Exploring National Certificate Vocational (NCV) level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning

By

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DECLARATION

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DECLARATION:

In accordance with Rule G5.6.3, I hereby declare that the above-mentioned treatise/ dissertation/ thesis is my own work and that it has not previously been submitted for assessment to another University or for another qualification.

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DEDICATIONS

I would like to thank the Lord, Jesus Christ for giving me the ability, the strength, and the courage to pursue and complete this study.

I dedicate this dissertation to my father, **Falakhe Antony Ngwabe**.

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LIST OF ABBREVIATIONS

NCV :	National Certificate Vocational
TVET :	Technical Vocational Education and Training
ZPD :	Zone of Proximal Development
GDS :	GeoGebra Dynamic Software
ICT :	Information and Communications Technology

ABSTRACT

This study focuses on exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra Dynamic Software during teaching and learning. The research investigates how the integration of GeoGebra during teaching and learning algebraic functions influenced learners' misconceptions in algebraic functions. Vygotsky's sociocultural view of learning underpinned the process of teaching and learning during the study. The research was carried out at the TVET College in Port Elizabeth. The data was collected by means of pre-test and post-test, focus group interviews, GeoGebra intervention and observations. Created GeoGebra applets and a worksheet was used during the integration process. The data collected was analyzed and used to answer the research questions of this study. Research findings showed that the integration of GeoGebra during teaching and learning enhanced learners' conceptual understanding in algebraic functions. There was a significance increase in the number of learners who showed ability to interpret algebraic functions based concepts after the engagement with GeoGebra applets.

KEY WORDS*: Misconceptions, algebraic functions and GeoGebra software.*

CHAPTER 1: INTRODUCTION

1.1. RESEARCH CONTEXT

In general, learners' mathematics performance in South Africa has been regarded as very poor (Mji & Makgato, 2006; Graven, 2013). Several factors influence the poor mathematics performance of learners. Such factors include: limited alternative teaching strategies, and the insuffient understanding of subject content, concepts and knowledge by learners (Mji & Makgato, 2006). Literature has shown that teaching and learning mathematics in the Zone of Proximal Development where the child's learning is mediated by technological tools and scaffolded by an educator or a competent peer results in a learning process that is more meaningful, managable and effective (Fani & Ghaemi, 2011; Denhere, Chinyoka & Mambeu, 2013; Siyepu, 2013). In previous studies, it has been confirmed that integrating technological tools, specifically GeoGebra software, makes teaching and learning mathematics more effective and efficient (Yu-Wen, 2008; Preiner, 2008; Hohenwarter & Fuchs, 2004).

This study focused on integrating GeoGebra, a technological dynamic software, during teaching and learning of algebraic functions to support NCV level two learners to deal with misconceptions. GeoGebra is a dynamic mathematics software that was designed for teaching and learning algebra and geometry in secondary school up to the tertiary level (Preiner, 2008). Learners understand mathematics concepts much better when they can see how objects are related and changed dynamically and GeoGebra software is capable of facilitating such teaching and learning process (Yu-Wen, 2008).

According to Hohenwarter & Fuchs (2004), GeoGebra was specifically designed for learning purposes and can assist students to foster their mathematical learning. It provides a wide range of mathematics concepts that are dynamic and thus more accessible to pupils (Chrysanthou, 2008). The purpose of integrating GeoGebra in this study was to assist learners to develop mathematical thinking and conceptual understanding around algebraic functions. The study used GeoGebra as a dynamic, conceptual, problem solving and cognitive tool during teaching and learning algebraic functions. Therefore, the drawing of function graphs using software GeoGebra enabled learners to observe how graphs are constructed, moved dynamically and the relation between the intercepts (Bu Lingguo & Schoen, 2011). The process of teaching and learning mathematics and developing algebraic concepts through integrating GeoGebra is based on Vygotsky's sociocultural theory. .

1.2. STATEMENT OF THE PROBLEM

I have been teaching mathematics level two learners in the National Certificate (Vocational) also known as the NCV program for three years at a Technical Vocational Education and Training (TVET) College in Port Elizabeth. The NCV program was introduced in 2007 and is offered at NQF levels 2, 3 and 4 in the public TVET colleges and in a few private colleges (SETA, 2013/14).

Having taught mathematics for three years, I have realized that very few of my NCV level two learners pass the subject and that they do not perform very well in algebraic based assessments. It seems that learners experience misconceptions with algebraic functions and therefore, they cannot understand the basic concepts in algebraic functions. This prevented them from effectively progressing and mastering the concepts of algebraic functions.

As a TVET teacher, I have always been teaching algebraic functions given as linear $f(x) = ax + q$, parabolic $f(x) = ax^2 + q$ and hyperbolic $f(x) = \frac{a}{x} + q$ by sketching them on the white board. I would explain to learners the constant q as the y-intercept where the graph cuts the y-axis and also as the vertical shift of the graph. Furthermore, parameter a in a linear function is the value that represents the slope, in a parabolic function parameter a represents the shape of the graph and in a hyperbolic function parameter a determines which quadrants the graph will occupy. I would also explain how the functions can shift horizontally and vertically for each function. All that would take a lot of time to explain and some learners would still struggle to understand the concepts of algebraic functions.

Therefore, I decided to explore the integration of GeoGebra software during teaching and learning algebraic functions as the literature has confirmed the success of technology based teaching and learning methods in mathematics (e.g. Chai, Koh, Tsai, 2013; Lowerison, Sclater, Schmid & Abrami, 2004; Preiner, 2008). The integration of GeoGebra software in teaching and learning algebraic functions has been successfully implemented in previous studies (Yu-Wen, 2008; Preiner, 2008; Hohenwarter, Fuchs, 2004). Therefore, it seemed viable to explore its potentilal to support conceptual development in algebraic functions.

1.3. IMPORTANCE OF THE STUDY

The need for integration of GeoGebra software in the mathematics classroom is increasing rapidly (Dikovic, 2009). Mathematics is one of the subjects that are in high demand in the country. Learners are expected to pass the subject in order for them to further their studies. Yet, the majority of NCV level two learners are not performing well in mathematics. The high failure rate in NCV level two mathematics challenges teachers to develop strategies to deal with this crisis. Therefore, it was important for this study to explore the potential of GeoGebra software to address NCV level two learners' misconceptions in algebraic functions.

The South African Education system encourages the development of the Information and Communications Technology (ICT) skills in schools to prepare pupils to be able to compete anywhere in the world. The South African Government further encourages educators to integrate technological resources during teaching and learning to enhance learning skills in the curriculum (Chigona & Davids, 2014). Integration of technology in education ensures that learners become exposed to technology which might help to develop skills that will be of use in the workplace. As a result, the Department of Education in South Africa had an initiative of distributing laptops and computers in schools for both teachers and learners to utilize effectively during teaching and learning processes.

Nowadays, learners are exposed to technologies. They explore the world through their technological devices. Learners own smart phones, iPads, TVs, computers, laptops, cameras, and many more devices. Therefore, integrating GeoGebra in a mathematics classroom, is an attempt to meet the needs of a 21st century learner.

GeoGebra software is capable of clearly demonstrating to students the dynamic graphical changes and is becoming a recognized part of mathematical knowledge (Bu Lingguo & Schoen, 2011; Hohenwarter and Jones, 2007). The utilization of GeoGebra can enable learners to engage in the potential that it brings, which includes observing dynamical movements, viewing patterns and making connections (Yu-Wen, 2008). In addition, the technology-based teaching and learning method, helps learners to be actively engaged and to improve conceptual knowledge that enables them to solve and construct algebraic equations and graphs (An & Reigeluth, 2011).

1.4. AIM OF THE STUDY

This study aims to determine how the integration of GeoGebra software during teaching and learning supports NCV level two learners to deal with misconceptions in algebraic functions.

1.5. RESEARCH QUESTIONS

This research aims to address the main question in 1.5.1. However, two further sub questions are posed in 1.5.2 to provide further insights into the main question.

1.5.1. Main research question

How does integrating GeoGebra software during the teaching and learning of algebraic functions support NCV level two learners in dealing with misconceptions?

1.5.2. Sub research questions

Sub-questions which assist in deriving answers to the main research question are as follows:

- What are NCV Level two learners' misconceptions in algebraic functions?
- In what way does engagement with GeoGebra support learners to develop a conceptual understanding in algebraic functions?

1.6. RESEARCH METHODOLOGY

This section provides a brief but more comprehensive discussion on the research methodology, data collection and analysis used in this study. The researcher explains the research instruments, data collection and analysis procedures that were followed in the study.

1.6.1. Research approach

The research method employed in this study is the mixed method approach. It is defined as the method of combining two or more theories, data collection sources, methods or investigators in one study of a single phenomenon (Yeasmin & Rahman, 2012). Both qualitative and quantitative research methods are used in the study to obtain valid and reliable data

1.6.2. Data collection

The research instruments used to collect data in this study include pre-test and post-test worksheets, GeoGebra integration worksheets, focus group interviews and observations. The structure of the worksheets and leading questions during focus group interviews allowed participants to express themselves effectively. This increased the validity and significance of results.

1.6.3. Data analysis

Data analysis was designed to provide a clear understanding of learners' misconceptions in algebraic functions and of how GeoGebra influenced learners' conceptual understanding while learning algebraic functions. The data obtained from the pre-test and post-test is analyzed with the help of tables and bar graphs. The focus group interviews were audio recorded and transcribed. In order to refine the actual study' research instruments, the researcher conducted a pilot study.

1.7. OUTLINE

Chapter 1 is the introduction of the study. It provides information about the background, the problem statement, the importance of the study, the research aim and the research questions. It provides a brief overview of the research methodology used, in data collection and analysis.

Chapter 2 presents the literature review and theoretical framework of the study. It provides an overview of the literature on misconceptions in mathematics and the role of GeoGebra software in teaching and learning algebraic functions. This chapter also gives insights on how Vygotsky's sociocultural theory underpins the process of teaching and learning.

Chapter 3 reports the research methods used to conduct this study. It gives details about the procedures followed to conduct the research which include the methods used to collect the data and how the data was analysed. Furthermore, it explains how the validity and reliability of the study were maintained and the considerations that were taken into account.

Chapter 4 presents the data and how it was collected and analysed.

Chapter 5 presents a discussion of the data and concludes the study. It also give some limitations experienced during the study and recommendations to future research studies.

CHAPTER TWO: LITERATURE REVIEW

2.1. INTRODUCTION

This chapter begins by reviewing literature based on algebraic functions in the NCV curriculum followed by a discussion of learners' mathematical misconceptions in algebraic functions. The literature also focuses on Vygotsky's' Sociocultural theoretical framework which underpins the teaching and learning of algebraic functions as discussed in this context. The focus will be mainly on the Zone of Proximal Development (ZPD) and specifically on scaffolding and mediation as teaching and learning strategies. Finally, the reviewed literature focuses on the role of GeoGebra as a dynamic modelling tool, problem solving and conceptual tool and cognitive tool.

2.2. ALGEBRAIC FUNCTIONS IN NCV CURRICULUM

The research has shown that the concept of function is regarded as very important in the mathematics curriculum, yet it has been discovered that learners struggle to plot points, analyze or interpret functions (Yuksel, 2006; Bush, 2011). Kalchman & Koedinger (2005, p.352) define function as a "set of ordered pairs of numbers (x, y) such that to each value of the first variable (x) there corresponds a unique value of the second variable (y)". Bush (2011, p.92) stated that:

> *Determining outputs from given inputs is a basic requirement knowledge for algebra while exploring functions using formulas, exploring rates of change of different functions, analyzing and comparing graphical presentations is proficient requirement knowledge for algebra.*

According to NCV level two mathematics guidelines (Appendix 10), a variety of techniques such as integration of technological tools should be used to sketch and interpret information from the graphs. Learners are expected to understand, interpret and analyze the effects of parameters a and q from graphical presentations of algebraic functions.

The following figure 1 shows NCV level two subjects outcomes based on functions and algebra.

Topic 2: Functions and Algebra.

Subject Outcome 2.1: Use a variety of techniques to sketch and interpret information from graphs of algebraic and transcendental functions.

Learning Outcomes

Students are able to:

- Generate graphs by means of point-by-point plotting supported by available technology.
- Use the generated graphs to make and test conjectures.
- Generalise the effects of the parameters a and q on the generated graphs of functions including the following:

$y=ax+q$

$y=ax^2+q$

$y=\frac{a}{x}+q$

$y=ab^x+q$, $b>0$

$y=a\sin x+q$

$y=a\cos x+q$

$y=a\tan x+q$

Figure 1: Topic 2 NCV level 2 subject guidelines (Appendix 10, p.5).

Usually learners learn mathematical concepts without understanding because their teachers use the same conceptual instruction that they used to master concepts when they were learners themselves (Makonye, 2014). Such teaching and learning is said to be traditional, where the teachers' goal is to find solutions to problems by applying formulae or using theorems without learners understanding mathematical concepts (Makonye, 2014).

NCV learners begin learning functions by substituting independent variables into a given function to calculate dependent variables and plot algebraic functions. Learners tend to memorize the whole procedure without understanding the basic algebraic function concepts such as equality, the effect of parameters a and q, and functional relationships. In addition, learners need to be actively engaged with algebraic function concepts being taught in order for mathematics learning and understanding to take place. Learners can be actively engaged through practical applications of mathematics that will help them to observe critically, to reason, compare, make and test conjectures, reflect, analyze, interpret and make connections between mathematical ideas (Uddin, 2011). Learning algebraic functions in a traditional way limits learners from understanding the connections between equations and their graphs, the effects of variables, interpreting, analyzing, reasoning, making applications and problem solving (Farmaki, Klaoudatos, & Verikios, 2013). Hence, the potential of GeoGebra as a tool to help learners overcome their misconceptions during the teaching and learning algebraic functions.

2.3. DEFINING MISCONCEPTIONS IN MATHEMATICS

The nature of mathematics requires active participation where learners make practical applications by making sense of the concepts learnt in class (Birhanu, 2010).

According to Godino (2015, p.3):

> *Mathematics is a human activity involving the solution of problematic situations. In finding the responses or solutions to these problems, mathematical objects progressively emerge and evolve. Mathematics is a symbolic language in which problem-situations and the solutions found are expressed.*

Mathematics can be referred to as an informally shared, yet reasonably structured conceptual system of concepts and mathematical procedures developed from ones' acts for solving some problem fields (Godino, 2015). Mathematical misconceptions occur when learners fail to make connection to what they already know (Hjh Roselizawati & Masitah, 2014) or when learners make use of the previously learned principles, rules, strategies or theories incorrectly when solving a new problem (Russell, O'Dwyer, & Helena, 2009). Bush (2011) compares the notion of conceptual mathematical understanding and procedural knowledge. She defines conceptual mathematics understanding as the ability to generalize, understand and connect mathematical ideas, while procedural knowledge is defined as procedures and skills that learners should apply methodically to solve mathematical problems. Therefore, misconceptions mostly occur when learners fail to make connections between mathematical fields of knowledge or when learners overgeneralize (Bush, 2011).

According to Egodawatte (2011) conceptions are students' beliefs, theories, explanations and meanings. Misconceptions occur when those conceptions are in conflict with the accepted theories, beliefs, and explanations in mathematics. Moschkovich (1998) claims that learners frequently develop conceptions-coherent, firm, and robust ideas that are different from professional conceptions in mathematics, which then become misconceptions and interfere with the learning of mathematical concepts. Olivier (1989) differentiates between errors and systematic misconceptions, regarding errors as wrong answers due to planning and regarding systematic misconceptions as errors that can occur repeatedly in the same circumstances over a period of time.

Dlamini (2014) concurred that learners show mistakes and challenges in their mathematical problem solving tasks; these mistakes result in systematic and unsystematic misconceptions.

2.3.1. Systematic misconceptions

Systematic misconceptions refer to the application of previously learned strategies in new situations where they do not apply (Ojose, 2015); for example, when learners believe that the gradient (a) in a linear equation $y = ax + q$ represents the x-intercept because parameter q represents the y-intercept. Luneta & Makonye (2010) argued that systematic misconceptions are false concepts assumed to form new concepts; for example, when a learner assumes that the changes in a vertical shift (q) of a particular algebraic graph affects the value of parameter a. This is an example of a conceptual error and is considered to be a systematic misconception (Luneta & Makonye, 2010).

2.3.2. Unsystematic misconceptions

Unsystematic misconceptions are displayed unintentionally and learners do not repeat these misconceptions; instead, they correct them (Olivier, 1989). Lack of strategic competence appears in unsystematic misconceptions (Ojose, 2015). These are slips, lapses or unintended errors (Luneta & Makonye, 2010) for example assuming that a parabola graph of $y = x^2 - 4$ has the same graphical representation as $y = x^2 + 4$.

2.4. LEANERS' MISCONCEPTIONS IN ALGEBRAIC FUNCTIONS

Based on NCV mathematics guidelines (Appendix 10), learners are not only expected to understand how to construct and interpret the graphs, they are also expected to make meanings or connections between the graphs. That is the ability to relate graphs with their equations and vice versa. Learners should also understand the role and significance of each and every feature presented by the graph and equation. Generally, in algebra, graphs and equations have the same meaning but differ in presentation: equations are symbolic representations of the relationship between the equation and the graph while graphs are visual representation of the same relationship (Knuth, 2000). Algebraic misconceptions amongst learners could result in learners' struggling to make sense of function transformations, i.e., struggling to understand the relationship between equations and their graphs (Makonye, 2014).

According to (Glenda & Walshaw, 2009), learners need to be able to connect a new concept or skill to their existing mathematical understandings. For learners to make such a connection, teachers need to support them by emphasizing links between mathematical ideas and make those ideas accessible by gradually introducing adjustments that build on learners' understandings. Glenda & Walshaw (2009, p.15), further claim that:

> *Different mathematical patterns and principles can be highlighted by changing the details in a problem set; for example, a sequence of equations, such as y =2 x + 3, y =2 x + 2, y =2 x and y = x + 3, will encourage students to make and test conjectures about the position and slope of the related lines.*

Learners are expected to develop conceptual understanding in algebraic functions through making connections between mathematical ideas. In Glenda & Walshaw's (2009) quote above, about making and testing conjectures about the position and slope, the first three equations are based on the effect of the y intercept q in the linear equation $y = ax + q$. As much as these equations could be used to encourage learners to make and test conjectures, they could also be used as an important item to identify learners' misconceptions (Christou & Elia, 2004) related to the slope (a) and the y intercept (q). For example, learners could be given a set of linear equations to match them to a correct graph, where the value of the y intercept or slope differs. Therefore, learners' misconceptions concerning sketching of a linear function, gradient (a) and y-intercept (q) can be easily identified when learners are matching multiple equations to their graphs and vise versa (Long, 2005). This will be the basis of the test items used in the pre-test worksheet of this study.

Additionally, activities that involve multiple representations for learners to make multiple connections within and across topics help learners develop conceptual understanding and computational flexibility (Glenda & Walshaw, 2009; Christou & Elia, 2004; Long, 2005). Algebraic conceptual knowledge is not only about identifying the symbols or carrying out an operation during problem solving but it is more about understanding the function of each feature in the equation and how shifting the location of the feature affects the whole equation or expression (Anne, 2007; Koedinger & Booth, 2008). Furthermore, literature has confirmed the importance of integrating a range of representations and tools to support learners' mathematic conceptual development; such can include technological tools like GeoGebra dynamic software (Linggou & Robert, 2011; Glenda & Walshaw, 2009; Hohenwarter & Fuchs, 2004).

2.4.1. Misconceptions under the effects of x intercept(s) and $a\&q$ parameters

NCV learners are expected to develop conceptual knowledge concerning x intercept, parameter a and parameter q or y intercept of algebraic functions given as linear $y = ax + q$, parabola $y = ax^2 + q$ and hyperbola $y = \frac{a}{x} + q$ (Appendix 10). Understanding the transformation and construction of parameter a, cognitively challenges learners' capability than understanding the transformation and construction of parameter q (Hattikudur, Prather, Asquith, Alibali, Knuth & Nathan, 2012). For example, when learners are given a linear graph to determining parameters a and q ($y = ax + q$) as given in the figure 2 below.

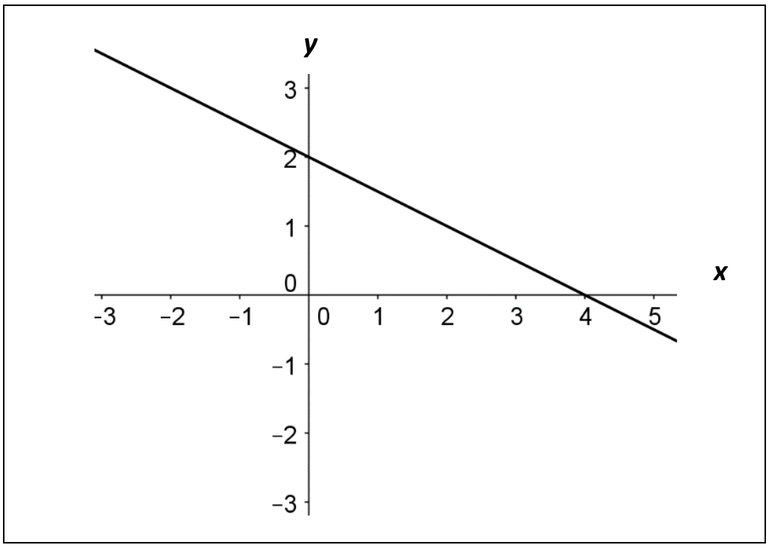


Figure 2: Graphical representation of a linear function.

Usually, learners find it challenging to identify the value of a because it requires that a learner must be able to track points and the nature of their relationship, while on the other hand finding q involves identifying one point where the line cuts the y axis (Hattikudur, et al., 2012). For example, because the y intercept $q = 2$ can be seen easily as it is visual in the graph in figure 2, learners are likely to assume that the value of parameter $a = 4$ because according to Hattikudur et al. (2012) they will confuse the gradient with the x intercept.

With GeoGebra software, learners can easily identify x intercepts, parameters a and q and dynamically transform a function through regulating parameters a and q (Dikovic, 2009). Using a linear function $y = ax + q$ as an example, learners are usually looking for a slope (a), y intercept (q) and x intercept to generate the equation (Moschkovich, 1998). Some learners would find it difficult to imagine how the y intercept q changes if the line moves left and right while others would expect the x intercept to appear in the equation as it can be readable from the graphical presentation. Moschkovich, (1998) conducted a study based on learners' misconceptions regarding the x intercept from a linear function $y = ax + q$. Learners believed that x intercept should appear in the equation $y = ax + q$, like parameters a and q. From his study, learners showed misconceptions when generating x intercept. Most learners identified x intercept as values of a or q and others would and believed that the x intercept should reflect on the $y = ax + q$ equation.

2.4.2. Misconceptions on function transformations

According to NCV subject guidelines (Appendix 10), learners should be able construct and make transformations between a single function by changing values of parameters a and q from equations $y = ax + q, y = ax^2 + q$ and $y = \frac{a}{x} + q$. Graphical representations of functions are essential in learners' mathematics education. However, learners experience misconceptions in understanding of graphs (Hattikudur, et al., 2012). For example, Postelnicu (2011) reviewed literature on a study that was conducted amongst the grade 8 and grade 9 learners who enrolled in algebra. The purpose of the study was to gain insights into learners' difficulties concerning linear functions. Postelnicu (2011) found that students failed to connect the sign of the slope, positive, with the graphed line representing an increasing function. Furthermore, learners calculated a negative slope, although the linear function is increasing. In addition, learners experience difficulties in understanding function transformations, mainly in the visualization of transformations (Uddin, 2011). Identifying, extracting and conveying information with graphs is often challenging for learners (Hattikudur, et al., 2012).

The research has shown that learners' ability to construct graphs by a traditional method (i.e. to create a graph by hand, developing a table with intercepts) has an influence on learners' misconceptions in understanding of graphical construction.

Learners find such graph construction difficult as it is highly possible to confuse the values on the table (using x values as y values) and confusing parameters a and q (Hattikudur, et al., 2012). Technological programs and software such as GeoGebra promote an understanding of function constructing and transformation (Bhesh, 2014).

2.5. THEORETICAL FRAMEWORK

This study aimed to integrate GeoGebra software during learning and teaching of algebraic equations and graphs to support NCV learners to develop deeper understanding of algebraic concepts. The following theoretical framework supports deeper stages of thinking around concepts and learning generally and specifically learning mathematics through integrating GeoGebra software.

2.5.1. Sociocultural theory

The study used sociocultural theory as the lens to understand the concept of cognitive learning during the integration of GeoGebra dynamic software during teaching and learning algebraic functions. Sociocultural theory is best known through the work of Vygotsky during the 20th century.

Goos (2008) defines Vygotsky's theoretical approach as the mediation of social origins of higher mental functions through signs and tools, which can include language, writing, methods for counting and calculating, diagrams, algebraic symbol systems and many more. Sociocultural theories define learning as a process where there is an interaction between people and materials or tools offered by the learning environment (Goos, 2009). A sociocultural approach forms a basis of the integration of GeoGebra dynamic software during teaching and learning algebraic functions, as the learners are working in collaboration using GeoGebra as a social tool to scaffold the learning process.

Vygotsky's sociocultural theory of human learning describes learning as a social process and the origination of human intelligence in society (Vygotsky,1978). Vygotsky believed that learners have their own mathematical knowledge, beliefs and understanding from their experiences; however, it is the competent adult's role to assist learners' thinking to understand mathematical concepts (Nelson, 2002).

Technology provides new and innovative methods to create social learning environments and the use of technological tools in the sociocultural aspect is that they support active learning and engagement among learners (Koc, 2005). GeoGebra based teaching facilitates social learning between learners as they learn from one another in the involvement of technological tools.

Further, the sociocultural approach argues that knowledge cannot be injected into the students 'minds in order to get learners to know mathematical concepts. The teachers' role is to provide learners with rich classroom activities in a suitable manner (Radford, 2008). In this study, the researcher integrated GeoGebra into a mathematics classroom to support the development of mathematical concepts. In other words, according to sociocultural theory, learners learn mathematical concepts more proficiently using socially proper skills and technologies. In the NCV classroom, GeoGebra software was used as an appropriate technological resource to assist in the development of algebraic concepts.

2.5.2. Zone of Proximal Development (ZPD)

Vygotsky defined a concept of zone of proximal development (ZPD) as the distance between the level of actual development and the level of potential development (Wood, Bruner, & Ross, 1976). The level of actual development is determined by the independent problem solving and the level of potential development is determined through problem solving under guidance by a teacher or working together with a more capable learner (Shabani, Khatib & Ebadi, 2010).

