>
<th>NUMBER OF HOLES IN QRE</th>
<th>TOTAL RESERVES (lbs U₃O₈)</th>
<th>EXPERIMENTAL ERROR compared to final estimate</th>
<th>GEOSTATISTICALLY PREDICTED ERROR 68% confidence limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.5%</td>
</tr>
<tr>
<td>100</td>
<td>a</td>
<td>665</td>
<td>23,751,000</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>b</td>
<td>712</td>
<td>21,864,000</td>
<td>-6%</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>c</td>
<td>686</td>
<td>26,472,000</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>d</td>
<td>665</td>
<td>24,460,000</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>a</td>
<td>173</td>
<td>24,748,000</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>b</td>
<td>192</td>
<td>21,664,000</td>
<td>-8%</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>c</td>
<td>175</td>
<td>26,684,000</td>
<td>14%</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>d</td>
<td>162</td>
<td>23,729,000</td>
<td>2%</td>
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</tr>
<tr>
<td>400</td>
<td>a</td>
<td>40</td>
<td>17,469,000</td>
<td>-25%</td>
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<td>b</td>
<td>50</td>
<td>18,141,000</td>
<td>-22%</td>
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</tr>
<tr>
<td>400</td>
<td>c</td>
<td>42</td>
<td>30,773,000</td>
<td>32%</td>
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</tr>
<tr>
<td>400</td>
<td>d</td>
<td>38</td>
<td>28,452,000</td>
<td>22%</td>
<td></td>
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<tr>
<td>800</td>
<td>a</td>
<td>12</td>
<td>25,996,000</td>
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<tr>
<td>800</td>
<td>b</td>
<td>13</td>
<td>18,765,000</td>
<td>-20%</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>c</td>
<td>11</td>
<td>38,608,000</td>
<td>65%</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>d</td>
<td>8</td>
<td>29,093,000</td>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>a</td>
<td>4</td>
<td>13,937,000</td>
<td>-40%</td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>b</td>
<td>3</td>
<td>19,615,000</td>
<td>-16%</td>
<td></td>
</tr>
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<td>c</td>
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<td></td>
</tr>
<tr>
<td>1600</td>
<td>d</td>
<td>3</td>
<td>10,498,000</td>
<td>-55%</td>
<td></td>
</tr>
</tbody>
</table>

* Since the subset process chooses certain holes within the available drilling data, four cases were run selecting a different set of holes in each case.

The mean and the number of samples. Assuming the samples values to be normally distributed about the mean, \( \bar{x} \), the precision of the mean, \( \sigma_m \), is given by:

$$\sigma_m = \frac{S}{\sqrt{n}}$$

where \( S \) = standard deviation of the samples, and \( n \) = number of samples. The standard deviation is given in Appendix V. The confidence limits attached to the estimate are determined by the proportion of the area beneath the Gaussian curve that lies within the precision range. The confidence limit for a precision of one standard deviation is 68 per cent, and for two standard deviations, it is 95 per cent.

The above equation can be justifiably used in the evaluation.
of orebodies where there is no correlation between samples, i.e. random samples. Samples from prophyry type mineralization are often random because the normal borehole samples are usually too small to be correlated by the larger scale structures. A comparison between this formula and those derived by Matheron (1971), and which are given in subsection 3.5.2, shows that it will overestimate the number of samples necessary for a given precision of the estimate, if the samples can be correlated. A calculation is given in subsection 3.5.2 which illustrates this overestimation.

The standard deviation used in the formula can only be determined from a set of samples, and it will only apply to those samples that have the same volume (see equation 19). Furthermore, different sets of samples will have different standard deviations which will be normally distributed about a mean standard deviation. This problem has been considered by Hazen (1967) and is illustrated in Fig. 27. Consider the

![Fig. 27. Standard error limits and expected confidence intervals for various values of N derived from a distribution of 153 NX samples.](From: Hazen, 1967)
following examples:

a) A set of 153 whole core samples has a mean 0.33 and a standard deviation of 0.114. The standard deviation has been plotted as a straight line and a dot corresponding to 153 samples has been plotted on the line. The position of this dot with respect to the confidence-interval curves indicates an interval of approximately ± 0.019. Thus the mean of the samples is 0.33 ± 0.2 at the 95 per cent confidence limits.

b) Assuming the standard deviation would remain the same for another set of samples, the number of samples can be estimated which will have a mean with a confidence-interval of 0.025. The estimation is made by following the horizontal line corresponding to the standard deviation of the 153 samples until the curve for the 0.025 confidence-interval is reached, and dropping a perpendicular line onto the abscissa. This gives approximately 90 samples.

c) The assumption that the standard deviation is the same for both sample sets is not altogether justified. Consequently the assumption generates an error. Curves corresponding to one standard error and two standard errors have been drawn on either side of the line for the standard deviation of the 153 samples. Using these curves it can be seen that the 95 per cent confidence-interval for the mean of 90 samples will lie somewhere between ± 0.029 and ± 0.023. Points representing samples in random sets of 10, 25, 50 and 100 samples, drawn from the 153 samples, have been plotted to demonstrate the manner in which the standard deviations fall within the limits established by the standard error of the standard deviation.

If any two samples are unrelated because they lie outside of their volumes of influence, then their relative positions are unimportant
so long as they lie within the sampling stratum. The required number of samples can therefore be distributed in either random or regular grid patterns with no effect on the precision of the estimate. However, unpredictable correlation between supposedly random samples may arise when the mineralization is related to a haphazard distribution of ore-bearing structures which may or may not be common to two or more samples. In other words there is a high variability to the volumes of influence of the samples. Kuhn and Graham (1972) have proposed a technique to detect an unpredictable correlation between samples, and which was developed from a method evolved at Christmas mine, Arizona (Fig. 28).

Fig. 28. Drill-hole pattern for large disseminated orebodies, based on a method developed at Christmas mine, Arizona.

(From: Kuhn and Graham, 1972)

Essentially the method involves measuring the linear correlation between equivalent samples, composited to fixed lengths, along adjacent boreholes. If there is significant correlation between the samples of the two holes, Kuhn and Graham suggest that it is reasonable to assume that the areas of influence of each hole overlap. Correlation is considered significant.
at 95 per cent confidence. The technique was designed for prophyry type orebodies and is not applicable to discontinuous mineralization.

Fig. 28 represents a 122 m square grid designed from geological information and experience, with one borehole in the centre of each square. Grade estimates were needed at a precision of 0,08 per cent Cu. An initial series of holes (1 to 9) was drilled on 244 m centres, twice the designed grid size. The assays were composited to standard intervals in each hole, and a linear correlation was sought along diagonals in statistical runs 1 to 8. No significant relationship was found, so a second series of holes, A to D, was drilled, and before going to the next series of holes, a to 1, correlations were sought in runs 1 to 24. In run 11, a suitable correlation was found between holes 4 and 5; an acceptable average confidence-interval of ± 0,08 per cent was also calculated for the two holes. On this basis, hole f was cancelled; after drilling the rest of the holes in the series new statistical runs were used to test the need for holes in the centre of each 122 m square. Diagonal runs indicated significant correlations between holes B and 2, and between d and b; hole 102 was cancelled. Another run between holes 5 and D showed that hole 110 was not needed. If further evaluation was to be considered on the basis of information from the completed pattern, the statistical process could have been carried down to 61 m grid, scheduling only those holes actually needed.

3.6. CONCLUSIONS

When sampling an ore deposit, it is important to take samples that have a minimum population variance. This can be done by first subdividing the deposit into sampling strata that contain similar types of mineralization. The subdivisions should be made according to the geology of the deposit and their boundaries should be significant geological discontinuities in the mineralization. The best samples in terms of minimum population variance are those samples which are taken right across the small-scale structures in the sampling stratum, so that they average the local variabilities of grade. The population variance of samples that are smaller than the sampling stratum, and which have been taken within the sampling stratum, will be larger than the variance of samples
taken across the whole stratum. Samples should not be taken across major geological boundaries because this will introduce a heterogeneous variance into the population.

Sample variance and bias are caused by the inability of samples to average out the inherent variability in the distribution of grade within a sampling stratum. Sample variance reduces the precision of the ore reserve estimate based on the sample values. Where the small-scale structures causing the variance are smaller or of the same order of magnitude as the sample size, sample variance can be reduced by increasing the sample size. The positive skewness to the distribution of sample values decreases with an increase in sample size, consequently small samples tend to be negatively biased with respect to the true grade of the mineralization. Where it is not feasible to take large bulk samples, a method of composite sampling that determines the equivalent of a bulk sample, can be applied in certain situations.

The volume of influence of a sample value depends on the size of the samples with respect to the small-scale structures that control the distribution of mineralization in the sampling stratum. Samples can be taken so that they either lie in or out of the volumes of influence of adjacent samples. If samples lie inside the volumes of influence of adjacent samples, then the correlation of values must be taken into account and their relative positions are important. Geostatistics has been evolved to statistically quantify the correlation between samples and can be effectively used to plan sampling programmes and assess the efficiency of a sampling programme. Where samples lie outside their volumes of influence but within the same sampling stratum, their values can be considered independent. The relative positions of samples is not important and statistics can be used to evaluate the efficiency of a sampling programme. If samples from the same sampling stratum have highly variable volumes of influence, there is a statistical technique to take advantage of the occasional correlation that may occur between them. Otherwise the volume of influence of each sample value will have to be determined independently, based on geological interpretation.
4. A SELECTION OF ORE DEPOSITS AND THEIR SAMPLING CHARACTERISTICS

The purpose of this chapter is to select a number of different types of ore deposits and identify some of the sampling problems that are typical of their geology. In selecting the deposit types, an attempt has been made to provide a cross-section of sampling problems, but the selection has been restricted to the documentation that exists for specific sampling problems. Sampling information from both drilling and underground programmes has been used. This chapter does not, therefore, attempt to present an exhaustive review of ore deposits and their sampling problems. The order in which the deposits have been considered is approximately in order of increasing complexity of their mineralization. Some ideas are given as to how the concepts and techniques discussed in the previous two chapters can be applied to some of the sampling problems.