The ZPD allows the teacher to be in control of the concept for the learner until the learner is able to internalize the external knowledge (Cottrill, 2003). Vygotsky understood the ZPD to describe the actual level or performance of development of the learner and also the next level through the use of environmental tools and the competent adult or peer person (Siyepu, 2013; Shabani, Khatib, & Ebadi,2010). A learner learns better when working together with others in collaboration. Such collaboration work involves competent individuals, to assist learners to learn and internalize new concepts, skills and psychological tools (Shabani, Khatib, & Ebadi, 2010). In this study, GeoGebra software was used as a scaffolding tool to assist and guide NCV learners to develop algebraic concepts.

Scaffolding refers to the guidance provided for one to reach the ZPD (Shadaan & Kwan, 2013). Therefore, the researcher introduced the GeoGebra tool and allowed learners to be actively engaged with GeoGebra, which helped them to internalize algebraic concepts. The researchers' role was not replaced by the GeoGebra tool. The researcher continued to facilitate learning by encouraging learners to be actively engaged during the lesson. ZPD was useful in describing the difference between what NCV level two learners could achieve prior the integration of GeoGebra tool and what they could achieve independently after integrating GeoGebra. The following figure 3 indicates and describes the four stages in the Zone of Proximal Development.

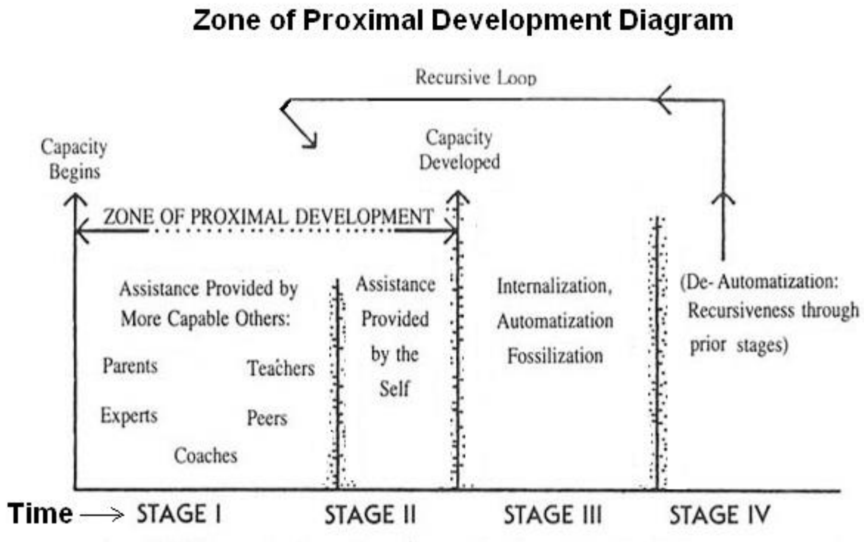


Figure 3: Four stages of proximal development. (Siyepu, 2013, p. 5)

There are four stages of the Zone of Proximal Development (Siyepu, 2013). Capacity initiates at stage I where assistance is provided by more competent individuals. The competent individuals can include teachers, parents or peers. NCV learners who seem to be more skilled than others assisted their peers with information based on the transformation of algebraic functions, which means that those who understand will assist those who find it difficult to understand. Shadaan & Kwan (2013, p. 3) stated that social interaction between peers benefit both higher ability or skilled leaners and less skilled learners. The highly skilled learners assist their peers in the Zone of Proximal Development, while the highly skilled learners benefit through the new ideas and views of their peers.

Consistent practice and assistance allow learners to move on to stage II, where they can provide their own assistance. To help with this process means providing learners with a tool or clear instructions that provide a systematic description of how to properly perform the required skill (Siyepu, 2013).

During stage III, learners grow in knowledge through practice. It is a stage where learners no longer need assistance. The action is internalized, and no longer requires extra effort. Cognitive learning can become complex, such as when there is new requirement or unfamiliar contexts. This is where the learner loops back (shows de-automatization) to the beginning and moves through all the stages again (Vygotsky, 1978). In this study, ZDP is explained in-depth considering the two interrelated aspects of mediation and scaffolding.

2.5.3. Mediation

Turuk (2008, p. 250) states that “Mediation according to Vygotsky refers to the part played by other significant people in the learners’ lives, people who enhance their learning by selecting and shaping the learning experiences presented to them”. Mediation involves interaction between two or more people who vary in skills and the level of knowledge (Denhere, Chinyoka, & Mambeu, 2013). For example, this involves an interaction between a learner and a teacher or a competent peer during the teaching and learning process. Mediation could result in effective learning when learning tools are involved as a support to enhance the learning process (Thompson, 2013). Furthermore, teachers should provide a range of social interaction through utilizing mediating tools to help scaffold learners’ learning development (Thompson, 2013). GeoGebra is also used as a mediation tool because of its potential to enhance the process of learning concepts in algebraic functions.

2.5.4. Scaffolding

The Zone of Proximal Development is where scaffolding is provided. Scaffolding refers to guidance or support offered by an adult or more competent peer to enable a learner to work within the ZPD (Denhere, Chinyoka, & Mambeu, 2013). Scaffolding involves effective support from a teacher to learners. Typically, the teacher guides learners by asking systematic questions that lead them to the solution. A teacher should begin at the learner’s level of knowledge and build from there in order for a scaffolding to be effective (Turuk, 2008). Learners that complete cognitive challenging tasks with scaffolding support from a teacher have an advantage to grow in knowledge. Such knowledge could help learners to tackle advanced tasks even when the scaffold has been removed.

2.6. GEOGEBRA DYNAMIC SOFTWARE

GeoGebra is an advanced technology, free of charge and accessible, that can be downloaded freely from http://www.geogebra.org/cms/ (Hohenwarter & Fuchs, 2004). It is an open, free source or a technological tool that requires no licensing and which provides opportunity for both teachers and learners to utilize it in the classroom and at home without being charged (Escuder & Furner, 2012).

GeoGebra does not require any user to be a master in computer literacy; it is suitable for anyone and designed to be utilized at all levels of mathematics education (Preiner, 2008). According to Hohenwarter & Fuchs (2004), GeoGebra is specifically designed for learning purposes and can assist students to foster their mathematical learning and it provides a wide range of mathematics concepts that are dynamic and thus more accessible to pupils (Chrysanthou, 2008).

Goos, Galbraith, Renshaw, & Geiger (2003), declared that a rapid increase of technologies that todays' learners' find themselves utilizing, such as smart phones, tablets and computers could be used to help students to communicate and analyze their mathematical thinking.

GeoGebra can be used to support the development of mental models suitable for solving mathematical complex problems (Linggou & Robert, 2011). Furthermore, GeoGebra is a dynamic software, for it incorporates various dynamic representations, various domains of mathematics, and a rich variety of computational utilities for demonstrating and simulations (Linggou & Robert, 2011). It consists of various valuable features that accommodates solving both algebra, geometry and calculus problems within mathematics.

In addition, GeoGebra provides a range of digital resources that allow learners to mathematize realistic problem situations, create and test conjectures with meaningful models using several demonstrations and modelling tools; and further proceed to formulate increasingly abstract mathematical ideas (Linggou & Robert, 2011).

Algebraic functions, equations and co-ordinates can be entered directly, defined algebraically and changed dynamically using the GeoGebra dynamic algebraic system (Escuder & Furner, 2012). Furthermore, GeoGebra has the ability to dynamically indicate values of gradients, turning points, roots, points and reflect functions using a computer algebra system (CAS) found in the top of the main menu window (Lavicza, Hohenwarter, Jones, Lu, & Dawes, 2009; Escuder & Furner, 2012). GeoGebra also offers a spreadsheet view that enables teachers and learners to enter data of a particular function while viewing its graphical representation in the graphic and algebra views (Escuder & Furner, 2012).

In the light of the above, mathematics functions can be effectively learned using GeoGebra views of algebra and graphs. The figure 4 below indicates a GeoGebra illustration showing a menu bar, tool bar, undo/redo input bar, algebra and graphic views.

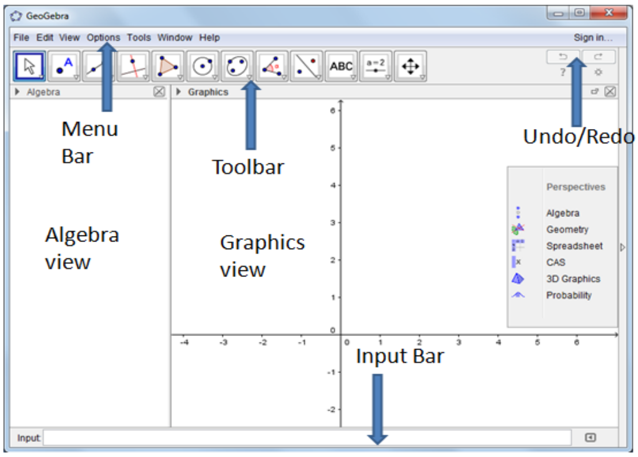


Figure 4: Screenshot from GeoGebra window.

A GeoGebra graphic view dynamically indicates a graphical presentation of a function while the algebra view indicates the equation of the same function. GeoGebra dynamic software helps both learners and teachers to make changes of parameters a, q, x and y intercepts graphically that simultaneously change the equation from the algebra view.

The drawing of function graphs using software GeoGebra saves time and therefore creates space for other activities in class (Kotrikova, 2012). Furthermore, Hohenwarter & Fuchs (2004) claim that GeoGebra software can be used for multipurpose such as demonstration and visualization. It can be used as a construction tool where function graphs are created. Students can use GeoGebra to discover information and organize knowledge on their own. GeoGebra can be used for teacher preparation of lessons as a cooperation and communication tool (Hohenwarter & Fuchs, 2004).

Benefits of using technological resources such as GeoGebra during teaching and learning mathematics can help in learning mathematical procedures and skills and improvement of mathematical abilities such as problem solving, justifying and reasoning (Gadanidis & Geiger, 2010). Technology in the mathematics classroom enables fast, accurate computation, collection and analyzing of data and the demonstration of multiple forms such as numerical, symbolic or graphical forms (Goos, Galbraith, Renshaw, & Geiger, 2003). GeoGebra software can be used as modelling, conceptual, and cognitive tools to enhance mathematics concepts and to solve mathematically based problems (Hohenwarter & Fuchs, 2004; Linggou & Robert, 2011).

2.6.1. GeoGebra as a dynamic modelling tool

Dejene, (2014) defines the term dynamic as the process of action or motion; he further claims that such a dynamic modelling tool provides an active and energetic teaching and learning process in a mathematics classroom. GeoGebra, as a dynamic modelling tool, enables teachers and learners to demonstrate and visualize algebraic objects (Dejene, 2014). Teachers can use a GeoGebra graphic window to demonstrate how the values of parameters a and q from algebraic equations change and what effects the changes have on the shape of the graphs.

2.6.2. GeoGebra as a problem solving and conceptual tool

A mathematical conceptual tool provides the ability to understand mathematical concepts, ideas and enhances mathematical proficiency (Mehdiyev, 2009; Dejene, 2014). At the same time, problem solving refers to the ability to identify mathematical problems and develop possible multiple solution techniques to solve them (Dejene, 2014). GeoGebra provides learners with the ability to construct mathematical models of a problem and help learners to develop problem-solving skills (Furner & Marianas, 2013).

Furthermore, GeoGebra as problem solving and conceptual tool, provides more concrete and visual approaches of learning mathematics and it also helps to minimize memorizing the algebraic operations and formulas and provides more conceptual understanding of mathematics (Mehdiyev, 2009; Bhesh, 2014). In addition, Furner & Marianas, (2013) concur that GeoGebra provides learners with the ability to visualize mathematical problems and deveolop techniques to solve algebra based problems.

2.6.3. GeoGebra as a cognitive tool

In the context of using GeoGebra as a cognitive tool, Velichova (2011, p. 108) explains that “cognitive tools allow users to explore mathematical concepts dynamically”. Hirono & Takahashi (2011, p.1), define a cognitive tool as a “tool for embodying the image of an outer object that appears in the consciousness on the basis of the human perception”. This means that a cognitive tool must be a tool that offers relevant support to a particular activity in a mathematics classroom. Further, Hirono & Takahashi (2011) claim that when using GeoGebra as a cognitive tool in mathematics, teachers should develop activities that will enable learners to observe, construct, solve and critically reflect. For example, in this study, learners observed teacher’s demostrations of algebraic functions using sliders from the GeoGebra applet; which enabled learners to solve and construct mathematics problems. In addition, teachers can use GeoGebra as a cognitive tool to clarify, explore and to model mathematics concepts and to determine the connections between these concepts (Linggou & Robert, 2011 & Velichova, 2011).

2.7. PEDAGOGICAL FRAMEWORK FOR GEOGEBRA INTERVENTION

A Pedagogical framework focuses on integrating technology in a mathematics classroom (Lavicza, Hohenwarter, Jones, Lu & Dawes, 2009). According to the pedagogical framework, teachers originally used technology, specifically GeoGebra, as an instruction tool. Thereafter, teachers learnt how to utilize GeoGebra software effectively during teaching and learning in a mathematics classroom. Once teachers became familiar with utilizing GeoGebra software in the classroom, they allowed learners to interact more directly with the software. (Prodromou & Lavicza, 2015).

Figure 5 below indicates the consecutive phases of pedagogical approaches with GeoGebra.

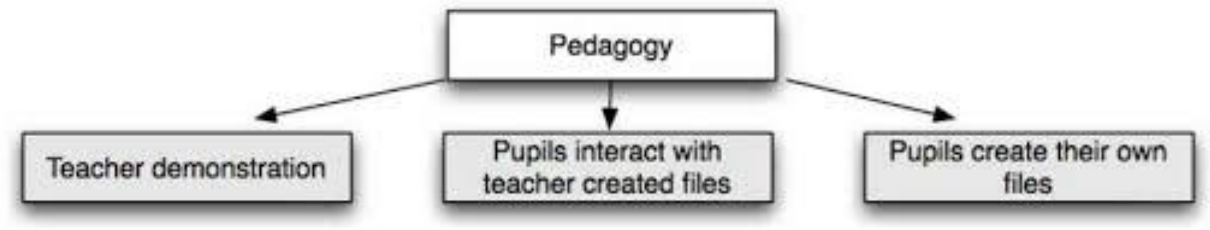


Figure 5: Pedagogical approaches with GeoGebra. *(Lavicza, Hohenwarter, Jones, Lu & Dawes, 2009, p. 176)*.

Lavicza, Hohenwarter, Jones, Lu, & Dawes (2009) proposed a theoritical model that involves three phases, which are: a teacher demonstration phase, learners interact with teacher-created files and pupils create their own files. The first phase, the teacher demonstration phase, allows teachers who have litle experience in using technology to experiment with technologies which have small risks where fewer technological resources are used. In the second phase, teachers became more comfortable and familiar with the utilization of technologies and therefore a technology based lesson was provided for learners to work on. Learners also became familiar with using GeoGebra software. In the third phase, learners' roles shift from being knowlegde transmitters to becoming more like facilitators. They developed their own solution for the task and discussed their ideas. Therefore, this study follows the pedagogical framework phases during a GeoGebra intevention.

2.8. THE ROLE OF GEOGEBRA APPLETS IN THIS STUDY

Uddin (2011, p. 18) defines applets as "visual representations that are used as models for mathematical concepts within which learners can work on the basis of their own ideas and experiment freely". GeoGebra applets can improve the understanding of mathemathical concepts obtained during graphical presentations enabling learners to dynamically visualize graphical transformations (Morphett, Gunn, & Maillardet, 2015).

Figure 6 below shows an example of a GeoGebra applet showing a linear function $f(x) = -x + 4$ with sliders a and q.

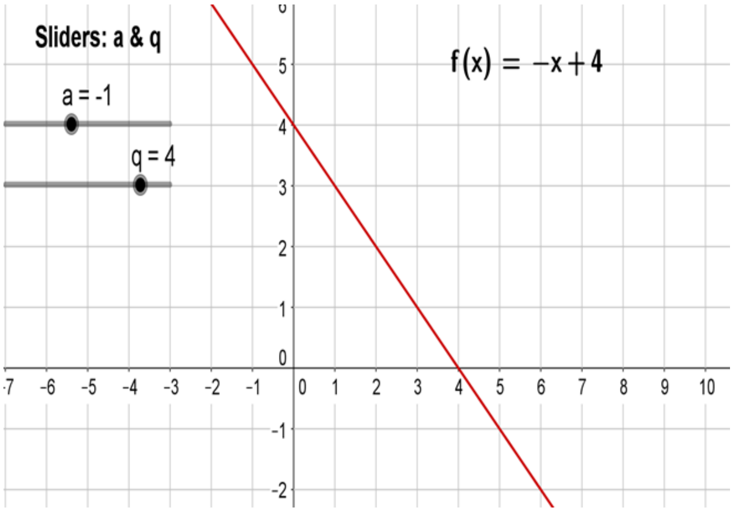


Figure 6: GeoGebra applet showing linear function and sliders.

GeoGebra applets can make learning an abstract concept much more meaningful and enhances conceptual understanding (Shadaan & Kwan, 2013). Both teachers and learners can use applets to visualize related concepts and observe how they affect each other. For example, given a GeoGebra applet showing graphical presentation of a quadratic function $y = ax^2 + q$ and sliders a and q. Moving of sliders $a \; or \; q$ will affect the original graph depending on the values of the sliders. Therefore, applets enable learners to drag graphical objects while observing the changes, which will enhance the understanding of mathematical concepts in algebraic functions. Furthermore, Uddin (2011) states that the applet's nature of flexibility makes mathematics more easy to learn for learners can visualize, interact and practise until they develop a deeper understanding of mathematical concepts within graphical transformation of functions.

To sum up, this chapter provided reviewed literature based on learners' misconceptions in algebraic functions, followed by literature based on Vygotsky's theoretical framework employed in this study and lastly, literature based on the role of GeoGebra software in algebraic functions was reviewed.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1. INTRODUCTION

This chapter reports the research methods used to conduct this study. It gives detailed information about the research approach, research design, research setting, data collection instruments, data collection procedures and data analysis. It also gives information on how the research instruments were developed and what they intended to bring to the study. Furthermore, it explains how the validity and reliability of the study were maintained and the considerations that were taken into account.

3.2. METHODOLOGY

Research approach refers to plans and the procedures for research that provides clear information from broad assumptions to detailed methods of data collection, analysis, and interpretation (Creswell, 2014). The process of choosing the appropriate methodology for the study is done through considering the main research question and sub-questions. In order to address the questions of this study, the study used a mixed methods research, which involves both collecting and analyzing quantitative and qualitative data into a single study (Onwuegbuzie & Combs, 2011). Mixed method research design offers a better understanding of a research problem than either research approach alone (Creswell, 2003; De Vos, Strydom, Fouche, & Delport, 2013).

This study intends to collect valid and reliable results concerning learners' misconceptions in algebraic functions and how the integration of GeoGebra could approach those misconceptions. Therefore, the researcher found mixing both qualitative and quantitative data collection instruments suitable for this study because of its potential to bring rich and reliable data. Both quantitative and qualitative data collection methods assisted to identify the actual misconception learners have in algebraic functions and provided how the integration of GeoGebra software in algebraic functions influenced learners' misconceptions.

3.2.1. Benefits of mixed method research

Mixed method research design has the potential to provide valid reference as it provides strengths that overcome all the weaknesses of the other two research designs which are qualitative and quantitative (De Vos, Strydom, Fouche, & Delport, 2013, p. 436). Addam (2014) concures that a mixed method approach assits in answering research questions that qualitative or quantitative appoaches alone cannot answer. Thus, it gives rich and valid results about the research.

The quantitative research method employed in this study includes collection of data using questionnaires. Therefore, some of the data was collected by using developed worksheets. The worksheets were created to collect data from all the three stages of data collection in this study. Thus, before the integration of GeoGebra there was a "pre-test", then there was GeoGebra intervention and after the integration of GeoGebra, there was "post-test". Quantitative methods emphasize objective measurements and the statistical, mathematical, or numerical analysis of data collected through polls, questionnaires, and surveys, or by manipulating pre-existing statistical data (Creswell, 2014). Quantitative method and numerical data mostly provide the information about "*what is happening*", rather than "*why or how*" (Yu-Wen, 2008). In light of Yu-Wen's (2008) insight, in this study, the worksheets enabled the researcher to identify learners' misconceptions and explored the effect of GeoGebra in learning algebraic functions.

Furthermore, Williams (2007) stated that quantitative data collected can be structured using graphs, tables or figures to give a clear information about the relationship between variables to readers. Following the advice of Williams (2007), the quantitative data collected in this study was controlled using graphs, tables and figures to give a readable analysis of results.

To provide more strength and validity of data collection in this study, the researcher also used qualitative approaches to explore learners' algebraic misconceptions by integrating GeoGebra software. Qualitative research is concerned with developing explanations of social phenomena (Hancock, 1998). Qualitative method focuses on helping people understand the world in which we live and why things are the way they are. It answers *"why"* and *"how"* questions (Creswell, 2003).

According to De Vos, Strydom, Fouche, & Delport (2013, p.308)

> *Unlike quantitative method, qualitative method requires the design of the research to be more than a set of worked out formulas. The qualitative researcher is concerned with understanding rather than controlled measurement, with the subjective exploration of reality from the perspective of an insider as opposed to that of an outsider predominant in the quantitative paradigm*.

Qualitative researchers usually collect data in the place where participants experience the issue (Creswell, 2014). The data was collected at the college in a mathematics classroom where learners were experiencing misconceptions when working with algebraic functions.

The study used qualitative data collection tools, which include focus group interviews and observations to collect data. Creswell (2014) concurs that qualitative researchers observe participants' behavior and that they interview participants during data collection. Qualitative researchers collect data and gather information on their own using certain tools or protocols, unlike quantitative researchers who tend to rely on questionnaires or instruments developed by other researchers (Creswell, 2014). Therefore, the research procedures followed and instruments used in this study were developed by a researcher with the aim of answering the research questions of the study.

In a qualitative research, the researcher becomes more interested in understanding researched issues from participants' perspectives (Struwig & Stead, 2011). Qualitative researchers do not depend on a single source of data; they normally collect several forms of data, such as interviews, observations, documents, and audiovisual information (Creswell, 2014).

Running focus group interviews have played a vital role in this study. The nature of focus group interviews tends to involve a small sample of participants. Therefore it was easy to develop a deeper understanding of learners' misconceptions. Questions based on algebraic functions were answered easily during the focus group discussions because the researcher could investigate in-depth about learners' understanding of algebraic concepts. In addition, this study seeks to explore the “why” and “how” in order to fully grasp the potential of GeoGebra. Therefore, the qualitative research method is appropriate in this study.

Furthermore, it was important for this study to use a mixed methods approach. The use of qualitative methods helped in evaluating in-depth details of questions that were not easily answered through quantitative methods. In addition, qualitative researchers use categories or themes to organize data after the researchers had reviewed and made sense of data (De Vos, Strydom, Fouche, & Delport, 2013; Creswell, 2014). In light of the above citation, NCV learners’ misconceptions identified are summarized into categories which enabled the researcher to develop an appropriate GeoGebra applet for each misconception category.

3.3. RESEARCH DESIGN

Research design is defined as a method a researcher uses to collect, analyse and interpret data. A research design provides a clear picture on how the entire project will be carried out and all the steps required to achieve the anticipated outcome (De Vos, Strydom, Fouche, & Delport, 2013) such as, how data collection will be conducted and how the observations will be made. Olsen (2004) concurs that mixing of qualitative and quantitative data methods in a single study such as collecting data by means of interviews, surveys, observations and questionnaires enhances the validity of results. Combining both qualitative and quantitative methods in this study is an attempt to improve the validity and reliability of results through triangulation. Yeasmin & Rahman (2012) define triangulation as a method of combining two or more theories, data collection sources, methods or investigators in one study of a single phenomenon.

3.4. RESEARCH SETTINGS

The research setting is defined as the physical, social, and cultural site in which the researcher conducts the study (Struwig & Stead , 2011). It is the environment in which the research is carried out.

3.4.1. Physical setting

The research study was conducted at a TVET college in Port Elizabeth. The college is well resourced with more than two computer labs. All mathematics classes have white boards and data projectors. Permission to utilize a computer lab and mathematics classroom for this study was requested and granted by the college campus head and college principal (Appendix 3 & Appendix 5). The research for this study occurred in the researcher's mathematics classroom and in a computer laboratory during break times and after school. The classroom was equipped with 25 chairs and desks, which was enough for each group during the data collection process. The computer lab has 20 computers, arranged in a comfortable manner and the computer lab is air-conditioned. All the facilities in the classroom as well as in the computer laboratory met the needs of GeoGebra intervention.

3.4.2. Target population

The target population is the entire combination of participants that meet the selected set of criteria (Addam, 2014). In this study, the target population are NCV level two mathematics learners (aged 17-28) that the researcher teaches. Learners were expected to utilize computers during integration of GeoGebra software. NCV level two learners are computer literate. They had obtained basic skills of computers as one of their compulsory subjects and, therefore, it was not a challenge for them to use a computer.

3.4.3. Sampling

A sample is defined as a set of participants selected from a larger population for the purpose of a survey (Struwig & Stead, 2011). Sampling is a process where the researcher selects who should or should not be part of the study. Sampling in a mixed method research is divided into probability and purposive sampling. Teddlie & Yu (2007) differentiate between probability and purposive sampling, claiming that probability sampling methods are basically applied in quantitative based studies and it involves selecting a large number of population or objects. Purposive sampling methods, on the other hand, are mostly used in qualitative studies for the selection of components such as individuals, institutions or groups of individuals based on purposes of answering research questions.

The researcher found a qualitative, purposeful random sampling relevant for this study. The researcher intended to determine the richness of the data, hence the sample was selected purposefully. Purposeful random sampling refers to a random selection of a small sample to obtain the in-depth or richness information; it avoids any criticisms that the researcher is biased in selecting the sample (Struwig & Stead , 2011). Therefore, in this study, out of approximately 600 NCV level two learners registered at the college, the researcher only teaches about 76 NCV level two learners (divided into four classes). All the 76 learners that the researcher teaches were selected to participate in the study. Learners who participated in the focus group interviews were selected with random purposive sampling. After the researcher had analyzed learners' results obtained from the pre-test; based on learners' performance, she selected the random 10 learners from the 76 learners to participate in the focus group interviews.

3.4.4. Pilot study

A pilot study is defined as "the phase in a research project in which the researcher collects data to test drive the research procedures, identify possible problems in the data collection and set the phase for the actual study" (De Vos, Strydom, Fouche, & Delport, 2013, p. 446).

I randomly selected six NCV level two learners from the other groups that I do not teach to pilot this study. These learners were already taught algebraic functions by their teachers in a traditional way of teaching, which includes teaching by explaining and sketching the algebraic function on the white board.