4.1 SAMPLING STRATIFORM DEPOSITS

The category of stratiform deposits as used here includes stratabound ores in both sediments (including volcanics) and layered igneous complexes. The deposits are normally shallow dipping, extensive, and easily accessible except where they have been deformed. The mineralization within them can be even to patchy, depending on the processes by which they were deposited, and can be confined between sharp geological boundaries.

4.1.1. Sampling stratiform deposits in which samples have large volumes of influence

Samples that are taken from deposits where the distribution of the mineralization is uniform and extensive will have very large volumes of influence. This type of mineralization is formed by the chemical precipitation of the valuable metals, and it includes the volcano-sedimentary polymetallic sulphide deposits normally found in low-energy, deep basin environments, the manganese seams of the Kalahari Manganese Field, the Lorraine Minette iron ore deposits and the Merensky Reef in the Bushveld Igneous Complex.

Choice of sampling strata in sedimentary stratiform ores depends on the distribution of mineralization with respect to clear and extensive
geological boundaries. The ore is usually distributed as thin laminae of ore minerals interspaced by laminae of clastic material. If the conditions of precipitation of the ore change rapidly, then the boundaries between ore and waste will be sharp. At Wessels mine in the Kalahari Manganese Field, the sampling strata are the manganese seams that are being mined and which are clearly sandwiched between iron formations. The contact between the iron formation and manganese is very sharp. Samples are taken underground from the hangingwall to the footwall of the seam by taking chips from between two vertical and parallel lines spaced 10 cm apart. The chip sample zone is subdivided into smaller units about thin but persistent jasperlitic marker bands, viz. red line marker and pencil marker in the Lower Seam at Wessels mine (Benfield, personal communication). Each sample weighs between 1 to 2 kg and is taken at a maximum distance of separation of 4 m. Had the marker bands been imper- sistent or related to local sedimentary features, they would have been unsuitable for subdividing the samples. A pitfall in sampling the very even mineralization that occurs in the Kalahari Manganese Field, is to overlook secondary alteration or deformation of the mineralization which will reduce the original volume of influence that a sample could have had in the ore. For example, a drill spacing of greater than 500 m to evaluate the manganese seams is justified in terms of the distribution of manganese in the seams, but it was too large with respect to areas of ferruginization, that were found to occur in the vicinity of one mine.

If the conditions that cause the precipitation of the mineralization change slowly, then the boundaries to the distribution of the mineralization will be determined by sampling and analysis. For example, the McArthur deposit in Australia has been subdivided into seven orebodies separated by dolomitic shales and inter-ore breccias, as shown in Fig. 29. The inter-ore beds are relatively metal-poor intervals, and the hangingwall of the top orebody has been described by Lambert (1976) as a relatively sharp boundary with Zn grades decreasing from greater than 5 per cent to less than 2 per cent over an interval of approximately 5 m. The ore of volcano-sedimentary deposits consists of alternating bands of sulphides and shales with the sulphides distributed as thin laminae within the bands. At Mount Isa mine, the mineralized bands are sampled (Fig. 30) with channels 10 cm wide and 1,2 cm deep. The lengths of the channels normally range
Fig. 29. Section through the McArthur deposit showing the seven orebodies, the inter-ore beds and the metal grades. The orebodies are numbered 2-8; a small area of stratiform mineralization has been located under the number 2 orebody in the northern part of the deposit.

(From: Lambert, 1976)

Fig. 30. Idealized cross-section of stratiform Ag - Pb - Zn ore, Mount Isa mine, showing channel and line chip sampling locations.

(From: Morrow, 1976)
from 1.0 m to 1.5 m (Morrow, 1976), and their limits are determined by the geology of the mineralization. Line chip samples are taken from the thick, subgrade shale bands between rich zones, because a precise estimation of their mineralization is not required. Experience has shown this method to be operator sensitive and even-sized contiguous chips must be taken (Morrow, 1976).

Disruption of the thin sulphide laminae by widespread slumping, and slide and scour structures can be expected to introduce a nugget effect into an otherwise even mineralization. Other small-scale structures include unmineralized micro-faults, nodular dolomite, and small, discordant sulphide-carbonate veinlets. If, in this type of mineralization, samples are taken so as to overcome the nugget effect, they should show a correlation over large distances that could be used to advantage in evaluation.

Sampling of the Lorraine Minette iron ore deposits illustrates how correlation between sample values can be achieved over large distances corresponding to large-scale structures, although the samples may have a high variance due to small-scale structures. The iron ore occurs in up to 3 of 13 layers in a ferruginous formation 10 to 65 m thick (Bubenicek and Haas 1969). The deposit is a littoral marine formation of an epicontinental sea in an area where a river flowed onto a tidal flat (see Fig. 31). The actual accumulations of ore correspond to banks of sand formed between channels of river and marine water and are made up of detrital deposits such as limonitic oolites, quartz grains and fragments of calcite shells. Laterally and vertically the sands go through complex transitions into argillaceous formations. Within the ore there is a diagenetic transformation from limonite into chlorite and siderite (Bubenicek and Haas, 1969). The high nugget effect is due to the various bedforms in which the oolites accumulated, and to the diagenetic alteration. The range of the spatial variance due to these small-scale structures is approximately 1 m and was reduced by cluster sampling (see subsection 3.5.1). The samples were spaced at distances greater than 1 m (a_2 in Fig. 24) so as to randomize the influence of the small-scale structures. The average values of the clusters were found to show a correlation over 30 to 200 m.
The platiniferous Merensky Reef in the Bushveld Igneous Complex is an example of a stratiform type orebody. However, the mineralization is not confined between sharp boundaries, but disseminated into the hangingwall and footwall of the Reef. The Reef consists of a hangingwall and a footwall chrome seam (each having a thickness that ranges from 1 to 50 mm) which are separated by pegmatoid that has a thickness of between 50 cm to 500 cm. The platinum occurs with copper, nickel and iron bearing sulphides that are disseminated about the chrome seams in the manner illustrated in Fig. 32.

The natural geological markers in the Merensky Reef are the chrome seams but because the platinum is distributed on either side of them, they are unsuitable as natural subdivisions for samples. If samples were to be split at the chrome seams they could be expected to have a very high variance due to the very friable nature of the chromite, and the high platinum values that occur within them. When cutting the sample, either underground or in drill core, disproportionate amounts of
Fig. 32. Section of the Merensky Reef in borehole ML 34 showing the vertical variation of Cu, Ni, and Cr and the platinum-group elements Pt, Pd, and Ru.

(From: Brynard, De Villiers and Viljoen, 1976)

chromite can be expected to enter either sample. The alternative is to cut one sample across the chrome seam with limits determined by the average distance that the platinum is disseminated into the hangingwall and footwall. It is very important that the samples are cut evenly across the chrome seam. This type of approach is adopted by mines on the Merensky Reef, and it is normal practice to subdivide samples into 10 cm to 20 cm units along a 10 cm wide zone, marked across the reef and perpendicular to the dip (Markus, 1979). The taking of a number of short samples across the reef aids the grade control.

The distribution of platinum in the Reef is very consistent and the average values obtained over the past 10 years differs with the average over 38 years by only 2 per cent (Markus, 1979). The sample values are normally distributed (Barry, 1979) which is a reflection of the fine grained nature and even distribution of the platinum minerals. The majority of platinum group minerals are developed along the outer edge of the sulphide grains and as separate minerals between interstitial silicates (Von Gruenewaldt, 1979). Samples have a large volume of influence due to the even mineralization, and underground channel samples have spacings of between 3 m to 10 m. The uniform mineralization has
also made it possible to introduce a photographic technique for grade control (Markus, 1979).

4.1.2. Volcanogenic massive sulphide deposits

The various types of this deposit include cupreous-pyrite, copper-zinc and Kuroko or polymetallic (Hutchinson, 1973). Deposition occurred around vents of brine exhalations in the vicinity of submarine volcanoes. The orebodies are lenticular in cross-section and overlie an alteration zone with stringers and veinlets of sulphide minerals (stringer ore). The ore is massive, banded and interbedded with tuff bands. The orebodies are often overlain with siliceous tuffs and cherts, and often zoned. Pyrite and chalcopyrite (if present) are concentrated at the base and chalcopyrite, or sphalerite and galena in Kuroko deposits, become concentrated towards the top, as shown in Fig. 33. In Kuroko deposits, barite shows highest concentrations in the uppermost part of the massive ore, and the overlying tuff or mudstone is hematitic and sometimes manganeous (Sato, 1977).
The vertical metal zonation from iron and copper rich ore in the footwall through copper, zinc and lead sulphides to barite and chert in the hangingwall represents one exhalation cycle and would therefore be the most suitable subdivision in establishing a sampling stratum. The interbedded tuff bands in the ore, if they exist, represent periods of volcanic activity and rapid deposition as opposed to a break in the brine exhalations. The exhalations are the result of a ground water and magmatic brine convection cell being set up in the crust as a result of the heat emitted from a cooling magma (Hutchinson, 1973), and will not be comprised of a number of short, sharp bursts. Consequently any tuff bands should be considered as waste inclusions in the ore rather than a significant break in sedimentation, unless the tuff bands are large and they isolate orebodies. Chert bands can be expected to mark the conclusion of an exhalation cycle, and where they isolate significant units of ore, they should be used as boundaries for sampling strata.

Small-scale structures in massive sulphide deposits are lenticular or banded, and not as extensive as the thin laminae found in the volcano-sedimentary sulphide deposits. For example, observations of existing exhalations on the East Pacific Rise show that the sulphides are deposited in the irregularities of the sea floor, and as cones or hornitoes which rise up to 10 m around the vents (Rise Project Group, 1980). As a result, samples from this mineralization can be expected to show a high nugget effect and variable volumes of influence, except towards the outer boundaries of the deposit where a greater uniformity may occur. Very large samples, or sample clusters where samples in the clusters are spaced at greater distances than the average size of the cones and other irregularities, could reveal a useful correlation between sample values due to larger structures. Davis (1962) found that the best estimations of grade at Kilembe mine were obtained from the averages of channel sample pairs taken on the opposite walls of cross-cuts.