To obtain reliable and valid results, the researcher carried out a pretest, focus group interviews and intervention of GeoGebra with these NCV learners. The purpose of holding a pilot study was to test the structure and wording of the worksheets and focus group interview questions. Furthermore, the pilot study was intended to test GeoGebra applets and to see whether they consisted of enough and relevant information to collect data for this study. A pilot study was carried out to improve data collection instruments, a pre-test worksheet was completed, and interaction with created applets and a 20 min long focus group discussion was held. The data obtained from a pilot study was used to refine a worksheet questionnaire, GeoGebra applets and the focus group interview questions. For example, the pilot study indicated that GeoGebra applets created were not fixed and that they made learners forget to recall the original graph after the shift. The researcher refined the GeoGebra applets by creating two same graphs and made one graph immovable and the other one flexible with the ability to be shifted. Additionally, conducting a pilot study was useful for this study because it helped a researcher to make some effective adjustments on the formatting of the worksheet.

3.5. DATA COLLECTION INSTRUMENTS

In this study, the researcher used multiple forms of data collection, namely questionnaires (worksheets), focus group interviews and observations. The data collection procedure was divided into the identifying of misconceptions (pre-test and focus group interviews), integration of GeoGebra software (intervention) and determining the influence of GeoGebra in algebraic misconceptions (post-test and focus group interviews). The National Certificate Vocational subject guidelines for mathematics NQF level two only consist of the basic standard forms of all algebraic forms (Appendix 10). The study was conducted using only three types of functions in the following standard forms: parabola function $f(x) = ax^2 + q$, linear function $f(x) = ax + q$ and hyperbola function $f(x) = \frac{a}{x} + q$.

3.5.1. Worksheets

There were three worksheets created for this study. The first worksheet was given at the beginning of the study to identify learners' misconceptions on algebraic functions prior the integration of GeoGebra software. The worksheet consisted of questions based on converting algebraic functions from equations to graphs and vice versa (Appendix 6). The second worksheet was given during GeoGebra intervention. Learners completed this worksheet while working with electronic applets. They observed graphical changes when moving sliders and completed the worksheet according to what they saw (Appendix 7).

The third worksheet was given at the end of the study; questions testing similar skills as of the first worksheet were given in this worksheet. The purpose of the last worksheet was to identify whether GeoGebra helped learners deal with algebraic misconceptions (Appendix 8). The researcher used the NCV level two class list to distribute these worksheets. Each worksheet had a code that corresponded with the name of each learner from the class list. The purpose of writing codes that correspond with a learner's name was to enable the researcher to track changes of individual learner performance from the pre-test, focus group interviews, intervention and the post test. Learners' codes also helped in the selection of learners to participate in the focus group interviews. The researcher used the first four letters of alphabets (A, B, C & D) as group codes. Learners from each group were given a group letter and a unique number as their code identities. For example, a random 5 learners from group A would have names as: A1, A7, A11 and A2. For the sake of anonymity, learners' actual names were never used.

3.5.1.1. Pre- test worksheet design and layout

The pre-test worksheet (Appendix 6) is divided into section A, section B and Section C.

Section A:

The first stage of data collection was based on identifying leaners' misconceptions towards algebraic functions. The questionnaire consisted of sixteen questions divided into three sections. Section A consisted of six multiple choice questions based on transformations between equations and graphs. Questions in section A consisted of four algebraic equations options and one graph given in each question. Learners were expected to match the graph to the equation it represents between the four given options of equations. The purpose of this section was to determine whether NCV learners understand the relationship between graphs and equations concerning the effects of parameters a and q. From the six questions of section A, three questions (one question for each function) were based on the interpretation of parameter q only, all options of equations had the same a value but different q value. The other three questions were based on the interpretation of parameter a where all options of equations had the same q value but varied in a value.

Each question in the worksheet was developed to test a particular skill. To give an overview of the questions in the pre-test worksheet section A, the following two selected examples of questions provides in detail the skills that the research intends to test.

Example 1:

The figure 7 below is question 1 in the worksheet based on matching a correct hyperbola equation to the given graph concerning the interpretation of parameter q.

1. Which one of the following equations defines the given graph?

A. $\boldsymbol{f(x) = \frac{2}{x} - 3}$

B. $\boldsymbol{f(x) = \frac{2}{x} + 3}$

C. $\boldsymbol{f(x) = \frac{2}{x} + 0}$

D. $\boldsymbol{f(x) = \frac{2}{x} - 2}$

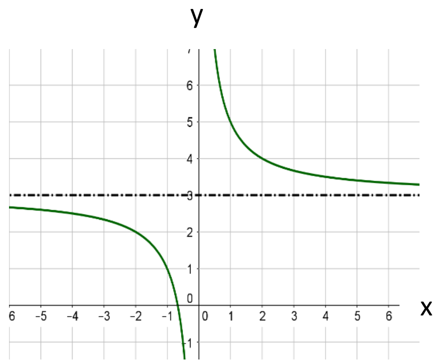


Figure 7: The effect of parameter q in a hyperbola based question.

The graph shows the horizontal asymptote as $y = +3$. Therefore, the correct option is B, given as $y = \frac{2}{x} + 3$. Similarly, each of the options was deliberately designed to point out a particular misconception. Selecting option A is the result of an unsystematic misconception. Learners understand that the line $y = q$ is cutting the y axis at 3 but confusing the sign of the y value. Option C indicates that horizontal and vertical asymptotes correspond with the x and y axis respectively. Therefore, selecting option C is the result of systematic misconceptions. It shows that learners are confused by the apparent absence of parameter q. Selecting option D is the result of systematic misconception; learners are unable to generalize, to understand or to connect mathematics ideas (Bush, 2011).

Example 2:

The figure 8 below is question 5 in the worksheet based on matching a correct parabola equation to a given graph concerning the interpretation of parameter a.

Which of the following equations defines the given graph?

A. $f(x) = 2x^2 + 5$

B. $f(x) = x^2 + 5$

C. $f(x) = -x^2 + 5$

D. $f(x) = -2x^2 + 5$

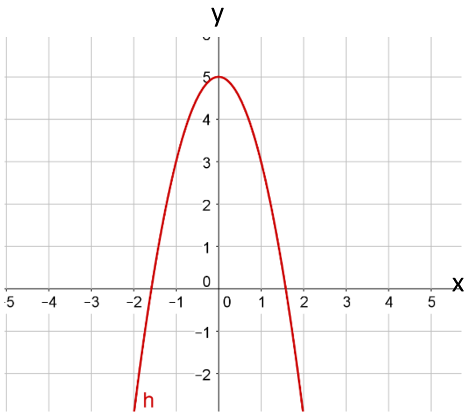


Figure 8: The effect of parameter a in a parabola based question.

This question intended to determine whether learners understand the interpretation of parameter a in a parabola function. Figure 8 shows a decreasing parabola curve turning at positive 5, which makes option D the correct answer given as $f(x) = -2x^2 + 5$. Generally, when $a > 1$ or $a < -1$, the parabola graph becomes narrow and when $-1 \leq a \leq 1$ the parabola graph becomes broader. Selecting option A is the result of unsystematic misconception, learners are unable to interpret parameter a sign effect. Selecting options B and C is the result of systematic misconceptions. Learners are unable to interpret the effect of the parameter a value in parabola function.

Section B:

Section B consisted of three multiple-choice questions based on converting algebraic graphs to equations. There were four choices of graphs given and one equation, learners were required to select a graph that represents the equation given. This section was intended to investigate whether learners are able to analyze or interpret the graphical presentation of each function to its equation; this involved the interpretation of both parameters a and q

To give an overview of the questions in the pre-test worksheet section B, the following two selected examples of questions provide in detail the skills that the research intends to test.

Example 1:

The purpose of the following question was to determine whether learners are able to match a single hyperbola graph to the given equation. Figure 9 below shows question 1 in section B based on the interpretation of both parameters a and q.

Which one of the following graphs defines the given equation: $y = \frac{3}{x} - 2$

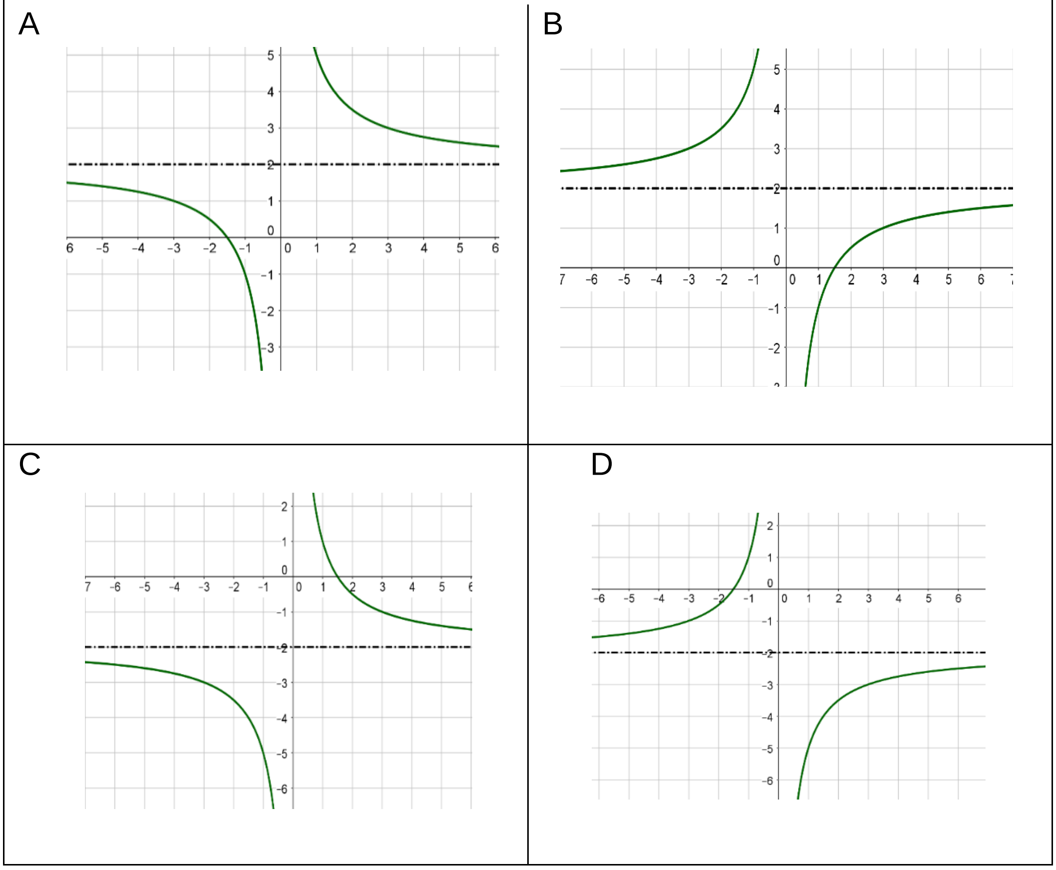


Figure 9: Effect of a and q in a hyperbola based question.

This question required learners to be able to connect the interpretation of both parameters a and q. The equation shows parameter a as $+3$ and parameter q as -2 , which means the graph should occupy first and third quadrants and shift 2 units down from the origin.

Options A & C both have their graphs on the first and third quadrants. Nonetheless, option A shows a horizontal asymptote of $+2$. Selecting option A is the result of unsystematic misconception. Learners have the ability to interpret the effect of parameter a but they confuse the significance of the sign of parameter q . Option C is the correct option. The horizontal asymptote cuts the y axis at -2.

Selecting options B is the result of systematic misconception. Learners are unable to understand the effects of parameters a and q. Selecting option D is the result of unsystematic misconception. Learners have the ability to interpret the effect of parameter q but are unable to interpret the effect of parameter a.

Example 2:

The purpose of the following question was to determine whether learners are able to match a single linear graph to the given equation. Figure 10 below is question 2 in sections B based on the interpretation of both parameters a and q.

Which one of the following graphs define the given equation: $f(x) = 2x - 4$

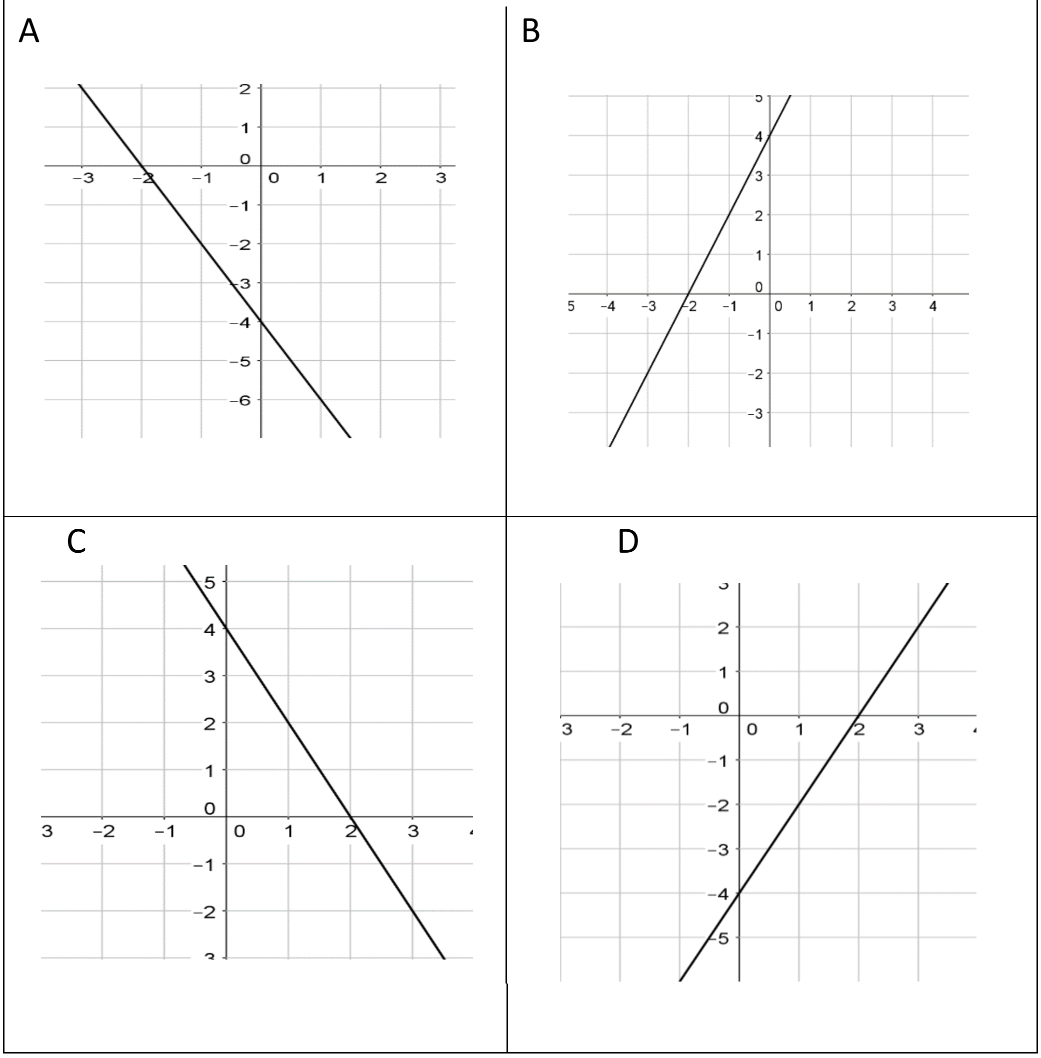


Figure 10: Effect of a & q in a linear based question.

The equation has parameters $a = 2$ and $q = -4$. This means that the graphical representation should be an increasing line cutting the y axis at -4

Options B and D are showing increasing linear functions, however the linear graph in option B is cutting the y axis at positive 4. Therefore, learners who selected this option showed unsystematic misconceptions related to understanding the effect of parameter q. Option D is the correct answer for this question; it is an increasing line cutting the y axis at -4. Options A and C are showing decreasing linear graphs. Learners who selected one of these options are experiencing systematic misconceptions based on transforming equations to graphs.

Section C:

Section C is a summary of sections A and B. The purpose of section C was to verify misconceptions obtained from both sections A and B. It consisted of seven general questions based on algebraic concepts. Questions were based on the understanding of x and y intercepts and the effects of a and q parameters on both algebraic equations and graphs. To give an overview of the questions in the pre-test worksheet section C, the following two selected examples provide in detail the skills that the research intends to test.

Example 1:

The purpose of the following question in figure 11 was to verify learners' misconceptions regarding the understanding of parameters a and q in a linear function. Figure 11 below is question 5 in section C about the substitution of parameters a and q.

Which one of the following equations represents a linear function $y = ax + q$ with a slope/gradient given as 2, y intercept $(0,4)$ and x intercept $(-2,0)$?

A. $y = -2x + 2$
B. $y = 4x - 2$
C. $y = 2x - 2$
D. $y = 2x + 4$

Figure 11: Interpretation of parameters a and q in a linear function.

This question is about understanding the difference between the parameter a, x and y intercepts q. In a linear function, it is the parameter a and y intercept q that reflects exactly on the equation. Therefore, option D is given as $y = 2x + 4$ is the correct answer. The selection of option A is the result of a systematic misconception. Learners are confusing parameter a with the x intercept and confusing parameter q with parameter a. Options B and C are also showing systematic misconceptions and learners are unable to substitute parameters a and q.

Example 2:

The purpose of the following question in figure 12 was to verify learners' misconceptions regarding the understanding of the x intercept(s) in algebraic equations. Figure 12 is question 2 in section C about defining x intercept (s).

x intercept(s):

A. Is identified as a from linear equation $y = ax + q$.
B. Are point(s) on the x axis that the graph cuts, where y is zero and are called roots from quadratic function $y = ax^2 + q$
C. Is identified as q from hyperbolic function $y = \frac{a}{x} + q$
D. All the above descriptions about the x intercepts are correct.

Figure 12: Defining x intercept(s) in algebraic functions.

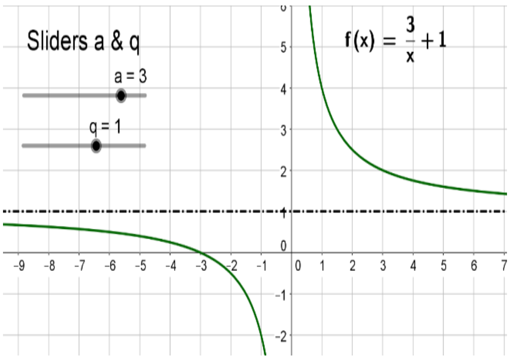
Option B is the correct option. Learners who have selected this option understand the meaning of the x intercept. Options A & C can be referred to as systematic errors. Learners probably believed that x Intercept should reflect on the algebraic equations like the y intercept (q) and the slope (a). Leaners are confusing the x intercept with parameters a and q. Learners are using previously learned principles incorrectly. This type of misconception could be referred to be systematic. The selection of option D results in systematic misconceptions about the x intercept because options A, B & C have different meanings and therefore they cannot all be correct.

3.5.1.2. Post-test worksheets design and layout

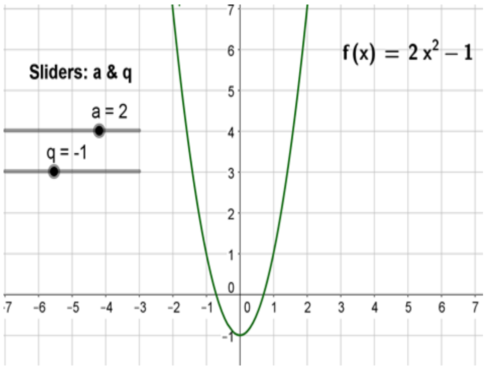
As mentioned earlier in the study, the post-test worksheet assessed similar skills as those of the pre-test. The format of this worksheet is similar to that in the pre-test worksheet. However, the format of the questions is twisted or exchanged to avoid learners from seeing a pattern (Appendix 8). Post-test worksheets were given to learners after the integration of GeoGebra software to determine whether GeoGebra helped learners deal with algebraic misconceptions identified during the pre-test.

3.5.1.3. GeoGebra intervention worksheet design and layout

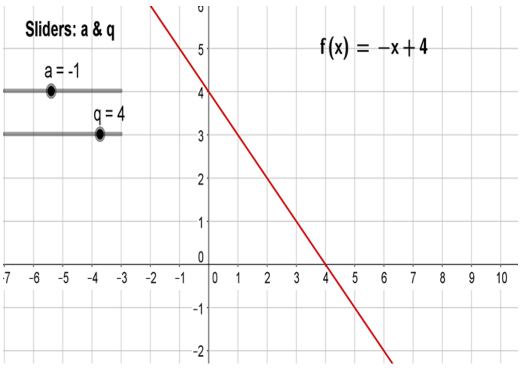
After algebraic misconceptions had been identified during pre-test worksheet, the researcher integrated GeoGebra software. This worksheet (Appendix 7) is divided into section A, Section B and Section C based on the created electronic GeoGebra applets. Learners answered this worksheet while utilizing created GeoGebra applets with algebraic functions. The figure 13 below indicates electronic GeoGebra applets that learners used on sections A, B and C of the worksheet.



Hyperbola function



Parabola function



Linear function

Figure 13: Electronic GeoGebra applets showing parabola, hyperbola and linear functions.

Section A of the worksheet consisted of nine questions. These were, three questions each per linear $f(x) = ax + q$, parabola $f(x) = ax^2 + q$ and hyperbola $f(x) = \frac{a}{x} + q$ function. In this section, questions were based on understanding the effect of variable a of each function. Learners were given three different values of variable a in each function and they were required to plot the results of the graph for each a value given. They were also required to explain the effect variable a has on each graph. The changing of a variable a was done by moving a slider created for this variable. The aim of this section was to help learners to view the changes dynamically in each function when the a value changes. Section B of the worksheet consisted of nine questions based on understanding the effect of variable q in each function. Section B format and expectations are similar to section A except that section B focuses on the variable q (y intercept).

Section C of the worksheet consists of open-ended questions. NCV learners were expected to create their own functions using electronically created GeoGebra applets, sliders (without functions given) and reflect on those functions.

The figure 14 below is an example of GeoGebra applet with sliders a and q that learners used to create algebraic functions

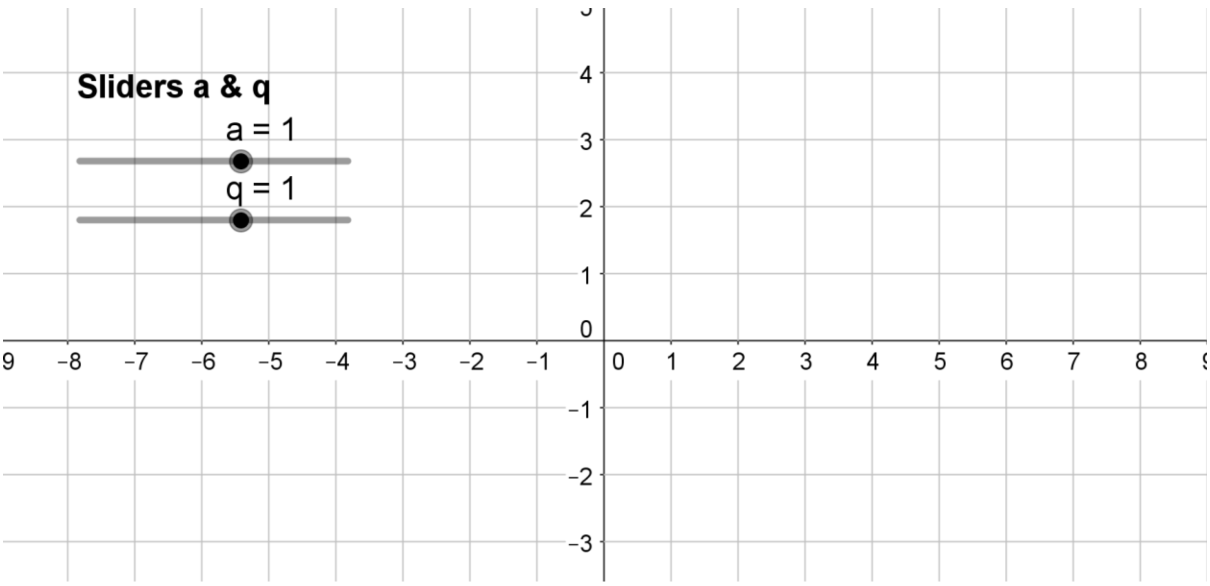


Figure 14: GeoGebra applet with sliders a and q

Questions were based on creating linear function using a GeoGebra applet. Learners were asked to reflect on the functions they created, such as explaining their observations when a & q values change. Learners were also required to state whether there is any relationship between these two variables. Does changing of variable a has any effect on variable q and vice versa? These questions required learners to use GeoGebra as dynamic, problem solving, conceptual and cognitive tool. For NCV level two learners to engage with this section, they were required to critically observe the changes from GeoGebra applets as they dynamically move variables a and q.

3.5.2. Focus group interviews

Focus group interviews are defined as a wisely planned discussion intended to obtain perceptions on a defined area of interest in a non-judgmental and non-threatening environment (Struwig & Stead , 2011).

According to Freitas, Oliveira, Jenkins, & Popjoy (1998, p.2):

> *Focus Group is a type of in-depth interview accomplished in a group, whose meetings present characteristics defined with respect to the proposal, size, composition, and interview procedures. The focus or object of analysis is the interaction inside the group. The participants influence each other through their answers to the ideas and contributions during the discussion. The moderator stimulates discussion with comments or subjects. The fundamental data produced by this technique are the transcripts of the group discussions and the moderator's reflections and annotations.*

The purpose of focus group interviews is to give a better understanding on how participants feel or think about an issue (De Vos, Strydom, Fouche, & Delport, 2013). Therefore, the issue in this study was learners' identified misconceptions.

This study conducted two focus group interview sessions; the first focus group interview was conducted prior to the integration of GeoGebra software, yet after the pre-test (Appendix 11). The purpose of conducting the first focus group interview was to verify misconceptions identified from the pre-test worksheet. About ten learners were selected to participate during the first focus group session; five learners from the selected ten were interviewed separate from the other group of five learners. The first group of five learners that participated in the focus group interview were learners who mostly showed similar misconceptions related to effect of variable a in the pre-test. While the other group of five learners that participated in the focus group interview mostly showed misconceptions related to a variable q in the pre-test. The reason for grouping learners according to their misconception is to obtain enough data under

each misconception rather than mixing up students showing different misconceptions together. As mentioned earlier, worksheets had codes corresponding with learners' names from the class list. This helped the researcher to distinguish which learner showed what misconception without disclosing their identities.
The focus group interviews were held in a mathematics classroom during break time or after school. The researcher and learners selected negotiated a convenient time between break and after school to conduct the focus group interviews. The classroom where focus group interview were conducted had all the graphs that were to be discussed drawn on the white board. The researcher referred to those graphs when asking certain questions during the interview session.