The ores from volcanogenic massive sulphide deposits are made up of a selection of minerals with extreme contrasts of hardness, e.g. chlorite, sulphide minerals such as chalcopyrite, sphalerite and galena, and chert. The hardness contrasts can lead to significant bias being introduced into the sampling procedure. For example, soft sulphide minerals in the
presence of chert will introduce a positive bias into the sample values because of the tendency to oversample the softer mineral. Alternately, soft chlorite minerals in the presence of sulphide minerals can be lost from the drill core, especially if the rock has been sheared, and so introduce a positive bias to the sample value (Davis, 1962). Individual samples should be taken according to the distribution of grade in the small-scale structures, i.e. alternating bands of massive sulphides and tuff, so as to avoid bias in the sampling procedure (Davis, 1962, Cox, 1968b). However, the values must be weighted by their lengths and specific gravities if they are to be combined into larger units.

Sampling problems encountered in the evaluation of volcanogenic massive sulphide ores are therefore found in the recognition of sampling strata, the irregular distribution of grade, and the contrasting hardness in the ore and gangue minerals. It is important to recognize the boundary that marks the end of an exhalation cycle and be able to distinguish between it and ore dilution by tuff layers. A high nugget effect can be effectively reduced by cluster sampling, and bias arising from contrasting hardness in the ore minerals can be avoided by meticulous care in the sampling procedure.

4.1.3. Sampling deformed massive sulphide deposits

Massive sulphide orebodies are easily deformed and their plasticity often encourages fold closures to develop in them. Mild deformation will attenuate the primary structures in an orebody, but the structures will maintain their positions relative to one another. Consequently, samples taken according to geological boundaries may be correlated between those boundaries, regardless of the thickness of the orebody. The distance over which samples can be correlated will increase in the direction of attenuation. With more severe deformation, the orebodies may become boudinaged or even displaced by faulting and shearing. If a block or boudin of ore is large enough to be taken as an orebody on its own, then its grade may be determined by a sampling programme depending on the structures within it. The Gamsberg zinc deposit has resulted from the closure of two limbs of a drag-fold and is an example of how attenuation can extend the volume of influence of a sample. Split AXT core samples
from boreholes spaced from 100 m to 300 m apart were found to be adequate for the evaluation of the orebody (The Staff, O'okiep Copper Company 1977). Bulk samples obtained in exploratory mining showed that split core samples from coaxial holes were reliable. Similar continuity of grade was found in the less deformed orebodies at the Hilton mine (Morrow, 1976). If, however, the orebody has been dismembered by the deformation, then there can be an apparent correlation between samples which is unjustified, because of the discontinuities between the segments of ore.

If the segments of the dismembered orebody are too small to constitute individual blocks of ore, they must be evaluated in combination with the waste ground in which they occur. Their relative positions become important, and they make up a new pattern to the distribution of mineralization which would be confined between structural boundaries related to the axes of deformation. The structural boundaries could, for example, include faults, shears, or slip planes between strata of two different ductilities in a fold closure.

Small samples taken according to the geology of the ore segments and waste rock will have almost indeterminable volumes of influence, because they would be constrained by the boundaries of the segments. However, if samples larger than the average size of the segments could be taken, more reliable grade estimations could be made. Morrow (1976) shows how pattern chip sampling (see subsection 2.1.1) can be applied to this type of problem encountered at Hilton mine, in Australia. A reliable grade could also be obtained from composite samples, as discussed in subsection 3.4.3.

4.2. SAMPLING NICKEL-SULPHIDE DEPOSITS

Nickel-sulphide deposits form from the gravity settling of an immiscible sulphide liquid melt that develops during the cooling of a tholeiitic or komatiitic magma (Naldrett and Cabri, 1976). The sulphide droplets accumulate on the floor of the magma chamber or lava, and become trapped in depressions or in the lee of protrusions that cause eddies in the magma flow. Fig. 34 illustrates a typical distribution of mineralization after the magma has solidified. The highest grades
occur as massive sulphide ore in the base of the lava flow or intrusion and are concentrated into lenticular bodies by the irregularities in the floor. Overlying the massive ore is a zone of disseminated sulphides resulting from two processes. One process is the downward pressure of the overlying pile of cumulate olivine crystals, which pushes the lowest crystals into the sulphide liquid (Usselman et al., 1979). The other process is the trapping of sulphide droplets that are still settling as the magma solidifies. The concentrations of nickel decrease into the hangingwall due to the increasingly disseminated sulphides, but copper tends to concentrate relative to nickel towards the top of the mineralization, because of the different chemical characteristics of copper compared to nickel.
An account of sampling the Perserverance nickel deposit in Australia is given by Martin and Allchurch (1976). The mineralization occurs in a partly serpentinized dunite lens that strikes north-northwest, and dips sharply to the west (Fig. 35). There are three types of mineralization, viz., massive ore, heavily disseminated mineralization and sparsely disseminated mineralization. The massive ore consists of a breccia formed by the inclusion of footwall rocks set in a sulphide matrix. Its composition is relatively uniform but it depends on the number of inclusions. The massive ore follows the contact between the dunite and the metasediments, or lies in fault zones where it forms tabular bodies with strikes of up to 800 m. Grades are normally in excess of 4 per cent Ni. The heavily disseminated mineralization consists of sulphides evenly scattered amongst equigranular silicates. The mineralization occurs in oreshoots that reach 600 m in length and 100 m in width. Grades range between 1 per cent to 4 per cent Ni. Sparsely disseminated mineralization is very similar to the heavily disseminated variety, but grades
range between 0.4 per cent to 1.0 per cent Ni. This mineralization is found on the western margin of the dunite lens. The three types of mineralization are natural subdivisions of the deposit into sampling stratum, and this is confirmed by Martin and Allchurch (1976) who found that the sample values plotted onto log-probability paper showed three distinct populations with averages of 5.2 per cent Ni (massive ore), 1.74 per cent (heavily disseminated ore) and 0.47 per cent (sparsely disseminated ore).

Undisturbed precipitation of immiscible sulphide droplets can be expected to be uniform with possibly some banding in the liquid at the bottom of the magma body, if the liquid was stationary and there was no mixing. Mixing would occur if accumulations of sulphide liquid flowed independently to the magma, down the irregularities in the floor of the magma body. The disseminated mineralization is unlikely to show many irregularities because the pile of cumulate olivines can be expected to resist the turbulence from flow in the underlying sulphide liquid or overlying magma. However, increasing disturbances can be expected towards the top of the disseminated mineralization. Therefore sample spacing in nickel-sulphide deposits is more likely to be determined by external influences on the distribution of the massive ore, e.g. floor irregularities, than it is on internal structures. The Perserverance deposit was evaluated by drilling inclined holes (60°) so that intersections were made approximately every 60 m along strike and down dip. The spacing was chosen from experience of other similar deposits (Martin and Allchurch, 1976). The drill core size was BQ wire line.

Samples were taken by cutting a segment from the core with a diamond saw. The segment approximated to one third of the total core so that the remaining core could be used for metallurgical testing. The sample limits were established according to the lithogy and mineralization. The sample values were found to be lognormally distributed indicating that the distribution of grade was determined by structures larger than the size of the samples taken. A comparison between the values of core samples and the values of channel samples that weighed approximately 10 kg each suggests that this negative bias can be substantially reduced by taking larger samples.
The core samples from the massive ore breccia are probably negatively biased by the coarse fragments of metasediments. The reasons for the negative bias in the heavily disseminated ore are not clear, but the bias may be caused by the distribution of nickel in coarse-grained sulphides. Whatever the causes are for the negative bias in the sample values, they are likely to be uniform over large parts of the deposit, because the behaviour of settling droplets of nickel sulphide liquid is uniform. Consequently Ingamells' approach to determining an optimum sample weight could be successfully applied here (see subsection 2.2.1) and the negative bias in the sample values eliminated should the indicated sample size be practical to obtain.

4.3. SAMPLING PLACER DEPOSITS (ESPECIALLY THOSE OF THE WITWATERSRAND GOLDFIELDS)

Placer deposits occur in fluvial and littoral sedimentary facies, and contain heavy minerals concentrated by the interaction of flowing water with gravels and sands containing the heavy minerals. The fluvial systems which form the placers are normally those of braided streams which have a bed load in excess of what can be carried in suspension by the flowing water. Consequently the channel becomes choked with sediment, or an alluvial fan is built where a stream loses its energy as it flows onto a flood plain or yoked basin. Particles of sediment move in short bursts when the flow of water is strong enough to dislodge a sand grain, pebble or boulder and redeposit it further downstream. Redeposition of the

Fig. 36. Block diagram showing the elements of a braided stream. Stippled areas exposed, all other features are underwater.

(From: Walker, 1976)
sediment particles produces bedforms (see Fig. 36) which include longitudinal, lingoid, transverse and point bars, and dunes and ripples as reviewed and discussed by Miall (1977) and Smith (1974). Bedforms are normally temporary features and are eroded as the channel migrates across the sediment pile, leaving scour surfaces onto which other bedforms will be deposited.

The behaviour of heavy minerals in this system is a function of their specific gravities and has been studied in some detail by Jenkins in Averill (1946), Jenkins (1964), McQuivey and Keefer (1969), Smith and Minter (1977) and Minter (1978). Grains of heavy minerals introduced into the fluvial system from a source area move downstream with the other sediment, but settle sooner than less dense grains, e.g. sand grains of approximately the same size. The heavy minerals become trapped and concentrated in the lee of bedforms such as pebbles, bars, dunes and ripples; concentrated by the winnowing of lighter sedimentary particles from between the heavy mineral grains; and trapped by falling into the spaces between unconsolidated sediment on the floor of the channel. When a heavy mineral concentration is eroded by the stream, it becomes the heavy mineral source for other traps further downstream. This is the process by which the heavy mineral grains are distributed throughout a placer deposit.