The second focus group interview was conducted after the integration of GeoGebra at the end of the study (Appendix 12); the same ten learners elected for the first interview were also be selected to participate during the second focus group discussions. Questions for this interview were based on learners' experiences of GeoGebra software. The purpose of this interview was to obtain learners' perspectives on the challenges and benefits they experienced with the utilization of GeoGebra software. The focus group interviews also intend to determine how the integration of GeoGebra software during teaching and learning supported learners to deal with misconceptions in algebraic functions.

3.5.2.1. Focus group interview questions

Questions for the first focus group interview were structured and open-ended; however, there were follow up questions to participants in cases where clarity was needed. There were four main questions with sub-questions for each focus group discussion (Appendix 11). Similar questions were asked during both sessions. The first question mainly focused on the effects of a variable q (y intercept). The remaining questions were based on the three algebraic functions. During the second session with learners who showed misconceptions related to variable a, questions asked were based on the effect of parameter a and learners' experiences on algebraic functions (Appendix 11).

The purpose of running focus group discussions was to deepen my own understanding of what had been identified as misconceptions in the pre-test. I therefore verified learners' misconceptions identified during interviews with the misconceptions identified in the questionnaire worksheet. Thereafter, I developed categories of the misconceptions identified for data analysis purposes. The second focus group interview that was conducted after the integration of GeoGebra, consisted of GeoGebra based questions and assisted to answer the second sub research question of this study (Appendix 12).

3.5.3. Observation

Observation is defined as a research instrument used during data collection where the researcher observes the interactions amongst participants or events as they naturally occur with the intention of answering research questions (Zohrabi, 2013). The researcher observed learners during the intervention stage and field notes were taken while learners were busy working on GeoGebra applets to complete the worksheet. The researcher mainly observed how learners were progressing when utilizing GeoGebra, how learners made use of GeoGebra as a modelling and problem solving tool and observed whether learners appeared to find GeoGebra effective or not.

The information obtained during observation assisted the research to determine whether the GeoGebra software is an effective tool to learn algebraic functions.

3.6. DATA COLLECTION PROCEDURE

The study occurred during break times or the first 40 minutes after school; the figure 15 below indicates the research procedure for this study. The research was conducted in the following order.

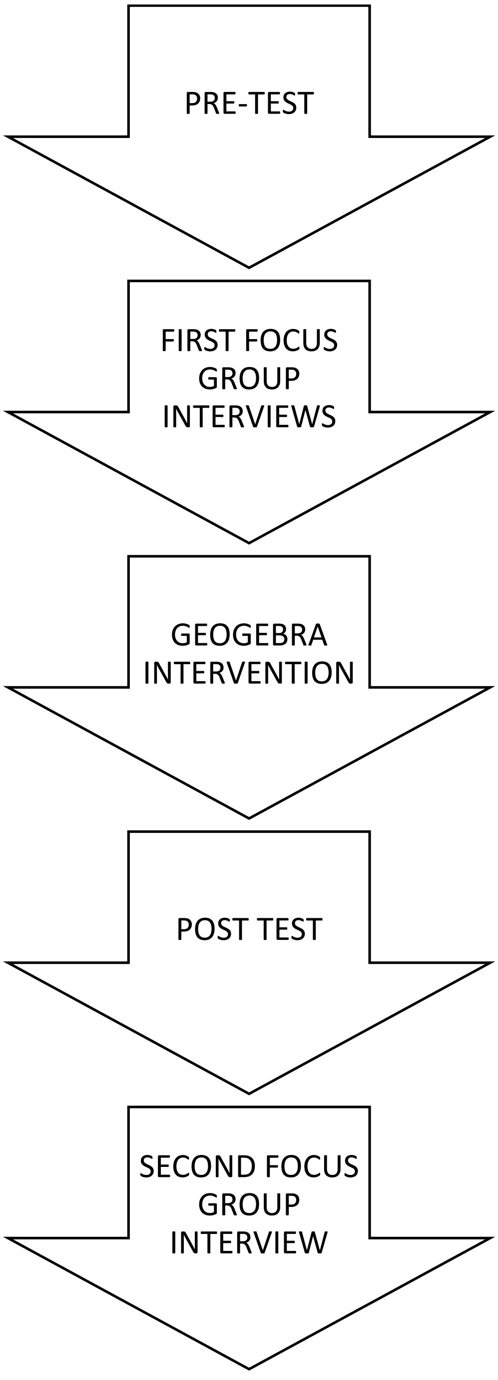


Figure 15: Illustrative view of the research procedure.

3.6.1. Pre-test

The first stage of data collection for this study was the identification of misconceptions, where learners were given a worksheet based on converting algebraic graphs to equations and vice versa. After learners completed the worksheet, they were invited to participate in the first focus group interview.

3.6.2. First focus group interviews

The researcher selected learners who showed misconceptions related to parameter q and parameter a to participate in the focus group interviews. The interview session was audio recorded. Misconceptions identified were transcribed and grouped into categories.

3.6.3. GeoGebra intervention

Figure 16 below indicates stages of GeoGebra integration for this study.

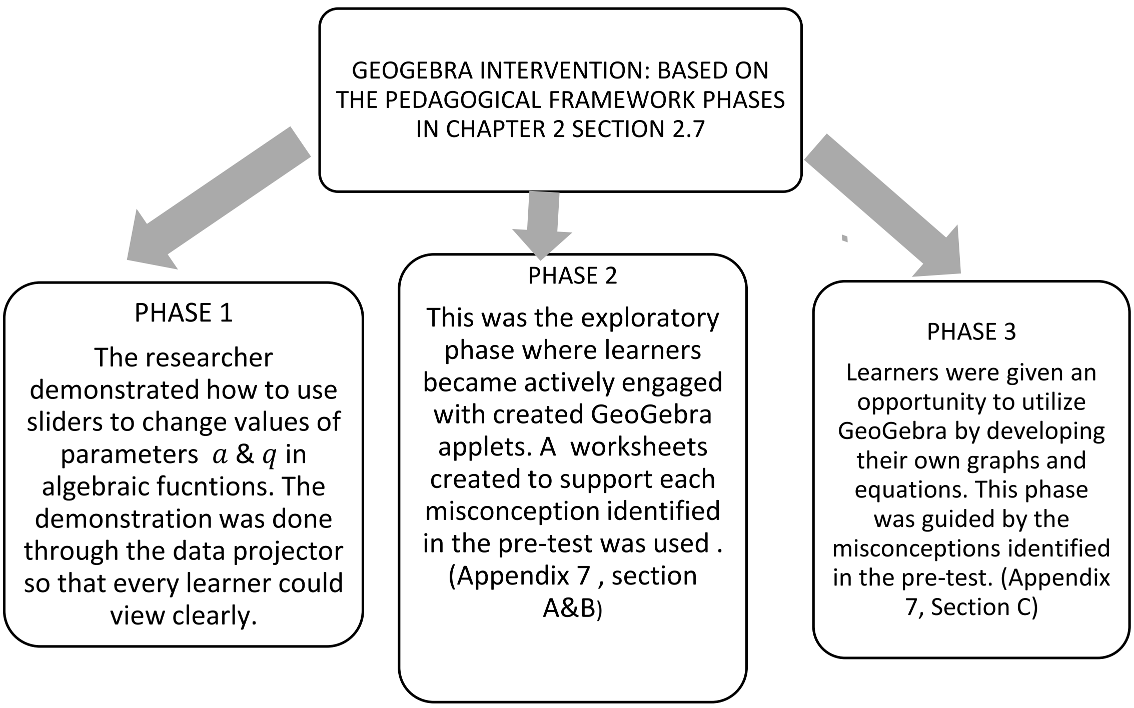


Figure 16: GeoGebra integration stages.

3.6.4. Post-test

Learners completed a worksheet based on algebraic functions with similar skills tested from pre-test. Learners completed the post-test worksheet without the use of GeoGebra software. The aim of post-test worksheet in this study was to determine whether GeoGebra helped learners deal with algebraic misconceptions or not.

3.6.5. Second focus group interviews

Finally, the researcher conducted the second focus group interviews based on learners' experiences on GeoGebra. The interview session was audio recorded and transcribed for data analysis purposes.

3.7. DATA ANALYSIS

Data analysis is defined as a process of making sense of raw data collected using methods such as questionnaires, observations and focus group interviews (Struwig & Stead, 2011). Data analysis in a mixed method study involves analysis of qualitative data using qualitative methods, and the quantitative data using quantitative methods and both qualitative and quantitative analysis strategies get combined (De Vos, Strydom, Fouche, & Delport, 2013). In this study, the analysis of data collected during the worksheets was done by developing themes and categories. The worksheets that were distributed to learners had unique codes that assisted the researcher in analyzing the data into categories. From the first worksheet, the researcher developed themes of common misconception identified. For example, misconceptions related to variable q or variable a of algebraic functions.

These categories of misconceptions guided the researcher during the intervention of GeoGebra; each category was addressed using created GeoGebra applets. The pre-test and post-test data collected analysis was done through grouping the three algebraic graphs into graphs and tables. Tables and graphs helped in comparing learners' responses or their performance from the worksheet.

The focus group interviews were audio recorded and transcribed. Audio recording focus group interviews provides rich data as it allows the researcher to access full information obtained during discussions (Rana & Latif, 2013).

3.8. ETHICAL CONSIDERATIONS

“Research should be based on mutual trust, acceptance, cooperation, promises and well-accepted conventions and expectations between all parties involved in a research project” (De Vos at al. 2013, p. 113). Involving other people in my study required ethical approval; therefore, I negotiated and asked permission from all parties involved in my research study. The Ethics Committee of NMMU has granted me an approval letter to run the study (Appendix 1). The sample of data to conduct this study is taken from NCV level two learners and Port Elizabeth TVET College Struandale Campus. I have written letters of consent to learners, Struandale Campus Head and Principal of the College (Appendix 5). All parties involved permitted me to run the study (Appendices 2, 3 and 4).

De Vos, Strydom, Fouche, & Delport (2013,p.116) claim that participation should be voluntary and no one should be forced to participate in a research study. All the parties involved were clearly informed that participation is voluntary. Participants were also informed that it is of their right to withdraw anytime during the process and their identity will be secured and their opinions and views will be respected.

3.9. CONCLUSION

This chapter provided detailed information about the research approach, research design, research setting, data collection instruments, and data collection procedures and data analysis. It also provided information on how the research instruments were developed and what they intended to bring to the study. Lastly, this chapter explained how the validity and reliability of the study were maintained and the considerations that were taken into account.

CHAPTER 4: DATA PRESENTATION AND ANALYSIS

4.1. INTRODUCTION

This chapter presents the quantitative and qualitative data collected during the study. Data was collected by means of focus group interviews, pre-test, intervention, observation and post-test questionnaires. In order to make the process of quantitative and qualitative analysis easier, the findings from different data sources are first tabulated into appropriate tables and presented graphically. During analysis of the findings, the relevant data from different sources are cross-referenced and combined in order to develop answers for the research questions. The analysis of data obtained from the pre-test and the first focus group interviews serves to answer the first research sub-question about identifying learners' misconceptions in algebraic functions. The analysis of data obtained from the intervention of GeoGebra, post-test and the last focus group interview serves to answer the second research sub-question about the effect of GeoGebra in algebraic functions.

4.2. PRE-TEST AND FOCUS GROUP DISCUSSION ANALYSIS

This stage of data collection occurred at the beginning of the study. Both quantitative and qualitative instruments were used to collect data during this phase. The data was collected by means of learners answering the worksheet and participating in the focus group interview. The researcher gave learners a three section worksheet based on the interpretation of parameters a and q in algebraic functions. The purpose of this stage was to determine learners' misconceptions in algebraic functions. Data presentation and analysis is categorized into three algebraic functions; i.e. hyperbola function $f(x) = \frac{a}{x} + q$, linear function $f(x) = ax + q$ and parabola function $f(x) = ax^2 + q$.

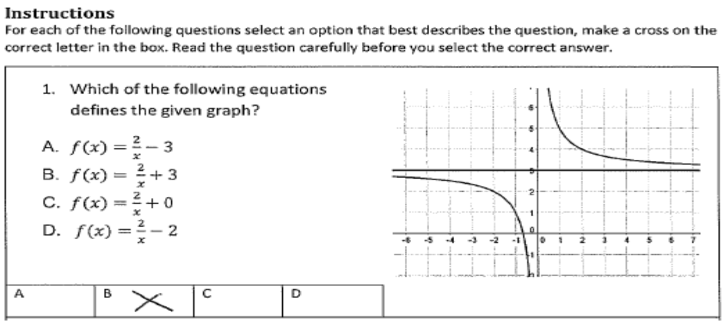
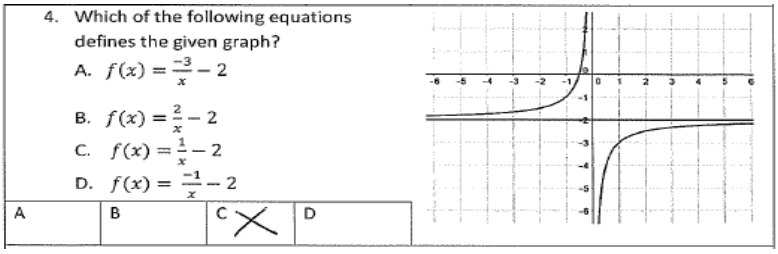
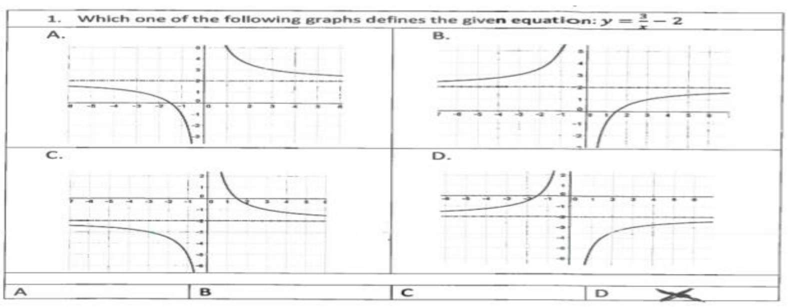
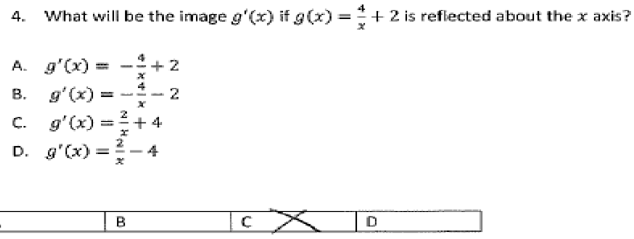
4.2.1. Pre-test sections A & B data presentation and analysis

4.2.1.1. Hyperbola function $f(x) = \frac{a}{x} + q$

The data obtained from questions based on the interpretation of parameter a and parameter q in a hyperbola function has shown that the majority of learners struggled to understand the effect of parameter a compared to the interpretation of parameter q

in a hyperbola equation. For example, table 1 below shows learner B15's solutions when working with hyperbola function.

Table 1: Learners' hyperbola function based extracts obtained from pre-test

Sections	Effects of parameters	Solutions
Section A Question 1	q	 **Instructions** For each of the following questions select an option that best describes the question, make a cross on the correct letter in the box. Read the question carefully before you select the correct answer. 1. Which of the following equations defines the given graph? A. $f(x) = \frac{2}{x} - 3$ B. $f(x) = \frac{2}{x} + 3$ C. $f(x) = \frac{2}{x} + 0$ D. $f(x) = \frac{2}{x} - 2$ A B C D
Section A Question 4	a	 4. Which of the following equations defines the given graph? A. $f(x) = \frac{-3}{x} - 2$ B. $f(x) = \frac{2}{x} - 2$ C. $f(x) = \frac{1}{x} - 2$ D. $f(x) = \frac{-1}{x} - 2$ A B C D
Section B Question 1	a and q	 1. Which one of the following graphs defines the given equation: $y = \frac{3}{x} - 2$ A. B. C. D. A B C D
Section C Question 4	a and q	 4. What will be the image $g'(x)$ if $g(x) = \frac{4}{x} + 2$ is reflected about the x axis? A. $g'(x) = -\frac{4}{x} + 2$ B. $g'(x) = -\frac{4}{x} - 2$ C. $g'(x) = \frac{2}{x} + 4$ D. $g'(x) = \frac{2}{x} - 4$ A B C D

The results of learner B 15 in table 1 above shows that he or she understood the effect of parameter q in a hyperbola equation but experienced misconceptions in understanding the effect of parameter a. Looking at his or her work, he or she managed to answer section A question 1 correctly. The question was based on understanding the effect of parameter q. However, in section A question 4, he or she was unable to understand that a negative value of parameter a results in a graph occupying second and fourth quadrants.

Section B question 1 confirms that learner B15 was experiencing systematic misconceptions towards the interpretation of parameter a in a hyperbola function. It is clear that this learner was unable to interpret the effect of parameter a; as he or she selected an option showing a graph occupying second and fourth quadrants with a horizontal asymptote as -2 yet the given equation had parameter a as $+3$ which means that the graphical representation should be a hyperbola graph positioned in first and third quadrants. Learner's B15 selection in section C question 4 verifies that he or she was experiencing systematic misconceptions related to the interpretation of parameter a. The table 2 below shows the summarized hyperbola function based results obtained from pre-test sections A and B, i.e., results based on the interpretation of parameter q , parameter a and parameters a & q for the whole sample (N=76). The correct responses are highlighted in boldface.

Table 2: Summary of hyperbola function based results from pre-test sections A and B

N=76	**SECTION A**								**SECTION B**			
	QUESTION 1 q				**QUESTION 4 a**				**QUESTION 1 a and q**			
Correct option	**B**				**D**				**C**			
	A	**B**	**C**	**D**	**A**	**B**	**C**	**D**	**A**	**B**	**C**	**D**
Number of learners' responses per option	6	**50**	11	9	12	17	19	**28**	3	7	**34**	32
Learners' responses in percentages	8%	**66%**	14%	12%	16%	22%	25%	**37%**	4%	9%	**45%**	42%

The results show that about 66% of learners had the ability to interpret the effect of parameter q, while about 37% of learners had the ability to interpret the effect of parameter a, and about 45% of learners had the ability to interpret the effect of parameter a and q. The figure 17 below shows hyperbola based summarized results obtained in the pre-test worksheet.

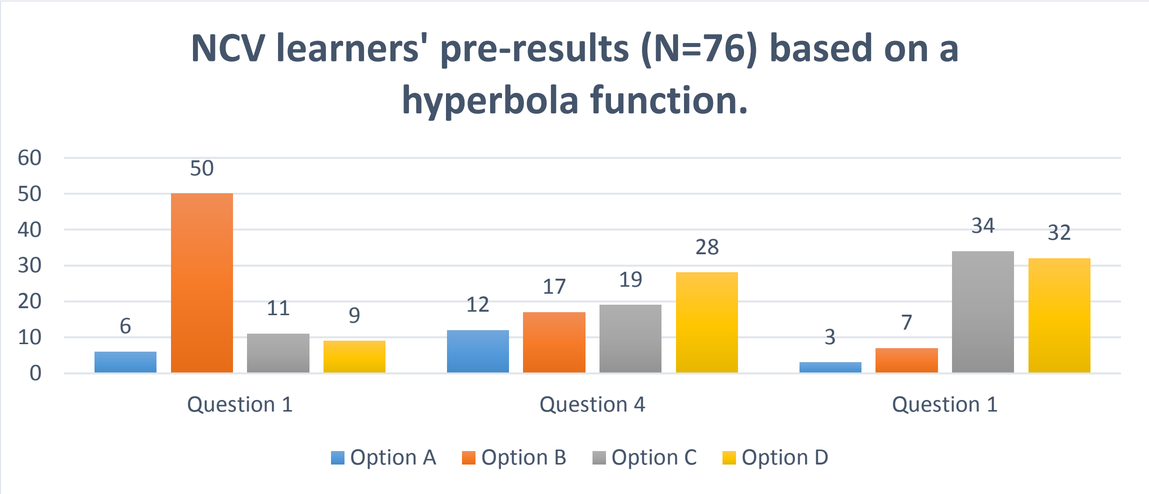


Figure 17: Showing a summary of hyperbola function results obtained in the pre-test.

If learners understand the combined effects of a and q, then it is reasonable to expect that they understand individual effects of a and q respectively. Learners who selected option C from section B question 1 should have selected the correct options in section A question 1 based on parameter q and question 4 based on parameter a. However, this is not necessarily the case, as the data has shown. The following table 3 indicates how many of the 34 learners (45%) understood the effect of q in question 1 and the effect of a in question 4.

Table 3: Learners' ability to interpret parameters a and q in a hyperbola function

Analysis of learners (N=34) that selected option C in section B question 1				
Questions selected	**Able to interpret:**	**Unable to interpret:**	**Number of learners**	**%**
Q1 & Q4	a and q effects		10	29%
Q4	a effect		2	6%
Q1	q effect		17	50%
Q1 & Q4		a and q effects	5	15%

The results in table 3 above shows that not all learners who answered question 1 of section B correctly have also answered questions 1 and 4 correctly. The analyses of results indicates that only 29% of learners had the ability to interpret both parameter a and parameter q. About 6% of learners showed the ability to interpret parameter a and 50% managed to interpret parameter q. The 15% of learners out of the 34 were unable to interpret either a and q parameters. However, they selected option C from section B question 1. Clearly, these learners were experiencing systematic misconceptions and they probably guessed option C.

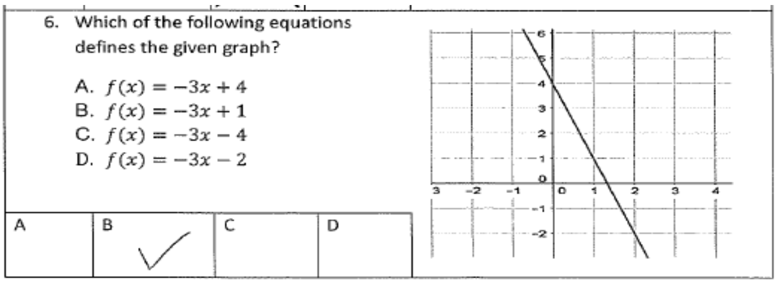
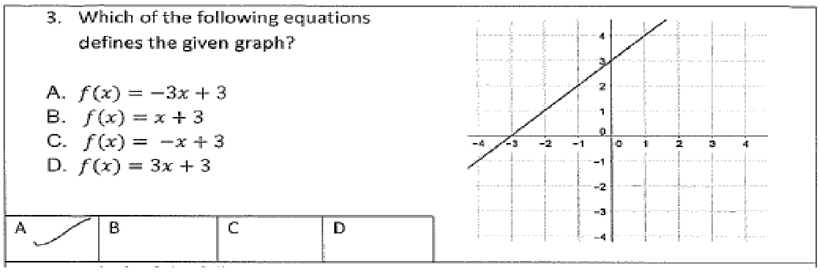
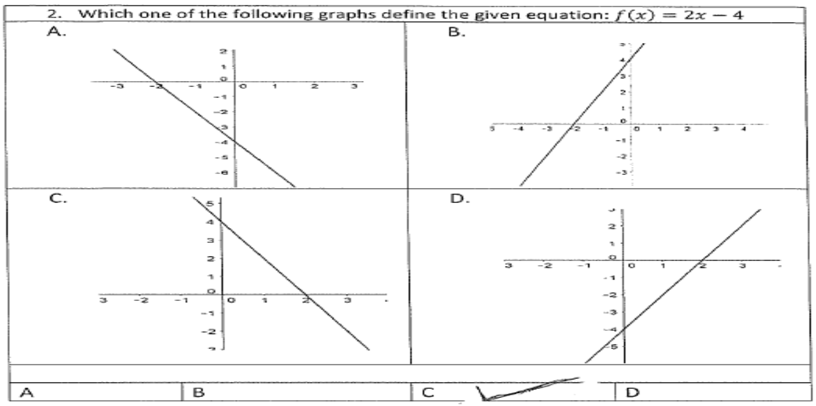
From the relation between the results in table 3 and table 2, it can be concluded that more learners experience systematic misconceptions related to the interpretation of parameter a than parameter q in a hyperbola function. Both figures are showing fewer learners understanding the effect of parameter a than parameter q. A possible reason for this is the fact that parameter q has only one effect, that of shifting the graph up and down which can be identified or observed graphically and algebraically. For example, if $q = -2$, the horizontal asymptote will cut the graph at $y = -2$ and also $q = -2$ will reflect in the equation, while the value of parameter a cannot be easily identified graphically. Parameter a is influenced by several factors which include the effects of sign and the quantity.

4.2.1.2. Linear function $f(x) = ax + q$

The data obtained from questions based on the interpretation of parameter a and parameter q in a linear function has shown that the majority of learners struggled to understand the effect of parameter a compared to the interpretation of parameter q in a linear function.

For example, table 4 below shows learner D12's solutions when working with linear function.

Table 4: Learner's linear function based extracts from pre-test

Sections	Effects of parameters	Solutions
Section A Question 6	q	 6. Which of the following equations defines the given graph? A. $f(x) = -3x + 4$ B. $f(x) = -3x + 1$ C. $f(x) = -3x - 4$ D. $f(x) = -3x - 2$ A | B | C | D
Section A Question 3	a	 3. Which of the following equations defines the given graph? A. $f(x) = -3x + 3$ B. $f(x) = x + 3$ C. $f(x) = -x + 3$ D. $f(x) = 3x + 3$ A | B | C | D
Section B Question 2	a and q	 2. Which one of the following graphs define the given equation: $f(x) = 2x - 4$ A. B. C. D. A | B | C | D
Section C Question 5	a and q	5. Which one of the following equations represents a linear function $y = ax + q$ with a slope/gradient given as 2, y intercept $(0,4)$ and x intercept$(-2,0)$? A. $y = -2x + 2$ B. $y = 4x - 2$ C. $y = 2x - 2$ D. $y = 2x + 4$ A | B | C | D

Learner D12's results of linear function showed that he or she experienced misconceptions towards the interpretation of both parameters a and q. Looking at his or her results in question 3, he or she selected option A in which parameter a is given as -3 yet, the given graph is an increasing linear function. Clearly, learner D12 was unable to distinguish between the effect of a negative parameter a and positive parameter a. His or her solution in question 6 also shows that he or she experienced difficulties when interpreting parameter q. The given linear graph in question 6 shows a decreasing line cutting the y axis at $+4$ which means that the equation for this function must reflect q value as $+4$. And yet, he or she selected an equation showing parameter q as $+1$. Furthermore, learner D12 experienced misconceptions when interpreting section B question 2 which was based on interpretation of both a and q parameters. His or her answer showed a decreasing linear graph cutting y at $+4$ yet, the given equation had $q = -4$. Lastly, Learner D12's results in section C question 5 verified that he or she experienced misconceptions towards the interpretation of linear function parameters. In this question, he or she could not identify both parameters a and q. The table 5 below shows the summarized linear function based results obtained from pre-test sections A and B, i.e., results based on the interpretation of parameter q, parameter a and parameters a and q for the whole sample (N=76). The correct responses are highlighted in boldface.