Placer deposits can be found in both consolidated and unconsolidated gravels. Consolidated placers include the gold and uranium deposits in the Witwatersrand Basin (Pretorius, 1975) and the uranium deposits at Blind River, Canada. Unconsolidated placers include the gold deposits in California (Averill, 1946) and the cassiterite deposits in Southeast Asia. The mines at Blind River and on the Witwatersrand are more accessible for geological investigation than the unconsolidated, and sometimes submerged placers in California and Southeast Asia.

Sampling practices for unconsolidated placers are reviewed by Fricker (1976). The sample values are characterized by a very high variance and bias, and final recoveries compared with grade estimates are known to range between 32 to 149 per cent. Final recoveries from cassiterite deposits in Malaysia, expressed as a percentage of the
grade estimates, range from 141 to 174 per cent. Both these comparisons show that a significant negative bias can occur in the evaluation.

Unconsolidated placer deposits are normally sampled at a traditional drilling density that ranges from one hole per 0.04 ha to 1.6 ha. This is a considerably wider spacing than that used to evaluate other types of ore deposits. Palmer (in Fricker, 1976) recommends that the drilling density on cassiterite placers in Malaysia should be increased until two groups of alternate boreholes are essentially similar. However, such prerequisites will obviously lead to overdrilling, which is an expense that could be avoided.

Normally a Keystone drill is used in the sampling of unconsolidated placers, and loose grains of heavy minerals can cause sample bias. For example, gold particles in the hole wall tend to fall into the hole and therefore the sample, leading to an overevaluation. Conversely, coarse grained gold will tend to concentrate at the bottom of the hole, giving a negative bias to the sample values. However, evaluation of placer deposits in the permafrost areas of the Yukon (Hester, 1970) has shown recoveries to differ from the estimates, so poor sampling technique is not the only source of error in obtaining representative values.

The Keystone drill bores a hole that has a diameter of 19.05 cm (285 cm²) but the negative bias found in sample values indicates that this sample size is too small. This bias is caused by the gold concentration in a placer resulting from relatively few grains of gold, all of which may be located in a few traps in the bedforms and floor irregularities. Hester (1970) shows that if the gold particles are larger than 1 mm, then a hole of this diameter cannot possibly deliver a representative sample. Samples should therefore be taken with reference to the size and distribution of gold particles in the placer.

Accounts of the sampling practice on the Witwatersrand are given in Storrar (1977), and Hallbauer and Joughin (1974) with earlier investigations by Sichel (1947) and Sichel and Rowland (1961). During the exploration of a gold reef, boreholes are drilled from the surface and
intersections made with B-standard core which has a cross-sectional area of $20\text{ cm}^2$. The boreholes are deflected a few times to obtain more intersections of the reef in a relatively small area. The holes are drilled at a density of approximately one every $3\text{ km}^2$.

Oreblocks in existing mines are evaluated with chip or channel samples at intervals of between 1.5 to 5 m along the reef exposure. The samples are taken up to 2 cm deep between two lines approximately 10 to 15 cm apart and drawn across the reef sections. The projected areas of these samples are about $20\text{ cm}^2$, and therefore comparable to the B-standard core. The sections are subdivided into smaller samples corresponding to approximately 15 cm lengths, if the width of the reef exceeds 15 cm. The sample values show a very strong negative bias. The spread of sample values approximates to a lognormal or three parameter lognormal distribution (Krige, 1978) as shown in Figs 23 A and 23 B. The reasons for the negative bias in the sample values from the Witwatersrand are essentially the same as those found in the values of samples taken from unconsolidated placer deposits. However, the good underground exposures of the gold bearing reefs in the Witwatersrand provide an opportunity to assess the detailed distribution of the gold particles, so that the problem of placer sampling can be resolved.

Hallbauer and Joughin (1974) have shown how the distribution and concentration of gold particles in the Witwatersrand reefs can vary markedly over very short distances. Smith and Minter (1977) and Minter (1978) have shown how gold and uranium have been deposited in relation to bedforms. Therefore any sampling of the gold and uranium should be done in relation to these bedforms. Furthermore, if samples larger than the present size were taken, the bias in the sample values would be eliminated. This is because the high sample values, resulting from large gold grains and their local concentrations in the reef, would be diluted to a more representative figure. Hallbauer and Joughin suggest that a suitable sample would measure a few metres in extent within the footwall and hangingwall boundaries of the reef. Taking a sample of this size would be too cumbersome to become normal practice.

A sampling method based on the chip sampling technique described
by Davis (1962) and given in subsection 2.2.1, could be used to obtain a sample value that is equivalent to a bulk sample. The operator would have to have a good knowledge of bedforms and the distribution of heavy minerals in braided stream channels. Once the reef has been mapped, it can be subdivided into sampling strata depending on the variety of bedforms present, as shown in Fig. 37. Samples can then be taken from the sampling strata by either channels or chip sampling or with the portable gold-analyser (see subsection 2.1.3). The samples could either cross or follow the mineralized bands that occur on the scour surfaces and in the lee of the bedforms. The remaining parts of the reef that do not show any indications of mineralization can be estimated to have a negligible gold value. With the aid of geological mapping, the bedforms could be projected into a volume corresponding to the dimensions of the large sample and the volumes estimated for each of the sampling strata.

Fig. 37. Cross-section of gravel bar and overlying sandstones, showing pyrite-rich and other heavy-mineral bands and distribution of gold and uranium concentrations. Suggested sampling strata have been added.

(Modified from: Smith and Minter, 1977)
The values obtained from the samples must then be diluted into the volumes of their respective strata, in accordance with the fluvial processes that occurred. These diluted values must then be weighted by the volumes of the strata within the limits of the large sample to obtain the final value. This method could be tested against bulk samples.

The problem of placer sampling is the strong negative bias to the distribution of values obtained from small samples. This can only be counteracted by taking larger samples or their equivalents. Where access to the gold bearing strata is good, a larger sample or its equivalent can be taken. However, access to unconsolidated or submerged placers may prevent an accurate and unbiased sample from being taken.

4.4. SAMPLING CALCRETE AND ROLL-FRONT URANIUM DEPOSITS

The sampling of calcrete and roll-front uranium deposits has been discussed in earlier sections to illustrate the properties of sample variance with respect to sample length and sample spacing. They are discussed here as an illustration of the problems encountered in sampling highly variable and complex distributions of mineralization. Roll-front uranium deposits occur in sandstones while calcrete uranium deposits are found in valleyfill calcretes; both types occur in arid to semiarid environments.

The deposits are formed by the permeation of uranium bearing ground-waters through a porous host rock, and the uranium is precipitated by redox reactions in the presence of carbon and carbon dioxide, or by evaporation of near surface ground-waters. Other factors which influence the distribution of grade include host-rock porosity, permeability, stratigraphic arrangement of the sandstones or calcretes, temperature, and the balance between Eh and pH.

A typical roll-front deposit is illustrated in Fig. 38 and shows the distribution of grade in a porous sandstone trapped between two relatively impervious layers of claystone. The overall geological boundaries to porosity would therefore isolate the sampling strata to
which the distribution of grade should be related. Samples from within the stratum should be taken according to the distribution of grade and local lithological irregularities, but the values should be averaged by weighting them according to the sample lengths and, if necessary, specific gravity. Fig. 38 shows that if samples within the strata were not averaged, they would retain the complexity of the mineralization that would not be fully revealed even by very close spaced drilling. Sandefur and Grant (1976) found that each roll-front orebody in the Shirley Basin area measured less than 5 ft thick and 15 ft wide, perpendicular to the strike of the roll, which could only be outlined by drilling on 10 ft spacings. They found that samples from boreholes spaced up to 200 ft apart showed a significant correlation between their values. This is shown by the semivariogram in Fig. 26. Details of the predictions that were made, as to the minimum precision of the ore reserve estimate that could be achieved from the drilling, are discussed in subsection 3.5.2 and given in Table 1.
Uranium in roll-front bodies usually occurs as fine grains of coffinite, pitchblende and uraninite. Their distribution is related to local porosity and reducing agents such as carbon, and is therefore uneven. Consequently, drill-hole samples can be expected to be negatively biased. Schottler (1971) found the distribution of values of uranium in drill core samples from a roll-front deposit to approximate to a lognormal distribution.

An account of the sampling of the Yeelirrie calcrete uranium deposit is given by Haycraft (1976). The deposit has reserves of approximately 32 million tonnes at a grade of 0.15 per cent \( \text{U}_3\text{O}_8 \). The ore is erratically distributed through the calcrete and clay-quartz (varieties of arkose, quartzose grits, and kaolinitic clay-quartz material) but has a tendency to occur as flat lying lenses between 5 m to 6 m thick, and which are located on or just beneath the water-table (Fig. 39).

Fig. 39. Idealized geological column of the Yeelirrie calcrete uranium deposit. (From: Haycraft, 1976)

The lenses contain isolated patches of high grade ore. The only uranium ore is carnotite, and was one of the last minerals to be deposited in the calcrete. It occurs as thin films on the cavity walls in the
porcellaneous calcrete varieties; it is dispersed through earthy calcrete, coats grains or remains as disseminations in the sandy clay, and follows fractures and fault planes in any lithology. Sampling experiments (Haycraft, 1976) at Yeelirrie involved testing various types of drilling, gamma ray logging, and pit and channel sampling in costeans.

Boundaries to a sampling stratum in a calcrete uranium deposit should be chosen so that they coincide with the upper and lower limits to the distribution of uranium. The limits will be determined by the flow of the ground-waters through the calcrete and the porosity of the calcrete. David (1976; 1977) has shown that minimum sample variance will be achieved by taking one sample through the variable mineralization from the top to the bottom of the sampling stratum. Samples taken according to the local lithologies and grade distribution only enhance the complexities of the mineralization.

The Yeelirrie orebody was drilled on a 200 x 50 m grid which outlined three high grade areas. These high grade areas were drilled on a 6,25 x 6,25 m grid but this detailed drilling only served to confirm the complexity of the mineralization. Haycraft concluded that drilling on a 100 x 50 m grid would have been adequate to outline the reserves.