Table 5: Summary of linear function based results from pre-test sections A and B

N=76	**SECTION A**								**SECTION B**			
	QUESTION 3 (a)				**QUESTION 6 (q)**				**QUESTION 2 (a and q)**			
Correct option	**B**				**A**				**D**			
	A	**B**	**C**	**D**	**A**	**B**	**C**	**D**	**A**	**B**	**C**	**D**
Number of learners' responses per option	58	**11**	4	3	**61**	12	1	2	7	3	5	**61**
Learners' responses in percentages	76%	**15%**	5%	4%	**80%**	16%	1%	3%	9%	4%	7%	**80%**

The results showed that about 80% of learners have the ability to interpret the effect of parameter q, while only 15% of learners have the ability to interpret the effect of parameter a and about 80% of learners showed the ability to interpret both parameters a and q in a linear function.

Figure 18 below shows a summary of linear function based results obtained in the pre-test worksheet.

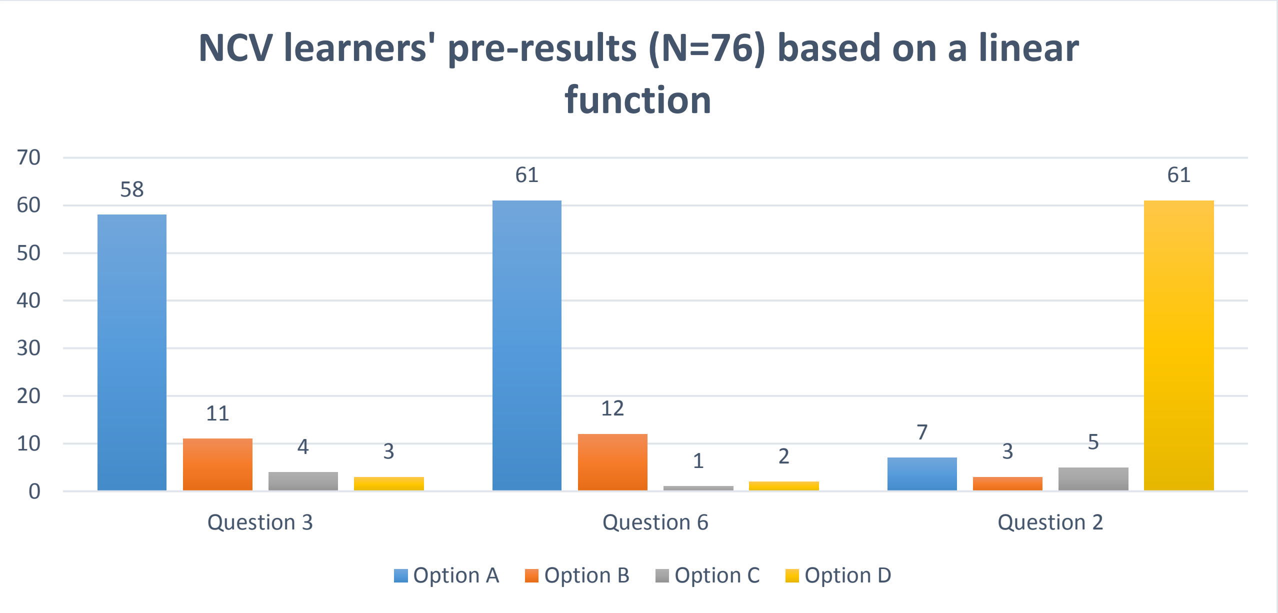


Figure 18: Showing a summary of a linear function results obtained in the pre-test.

Learners experienced systematic misconceptions towards the interpretation of parameter a. They could not distinguish between x intercept and parameter a. They assumed that the point at which the graph cuts at the x axis represents the value of parameter a. This type of misconception is systematic. Immediately a line passes a certain point in the x axis, learners think that the point represents parameter a. The possible reason for this misconception is the fact that parameter q is identified as a point that cuts the graph at the y axis.

Once again one expects that if learners understand the combined effects of a and q, then it is reasonable to expect that they understand individual effects of a and q respectively. For example, if 80% of learners managed to select the correct option in section B question 2 which was about the interpretation of both parameters a and q, they must have selected correct options in section A question 3 based on parameter a and question 6 based on parameter q. Once again, the data shows that this is not necessarily the case.

The table 6 below indicates how many of the 61 (80%) learners understood the effect of q in question 6 and the effect of a in question 3.

Table 6: Learners ability to interpret parameters a and q in a linear function

Analysis of learners (N=61) that selected option D from section B question 2				
Questions selected	**Able to interpret:**	**Unable to interpret:**	**Number of learners**	**%**
Q3 & Q6	a and q effects		9	15%
Q 3	a effect		2	3%
Q 6	q effect		42	69%
Q 3 & Q 6		a and q effects	8	13%

The results showed that from 61 learners, only 15 % showed the ability to interpret both parameters a and q in a linear function. About 3% of learners who showed the ability to interpret parameter a and 69% of learners were able to interpret parameter q. About 13% of learners selected section B question 2 correctly yet, they were unable to interpret neither parameter a or q from questions 3 and 6. Certainly, these learners experienced systematic misconceptions and they probably have guessed the answer in question 2 section B.

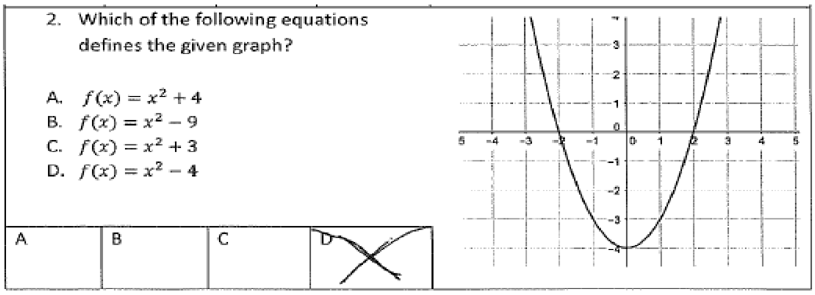
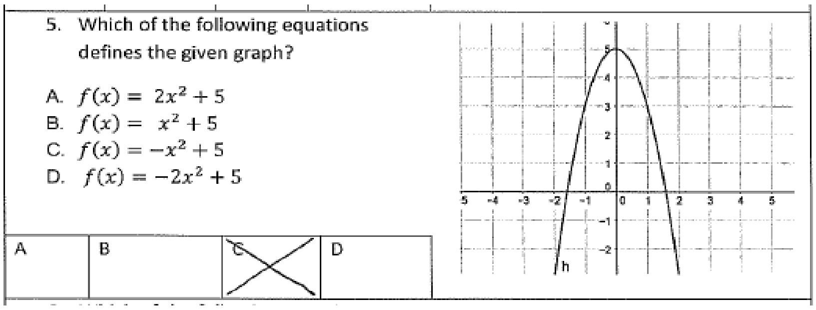
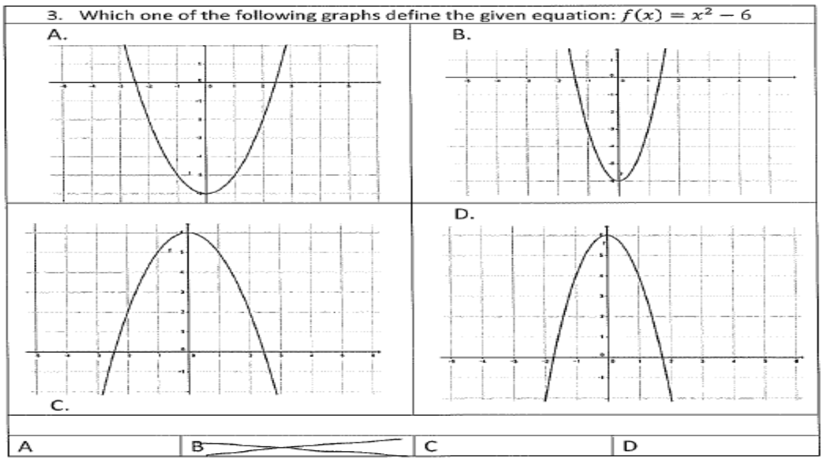
Comparing the results in table 5 and results in table 6 above, it is clear that more learners experienced misconceptions related with the interpretation of parameter a than parameter q in a linear function. The relation is that both tables show the smallest percentages of learners who showed the ability to interpret parameter a than parameter q in a linear function. Learners seem to be unable to distinguish between x intercept and parameter a. They assume that the point at which the graph cuts at the x axis represents the value of parameter a.

4.2.1.3. Parabola function $f(x) = ax^2 + q$

The results obtained from the parabola function showed that learners mostly experienced misconceptions when working with questions based on interpreting both the effect parameters a and q than interpreting a single parameter a and parameter q.

For example, table 7 below shows learner C13’s solutions based on parabola based questions.

Table 7: Learner's parabola function based extracts from pre-test

Sections	Effects of parameters	Solutions
Section A Question 2	q	 2. Which of the following equations defines the given graph? A. $f(x) = x^2 + 4$ B. $f(x) = x^2 - 9$ C. $f(x) = x^2 + 3$ D. $f(x) = x^2 - 4$ A | B | C | D
Section A Question 5	a	 5. Which of the following equations defines the given graph? A. $f(x) = 2x^2 + 5$ B. $f(x) = x^2 + 5$ C. $f(x) = -x^2 + 5$ D. $f(x) = -2x^2 + 5$ A | B | C | D
Section B Question 3	a and q	 3. Which one of the following graphs define the given equation: $f(x) = x^2 - 6$ A. B. C. D. A | B | C | D
Section C Question 6	a and q	6. The graphical representation of a quadratic function $y = x^2 - 9$ will have a: A. maximum turning point $(0; 9)$ B. minimum turning point $(0, -9)$ C. maximum turning point $(0; -9)$ D. minimum turning point $(0; 9)$ A | B | ~~C~~ | D

Learner C13 showed the ability to interpret parameter q in question 2 of section A. Furthermore, learner C 13 showed misconceptions related to the interpretation of parameter a and q in section B question 3. Option B that he or she selected shows that he or she struggled to relate the correct a value from the equation to the graph. The given equation has $a = 1$ and $q = -6$ which means that the graphical representation must be an increasing and broader parabola turning at $y = -6$ and cutting the x axis between ± 2 and ± 3. Learner C13's solution in this question verifies that he or she was experiencing systematic misconceptions related to the interpretation of parameter a in a parabola function.

The given graph is a decreasing parabola graph in section A question 5, which means that the value of parameter a is negative. Option C that learner C13 selected doesn't define the given graph. In a parabola function when $-1 \leq a \leq 1$ the graph becomes broader, which means that the graphical representation for option C would cut the x axis between ± 2 and ± 3. However, the given graph is cutting the x axis between ± 1 and ± 2 which makes option D correct.

Additionally, learner C13 was unable to transform a parabola equation to its graphical representation in section C question 6. His or her results showed that he or she experienced misconceptions related to the interpretation of parameter a. The table 8 below shows summarized parabola function based results obtained from pre-test sections A and B i.e. results based on the interpretation of parameter q , parameter a and parameters a & q; for the whole sample (N=76). The correct responses are highlighted in boldface.

Table 8: Summary of parabola function based results from the pre-test sections A and B

N=76	**SECTION A**								**SECTION B**			
Correct options	**D**				**D**				**A**			
	QUESTION 2 (q)				**QUESTION 5 (a)**				**QUESTION 3 (a and q)**			
	A	**B**	**C**	**D**	**A**	**B**	**C**	**D**	**A**	**B**	**C**	**D**
Number of learners' responses per option	4	1	1	**70**	2	2	1	**71**	**26**	44	3	3
Learners' responses in percentages	5%	1%	1%	**92%**	3%	3%	1%	**93%**	**34%**	57%	4%	4%

The results in the table 8 above show that about 92% of learners had the ability to interpret the effect of parameter q , 93% of learners had the ability to interpret the effect of parameter a and about 34% of learners had the ability to interpret the effect of parameters a and q. The figure 19 below shows a summary of parabola function based results obtained in the pre-test worksheet.

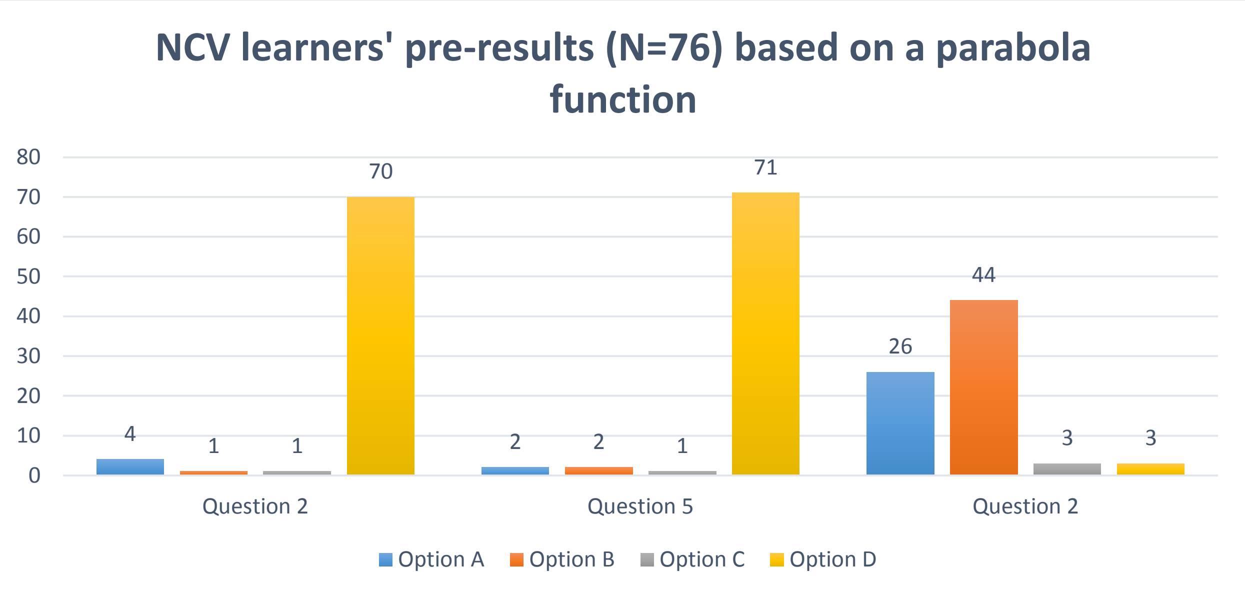


Figure 19: Showing a summary of parabola function results abstained in the pre-test.

Looking at table 8 and figure 19 above, the results obtained in section B question 3 show that the majority of learners experienced systematic misconceptions; only 26 (34%) learners managed to interpret both parameters a and q. Therefore, the researcher further investigated the 26 (34%) learners that selected option A in section C question 3 to determine the actual misconception learners' experienced. The researcher found that in section B question 3 the majority of learners selected option B instead of the correct option A. These options both show an increasing parabola curve turning at $y = -6$ but, option A is cutting the x axis between ± 3 and ± 2 and option B is cutting the x axis between ± 2 and ± 1. This means that most learners got confused with the effect of parameter a value.

The table 9 below indicates how many of the 26 (34%) learners understood the effect of q in question 2 and the effect of a in question 5.

Table 9: Learners' ability to interpret parameters a and q in a parabola function

Analysis of learners (N=26) that selected option A from section B question 3.				
Questions selected	**Able to interpret:**	**Unable to interpret:**	**Number of learners**	**%**
Q5 & Q2	a and q effects		3	12%
Q 5	a effect		13	50%
Q 2	q effect		10	38%
Q 5 & Q 2		a and q effects	0	0%

The results in table 9 above show that about 12% of learners selected correct options in section A question 2 and question 5. This means that, about 12% learners understood the concept of interpreting both parameters a and q in a parabola function. 50% of learners showed the ability to interpret parameter a and 38% of learners showed the ability to interpret parameter q. The relation of the results in table 8 and table 9 above is that both tables show that the majority of learners experienced systematic misconceptions when working with questions requiring the interpretation of the combined parameters a and q. Both table 8 and table 9 also show that fewer learners experienced misconception when interpreting parameter a and parameter q separately.

4.2.2. Pre-test section C data presentation and analysis

To verify the misconceptions identified from section A and section B, the researcher developed section C. This section is based on understanding the general concepts related to a and q parameters and also x and y intercepts in algebraic functions. The following table 10 shows a summary of results obtained in questions 1, 2 and 7. These questions were based on understanding general concepts in algebraic functions; thus, the concept of y intercept, x intercept and asymptotes. The correct responses are highlighted in boldface.

Table 10: Summary of results obtained in pre-test section C based on algebraic concepts

N =76	**SECTION C**											
	QUESTION 1				QUESTION 2				QUESTION 7			
Question	**Concept of y intercept q**				**Concept of x intercept**				**Concept of asymptotes**			
Correct option	**C**				**B**				**D**			
	A	B	**C**	D	A	**B**	C	D	A	B	C	**D**
Number of learners' responses per question	29	11	**27**	9	14	**39**	11	12	12	22	18	**24**
Learners, responses in %	38,2%	14,5%	**35,5%**	11,8%	18,4%	**51,3%**	14,5%	15,8%	15,8%	28,9%	23,7%	**31,6%**

Results from question 1 shows that 35.5% of learners understood the concept of y intercept. About 38.2% of leaners selected option A and 14.5% learners selected option B. These learners were experiencing unsystematic misconceptions They understood the idea of y intercept but were unable to differentiate between an intercept and a value. Algebraically, there are two intercepts x and y. To determine the x intercept, the value of y must be zero $(x, 0)$ and to determine the y intercept the value of x must be zero $(0, y)$. Furthermore, about 11.8% of learners selected option D. These learners experience systematic misconceptions when interpreting the y intercept.

The results from question 2 show that about 51.3% of learners who selected option B understood the concept of x intercept. Learners who selected options A, C and D were experiencing unsystematic misconceptions towards the understanding of x intercepts. Learners confused the x intercept with parameters a and q. The data obtained from this question relates to the data obtained in the interpretation of linear function in section A and section B. Learners understood parameter a as a point where the graph cuts the x axis.

The results from question 7 shows that about 28.9% of learners selected option B and 31.6% of learners selected option D and understood the concepts of asymptotes in a hyperbola function. Both options are correct. Therefore an average of 30.3% of learners understood the concept of asymptotes. About 15.8% of learners selected option A. This option only refers to a hyperbola graph that is positioned at the origin. Therefore, learners who selected option A were experiencing unsystematic misconceptions. About 23.7% of learners selected option C, which means that they were experiencing systematic misconceptions towards the understanding of the asymptotes in a hyperbola function. These learners confused the horizontal asymptote with parameter a in a hyperbola function.

The following table 11 shows a summary of results obtained from questions 3, 4, 5 and 6. These questions are based on the interpretation of parameters a and q in algebraic functions namely: linear function, hyperbola function and parabola function.

Table 11: Summary of results obtained in pre-test section C based on parameters a and q.

N =76	**SECTION C**															
	QUESTION 3				QUESTION 4				QUESTION 5				QUESTION 6			
Question	**q in a parabola**				**a and q in a hyperbola**				**a and q in a linear**				**a and q in a parabola**			
Correct option	**C**				**B**				**D**				**B**			
	A	B	**C**	D	A	**B**	C	D	A	B	C	**D**	A	**B**	C	D
Number of learners' responses per question	13	23	**35**	5	24	**30**	15	7	7	19	15	**35**	6	**35**	27	8
Learners, responses in %	17%	30.3%	**46.1%**	6.6%	31,6%	**39,5%**	19,7%	9,2%	9,2%	25%	19,7%	**46,1%**	7,9%	**46,1%**	35,5%	10,5%

The results from question 3 shows that about 46.1% of learners understood the effect of parameter q in a parabola function. About 17% of learners selected option A which means they were experiencing systematic misconceptions. These learners confused the effect of parameter a with parameter q. The 30.3% of learners selected option B experienced unsystematic misconception. These learners understood that when a parabola function shifts 3 units down, the value of q also reduces with 3 units.

However, they were unable to consider the original value of q. Instead they calculated from the origin. Selecting option D was the result of systematic misconception; about 6.6% of learners were unable to understand the effect of parameter q. These results relate to data obtained from parabola based questions in sections A and B. The majority of learners understood the effect of parameter q in a parabola function.

The results from question 4 show that 39.5% of learners selected option B. They understood that reflecting a hyperbola graph about the x axis impacts the sign of both parameters a and q. About 31.6% of learners selected option A and experienced unsystematic misconceptions, they struggled to interpret the effect of a sign. The 19.7% and 9.2% of learners who selected options C and D respectively, experienced systematic misconceptions. They were unable to reflect a hyperbola function about the x axis.

The results from question 5 show that about 46.1% of learners understood the role of parameters a and q in a linear function. About 9.2% of learners who selected option A and 19.7% of learners selected option C were experiencing systematic misconception. Learners confused the value of parameter a with x intercept. They also confused the value of parameter q with the value of parameter a. The 25% of learners that selected option B were experiencing systematic misconceptions by substituting the y intercept in a place of parameter a and replacing the value of parameter q with the x intercept value.

The results from question 6 show that 46.1% of learners understand the concept of transforming a quadratic equation to its graphical representation. About 10.5% of learners selected option D and experienced unsystematic misconception. These learners understood that the positive a value results in a parabola graph with a minimum turning point. However, they were unable to understand the effect of the sign of parameter q. About 7.9% of learners and 35.5% of learners selected options A and C respectively and experienced systematic misconceptions. Learners were unable to understand the effect of parameters a and q in a parabola function.

In summary, section C results clearly verified that learners experienced misconceptions related to the interpretation of algebraic functions.

4.2.3. The first focus group interview data transcription and analysis

As mentioned previously, to validate the misconceptions identified in the pre-test, the researcher conducted focus group interview considering parameters a and q in each function. Ten learners participated in focus group discussions that occurred immediately after the researcher had analyzed pre-test worksheet; learners were separated into two groups of five. The first group participated in the focus group interview based on understanding the concept of parameter q from all the three functions. The other five learners participated in the focus group interview based on understanding the concept of parameter a in each function. The following table 12 shows times and dates in which focus groups discussions took place.

Table 12: The first focus group interview scheduled times and dates

FOCUS GROUP INTERVIEW TIME TABLE		
Group 1		
Dates	**Themes**	**Duration**
20/6/2017	Effect of parameter q in general	11 minutes 27 seconds
	Effect of parameter q in linear function	3 minutes 51 seconds
	Effect of parameter q in parabola function	3 minutes 8 seconds
	Effect of parameter q in hyperbola	6 minutes 33 seconds
Group 2		
21/06/2017	Effect of parameter a in linear function	5 minutes 30 seconds
	Effect of parameter a in parabola	8 minutes 55 seconds
	Effect of parameter a in hyperbola	10 minutes 35 seconds

4.2.3.1. Focus group interview based on parameter q

Based on the pre-test results, learners experienced unsystematic misconceptions related to the interpretation of parameter q particularly when working with linear function and parabola function. In all the data obtained from the three algebraic functions, most learners experienced systematic misconceptions when working with parameter a.

One of the few selected learners who frequently showed misconceptions related to the interpretation of parameter q included: learner C15, learner A8, learner B3, learner B9 and learner D9. All learners were provided with a copy of focus group interview questions and a copy of a pre-test worksheet to refer to during the discussion. The focus group discussion began with the effect of q in general, thereafter the effect of parameter q in each algebraic function. Each discussion interview took a maximum of 23 minutes. The focus group interview was conducted in the following order.

Introduction: The general effect of parameter q in algebraic functions

The researcher asked learners to discuss about the y intercept in general by referring to the graphs on worksheets. The following are samples of the learners' responses.

B9: "I don't know…"

A8: "q represents a turning point"

C15: "I think q represents axis of symmetry"

The researcher tried to breakdown the question by referring the learners to algebraic graphs provided in the worksheet. However, the misunderstanding amongst learners persisted. It came to the researcher's attention that perhaps learners were lacking basic knowledge towards parameter q, such as the understanding of the Cartesian system of co-ordinates. The researcher asked learners in which line between the x-axis and y-axis represents parameter q. This question helped learners to be able to identify the value of parameter q graphically. The parameter q based discussion continued further. Learners were also referred to each function provided in the worksheet to identify the value of parameter q. It was clear that learners mostly experienced misconceptions related to the interpretation of parameter q in a hyperbola function compared to linear and hyperbola functions. The possible reason for this could be the fact that the hyperbola function has no intercepts. The graph does not touch both x and y intercepts unless it has shifted. Therefore, the horizontal asymptote represents parameter q unlike in a linear or hyperbola where q is the y value where the graph cuts the y axis.

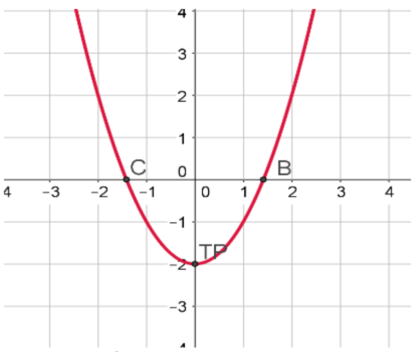
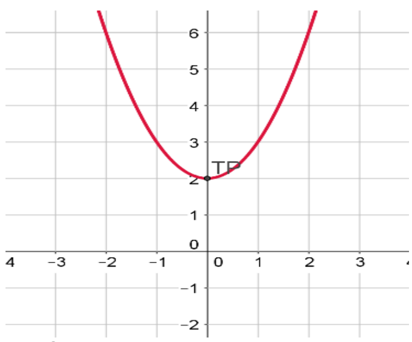
The effect of parameter q in a linear function $y = ax^2 + q$:

The researcher introduced the interview based on y intercept q in a linear function by asking learners to give their views about the effect of q. Learners were unable to describe the effect of parameter q in a linear function. However, they could identify the value of parameter q graphically and algebraically. Therefore, learners had the ability to mention the value of q from the linear equation or from the graph but, they were unable to explain the effect of q in a linear function.

The parameter q in a parabola function $y = ax^2 + q$:

Learners seem to recognize that parameter q is the point in the y axis at which the graph is cutting or turning. However, some learners confused the effect of parameter q with the effect of parameter a in a parabola function. During the interview, some learners mentioned that the negative value of q results in a decreasing parabola graph. These learners experienced systematic misconceptions concerning parameter q. The researcher sketched increasing parabola graphs with different turning points to make learners understand the effect of parameter q in a parabola function. The following table 13 shows the two increasing parabola graphs with different turning points.

Table 13: Two increasing parabola graphs with different turning points

 A. $y = x^2 - 2$	 B. $y = x^2 + 2$

The researcher instructed learners to refer to graphs in the figure above concerning to the effect of parameter q towards the shape of the graph. Learners realized that changes in the sign of parameter q do not turn the graph upside-down instead it shifts the graph up or down.

The parameter q in a hyperbola function $y = \frac{a}{x} + q$:

The researcher began the interview based on the effect of parameter q in a hyperbola function by referring learners to hyperbola graphs in the worksheet, asking learners to identify the value of q from the graph. Learners were able to recognize the correct value of q from the graph. Thereafter, the researcher asked learners how they could identify parameter q considering that the hyperbola function does not touch both x and y intercepts (q). They were unable to explain how to identify parameter q from a hyperbola function. The researcher had to break down the question by referring them to hyperbola graphs in the pre-test worksheet.

> **Researcher:** "Identify the value of q from section A question 4 and explain why do you think the value represents q"
>
> **A3**: "I saw the line $y = 2$, I think it represent parameter q".
>
> **B9:** "the asymptote line represent the value of q"
>
> **C15:** "the line is cutting $y = -2$ which mean $q = -2$"

Learners seem to hesitate about understanding the concept of a horizontal asymptote in a hyperbola function. Only 3 learners showed the ability to interpret the effect of parameter q in a hyperbola function.

In summary, the focus group discussion results from the above transcriptions helped the researcher to verify learners' misconceptions experienced during the pre-test worksheet based on interpreting parameter q in algebraic functions. The focus group interview result regarding parameter q shows that learners could identify the value of parameter q; however, they were unable to explain and interpret the effect of q in algebraic functions. Therefore, these results confirm that learners experienced unsystematic misconceptions based on interpreting parameter q.

4.2.3.2. Focus group discussion based on parameter a

The researcher randomly selected only five NCV learners who frequently experienced misconceptions related to the interpretation of parameter a from a pre-test worksheet to participate in focus group discussion. The discussion took a maximum of 25 minutes and was based on the effect of parameter a in each algebraic function. The following learners participated in a focus group discussion: C10, A7, B20, D25 and D2. The focus group interview was conducted in the following order.

The effect of parameter a in a linear function $y = ax + q$:

The researcher began the interview by asking learners to explain what effect negative and positive a has in a linear graph. Learners were able to respond to this question without showing any cognitive challenge. Learners mentioned that when parameter a is positive, the graph increases and when parameter a is negative, the graph decreases. However, learners struggled to transform linear function during pre-test activity. The researcher wanted to ratify whether learners understand the effect of parameter a in a linear function by asking them to differentiate between the increasing and decreasing linear graphs given in the pre-test worksheet. They were unable to identify the difference between an increasing and a decreasing straight line graph.

Therefore, it is possible that learners were only associating a negative a value with decrease and associating a positive a value with increase without understanding the effect of parameter a graphically.

The effect of parameter a in a parabola function $y = ax^2 + q$:

During a parabola based discussion, learners agreed that the sign of parameter a has an impact on the shape of the graph. The researcher asked what effect does parameter a has on a parabola function. The following are learners' responses:

> **B20**: "when a is negative, the curve faces up and when $a$is positive, the curve faces down".
>
> **A7**:"when a is negative is a smile face and when a is positive is a sad face"

In the above transcripts, it is clear that learners were showing misconceptions related to the effect of the sign of parameter a in a parabola function. The researcher referred learners to a parabola graph facing downwards in the worksheet and asked what they thought was the sign of parameter a in the given decreasing graph. Learners continued to have arguments about this question. Observing learners during this session, I found that learners understood that the sign of parameter a can result in a parabola graph facing up with a minimum turning point or facing down with a maximum turning point.

However, learners were unable to relate the correct sign of a to the relevant parabola graph.

The effect of parameter a in a hyperbola function $y = \frac{a}{x} + q$:

The researcher began by asking learners to explain what effect a value has in the hyperbola graph. Learners struggled to explain the effect of parameter a in a hyperbola function. The researcher had to refer learners to some hyperbola questions from the worksheet and asked learners to identify the value of a from each equation. Eventually, learners were able to identify values of a from hyperbola equations; however, they struggled to relate a values to their graphical representations. For example, the researcher asked learners to explain what effect a negative and positive a value has on the shape of the graph. They struggled to explain this. Furthermore, learners were asked to tell the difference between a graph with $a = 1$ and a graph with $a = 3$. Learners experienced systematic misconceptions towards the interpretation of parameter a in a hyperbola function. The results obtained from the focus group discussion verified learners' misconceptions identified from the pre-test worksheet.

To sum up, linking the results obtained from a pre-test worksheet with the results from the focus group discussion, it is clear that learners experienced misconceptions based on interpreting algebraic functions. The results confirm that learners experienced systematic misconceptions when interpreting parameter a in algebraic functions. Most learners showed the ability to identify parameter q but, they were unable to explain the effect of parameter q in algebraic function.

Therefore, the researcher introduced sociocultural learning to approach learners' misconceptions towards algebraic functions. The researcher integrated GeoGebra supported by scaffolding and collaboration. The following section explains in depth about how learning occurred when GeoGebra software was integrated with the aim of dealing with learners misconceptions in algebraic functions.

4.3. GEOGEBRA INTEGRATION ANALYSIS

According to Siyepu (2013), Vygotsky's sociocultural theory defines effective learning environment as the one in which there is an active interaction between teachers and learners. In an active classroom, teachers provide all possible tools to assist learners to develop meaningful understanding of the subject content. Vygotsky's theory further specifies that learners become challenged when they reach a stage where they are unable to solve problems. Through scaffolding, teachers provide support by means of introducing tools to assist in structuring ideas and concepts (Denhere, Chinyoka, & Mambeu, 2013). Based on the misconceptions identified, the researcher found it significant to integrate GeoGebra software as a dynamic modelling, problem solving, conceptual and cognitive tool to assist learners to develop algebraic based concepts.

The researcher summarized the misconceptions identified into categories. The following figure 20 shows categorized misconceptions identified in a linear function $f(x) = ax + q$, parabola function $f(x) = ax^2 + q$ and hyperbola function $f(x) = \frac{a}{x} + q$ during the pre-test and how GeoGebra was used as an approach to each of these misconceptions.

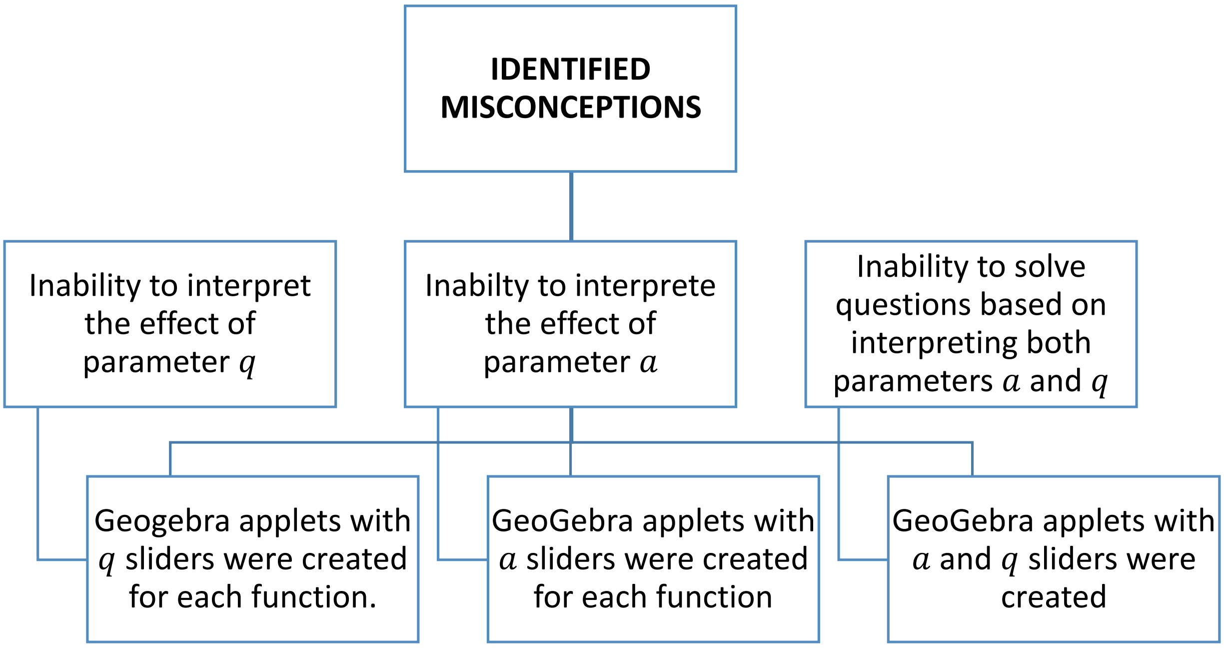


Figure 20: Categorized learners' misconceptions.

The figure 20 above clearly shows that there were three misconceptions and each of these included both systematic and unsystematic misconceptions.

The process of integrating GeoGebra took a period of a month (Appendix 9). Lessons took place every Tuesday and Friday. One of the four groups attended every Tuesday of the week and the other three groups had their sessions on Friday. For each group, the integration process took a maximum of 4 days. The data collected from the process of integrating GeoGebra answered the second research question of this study. The integration stage was guided by a three section worksheet with 26 questions. The researcher followed the three phases of pedagogical framework during the integration process. During session 1, the researcher introduced GeoGebra by means of demonstrations. Session 2 and session 3 learners were actively utilizing GeoGebra applets and finally, during session 4, learners were creating their own algebraic functions and reflecting on algebraic concepts learnt in previous sessions.

4.3.1. Session 1

The first lesson of integrating GeoGebra was the introduction phase, where the researcher introduced GeoGebra software to NCV learners. Learners were all seated in front of their computers in a computer lab. Each and every computer had a GeoGebra shortcut created on the desktop after installation and a folder with different GeoGebra applets arranged according to the intervention worksheet. However, during the first sessions, learners did not utilize GeoGebra applets that were created. The researcher began the introduction stage by showing learners the GeoGebra shortcut through demonstrating on a data projector big screen. The researcher prepared an applet showing all the three algebraic functions on an algebra window and a and q sliders.

The following figure 21 shows an introduction applet displayed on a data projector screen.

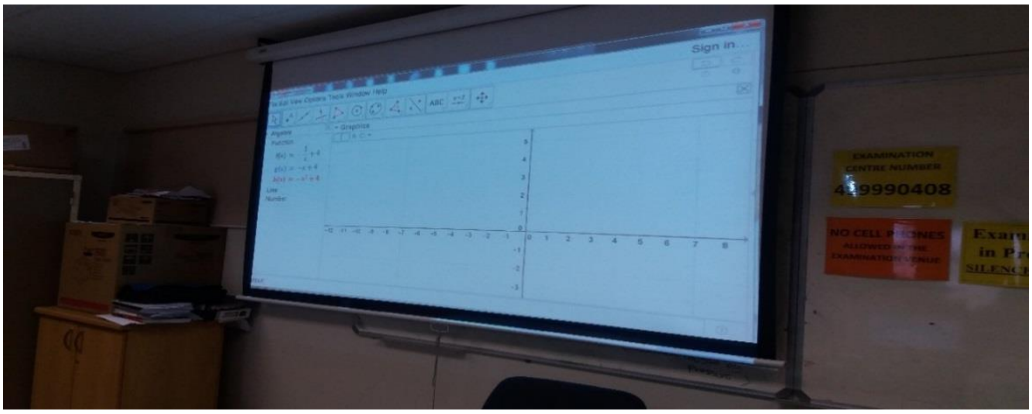


Figure 21: Introduction applet created

Therefore, the researcher demonstrated to learners the difference between algebra view and graphs view and how the manipulation of sliders can simultaneously affect the equation together with the graph. The demonstration started by showing linear function while other functions were hidden from the graphics view. I moved sliders a and q changing their values and asked learners what they could observe. One of the learners' responses was that changing the value of parameter q or a affects both graph and equation. The data projector was used to show learners how to manipulate sliders to create different algebraic functions.

The following figure 22 is the created applet showing hyperbola function reflecting on a data projector screen.

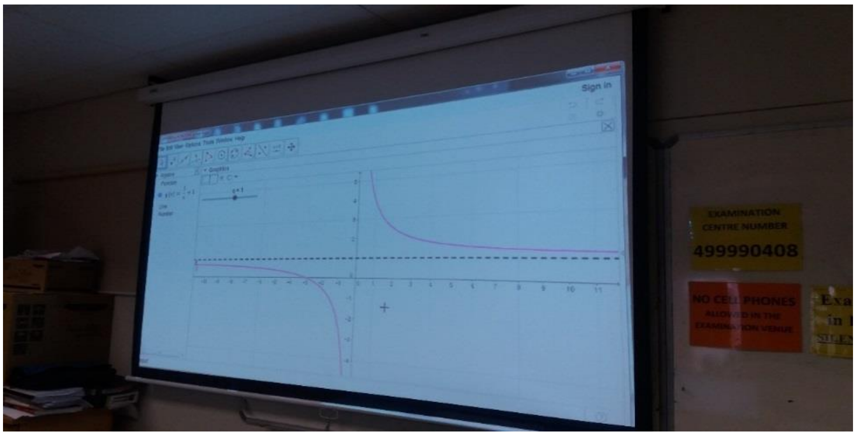


Figure 22: A hyperbola function on a data projector screen.

Furthermore, while demonstrating algebraic functions on a data projector screen, the researcher constantly kept learners' attention by asking them general questions such as what effect the parameter a or q has in each function. Learners showed enthusiasm during this phase. They were excited to observe the dynamical transformation of functions.

4.3.2. Session 2 and Session 3

Learners began to actively engage with GeoGebra applets. This process took them two different days to complete. Learners were required to engage with worksheet instructions in order to work with applets created. The reason for providing a worksheet was to support learners' engagement with GeoGebra applets (Appendix 7). The questions in the worksheet were developed considering learners' misconceptions identified from a pre-test activity. Therefore, the nature of the applets was intended to influence learners' understanding of algebraic concepts. The researcher separated sliders a and q based questions in sections A and B because she wanted learners to spend enough time working on each parameter and understand the effect of each parameter. This resulted in learners taking two different days to complete sections A and B.

Section A of the worksheet was based on manipulating parameter q only and we observed the changes in each function. An applet for each function was created for section A. The applets only showed parameter q, equation and the graph. For each function, learners were asked to change the value of q three times and record the effect of q for each function.

For example, learners would be given a parabola function as $y = x^2 - 4$ and be required to make $q = -2$, $q = 0$ and $q = 4$. For each resulting function per q value, learners were instructed to plot and record their observations.

The following figure 23 shows a snapshot taken from Learner C 20’s intervention worksheet.

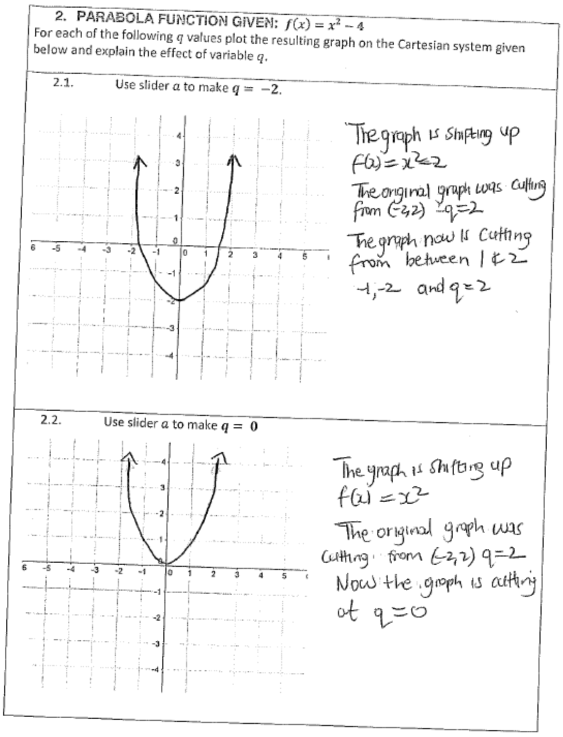


Figure 23: Learner C20’s response when engaging with applets.

The above extract shows learner C 20’s response when asked to make $q = -2$. This learner drew the resulting graph and explained his observation. He stated that the graph has shifted up from $q = -4$ to $q = -2$, he or she also mentioned that the new equation is $f(x) = x^2 - 2$.

Section B of the worksheet was based on manipulating parameter a in each algebraic function. Each function had three different a values provided; learners were required to draw the resulting function after changing the value of a and record their observations.

During the lesson, the researcher moved around to observe learners and also provided assistance where needed. Such assistance would include help with technical problems learners experienced. The researcher instructed learners to assist each other freely where needed. Learners appeared to be excited when working with the applets. Those who completed their task first helped others with excitement.

The following figure 24 shows learners working on the GeoGebra applets and recording their observations.

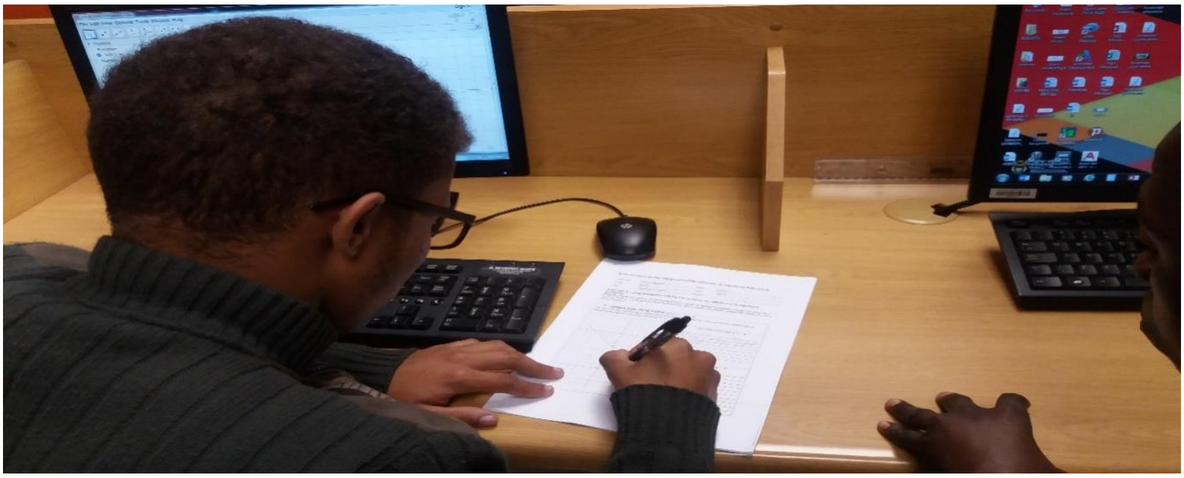


Figure 24: NCV learners utilizing GeoGebra applets.

4.3.3. Session 4

During the fourth session, learners continued with their worksheets, most learners began their section C. In section C, learners were expected to create their own functions using electronically created GeoGebra applets sliders (without functions given) and reflect on those functions. The researcher guided the process of creating algebraic functions. She instructed all learners to open section C folder with GeoGebra applets created. Furthermore, the researcher also displayed the same applet on the data projector screen and created the first function according to the worksheet instructions. Meanwhile, learners were also creating their function as per worksheet instructions. Furthermore, section C required learners to reflect on the functions they created such as explaining their observations when a and q values change. Learners were also required to state whether there is any relationship between these two variables. They had to ask the question: does changing of variable a have any effect on variable q and vice versa? These questions required learners to use GeoGebra as a dynamic, problem solving, conceptual and cognitive tool. This section of the worksheet required learners to observe the changes critically from GeoGebra applets as they dynamically move parameters a and q.

While observing learners during the integration stage, they seem to be excited in engaging with GeoGebra. They spent their time dragging functions through using a and q sliders and identifying the effects of each parameter in both graphic and algebra views. This stage can be linked with the third phase of the pedagogical framework wherein the role of learners shifts from being knowledge transmitters to becoming more like facilitators for each other. Learners reflected from the concepts learnt in section A and section B. Furthermore, the learning environment enables the sociocultural learning Learners are working together actively utilizing GeoGebra developing algebraic based concepts. Observing learners at this stage, I found that when learners were dynamically manipulating applets, they were developing an understanding towards concepts around algebraic functions.

For example, in this section, learners were also asked to draw rough sketches of each function by considering the value of a and q from the equation given. This question was developed in such a way that learners could not use sliders because the values of a and q were limited from the sliders created.

At this stage of learning, learners were expected to have internalized the concepts and skills that enable them to work without assistance; such as using mediation tools and working in collaboration. Learners should be able to complete the task independently after they have mastered concepts with the assistance from competent others (Denhere, Chinyoka, & Mambeu, 2013). Learners showed the ability to construct functions without using sliders, which provides evidence about the effectiveness of GeoGebra in developing algebraic concepts.

The following figure 25 shows learner's response in the intervention worksheet question 3.2 of section C (Appendix 7).

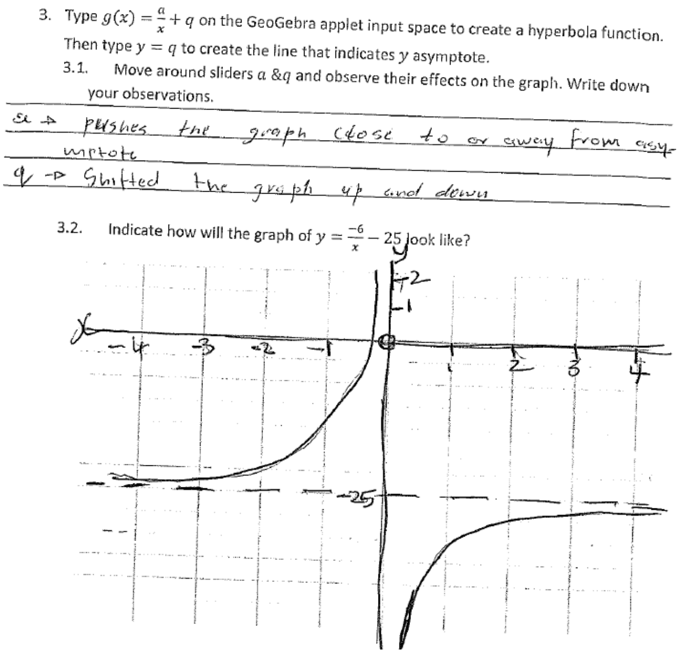


Figure 25: Learner A5's response in Section C question 3.2.

Learners were asked to draw a rough sketch of the hyperbola equation given as $y = -\frac{6}{x} - 25$. The researcher expected a hyperbola graph cutting $y = -25$ and occupying the second and fourth quadrants. Parameters a and q sliders did not go as far as ± 6 and ± 25 respectively. In that way, scaffolding was taken away by not having been able to use the sliders. Yet, learner A5 showed ability to understand the impact of parameters a and q even though the learner was limited from using GeoGebra when answering this question (figure 25).

This stage provided further evidence of whether the integration of GeoGebra enhanced learners' understanding in function transformation or not. The data shows that learners were able to transform functions without using GeoGebra as an assistance tool. In addition, this stage can be associated with the third stage of the Zone of Proximal Development (ZPD). During stage III, learners grow in knowledge through practice It is a stage where learners no longer need assistance. The action is internalized, and no longer requires extra effort (Vygotsky, 1978).

Additionally, during the integration of GeoGebra, learners were effectively working with created sliders to understand the concepts around algebraic functions. At this stage, learners were working in collaboration. Those who understood better helped those that were still struggling. The following figure 26 show learners working in colloboration while engaging with GeoGebra applets.

Figure 26: Learners engaging with applets and working in collaboration.

Denhere, Chinyoka and Mambeu (2013, p.373) state that “it is important to understand that a learner is able to perform a certain task alone, while in collaboration, is able to perform a greater number of tasks”. Observing learners during the GeoGebra intervention, they seem to understand algebraic concepts mostly when working with their peers in collaboration.

During the integration of GeoGebra software, learners seem to understand the interpretation of parameters a and q and the dynamic transformation between algebraic functions. Based on researcher’s observations during GeoGebra integration, effective learning was taking place. GeoGebra applets helped learners to understand algebraic concepts that they did not understand prior the integration. Learners transformed algebraic functions and they used sliders a and q to dynamically to shift, increase or decrease algebraic graphs.

Comparing the integration stage with a pre-test stage, the integration of GeoGebra enabled learners to work effectively in collaboration and to understand the effect of each parameter and dynamically transform algebraic functions.

4.4. POST-TEST ANALYSIS

This stage of data collection occurred towards the end of the study. Both quantitative and qualitative instruments were used to collect data during this phase. The data was collected by means of learners answering the post-test worksheet and participating in the focus group interviews.

4.4.1. Data presentation obtained in the post-test

The researcher used tables and charts to represent data collected from a post-test questionnaire. The post-test was completed by learners after they had engaged with GeoGebra applets in the computer lab. The aim of the post-test questionnaire was to determine whether learners were able to apply algebraic concepts learnt during GeoGebra intervention. Furthermore, the post-test questionnaire was intended to identify whether the integration of GeoGebra helped learners to develop conceptual understanding of algebraic functions and to overcome their misconceptions. The questions in the post-test worksheet are numerically different from pre-test worksheet questions; however, both worksheet questions were testing the same cognitive skills. Similar to the pre-test, the post-test worksheet consisted of three sections. Section A consisted of three questions based on transforming algebraic equations to their graphical representations concerning the effects of parameters a and q. Section B consisted of six questions based on transforming graphical representations to their correct equations concerning the effects of parameter a and q. Section C consisted of three questions based on algebraic concepts related to the understanding of parameters a and q.

Generally, the results obtained from a post-test questionnaire showed evidence that the engagement of GeoGebra helped learners develop better understanding of algebraic concepts. Unlike in the pre-test, only 75 learners participated in a post-test because learner D14 dropped out.

The following table 14 presents a summary of data obtained from a post-test worksheet based on linear function $f(x) = ax + q$. The correct responses are highlighted in boldface.