Haycraft (1976) found that reverse circulation drilling was the most suitable method for obtaining samples. Samples from conventional auger drilling became highly contaminated below the water-table, with the result that the thickness of the mineralization was overestimated by 70 per cent. Dry stick auger drilling was found to give more accurate measurements of thickness which were comparable with those of the reverse circulation drilling. However, the latter method was used for the drilling programme because there was less chance of contaminating samples by withdrawing the bit each time a sample had to be taken. Samples were taken over 1,5 m lengths. Diamond drill core was sampled according to the lithologies and grade variations, with a maximum sample length of 0,5 m. Core recovery was estimated at 91 per cent with losses attributed to the friable nature of the calcrete. A proportion of these losses may have been caused by cavities in the calcrete.
The sample results from the drilling are compared, in Table 2, with the grades of the stockpile from one of two costeans excavated during a trial mining programme. The values of the reverse circulation drill samples show a distinct negative bias and underevaluation, which can be attributed to the patchy distribution of the mineralization. An even stronger negative bias is suggested by the comparison, in Table 2, of the gamma logging of the holes compared with the X-ray fluorescence analysis of the chips. Pit and channel sampling cut to suit the mining dimensions were also found to undervalue the mineralization. Pits 0.76 m deep on 7.6 m centres were dug with a backhoe. Channel samples were cut 3.8 m apart, 7 to 8 cm wide, and 4 cm deep over the full bench height of 0.76 m.

The problem of sampling calcrite and sandstone deposits is therefore one of sample spacing with respect to the complexities of the mineralization. Sandefur and Grant (1976) have shown that the problem can be solved by averaging the variable grades so that a correlation between sample values can be established over a large area. In addition, even large samples from the Yeelirrie calcrite uranium deposit are negatively biased because of the variable distribution to the grades. Schottler (1971) counteracted this bias in samples from a roll-front uranium deposit by approximating the spread of values to a lognormal distribution.

<table>
<thead>
<tr>
<th></th>
<th>Stockpile Grade</th>
<th>Reverse Circulation Drill Grade</th>
<th>Pit Sample Grade</th>
<th>Wall Sample Grade</th>
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</thead>
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<td></td>
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<td>X.R.F. %\text{U}_3\text{O}_8</td>
<td>Gamma X.R.F. %\text{U}_3\text{O}_8</td>
<td>X.R.F. %\text{U}_3\text{O}_8</td>
</tr>
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<td>North Section</td>
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<td>0.15</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Central Section</td>
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<td>0.14</td>
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</tr>
<tr>
<td>South Section</td>
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<td>0.17</td>
<td>0.15</td>
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</tr>
<tr>
<td>Average</td>
<td>0.19</td>
<td>0.16</td>
<td>0.15</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 2. Comparison of sampling techniques, Yeelirrie uranium deposit.

(From: Haycraft, 1976)
4.5. SAMPLING PORPHYRY DEPOSITS

Porphyry deposits are low grade but high tonnage enrichments of copper, molybdenum, and less commonly tungsten and tin. They have been categorized by Hollister (1978) on a structural basis into breccia pipes, or stockworks, or combinations of the two. Breccia pipes in deposits with a dominant stockwork structure to the distribution of mineralization, are generally small, and play a subordinate role in the localization of metal sulphide. Large breccia pipes, as shown in Fig. 40,

Fig. 40. Lowell and Guilbert model, breccia type porphyry deposit. Arrows in this diagram indicate fluid flow at the time of coincident development of potassic and phyllic zones.

(From: Hollister, 1978)

may have stockwork structures ringing their periphery, but the distribution of metal sulphide is primarily within the pipe (Hollister, 1978). Stockwork deposits, shown in Fig. 41, are those porphyry deposits where the mineralization occurs predominantly in veins and veinlets formed by hydraulic fracturing. The density of mineralized fractures varies from one deposit to another, but common to most deposits are:

a) their occurrence on a major and normally strike-slip fault;
b) a principle set or trend to the veinlets; (see Fig. 42)

c) the stockwork fractures are tectonic voids filled with minerals, and the hydrothermal fluids were channelled along the stockwork fractures producing zoning as a function of distance from the heat source.

Precipitation of the ore minerals is caused by retrograde boiling, which tends to produce dome shaped orebodies centered on the heat source. Consequently, the orebodies are distributed in an annular configuration with

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Fig. 41. Lowell and Guilbert model, stockwork type porphyry deposit. Arrows in this diagram indicate fluid flow during the coincident development of both potassic and phyllic zones.

(From: Hollister, 1978)

a low grade core. Orebodies within breccia pipes adopt the shape of the breccia pipe. Alteration zones in the country rock "halo" the heat source and, for the Lowell and Guilbert model applicable to calc-alkalic plutons of the granodiorite-quartz monzonite range (Lowell and Guilbert, 1970), consist of a potassic core (orthoclase-biotite), a phyllic zone (quartz-sericite-pyrite), an argillic zone (kaolin-illite-montmorillonite-pyrite), and an outer propylitic zone (epidote-chlorite).
A similar zoning has been described by Hollister (1974) for porphyry deposits associated with quartz-free plutons and consists only of the potassic and propylitic mineral assemblages. Grades of the primary mineral deposition in porphyry deposits is often subeconomic and ore grades are only reached with supergene enrichment.

Programmes for sampling porphyry deposits begin with an extensive drilling operation which is often followed by limited underground bulk sampling if the results from the drilling are favourable (Reichhard Barends, 1980). The drilling methods most commonly used are diamond drilling, rotary drilling and percussion drilling, although churn drilling was popular (Hazen, 1963). The holes are either vertical or inclined so as to intersect any dominant fracture trend in the deposit at a steep angle (Fig. 42).

![Fig. 42. Surface geological map, Chaucha. Intersection of the Chaucha and Cordillera faults is in the centre of alteration and mineralization for Chaucha. The stockwork is largely composed of a parquetry of mineralized segments developed during simultaneous displacement along both faults.](From: Hollister, 1978)

The volume of influence of drill samples in porphyry deposits
is usually very small, but it depends on the extent of the mineralized structures. Larger volumes of influence can be expected in deposits where there is a dominant structural trend, as opposed to those deposits where the mineralization is distributed in a parquetry of fractures and veinlets. Hazen and Berkenkotter (1962) found that the correlation between the two halves of a split core from the Climax molybdenum deposit is very high, but the correlation between two drill holes parallel to each other and only 12 in apart is negligible. They also found that the influence of successive samples along a drill hole is also negligible when 2 ft or larger sample intervals are used. Up to 50 per cent grade difference between adjacent holes is acceptable (Reichhard Barends, 1980). Correlation can be achieved between samples if they are clustered to reduce the nugget effect. David (1976, 1977) found that there was no correlation between 10 ft lengths of split core samples taken from 96 holes drilled in fan patterns in a molybdenum porphyry deposit. The semivariogram for these values is given in Fig. 43 A and it shows an almost pure nugget effect. Compositing samples over 50 ft intervals gave a similar picture (Fig. 43 B) although the nugget effect has been reduced. The samples were then combined into groups of five and their average values assigned to 200 ft blocks in which the samples occurred. The computed semivariogram for these blocks is given in Fig. 43 C and shows a very low nugget effect and well defined sill.

Drill spacing in a porphyry deposit is normally established on
a square grid because there is no correlation between samples and their relative positions are unimportant. Drilling at Sar Cheshmeh began at 200 m centres, followed by 141 m centres, 100 m centres and finally irregularly distributed holes to evaluate local discontinuities in the mineralization such as dyke orientation and thickness. At the Panguna copper porphyry deposit, New Guinea, a 122 m spacing to the boreholes was found to be too wide for the local discontinuities in the mineralization, e.g. breccia pipes (Baldwin, et al., 1978).

Samples of drill core can be taken between geological boundaries and over lengths of essentially similar grade distribution. At Sar Cheshmeh, samples were taken over 1.5 m to 3.0 m lengths except where a geological boundary made a shorter sample length desirable (Lock, 1977). However, this was found to give an excess of sample data, so the values were composited to 12.5 m lengths as this was the proposed bench height. Samples from percussion and rotary drilling are taken over suitable lengths, e.g. 3 m at Sar Cheshmeh. Major geological boundaries must be taken into account when sampling porphyry deposits. For example, at the Panguna copper porphyry deposit, sample grades were correlated across geological boundaries, with the result that low grades within the biotite granodiorite were averaged with the higher grade samples outside the intrusion. Recalculation of the reserves by assigning this part of the
orebody to waste, decreased the tonnage but increased the overall grade of the ore (Baldwin et al., 1978).

Statistical evaluation (see subsection 3.5.3) of a drilling programme in a porphyry deposit is justified because the samples are virtually random with respect to one another. Hewlett (1965) has investigated the cost effectiveness of drilling porphyry deposits, based on statistical theory. Where there is unpredictable correlation between samples, the statistical method proposed by Kuhn and Graham (1972) can be used to eliminate unnecessary holes and cut drilling costs. Major structural trends that could allow correlation between samples over distances in the order of 100 m seldom exist in porphyry deposits because these are detrimental to the development of good mineralization. Therefore, geostatistical evaluation of drilling porphyry deposits is not really suited to the problem. Although correlation between samples can be achieved by cluster sampling, the 200 ft square blocks may not be practical in terms of the distribution of significant geological boundaries within the deposit.

Core loss is a problem with diamond drilling in porphyry deposits because of the soft and incoherent nature of the altered host rocks in which the mineralization occurs. At Chuquicamata, the mineralization consists of quartz, oxide and sulphide veinlets in rock conditions that vary from intensely sericitized (soft and porous), through clay altered ground, to almost fresh granodiorite. Consequently very inconsistent drilling results were obtained (Waterman, 1955). Core generally breaks along mineralized fractures and this leads to a loss of mineral and hence a negative bias to the sample value. Some soft and mineralized veinlets can also lose mineralization without any core loss. However, not all core loss will lead to a biased sample result, for example, a 75 per cent core recovery was obtained in soft and well fractured rocks at Sar Cheshmeh, but samples from raise boring (15 in diameter) along six diamond drill holes had average values within 1 per cent of the split core samples (Lock, 1977). This showed that there was no negative bias in the core samples. Core recovery of 95 per cent to 65 per cent from diamond drill holes in the Climax molybdenum deposit created a positive bias, while core recovery of less than 65 per cent created a negative bias (Hazen and
Berkenkotter, 1962). Positive sample bias also occurred at Questa (Silman, 1965). The bias of sample values caused by core loss in porphyry deposits is therefore difficult to predict because it can be either positive or negative. Usually a large core size, e.g. AX, is used as a precaution against core loss (Reichhard Barends, 1980), but NX core was mostly used at Sar Cheshmeh, although it ranged from H to IX (Lock, 1977). Alternatively, percussion and rotary drilling techniques can be used. Attempts to remedy core loss by sludge sampling have been made (see subsection 2.1.2), but can be hampered by porosity in the altered zones.

Where the mineralization in porphyry deposits tends to be distributed in high grade patches, sample values will become negatively biased. At the Panguna porphyry deposit, bulk sampling indicated that the diamond drilling had underevaluated the grade by 12 per cent to 29 per cent (Baldwin et al., 1978). Negative bias does not always occur in the sample values from other porphyry deposits, e.g. at Sar Cheshmeh, the borehole sample values were found to give an unbiased estimate of the grade (Lock, 1977).

Bulk sampling is becoming increasingly popular as a conclusion to a successful drilling programme, and as a check against bias in the drill sample values. Bulk sampling is also a check for the continuity of grade in different directions through a porphyry deposit (Lock, 1977), and is necessary because of the lack of correlation between samples. Bulk sampling at the Berkeley porphyry deposit totalled 20,000 to 30,000 tonnes (Reichhard Barends, 1980). Underground development for bulk sampling at the Sar Cheshmeh deposit was laid out in the supergene zone, and planned so that it conformed with the 200 m drilling grid. In addition to bulk samples, chip and channel samples were taken from the side walls. Two sizes of chip samples (5 kg and 30 kg) were taken from the same locations over a limited section of development and their average values did not differ by more than 1 per cent. This served to confirm that small samples from the deposit were not biased.

The problems of sampling porphyry deposits are, therefore, mainly the lack of correlation between samples, the local irregularities to the mineralization and core loss. Sample bias caused by patchy high
grade mineralization can occur but is not necessarily present in all types of porphyry deposits.

4.6. SAMPLING VEINS AND STOCKWORKS

Veins and stockworks are open spaced fracture fillings which normally have sharp boundaries with the enclosing country rock. The shape and distribution of the fillings depends on the nature of the fractures which in turn is related to the regional structure of the area in which they occur. Open space fracture fillings normally have two very large dimensions which can extend for many hundreds of metres along strike and down dip, and a third dimension which is narrow with respect to the other two. Many variations of this pattern exist, and include pinching and swelling or dilation veins (Fig. 44B), bifurcation (Fig. 44E), branch veins related to second and third order jointing, saddle veins in fold closures and stockworks where the controlling fractures are small, numerous and interconnecting. Discontinuous fillings include ladder veins, sigmoidal tension fissures (Fig. 44D) and fracture intersections, and normally have only one long dimension and two comparatively short ones.

Fig. 44. Varieties of veins. A, chambered vein; B, dilation veins in schist; C, sheeted vein; D, en echelon veins in schist; E, linked or bifurcated veins.

(From: Jensen and Bateman, 1979)

Veins are natural sampling strata if their contacts are sharp and they have widths of the same size as the minimum stoping width, e.g.
as shown in Fig. 44 A. However, as the fractures which the veins occupy become more complex, a sampling stratum that will minimise heterogeneous variance in the sample data is more difficult to define (see Fig. 44 C). The choice of a sampling stratum will depend entirely on the nature of the fractures, and the distribution of mineralization within them, but if geological boundaries exist they should be used. Whatever boundaries are established for the sampling, they should be carefully adhered to.

Samples from veins can be collected by drilling, underground channel, or chip sampling. Diamond drilling is most suitable because the geological boundaries to the sampling stratum can be accurately determined. Channel samples should be taken at a steep angle but preferably vertical to the structures in which the mineralization occurs. Samples can be taken along narrow veins and not across them, e.g. at Cobalt, Ontario, where the veins are rich and seldom exceed a few centimetres in width. However, auxiliary samples of the wall rock must also be collected (McKinstry, 1948). Subdividing samples according to lithologies and grade is advisable, especially where there is contrasting hardness between vein and wall rock, or between minerals within the veins, e.g. crustiform mineralization (Fig. 45). At the Kerr-Addison mine, samples of gold

![Fig. 45. Examples of crustified veins.](From: Jensen and Bateman, 1979)
bearing quartz veinlets were collected underground by taking chips of the veins and carbonate country rock, in proportions visually estimated by the sampler to be equal to those exposed in the face (The Staff, 1951). Visual grade estimates can also be made within acceptable limits of precision where there is suitable mineralization. Such estimates have been successfully applied at the Kerr-Addison mine, where the gold mineralization is directly proportional to the concentration of pyrite in some orebodies.

Mineralization within veins and stockworks is usually concentrated within shoots that plunge either in directions controlled by the regional structure, or in directions followed by the channelling of the ore forming fluids. The open spaces may be filled by a series of fluid pulses and only a few may be mineralized, consequently the grade may be confined to certain portions of the body, or to certain veins within a stockwork. Ore minerals within a vein can be disseminated or patchy, or concentrated along the edges either erratically or crustified.

If mineralization in a vein is patchy, the volumes of influence of samples will be small. Consequently the evaluation of such mineralization by widely spaced samples, e.g. diamond drilling, has to rely on a certain amount of geological interpretation. For example, a low sample value in an oreshoot is less likely to indicate subgrade mineralization than a low sample value lying outside an oreshoot. Furthermore, sample values from patchy mineralization can be expected to have a strong, negative bias, and may even be so distorted that an accurate estimation of grade cannot be made. Historic data from the exploitation of similar mineralization in other parts of the mine can be invaluable in applying the correct interpretation to erratic values.

Underground sampling of veins indicates that more reliable data can be obtained from relatively closely spaced samples compared with the normal spacing for diamond drill holes. For example, at Butte, Montana, chip sampling and ore reserve estimation for Cu and Ag were found to be accurate for well defined veins, with errors of approximately 5 per cent when checked against broken ore samples, but churn and diamond drill
sampling was not found to be satisfactory (Daly et al., 1925). At the Mogollan mine, New Mexico, channel samples at 5 ft to 10 ft intervals from gold bearing quartz veins gave values which lay within 3 per cent to 4 per cent of the mill results (Kidder, 1925). Sample spacings for most vein deposits described by Jackson and Knaebel (1932) do not exceed 10 ft.

A geostatistical approach to the problem of sampling the Eagle copper vein has been followed by Sinclair and Deraisme (1974). The vein occurs in a highly folded sequence of shale, limestone and dolomite. The contacts between the vein and wall rocks are sharp. Sulphides occur in disseminated and massive form with chalcopyrite comprising 95 per cent of the total sulphide minerals present. Samples were taken across the vein at irregular intervals, but generally 8.5 ft. Each sample weighed approximately 2 kg. The semivariogram from the sample data indicated that the sample spacing could be increased to 20 ft without loss of precision for the overall evaluation of the vein deposit, but a spacing of 10 ft was needed for the evaluation of ore blocks.

A deposit of comparable geology to a stockwork is the Rössing uranium deposit in Namibia. The uranium occurs in alaskite bodies which intrude intensely folded gneisses, schists and marbles along axial plane foliation which trends east to northeast, and dips in a southerly direction at 70° (see Fig. 46). The alaskite bodies range from stringers

Fig. 46. Geological section of the Rössing uranium deposit showing geology, boreholes and bulk sampling crosscut. (From: Berning et al., 1976)
through dykes to large masses formed by the coalescing of dykes, and form an orebody approximately 3.5 km long by 1.5 km wide. Distribution of the uranium is largely confined to a zone of smaller extent than the distribution of the alaskites, although rich spots may be found beyond the limits of the orebody (Berning et al., 1976). Uranium minerals within the uraniferous zone are concentrated in biotite-rich selvedges of the alaskite; in spots where robust alaskite bodies display sharp upward narrowing to form dykes or veins; in alaskite emplaced along fold axial planes of folds; and in those localities where amphibolite has been replaced by alaskite. The uranium minerals are predominantly uraninite and minor amounts of betafite. Secondary uranium minerals account for approximately 25 per cent of the mineralization, and are largely confined to the alaskite.

The deposit was sampled by diamond drilling (BX core) on a 60 m x 122 m grid orthogonal to the geological structure, with holes inclined at 45° north. Some percussion drilling to test the near surface distribution of the uranium was run concurrently with early stages of the diamond drilling (Fig. 47). Core recovery from the diamond drilling was

![Fig. 47. Section of Rössing uranium deposit showing drill pattern.](From: Berning et al., 1976)
good and split core samples were taken over 5 ft lengths (Berning et al., 1976). Grades determined by the drilling, in addition to those determined by pilot holes for exploratory underground development, agreed closely with bulk sample grades (Reid, 1977). The bulk samples were taken from a cross-cut which followed a north-south drill section line. The grade was estimated by dividing the orebody into polygonal blocks extending 60 m along strike either side of each borehole. However, grade in the open pit mine was found to depend on the distribution of a few high grade patches within the block. It was found that better estimates of the grade were achieved by percussion drilling within the pit on a 20 m x 20 m grid, but that this was further improved by sampling chips from blast holes drilled on an approximate 8 m x 8 m grid.

The problem of sampling vein and stockwork deposits is one of establishing the continuity of mineralization between samples, especially if the samples are spaced at distances that are far greater than the volumes of influence of those samples. The definition of a sampling stratum is normally easy when the vein is wide and has sharp boundaries, but the boundaries to complex structural patterns are more difficult to define. Samples should be taken from within the boundaries of the vein or veinlet because sampling over a minimum stope width will include a certain proportion of wall rock. This will introduce a heterogeneous variance to the sample values and reduce the already small volume of influence of the sample. Sample values from patchy mineralization can be expected to have a negative bias.