Table 14: Linear function post-test results

N=75	**SECTION A: Transfer of graph to an equation**				**SECTION B: Transfer of equation to a graph**								**SECTION C: Transfer between equations & graphs**			
	QUESTION 1: $\boldsymbol{a}$ & $\boldsymbol{q}$				**QUESTION 3: $\boldsymbol{a}$**				**QUESTION 6: $\boldsymbol{q}$**				**QUESTION 2: $\boldsymbol{a}$ & $\boldsymbol{q}$**			
Correct option	**B**				**A**				**B**				**C**			
	A	**B**	C	D	**A**	B	C	D	A	**B**	C	D	A	B	**C**	D
Number of learners' responses per option	0	**72**	3	0	**63**	1	0	11	3	**61**	6	5	5	2	**68**	0
Learners' responses in percentages	0%	**96%**	4%	0%	**84%**	1%	0%	15%	4%	**81%**	8%	7%	7%	2%	**91%**	0%

The results show that 96% of learners in section A and 91% of learners in section C understood the interpretation of parameters a and q in linear function based questions. Therefore, these two percentages make an average of 93.5% of learners who showed the ability of interpreting parameters a and q. About 84% of learners understood the effect of parameter a and 81% of learners understood the effect of parameter q in a linear functions.

The following table 15 presents a summary of data obtained from a post-test worksheet based on a parabola function $f(x) = ax^2 + q$. The correct responses are highlighted in boldface.

Table 15: Parabola function post-test results

N=75	**SECTION A: Transfer of graph to an equation**				**SECTION B: Transfer of equation to a graph**								**SECTION C: Transfer between equations & graphs**			
	QUESTION 2: a & q				**QUESTION 2: a**				**QUESTION 5: q**				**QUESTION 1**			
Correct option	**D**				**C**				**D**				**A**			
	A	B	C	**D**	A	B	**C**	D	A	B	C	**D**	**A**	B	C	D
Number of learners' responses per option	1	2	6	**66**	3	0	**70**	3	0	9	4	**69**	**71**	1	1	2
Learners' responses in percentages	1,3%	3%	8%	**88%**	4%	0%	**93%**	4%	0%	12%	5,3%	**92%**	**95%**	1,3%	1,3%	2,7%

The results showed that 88% of learners in section A and 95% of learners in section C understood the interpretation of parameters a and q in parabola function based questions. Therefore, two percentages make an average of 91.5% of learners showed the ability of interpreting parameters a and q. About 93% of learners understood the effect of parameter q and 92% of learners understood the effect of parameter q in a parabola function.

The following table 16 presents a summary of data obtained from a post-test worksheet based on a hyperbola function $y = \frac{a}{x} + q$. The correct responses are highlighted in boldface.

Table 16: Hyperbola function post-test results

N=75	**SECTION A: Transfer of graph to an equation**				**SECTION B: Transfer of equation to a graph**								**SECTION C: Transfer between equations & graphs**			
	QUESTION 3: a & q				**QUESTION 1: a**				**QUESTION 4: q**				**QUESTION 3: a & q**			
Correct options	**D**				**A**				**B**				**D**			
	A	B	C	**D**	**A**	B	C	D	A	**B**	C	D	A	B	C	**D**
Number of learners' responses per option	0	0	7	**68**	**67**	3	0	5	2	**73**	0	0	0	4	1	**70**
Learners' responses in percentages	0%	0%	9,3%	**90,7%**	**89,3%**	4%	0%	6,7%	2,7%	**97,3%**	0%	0%	0%	5,3%	1,3%	**93,3%**

The results showed that 90.7% of learners from section A and 93.3% of learners from section C understood the interpretation of parameters a and q in hyperbola function based questions. Therefore, two percentages make an average of 92% of learners showed the ability of interpreting parameters a and q. About 89.3% of learners understood the effect of parameter a and 97.3% of learners understood the effect of parameter q in a linear functions.

4.4.2. Data analysis obtained in the post-test

As mentioned previously, after learners had worked in collaboration using GeoGebra applets in a computer lab and mastered algebraic concepts, they completed a post-test worksheet independently without getting help from others. Effectively, this takes away the scaffolding in the form of help from the competent peers. The researcher intended to determine the effectiveness of GeoGebra software i.e. the difference between what learners could do after the integration of GeoGebra software and what they could not do before GeoGebra integration. The following three examples compare the pre-test and post-test responses of individual learners to show the difference GeoGebra has made. The following table 17 shows learner A11's pre-test and post-test solutions based on a linear functions.

Table 17: Learner A11 pre-test & post-test results based on 2 linear function questions

<table>
<tr>
<td>5. Which one of the following equations represents a linear function $y = ax + q$ with a slope/gradient given as 2, y intercept $(0,4)$ and x intercept$(-2,0)$?
A. $y = -2x + 2$
B. $y = 4x - 2$
C. $y = 2x - 2$
D. $y = 2x + 4$

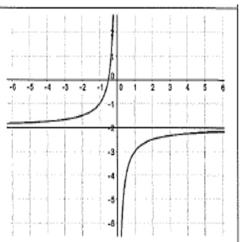
| A | B | C X | D |</td>
<td>2. The following defines the graphical representation of a linear equation given as $y = -3x + 5$:
A. Increasing line cutting at $y = 5$
B. Decreasing line cutting $y = -5$
C. Decreasing line cutting $y = 5$
D. Increasing line cutting $y = -5$

| A | B | C X | D |</td>
</tr>
</table>

Regarding a pre-test extract, learner A11 showed misconceptions related to the understanding of intercepts. He or she was unable to distinguish between x and y intercepts. Learner A11's performance clearly indicated that she/he needed assistance when working with a linear function, hence he or she interacted with GeoGebra applets and worked with others in collaboration. After learner A11 had interacted with GeoGebra, she showed the ability to interpret the effects of parameters a and q from a linear function. Looking at her post-test solution, he or she was able to analyze and interpret the effects of parameters a and q. The following table 18 shows learner D16's pre-test and post-test solutions based on a hyperbola function.

Table 18: Learner D16 pre-test & post-test results based on 2 hyperbola function questions

<table>
<tr>
<td>4. Which of the following equations defines the given graph?
A. $f(x) = \frac{-3}{x} - 2$
B. $f(x) = \frac{2}{x} - 2$
C. $f(x) = \frac{1}{x} - 2$
D. $f(x) = \frac{-1}{x} - 2$

| A | B ✓ | C | D |
</td>
<td>3. The following defines the graphical representation of a hyperbola equation given as $y = -\frac{1}{x} - 4$:
A. The graph occupies second and fourth quadrants with horizontal asymptote cutting $y = 4$
B. The graph occupies first and third quadrants with horizontal asymptote cutting $y = -4$
C. The graph occupies first and third quadrants with horizontal asymptote cutting $y = 4$
D. The graph occupies second and fourth quadrants with horizontal asymptote cutting $y = -4$

| A | B | C | D ✓ |</td>
</tr>
</table>

Comparing learner D16's pre-test and post-test solutions based on a hyperbola function; it is clear that she/he experienced misconceptions related to the interpretation of parameter a during a pre-test task. Learner D16 was unable to interpret the effect of parameter a in a hyperbola function. His or her calculations confirm that there was a need for a researcher to come up with alternative interventions in order to assist him to develop algebraic concepts. Through consistence, group work and use of GeoGebra applets, learner D16 moved to the third stage of ZPD. In the absence of scaffolding, she/he independently showed the ability to solve hyperbola based problems even without utilizing GeoGebra applets. Learner D16 has now developed the concepts related to the interpretation of a hyperbola function.

The following table 19 shows learner D21's pre-test and post-test solutions based on a parabola functions.

Table 19: Learner D21 pre-test & post-test results based on 2 parabola function questions

6. The graphical representation of a quadratic function $y = x^2 - 9$ will have a: A. maximum turning point $(0; 9)$ B. minimum turning point $(0, -9)$ C. maximum turning point $(0; -9)$ D. minimum turning point $(0; 9)$ A | B | C | D	1. The following defines the graphical representation of a parabola equation given as $y = -2x^2 - 8$: A. Decreasing graph turning at $y = -8$ B. Increasing graph turning at $y = 8$ C. Decreasing graph turning $y = 8$ D. Increasing graph turning at $y = -8$ A | B | C | D

Learner D21's pre-test solution indicate that she/he experienced misconception based the interpretation of parameter a and she/he was able to interpret the effect of parameter q. Judging from learner D21's post-test solution, she/he had probably put much focus on understanding the effect of parameter a when interacting with GeoGebra applets. The post-test results showed that she/he developed the understanding of parameter a effect in a hyperbola graph however, she/he could not understand the influence of parameter q. Learner D21's post-test solution confirms that as much as she/he had showed understanding of parameter a effect, the learner needed more assistance to develop conceptual understanding.

4.5. PRE-TEST AND POST-TEST DATA ANALYSIS

Generally, comparing the results obtained from a pre-test and post-test; the post test results showed an increase in the number of learners who showed conceptual understanding in algebraic functions. The following table 20 shows a summary of results obtained from a pre-test and post-test based on a linear function.

Table 20: Comparison of pre-test and post-test results based on a linear function

Linear Function	Pre-test	Post-test	Variance
Effect of a and q	80.3%	93.5%	+13.2%
Effects of a	14,5%	84%	+69.5%
Effects of q	80.3%	81%	+1%

The 80.3% of learners that managed to interpret the effect of both parameter a and q in a linear function increased to 93.5% in the post-test. The results showed an increase from 14.5% to 84% of learners that understood the interpretation of parameter a in a linear function. The number of learners who showed the ability to interpret parameter q increased from 80.3% to 81.3%. The possible reason for the +1% increase in the interpretation of parameter q is that learners understood the effect of parameter q better than parameter a. The pre-test results seem to confirm that. Therefore, the +69.5% increase confirms that learners' engagement with GeoGebra helped them to understand parameter a better.

The increase in the number of learners who showed the ability to interpret the effects of parameters a and q in a linear functions confirms the effectiveness of GeoGebra. The Vygotsky's sociocultural theory had a positive influence; through working in collaboration and interacting with GeoGebra applets, learners developed cognitive skills to understand algebraic concepts.

The following 27 indicates a summary of pre-test and post-test results based on a linear function.

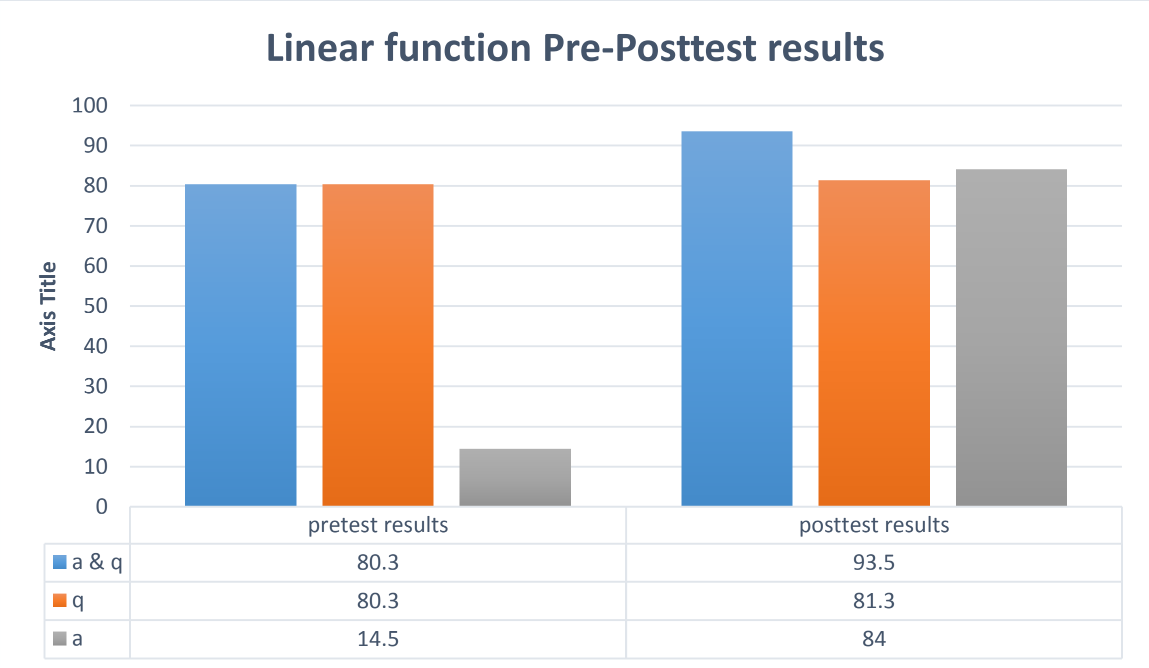


Figure 27: Comparison of pre-test and post-test results based on a linear function.

The following table 21 shows a summary of results obtained from a pre-test and post-test based on a hyperbola function.

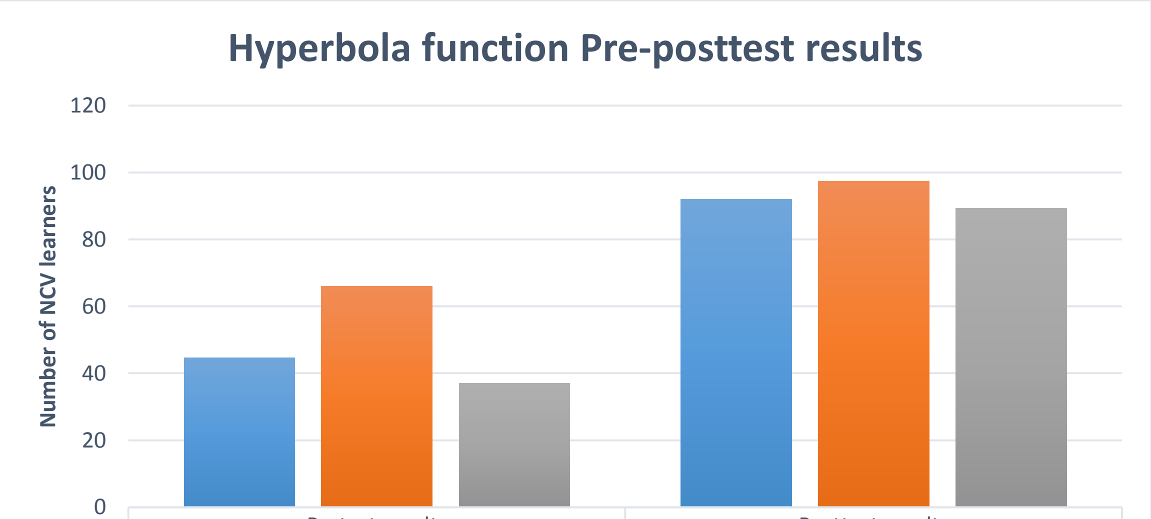
Table 21: Comparison of pre-test and post-test results based on a hyperbola function

Hyperbola function	pre-test	post-test	Variance
Effect of a and q	44.7%	92%	+47.3%
Effects of a	37%	89.3%	+52.3%
Effects of q	66%	97.3%	+31.3%

The majority of learners had difficulties when working with hyperbola based problems prior the integration of GeoGebra; they mostly struggled to interpret the effect of parameter a. The results showed an increase from 37% to 89.3% of learners who developed concepts related to the interpretation of parameter a. The results show a significant increase from 44.7% of learners who showed the ability to interpret parameters a and q . This ability increased to 92% after the integration of GeoGebra. The 66% of learners who showed the ability to interpret parameter q increased to 97.3% in the post test. These positive results confirm the effectiveness of GeoGebra and the teaching and learning improved considering the Zone of Proximal development.

Considering both pre-test and posttest results, most learners seem to experience difficulties when interpreting the effect of parameter a than parameter q, but after learners interacted with GeoGebra applets, they developed a sense of understanding all the possible effects of parameter a in a hyperbola function. The integration of GeoGebra software seems to have helped learners to understand the effects of parameters a and q in a hyperbola function.

The following figure 28 indicates a summary of pre-test and post-test results based on a hyperbola function.



	Pretest results	Posttest results
a&q	44.7	92
q	66	97.3
a	37	89.3

Figure 28: Comparison of pre-test and post-test results based on a hyperbola function.

The following table 22 shows a summary of results obtained from a pre-test and post-test based on a parabola function.

Table 22: Comparison of pre-test and post-test results based on a parabola function

Functions	pre-test	post-test	Variance
Parabola a and q	34.2%	91.5%	+57.3%
Parabola a	93%	93%	+0%
Parabola q	92%	92%	+0%

Learners showed misconceptions related to hyperbola questions regarding the interpretation of both parameters a and q. Teaching and learning hyperbola function concerning Vygotsky's sociocultural theory facilitated learners' conceptual understanding in a parabola function transformation. Thus, interacting with GeoGebra software and working in collaboration assisted the leaners understand the interpretation of a parabola function. Looking at the results, only 34.2% of learners understood the concepts related to the effects of both parameters a and q before the integration of GeoGebra.

The post test results show an increase of learners who showed an understanding related to the effects of parameters a and q. The number of learners increased from 34.2% to 91.5% after they had interacted with GeoGebra applets. When learners work in collaboration, they are able to perform a greater number of tasks. However, it is important that a learner is able to work alone (Denhere, Chinyoka, & Mambeu, 2013). The integration of GeoGebra seems to have helped learners in developing problem solving skills that enabled them to develop algebraic concepts.

Learners understood the effect of parameter a and the effect of parameter q in the pre-test better than when working with combined a and q parabola based questions. Therefore, the integration of GeoGebra played a positive role as a cognitive, mediation and problem solving tool in assisting learners sustain conceptual understanding in a parabola function. As much as the pre-test and post-test scores are the same towards the interpretation of parameter q (92%) and parameter a (93%), the difference is that after engaging with applets, learners developed conceptual understanding towards the interpretation of parameters a and q. In addition, during GeoGebra integration, learners showed the ability to justify, discover, investigate and solve parabola based questions without engaging with sliders in section C question 2.2. This means that GeoGebra facilitated learners to internalize the concepts; learners have the ability to solve problems without any assistance. The following figure 29 shows a summary of pre-test and post-test results based on a parabola function.

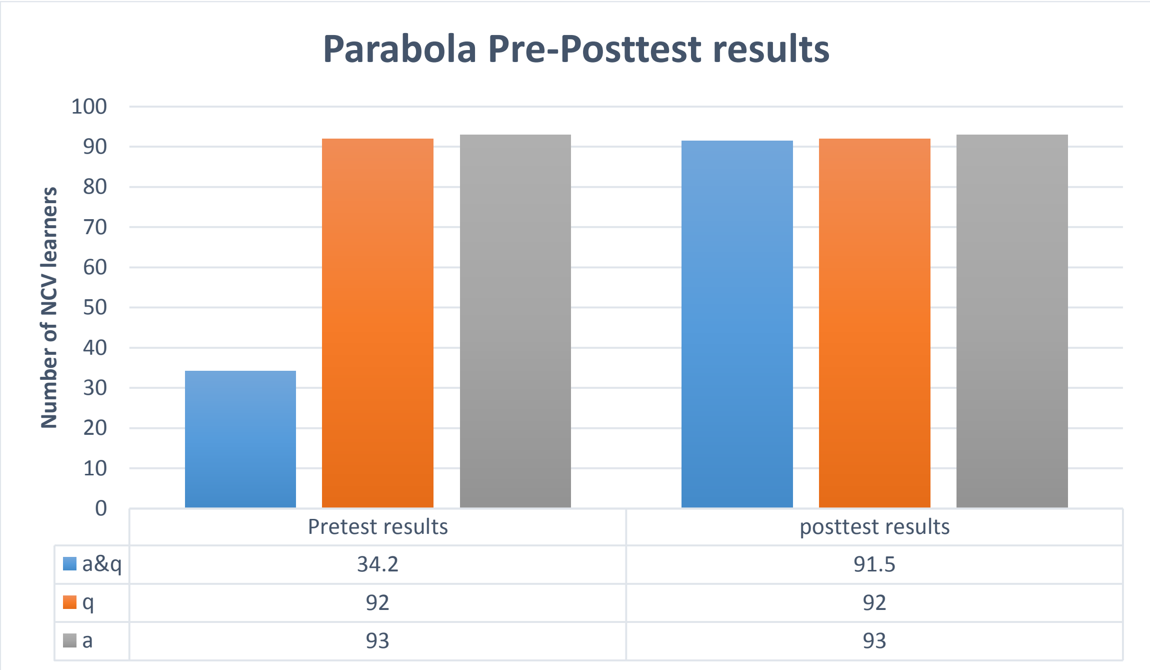


Figure 29: Comparison of pre-test and post-test results based on a parabola function.

4.6. DATA PRESENTATION OBTAINED FROM FOCUS GROUP DISCUSSION

The researcher conducted focus group discussion after the integration of GeoGebra. The researcher intended to obtain learners' experiences with GeoGebra software and their perceptions about the influence of GeoGebra in algebraic functions. The focus group interview occurred on the 31st August 2017, and the session took a maximum of 25 minutes.

Learners confirmed the effectiveness of GeoGebra when solving algebraic functions. From the ten learners who participated in a focus group discussion, about 8 learners gave positive feedback about GeoGebra software. Learners stated that GeoGebra helped them to have a deeper understanding of how algebraic functions get transformed dynamically. Almost all the learners stated that they enjoyed interacting with GeoGebra applets. They said that it helped them to visualize and drag graphs through manipulating sliders a and q in each function. They further stated that the manipulation of sliders helped them to observe changes in both graphical view and algebraic view of each function. Approximately 7 learners preferred that algebraic functions should be learned through using GeoGebra software. Yet, the other three learners felt like that they still needed more time to be confident with GeoGebra software. They reported that they experienced some minor challenges when working with GeoGebra applets. Learner **C10** stated that "at the beginning, I did not know how to drag a slider until my friend helped me out"

Learners further reported that they prefer to use GeoGebra to transform graphs instead of drawing graphs with pencil and pen.

Learner **B9** stated that:

> "*GeoGebra helped me develop deeper understanding of the effect of parameter a in each function. For example, it was difficult for me to differentiate between a hyperbola graphical representations with* $a = 1$ *and* $a = 3$*".*

Learners stated that through utilizing GeoGebra software when learning algebraic functions, they are able to test conjectures, analyze and critically interpret the effects of the parameters. Learners reported that given any kind of algebraic equation, they are able generally interpret its graphical representation.

The researcher asked learners to discuss about the graphical representation of a parabola equation given as $f(x) = -x^2 + 4$. Learners responded positively. Learner **A7**'s responded by saying that "the graph will be facing downwards and turning at $y = 4$". Learner **D2** agreed with learner **A7** by saying that "this equation will show a sad face graph with a turning point of (0;4)" In that regard, it is clear that the integration of GeoGebra helped NCV learners to analyze, interpret and reason critically without having to memorize concepts.

In summary, this chapter presented and analyzed data obtained from pre-test and post-test data and transcribed focus group interviews. This chapter also reported that data was collected prior the integration of GeoGebra where learners showed misconceptions related to interpretation of algebraic functions. Presentation and analysis of sociocultural learning and teaching where GeoGebra was integrated as a visual dynamic, conceptual development, problem solving and cognitive tool to support learners deal with algebraic misconceptions is presented. Finally, the chapter reported the post-test results and compared the difference between post-test and pre-test results. Learners' views and experiences with GeoGebra software is presented in this chapter.

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CHAPTER FIVE: CONCLUSIONS AND DISCUSSIONS

5.1. INTRODUCTION

This chapter discusses the findings that were presented in this study to answer the research questions. The purpose of this chapter is to show how the findings contribute to the current thinking. The research findings are discussed in relation to the research questions and literature of this study. The chapter also discusses the limitations of the study and my reflections on my role as a researcher. Finally, I recommend areas for further research study.

5.2. DISCUSSION OF FINDINGS IN RELATION TO THE RESEARCH QUESTIONS

This study began by investigating NCV level two learners' misconceptions in algebraic functions. Thereafter, I integrated GeoGebra dynamic software during teaching and learning algebraic functions as an approach to learners' misconceptions which were identified. My intentions of integrating GeoGebra during teaching and learning algebraic functions was to determine the effectiveness of it towards learners' conceptual development. This study is driven by two research sub-questions; the summary of the research findings are discussed in relation with these research questions as follows:

Research sub-question 1: What are NCV Level two learners' misconceptions in algebraic functions?

The findings show that the majority of NCV level two mathematics learners experienced misconceptions when working with algebraic functions For example, learners struggle to interpret the effects of parameters a and q in algebraic functions given as: Linear function $f(x) = ax + q$, parabola function $f(x) = ax^2 + q$ and hyperbola function $f(x) = \frac{a}{x} + q$. In a linear function, results showed that learners were unable to distinguish graphically between the effect of parameter a and the value of x intercept. Learners referred to x intercepts as parameter a when interpreting a linear graph. In a parabola function $y = ax^2 + q$, few learners showed misconceptions when interpreting parameter q when completing the pre-test especially in the first two sections. However, comparing the parabola based results of the first two sections with the results in section C, the majority of learners were unable to transform a parabola function by means of interpreting parameters a and q. Furthermore, the results obtained from the focus group interviews verified that learners were experiencing misconceptions towards the interpretation of a parabola function.

The results obtained from hyperbola based questions shows that learners experienced misconceptions when interpreting parameters a and q. Learners could not transform a hyperbola graph to its correct equation by means of interpreting parameter a. Learners' misconceptions towards interpreting a hyperbola function were verified in section B where the results showed that learners were unable to understand the effects of both parameters a and q. Basically, learners experienced more difficulties with questions based on interpreting the effect of a than with questions that were based on interpreting parameter q in a hyperbola function.

In summary, the findings from the relevant data sources disclose that a majority of learners who participated in this study experienced misconceptions prior to GeoGebra integration when transforming algebraic functions by means of interpreting the effects of parameters a and q.

Research sub-question 2: *In what way does engagement with GeoGebra support learners to develop conceptual understanding in algebraic functions?*

According to my classroom observations, learners worked collaboratively during the integration of GeoGebra. They effectively interacted with applets created to manipulate parameters a and q. Learners' engagement with created applets enhanced collaborative learning. GeoGebra integration also enabled learner centeredness. Learners used the worksheet and applets created which encouraged them to explore algebraic concepts without much need for the intervention of a teacher. Furthermore, learners used GeoGebra applets for visualization, demonstration and dynamic transformation of algebraic functions which helped them develop conceptual understanding in algebraic functions. Learners were able to identify the effects of parameters a and q through manipulations of sliders created. The post-test results verify that GeoGebra software played a positive role in helping learners develop algebraic concepts. During focus group interviews, the learners mentioned how effective the utilization of applets to transform algebraic function was. Learners found learning algebraic function using GeoGebra exciting and effective. The

GeoGebra applets played a role as a mediation tool and learners used applets as a scaffold to enhance conceptual development. Learners managed to solve problems in the post-test concerning the interpretation of parameters a and q that they were unable to solve prior to the GeoGebra intervention. Their inability to solve those problems were interpreted as misconceptions.