4.7. SAMPLING CARBONATE HOSTED LEAD-ZINC DEPOSITS

Lead-zinc sulphide orebodies fill cavities that have been formed by the dissolution of limestone or dolomite. The bodies can occur with their major axis parallel to bedding, or transgressive to bedding at any angle (see Fig. 48). They are found widely distributed throughout favourable beds, or confined either to the intersection of fissures and joints with these beds, or entirely to dominant structural features. Pipes, chimneys and fissure fillings show greater variation in trend than bodies confined to bedding (Prescott, 1925). Orebodies that transect bedding do not, as a rule, survive a change in the country rock but they
Fig. 48. Solution caves and cavities in limestone.  
A, B, open solution cavities lined with crusts of crystals.
C, gash vein or solution enlargement along a joint.
D, solution cave occupied by ore (black) and cave breccia on bottom, overlain by later breccia and ore, and by breccia fragments.  
(From: Jensen and Bateman, 1979)

often show great continuity, e.g. the Tsumeb pipe extends over a depth of greater than 1 000 m (Söhnge, 1964). Mantos which follow bedding can be twice this length.

The outlines of lead-zinc sulphide orebodies in carbonate are sharp and the grades drop rapidly into the wall rocks. The grade of mineralization in the cavities remains at a constant level, and is only diluted by particles of foreign material, e.g. fragments of dolomite or limestone and varying amounts of sand and clay. This is the "normal grade" of ore which is defined by Prescott (1925) as "... the average of all ores above that critical point where the grade commences to drop very suddenly to traces only". It is a critical value and independent of commercial considerations. This natural division between the mineralization and country rock provides the division for the sampling strata of these types of deposits.
Often carbonate hosted lead-zinc sulphide bodies have an oxide cap and the primary mineralization can be zoned. The oxide cap is usually enriched with oxide salts which can give very high and misleading values to the samples. Where there is strong vertical zoning, care must be taken not to oversample levels when combining these values with those from a disproportionately low number of samples taken from winzes and raises. In chimney shaped orebodies, horizontal zoning is stronger than the vertical zoning, and therefore emphasis should be placed on establishing a dominant horizontal component to the sample orientations (see Fig. 49).

Fig. 49. Section through Tsumeb pipe, showing ore classes and even spread of boreholes to determine grade from level to level.

(From: Söhnge, 1966)
The zoning is often caused by a succession of mineralizing events. Therefore, if discontinuities exist between zones they should be used as subdivisions for samples, because to sample across them will introduce a heterogeneous variance into the sample values, and reduce their volumes of influence.

Volumes of influence of samples are highly variable and almost impossible to determine. However, samples should be taken in the best possible localities and with the best possible procedure so as to maximise the chances of correlation between them. There is no optimum sample spacing that can be determined except by judgement as each sample is taken. Therefore, good geological knowledge of a deposit is essential for good evaluation. This knowledge should include an understanding of the ore occurrence, the loci of deposition, the infinite forms that the deposit may take, the deformation and alteration that may have occurred, and the origins and channels of ingress (Prescott, 1925).

The ores of carbonate hosted lead-zinc deposits usually exhibit great irregularity. The sulphide minerals occur in bunches and crustiform bands between and including carbonate layers and blocks. The metal content can be segregated into monometallic bands. Less usually, the ore is massive, and sometimes fairly uniform in shape and distribution of values. The grades of ore at Tsumeb mine, Namibia, fluctuate from 0 to 55 per cent Cu, 0 to 75 per cent Pb and 0 to 45 per cent Zn, and over 100 different minerals have been identified in the ore (Sühnge, 1966). A high nugget effect has been demonstrated in samples taken from the Colquijirca mine, Peru, where the values of adjacent samples differed by up to 50 per cent (McKinstry, 1936; 1948).

Samples can be collected by cutting a channel or boring a hole across the banding in the ore. However, hand-taken samples have been found to contain disproportionately high amounts of soft and friable minerals, which cause a positive bias to the sample values. At Tsumeb, hand taken samples were found to overvalue the ore by 10 per cent, but this was reduced to 5 per cent by careful sampling procedure. The sample values were reduced by a factor to counteract the positive bias (Sühnge, 1966). A sampling experiment at the Trepcă mine, Yugoslavia, which
involved cutting, by hand, a groove along an exposed diamond drill hole, showed that there was an estimated positive bias of 53 per cent Pb and a 36 per cent Zn in the hand-taken sample, compared with the diamond drill core (Hatch, 1931).

One method to avoid the positive bias on hand-taken samples is to collect samples underground by sludge hole drilling. Prescott (1925) found that holes up to 18 in deep gave excellent results. At Tsumeb, jack-hammer holes were drilled into the backs of stopes to estimate the payability of the ore and the degree of oxidation. Visual sampling (see subsection 2.1.3) is another method to avoid hand-taken samples and to which carbonate hosted lead-zinc ore is well suited (Prescott, 1925). Visual estimates have been successfully used at Mascot mines, Tennessee (Coy and Noble, 1925), and are presently being used by mines on the Viburnum Trend (B. McDonald, personal communication).

The porosity of carbonate hosted lead-zinc ores can be very high and should be taken into account to avoid overevaluation. Prescott (1925) shows how porosities in the primary sulphide ore can reach 12 per cent but in the oxide ore it can reach 44 per cent. Porosity due to cavities is relatively easy to estimate in underground exposures, but Hatch (1931) has shown that the estimation of cavities in borehole core was considerably less than that revealed when the hole was exposed in underground workings. Very careful measurement of borehole advance, which could be compared with core recovery, would be necessary to give an accurate estimate of porosity. The problems of porosity can be avoided by:

a) having a clear and definite idea of the primary processes of deposition, and secondary processes of oxidation;

b) careful comparison of the shipment record with the volume removed;

c) allowing for included limestone;

d) accurate determination of the specific gravity of the ore;

e) establishing accurate cross-sections at a close spacing (Prescott, 1925).
The problems of sampling carbonate hosted lead-zinc deposits are, therefore, largely due to the highly variable shapes of the orebodies, and the variable volumes of influence that the samples may have, but this can be overcome to some extent by good geological knowledge of the deposit. Collection of samples can generate very strong positive bias in the values, because of the preferential removal of the soft and friable sulphide minerals. Cavities can lead to an overevaluation, especially if the samples are obtained by drilling. The sharp contacts between the ore and wall rocks clearly define the outer limits of the bodies, but geological discontinuities within the ore should be used as sampling stratum where this is possible.

4.8. CONCLUSIONS

Where the mineralization of a deposit is confined between sharp geological boundaries, these boundaries define a sampling stratum which should be used in a sampling programme. Deposits that have well defined boundaries to their mineralization include the manganese seams of the Kalahari Manganese Field, most vein deposits and carbonate hosted lead-zinc deposits, while others occur within distinct stratigraphic units, e.g. roll-front uranium deposits. Sampling strata are more difficult to define where the boundaries to the mineralization are diffuse, e.g. porphyry deposits, and the Merensky Reef.

The volumes of influence of samples can be very large in stratiform ores, even where there is a high nugget effect from the local discontinuities in the mineralization. More variable volumes of influence are found in the volcanogenic ores and the carbonate hosted lead-zinc deposits. Although roll-front uranium deposits and calcrete uranium deposits have highly complex distributions to the mineralization, samples for which the grade is averaged over the total thickness of the sampling stratum often show a very large volume of influence. Deposits for which samples have small volumes of influence include porphyry deposits, and vein deposits. The volumes of influence of samples in a nickel sulphide deposit are potentially large but are controlled by external factors such as floor irregularities that trap the precipitating sulphide melt.
A negative bias to sample values due to the sample size being too small to average the irregularities in the mineralization, is found in the sampling of most ore deposits. It is particularly strong in the sampling of placer deposits where the low grades of gold rely on the distribution of a few relatively coarse grains of gold. A similar bias does not occur in sample values from the Merensky Reef because the grains of platinum minerals are small and their lateral distribution throughout the Reef is even. A negative bias exists in small samples from the Perserverance nickel deposit, roll-front uranium deposits, some porphyry deposits and is anticipated in vein deposits.

Sample bias generated by sampling procedure occurs where there are contrasting grades over short distances and contrasting hardness or friability between minerals in the same ore. A positive bias is often generated in collecting samples from massive sulphide deposits, especially those where some chert is present. Very high positive biases to sample values are generated in the sampling of carbonate hosted lead-zinc deposits. Core loss is a major problem in sampling porphyry deposits, but sample bias caused by poor sampling procedure is insignificant in ores where the grade is evenly distributed, e.g. manganese in the Kalahari Manganese Field.

In order to understand the meaning of sample values and predict their significance in the evaluation of an ore deposit, the sampling of any orebody should be conducted with constant reference to the geology of the structures which control the distribution of grade.
5. **RECOMMENDATIONS FOR THE PLANNING OF SAMPLING PROGRAMMES**

This chapter combines the previous three chapters and their conclusions into recommendations as to what considerations should be made when sampling an ore deposit for the purposes of evaluation.

An ore deposit can only be sampled successfully if the samples are taken according to the natural subdivisions, distribution, and nature of the mineralization. Therefore, before any evaluation programme is started, the deposit should be well explored by geological mapping, trenching and drilling. The eventuality of having to evaluate the deposit should always be borne in mind during the early exploration stages, and once an evaluation phase seems probable, experiments to test various sampling strata, sample spacing, size, methods of collection, reduction procedures and analysis should be conducted.

Experiments should first be aimed at establishing a suitable sampling procedure, i.e. sample collection, sample reduction and analysis, for the type of mineralization in the deposit. Incorrect or badly designed procedures can invalidate later and more expensive investigations to determine the volumes of influence of samples and optimum sample spacings. Errors due to this aspect of sampling are normally the easiest to control, as procedures can be changed and adapted to suit different sets of circumstances. However, in certain orebodies, e.g. carbonate hosted lead-zinc deposits and porphyry copper deposits, the nature of the mineralization may make it almost impossible to collect unbiased samples. Sample reduction must take the nature and distribution of ore bearing minerals into account, and the size of aliquot used in the analytical procedure must be established with a reference to the desired precision of the determinations, and the number and distribution of mineral particles in the subsample. Analytical error should be within acceptable limits, and the cumulative error from sample collection, sample reduction and analysis should be low, e.g. an order of magnitude less than the error caused by the size, number and location of samples within the sampling stratum.

Once a sampling procedure has been established to suit the
particular ore, the distribution of mineralization in the deposit should be subdivided into sampling strata. The strata must lie between natural geological boundaries which isolate units of the ore in which the character of the mineralization is essentially the same. Sampling strata need not be strata in the sedimentary sense of the word, but can be any subdivision of mineralization which stratifies the distribution of samples in a deposit. A good choice of boundaries will depend on a good geological understanding of the distribution of mineralization with respect to boundaries that occur. In some deposits, the boundaries are sharp and can extend over very large distances, while others have a relatively short extent or are diffuse. Once geological boundaries have been chosen as sampling stratum boundaries, they must be strictly adhered to during the sampling programme. Sample values must only be combined with others from the same sampling stratum because the sample values from different sampling strata will have different population parameters, i.e. mean, variance and skewness.

Variance between sample values is reduced as the size of sample increases. The way it is reduced depends on whether the variance is due to a random component or spatial component or both. If there is a significant random component or nugget effect in the variance, or if the spatial variance has a range close to the dimensions of the sample, then an increase in sample size, if practical, will decrease the variance between samples and increase the precision of the ore reserve estimates. An increase in sample size is, therefore, unlikely to significantly reduce the spatial variance component in values of samples taken from uniform stratiform ores, but may make a significant improvement to the distribution of values in samples taken from vein deposits and carbonate hosted lead-zinc deposits, where the mineralization is patchy. Choice of a sample size will therefore depend on the desired precision with which the grade of the orebody is to be estimated. If the variance between samples is excessive, and if it is impractical to increase the size of the sample, a certain amount of geological interpretation will have to be applied to each sample value.

Sample values are often negatively biased by the inability of a sample to average the irregularities in the distribution of grade within the deposit. The bias can be counteracted by approximating the spread of values to a lognormal or three parameter lognormal distribution. The
alternative way of avoiding, or reducing, a negatively biased distribution of sample values, is to take the samples according to the small-scale structures in which the mineralization occurs. If the volume of influence of small samples is considerably restricted, then larger samples must be taken. Where impractically large samples are needed, the equivalent of large samples can be obtained by taking composite samples, in favourable geology. The extent to which sample values will be negatively biased should be known at an early stage in an evaluation programme, and the effects of sample size on the bias should be tested. If there is significant contrast in sample bias between two different sample sizes, then their values should not be combined without adjusting for the bias, although the samples may come from the same sampling stratum.

The spacing between samples should be determined with reference to the volumes of influence of the samples. The volumes of influence are determined by the extent of the small-scale structures in the sampling stratum which controls the distribution of the mineralization. Where the mineralization is uniformly distributed over large areas, e.g. stratiform ores, samples will have large and similar volumes of influence. The dimensions of the volumes can be estimated with experimental semi-variograms. Samples that have large volumes of influence should be spaced so that they lie within the influence of adjacent samples, because this will improve the precision of the ore reserve estimates. Deposits in which samples have small volumes of influence, e.g. porphyry copper deposits, are best evaluated on a regular sampling or drilling grid and the samples can be considered independent of one another. Deposits in which samples have very variable volumes of influence can either be sampled on a fixed spacing, e.g. sections or drilling grids, or at sites selected by geological reasoning, e.g. in carbonate hosted lead-zinc deposits. Where a fixed spacing is used for variable volumes of influence, a certain proportion of samples will be uncorrelated with a corresponding decrease in the precision of the ore reserve estimate. Volumes of influence of samples can be altered by a change in sample size, if that sample size can extend beyond the range of the small-scale structures to which the volume of influence is related. Cluster sampling, or composite sampling can be used to establish a correlation between
samples over large distances.

The sampling of any orebody is dependent on the geology of that orebody, from the geological boundaries between which the mineralization is confined to the individual grains of valuable minerals in the aliquot of subsample taken for analysis. If geology is disregarded, an evaluation programme cannot claim to give an accurate estimate of ore reserves.
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APPENDIX I

Determination of L (see subsection 2.2.1.)

Ingamells (1974a) suggests five methods for the determination of L of which two are repeated here:

a) If a long series of small samples is taken there is a probability

\[ P = 1 - (1 - e^{-2})^N \]

that at least one of N samples will be free of grains of the minor mineral. If the lowest value (for an ore where the ore mineral is the minor constituent) is taken as an estimate of L and used to determine Z by

\[ Z = \frac{K - L}{H - L} \cdot \frac{W}{u^3 d} \]

then Z must give \( P \approx 0.98 \) for the estimate of L to be taken as the true value.

b) L can be determined by mechanical separation (e.g. with a magnetic separator or heavy liquid) of the ore mineral from the gangue. The mass of the ore mineral q from the sample can be used to estimate L by the relationship

\[ q = \frac{K - L}{H - L} \]

but errors can be caused by the non liberation of the ore mineral from the gangue.

(Note: Symbols defined in subsection 2.2.1)
APPENDIX II

Details for the practical application of Gy's equation (see subsection 2.2.2.)

The equation is:

\[ M = \frac{Cd^3}{S^2} \]

where:
- \( M \) = sample weight required (in grams);
- \( d \) = top particle size or screen size which passes 90 - 95 per cent of the material (in centimetres);
- \( S \) = standard deviation of values that can be expected from sample \( M \);
- \( C \) = sampling constant. It remains constant for a given ore but will be altered by the largest particle size as this will change the value of \( l \).

\( C \) is given by:

\[ C = f \times g \times l \times m \]

where:
- \( f \) = shape factor. Normally \( f = 0,5 \) but for gold ores, \( f = 0,2 \);
- \( g \) = particle size distribution factor. Normally \( g = 0,25 \) except for closely sized materials where \( g = 0,5 \). \( g = 0,2 \) for gold ores;
- \( l \) = liberation factor and has values which range from 0 to 1. For completely homogeneous material, \( l = 0 \). For completely heterogeneous material \( l = 1 \). \( l \) is determined from Table A having established the ratio: \( d/d_0 \)

where \( d_0 \) is the liberation size of the mineral in the ore.
Table A. Values of \( I \) for values of \( d/d_o \)

For gold ores \( I = 1 \) when the particles of gold are completely liberated, but \( I \) is difficult to determine when the gold particles are not completely liberated.

\( m = \) mineralogical composition factor. Its units are \( g \text{ cc}^{-1} \) and because it is the only function in the expression for \( C \), it too is expressed in \( g \text{ cc}^{-1} \).

\( m \) is given by:

\[
m = \frac{(1 - a)}{a} \left\{ (1 - a)r + at \right\}
\]

where: 
- \( r = \) density of ore mineral
- \( t = \) density of gangue mineral
- \( a = \) average mineral content (not metal content) expressed as a decimal part of 1 and estimated from earlier assays of the ore.

<table>
<thead>
<tr>
<th>( d/d_o )</th>
<th>(&lt; 1)</th>
<th>(1-4)</th>
<th>(4-10)</th>
<th>(10-40)</th>
<th>(40-100)</th>
<th>(100-400)</th>
<th>(&gt; 400)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>1,0</td>
<td>0,8</td>
<td>0,4</td>
<td>0,2</td>
<td>0,1</td>
<td>0,05</td>
<td>0,02</td>
</tr>
</tbody>
</table>
APPENDIX IV

Determination of the weighting coefficients $c_i$ and $c_j$ (see subsection 3.5.2)

The weighting coefficients $c_i$ and $c_j$ are determined by establishing $n + 1$ equations where $n$ is the number of samples in the proposed sampling programme. The simplest sample pattern to consider is a square grid where the samples have been taken at the corners of squares. A square grid can be made to represent all rectangular grids by applying the anisotropy modulus. The equations for nine samples are given below:

\[ c_1 \sigma_{i1} + c_2 \sigma_{i2} + c_3 \sigma_{i3} + \ldots + c_9 \sigma_{i9} + V = \sigma_{1B} \]

for all $i = 1, n$

\[ c_1 + c_2 + c_3 + \ldots + c_9 = 1 \]

The $\sigma_{ij}$ term on the left-hand side of the equation is the covariances between any two samples and are determined from the semivariogram by:

\[ \sigma_{ij} = \sigma^2 - \gamma(h)_{ij} \]

$\gamma(h)_{ij}$ is the mean variance between samples separated by distance $h$ while $\sigma^2$ is the sample variance which corresponds to the sill of a spherical semivariogram. $V$ is the Lagrange multiplier. The auto-covariance $\sigma_{iB}$ is determined from the expression:

\[ \sigma_{iB} = \sigma^2 - \overline{\gamma}_{iB} \]

where $\sigma^2$ is the sample variance, and $\overline{\gamma}_{iB}$ is the average value of $\gamma(h)$ when the tail of vector $\vec{h}$ is constrained to be at sample $i$, and the head allowed to move around within the polygon.
Determination of the Visman sampling constants (see subsection 3.4.1)

The Visman sampling constants are determined by analysing two series of samples of individual weights such that $Z$, roughly calculated from the following equation, is at least equal to 3, but preferably somewhere between 6 and 10. The samples should be taken randomly from the mass to be sampled:

$$Z = \frac{(K - L)^2}{S^2}$$

where $K$ = true overall content of the element of interest, $X$, in the mixture of two minerals (ore + gangue);

$L$ = $X$-content of the major component of the two mineral mixture; often the $X$-content of gangue;

$S^2_{1,2}$ = variance of sets of results using samples of weights $w_1, 2$.

The Visman sampling constants are then given by:

$$A = w_1 w_2 \left( S^2_1 - S^2_2 \right) / (w_2 - w_1)$$

$$B = S^2_2 - A / w_2$$
Standard deviation of a normally distributed sample population
(see subsection 3.5.3)

\[ S = \sqrt{\frac{1}{n-1} \left( \frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2 \right)} \]

where
- \( S \) = standard deviation
- \( n \) = number of samples
- \( x_i \) = individual values of samples
- \( \bar{x} \) = sample mean


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