In summary, the findings obtained during the intervention phase, post-test and second focus group interviews showed that the integration of GeoGebra helped learners to develop conceptual understanding of algebraic functions. The learners effectively utilized GeoGebra applets as a modelling, conceptual development, mediation and cognitive tool.

5.3. LIMITATIONS AND REFLECTIONS

The study's limitations are based on the exploratory nature of my research instruments. The nature of all the worksheets used in the study was restrictive. They did not allow learners to express themselves in terms of showing calculations. Perhaps, some learners had alternative ways to find solutions. This could have affected the study's results; learners could have estimated or guessed the answers. However, the researcher developed all possible methods to verify the validity of results. Such methods include development of sections that indirectly repeat the skills required from previous sections and the focus group discussions helped to verify the results obtained from the worksheets. Depending on their level of computer literacy, some learners took more time than others to adapt to GeoGebra. Some learners would struggle to place the mouse-cursor on the sliders. That was frustrating to these learners and time consuming.

The study has made me understand that creating opportunities for learners to utilize GeoGebra in mathematics provides a deeper conceptual understanding. Such learning enhances meaningful learning and enables the ability to discover, investigate, apply, prove and communicate mathematical ideas (Uddin, 2011).

5.4. RECOMMENDATIONS FOR FURTHER RESEARCH

For future research based on GeoGebra based learning, I recommend the following for future research. Certainly, the need for further research in the use of GeoGebra in mathematics at the TVET College is needed. This study was conducted at a particular college in Port Elizabeth. Therefore, the future study could be done in other TVET colleges in Port Elizabeth and perhaps, in other provinces of South Africa. I could possibly increase the number of participants and involve mathematics teachers as participants in the future research to obtain richer data. Teachers would be interviewed about their perspectives of using GeoGebra in mathematics. Obviously, some mathematics teachers would not be aware of GeoGebra software. Therefore, a proper GeoGebra training for teachers would be needed. This study only focused on integrating GeoGebra using the three algebraic functions namely: linear, parabola and hyperbola. In that regard, I would extend my future studies into other areas in mathematics.

5.6. CONCLUSIONS

The aim of this study was to determine how the integration of GeoGebra software during teaching and learning supports NCV level two learners to deal with misconceptions in algebraic functions. Therefore, the researcher focused on three interrelated aspects: GeoGebra as a dynamic modelling tool, GeoGebra as a problem solving and conceptual tool and GeoGebra as a cognitive tool. The GeoGebra applets that were used by learners were interactive, dynamic and designed to enable conceptual development. The results confirms that, through their engagement, learners developed a better understanding of algebraic concepts and they developed the ability to explain concepts showing in-depth understanding.

Enhancement in learners' scores regarding conceptual understanding after engaging with applets suggest that working with GeoGebra dynamic software showing visual interactive images has been a scaffold to learners' understanding. Learners' engagement with GeoGebra applets enhanced the ability to demonstrate their thinking and conceptual knowledge in algebraic functions. Utilization of applets offers learners the opportunity to observe dynamic visual representations when learning algebraic functions. For example, the manipulation of sliders a and q provided an opportunity for learners to generate immediate dynamic responses. This is useful because learning algebraic functions using GeoGebra enabled learners to explore concepts and saved a lot of time in comparison to learning in a traditional pen and paper method, which can be time consuming and can be an obstacle to the development of conceptual understanding.

During the focus group interview after learners had engaged with applets, they stated that GeoGebra helped them to obtain a deeper understanding of how algebraic functions get transformed dynamically. Learners gave positive responses about learning algebraic functions using GeoGebra software. My observations during the integration of GeoGebra, together with the focus group responses and the results of the post-test all verify the effectiveness of GeoGebra in algebraic functions.

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APPENDIX 1: NMU REC-H approval letter

• PO Box 77000 • Nelson Mandela Metropolitan University
• Port Elizabeth • 6031 • South Africa • www.nmmu.ac.za

Nelson Mandela Metropolitan University
for tomorrow

Chairperson: Research Ethics Committee (Human)

Tel: +27 (0)41 504-2235

NMU Ref: [H17-EDU-ERE-003 / Approval]

Contact person: Mrs U Spies

11 May 2017

Dr C Felix
Faculty:
Education
South
Campus

Dear Dr Felix

EXPLORING NATIONAL CERTIFICATE VOCATIONAL (NCV) LEVEL TWO LEARNERS' MISCONCEPTIONS IN ALGEBRAIC FUNCTIONS THROUGH INTEGRATING GEOGEBRA DURING TEACHING AND LEARNING

PRP: Dr C Felix
PI: Ms A Ngwabe

Your above-entitled application served at Research Ethics Committee (Human) for approval.

The ethics clearance reference number is **H17-EDU-ERE-003** and is valid for three years. Please inform the REC-H, via your faculty representative, if any changes (particularly in the methodology) occur during this time. An annual affirmation to the effect that the protocols in use are still those for which approval was granted, will be required from you. You will be reminded timeously of this responsibility, and will receive the necessary documentation well in advance of any deadline.

We wish you well with the project. Please inform your co-investigators of the outcome, and convey our best wishes.

Yours sincerely

Prof C Cilliers
Chairperson: Research Ethics Committee (Human)
cc: Department of Research Capacity Development
Faculty Officer: Education

APPENDIX 2: Informed CONSENT form to LEARNERS

Consent Form Template for Learner participants above the age of 18 years

TITLE: Exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning.

May 2017

Explanation of the Study (What will happen to me in this study?)

The purpose of this research is to use GeoGebra technological software when learning to deal with misconceptions in algebraic functions in NCV level 2. The study will be conducted during break times or during the first 40 minutes after school. You will be required to move to a computer lab during the integration of GeoGebra software when learning algebraic functions. You will also be expected to complete test questionnaires based on algebraic functions i.e. before, during and after the integration of GeoGebra. You will also be required to participate in focus group discussions about challenges experienced during pre-test and your experience of learning algebraic functions using GeoGebra software.

Risks or Discomforts of Participating in the Study (Can anything bad happen to me?)

There will be no risks or discomforts involved during participation in this study, except that the study will take place during your break times or it will be required that you stay for the 1st 40 minutes after school.

Benefits of Participating in the Study (Can anything good happen to me?)

The study may help you develop algebraic concepts and also provide an opportunity for you to learn algebraic functions using dynamic technological software. Based on previous studies, integration of GeoGebra in mathematics can improve learner performance and therefore there are chances that the study will help you to improve your mathematic performance.

Confidentiality (Will anyone know I am in the study?)

Your participation to this study will be kept confidential. The information that you give in the study will be handled confidentially. Your data will be anonymous which means that your name will not be collected or linked to the data. However, during focus group interviews, I cannot guarantee your data will be confidential and it may be possible that others will know what you have reported.

The information that you give in the study will be handled confidentially. Your information will be assigned a code number. The list connecting your name to this code will be kept in a locked file. When the study is completed and the data have been analyzed, this list will be destroyed. Your name will not be used in any report.

Contact Information (Who can I talk to about the study?)

Researcher: Miss A Ngwabe or Supervisor: Dr. Clyde Felix

Contact details: 0734295108 Contact details: 041 504 3030/4578

Voluntary Participation (What if I do not want to do this?)

Your participation to this study is completely voluntary. You have a right not to participate in this study and you can withdraw anytime during the process (i.e. pre-test, intervention, post-test and focus group interviews) without penalty. If you want to withdraw, you can leave the room immediately or tell the researcher to stop during the process.

Do you understand this study and are you willing to participate?

YES NO

______________ ____________________

Signature of a Learner Date

APPENDIX 2. 1: Informed CONSENT form to PARENTS

Consent Form Template for parents that their children are the under 18's participants

TITLE: Exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning.

May 2017

Explanation of the Study (What will happen to me in this study?)

The purpose of this research is to use GeoGebra technological software when learning to deal with misconceptions in algebraic functions in NCV level 2. Your child will be required to stay for 40 minutes after school or use break time during the study. He/she will be expected to complete test questionnaires based on algebraic functions i.e. before, during and after the integration of GeoGebra. He/she will also be required to participate in focus group discussions about challenges experienced during pre-test and your child's experience of learning algebraic functions using GeoGebra software.

Risks or Discomforts of Participating in the Study (Can anything bad happen to me?)

There will be no risks or discomforts involved during participation in this study except that your child will be required to stay for 40 minutes after school or sacrifice his/her break times.

Benefits of Participating in the Study (Can anything good happen to me?)

The study may help him/her develop algebraic concepts and also provide an opportunity for the child to learn algebraic functions using dynamic technological software.

Confidentiality (Will anyone know I am in the study?)

Your child's participation in this study will be kept confidential. The information that he/she gives in the study will be handled confidentially. The name of your child will not be mentioned during the study. Your child's information will be assigned a code number and handled confidentially. The list connecting your child's name to this code will be kept in a locked file. When the study is completed and the data have been analyzed, this list will be destroyed.

Contact Information (Who can I talk to about the study?)

Researcher: Miss A Ngwabe or Supervisor: Dr. Clyde Felix
Contact details: 0734295108 Contact details: 041 504 3030/4578

Voluntary Participation (What if I do not want to do this?)

Your child's participation to this study is completely voluntary. He/she has a right not to participate in this study and she/he can withdraw anytime during the process (pre-test, intervention, post-test & focus group interviews) without penalty.

Do you understand this study and are you willing to participate?

YES	NO

____________________________ _______________________

Signature of a parent Date

APPENDIX 2. 2: Informed ASSENT form to LEARNERS

Assent Form Template for Learner participants under the age of 18

TITLE: Exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning.

May 2017

Explanation of the Study (What will happen to me in this study?)

The purpose of this research is to use GeoGebra technological software when learning to deal with misconceptions in algebraic functions in NCV level 2. The study will be conducted during break times or during the first 40 minutes after school. You will be required to move to a computer lab during the integration of GeoGebra software when learning algebraic functions. You will also be expected to complete test questionnaires based on algebraic functions i.e. before, during and after the integration of GeoGebra. You will also be required to participate in focus group discussions about challenges experienced during pre-test and your experience of learning algebraic functions using GeoGebra software.

Risks or Discomforts of Participating in the Study (Can anything bad happen to me?)

There will be no risks or discomforts involved during participation in this study, except that the study will take place during your break times or it will be required that you stay for the 1st 40 minutes after school.

Benefits of Participating in the Study (Can anything good happen to me?)

The study may help you develop algebraic concepts and also provide an opportunity for you to learn algebraic functions using dynamic technological software. Based on previous studies, integration of GeoGebra in mathematics can improve learner performance and therefore there are chances that the study will help you to improve your mathematic performance.

Confidentiality (Will anyone know I am in the study?)

Your participation to this study will be kept confidential. The information that you give in the study will be handled confidentially. Your data will be anonymous which means that your name will not be collected or linked to the data. However, during focus group interviews, I cannot guarantee your data will be confidential and it may be possible that others will know what you have reported.

The information that you give in the study will be handled confidentially. Your information will be assigned a code number. The list connecting your name to this code will be kept in a locked file. When the study is completed and the data have been analyzed, this list will be destroyed. Your name will not be used in any report.

Contact Information (Who can I talk to about the study?)

Researcher: Miss A Ngwabe or Supervisor: Dr. Clyde Felix

Contact details: 0734295108 Contact details: 041 504 3030/4578

Voluntary Participation (What if I do not want to do this?)

Your participation to this study is completely voluntary. You have a right not to participate in this study and you can withdraw anytime during the process (i.e. pre-test, intervention, post-test and focus group interviews) without penalty. If you want to withdraw, you can leave the room immediately or tell the researcher to stop during the process.

Do you understand this study and are you willing to participate?

YES NO

_______________________ _______________________

Signature of a Learner Date

APPENDIX 3: Principal Invite

Nelson Mandela Metropolitan University

for tomorrow

• PO Box 77000 • Nelson Mandela Metropolitan University
• Port Elizabeth • 6031 • South Africa • www.nmmu.ac.za

Exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning

School Principal Consent Form

I give consent for you to approach NCV level two learners to participate in "**Exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning**" study.

I have read the Project Information Statement explaining the purpose of the research project and understand that:

- The role of the school is voluntary
- I may decide to withdraw the school's participation at any time without penalty
- NCV Level two learners will be invited to participate and that permission will be sought from them and also from their parents.
- Only learners who consent and whose parents consent will participate in the project
- All information obtained will be treated in strictest confidence.
- The learners' names will not be used and individual learners will not be identifiable in any written reports about the study.
- The school will not be identifiable in any written reports about the study.
- Participants may withdraw from the study at any time without penalty.
- A report of the findings will be made available to the school.
- I may seek further information on the project from Ngwabe Abongile on 0734295108.

W.C. Jonas

Principal

Signature

22/05/2017

Date

APPENDIX 4: Oral information

Faculty of Education
NMMU
Tel: +27 (0)41 504-4310 Fax: +27 (0)41-504-1610
E-mail Faculty Chairperson: carol.poisat@nmmu.ac.za

March 2017

Ref:

Contact person: **Miss A Ngwabe**
073 4295 108

Dear NCV level 2 learner.

You are invited to participate in a research study. The purpose of the study is to use GeoGebra technological software when learning to deal with misconceptions in algebraic functions in NCV level 2. We will provide you with the necessary information to assist you to understand the study and explain what would be expected of you. These guidelines would include the risks, benefits, and your rights as a study subject. Please feel free to ask the researcher to clarify anything that is not clear to you.

To participate, it will be required of you to provide a written consent/assent that will include your signature, date and initials to verify that you understand and agree to the conditions.

You have the right to query concerns regarding the study at any time. Immediately report any new problems during the study, to the researcher. Telephone numbers of the researcher are provided in the consent/assent form. Please feel free to call these numbers.

Furthermore, it is important that you are aware of the fact that the ethical integrity of the study has been approved by the Research Ethics Committee (Human) of the university. The REC-H consists of a group of independent experts that has the responsibility to ensure that the rights and welfare of participants in research are protected and that studies are conducted in an ethical manner. Studies cannot be conducted without REC-H's approval. Queries with regard to your rights as a research subject can be directed to the Research Ethics Committee (Human), Department of Research Capacity Development, PO Box 77000, Nelson Mandela Metropolitan University, Port Elizabeth, 6031.

If no one could assist you, you may write to: The Chairperson of the Research, Technology and Innovation Committee, PO Box 77000, Nelson Mandela Metropolitan University, Port Elizabeth, 6031.
Participation in research is completely voluntary. You are not obliged to take part in any research.
If you do partake, you have the right to withdraw at any given time, during the study without penalty or loss of benefits.
This informed consent statement has been prepared in compliance with current statutory guidelines.

Yours sincerely
Researcher: Abongile Ngwabe
Cell no. 073 429508
Supervisor: Dr. Clyde Felix
Tel: 041 5043 030

APPENDIX 5: Institutional permission

• PO Box 77000 • Nelson Mandela Metropolitan University

• Port Elizabeth • 6031 • South Africa • www.nmmu.ac.za

Nelson Mandela Metropolitan University

for tomorrow

May 2017

K. Matiso

Port Elizabeth TVET College Principal

Richmond Hill

Richmond Park Drive

Tel: (041) 509 6000

Fax: (041) 582 2017

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN IQHAYIYA CAMPUS

Dear Mr. Matiso

My name is Ms Abongile Ngwabe, and I am a master student at the Nelson Mandela Metropolitan University in Port Elizabeth and a lecturer at Iqhayiya campus. The research I wish to conduct for my master's involves **exploring NCV level two learners' misconceptions in algebraic functions through integrating GeoGebra during teaching and learning.** This project will be conducted under the supervision of Dr. Clyde Felix (NMMU, South Africa).

I am hereby seeking your consent to approach PE TVET Campus (Iqhayiya) to provide NCV level 2 mathematics students as participants to my study.

I have provided you with a copy of my dissertation proposal which includes copies of the measure and consent and assent forms to be used in the research process, as well as a copy of the approval letter which I received from the NMMU Research Ethics Committee (Human).

Upon completion of the study, I undertake to provide the College with a bound copy of the full research report. If you require any further information, please do not hesitate to contact me on 073 4295 108, abongilen@pec.edu.za. Thank you for your time and consideration in this matter.

Yours sincerely,

Ms Abongile Ngwabe

Nelson Mandela Metropolitan University

APPENDIX 6 : Pre-test worksheet

Instructor:	Miss A Ngwabe	Name:	
Class:	Group	Date:	
Title:	Algebraic functions	Results:	

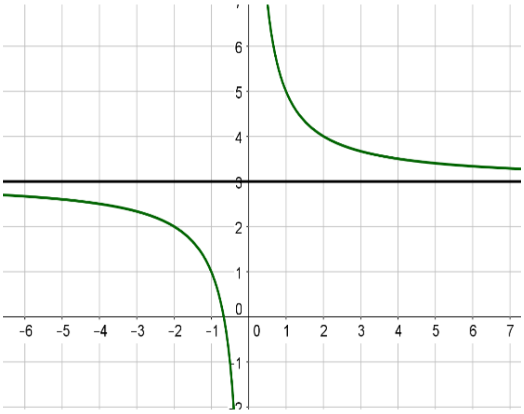
Section A: Converting equations to graphs

Instructions

For each of the following questions select an option that best describes the question, make a cross on the correct letter in the box. Read the question carefully before you select the correct answer.

1. Which of the following equations defines the given graph?

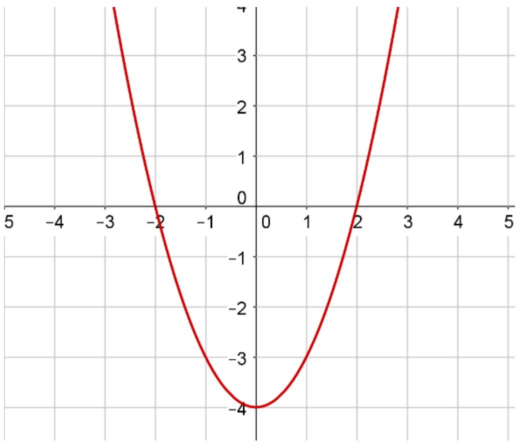
A. $f(x) = \frac{2}{x} - 3$
B. $f(x) = \frac{2}{x} + 3$
C. $f(x) = \frac{2}{x} + 0$
D. $f(x) = \frac{2}{x} - 2$



A	B	C	D

2. Which of the following equations defines the given graph?

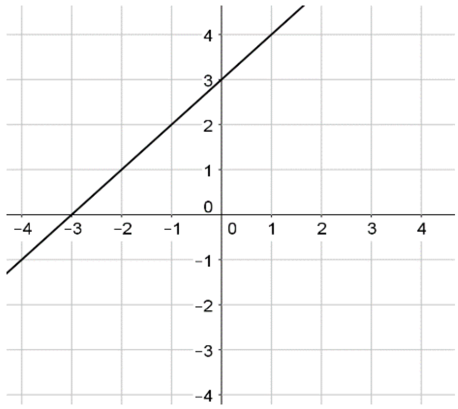
A. $f(x) = x^2 + 4$
B. $f(x) = x^2 - 9$
C. $f(x) = x^2 + 3$
D. $f(x) = x^2 - 4$



A	B	C	D

3. Which of the following equations defines the given graph?

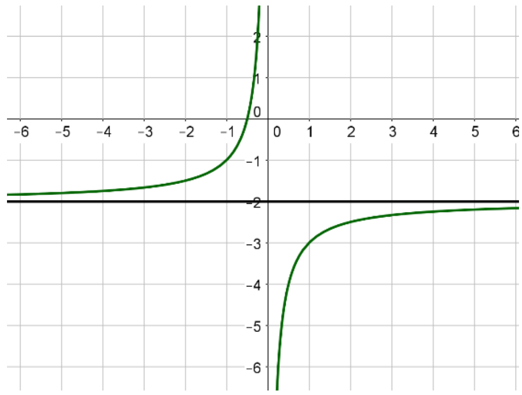
A. $f(x) = -3x + 3$
B. $f(x) = x + 3$
C. $f(x) = -x + 3$
D. $f(x) = 3x + 3$



A	B	C	D

4. Which of the following equations defines the given graph?

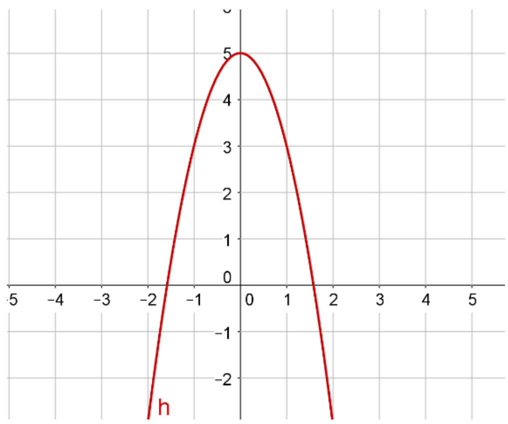
A. $f(x) = \frac{-3}{x} - 2$
B. $f(x) = \frac{2}{x} - 2$
C. $f(x) = \frac{1}{x} - 2$
D. $f(x) = \frac{-1}{x} - 2$



A	B	C	D

5. Which of the following equations defines the given graph?

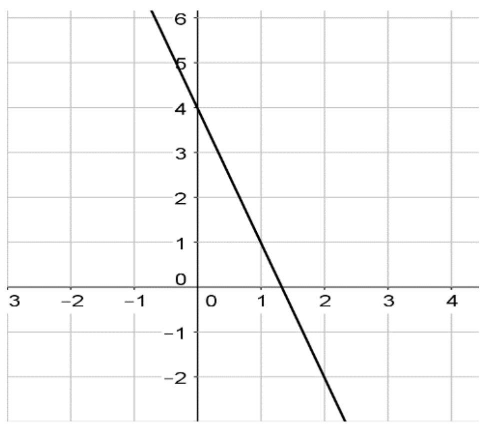
A. $f(x) = 2x^2 + 5$
B. $f(x) = x^2 + 5$
C. $f(x) = -x^2 + 5$
D. $f(x) = -2x^2 + 5$



A	B	C	D

6. Which of the following equations defines the given graph?

A. $f(x) = -3x + 4$
B. $f(x) = -3x + 1$
C. $f(x) = -3x - 4$
D. $f(x) = -3x - 2$

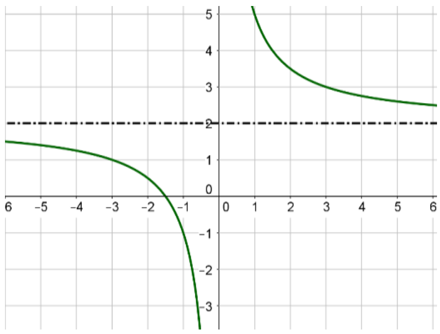
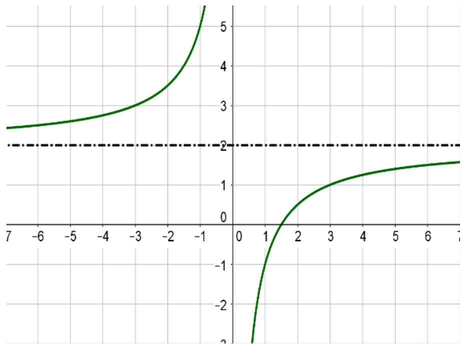
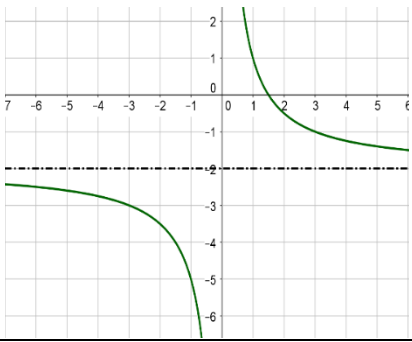
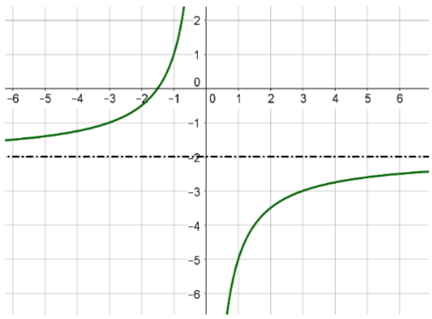


A	B	C	D

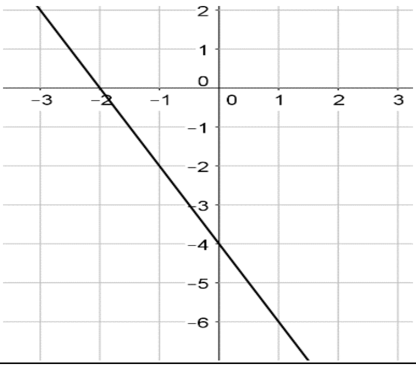
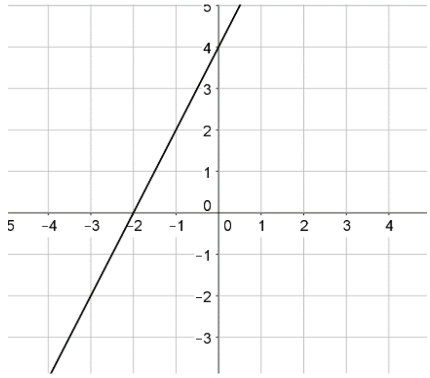
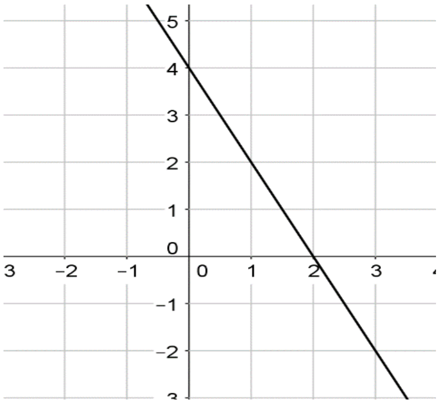
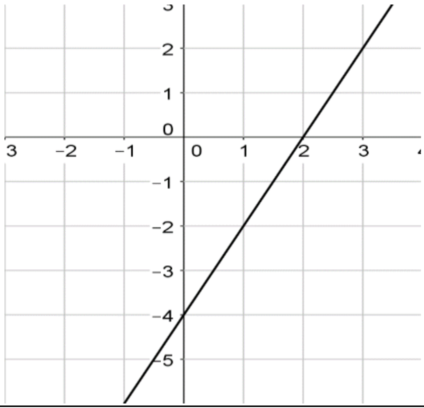
Section B: Converting graphs to equations.

Instructions: For each of the following questions select an option that best describe the question, make a cross on the correct letter in the box at the end of each question.

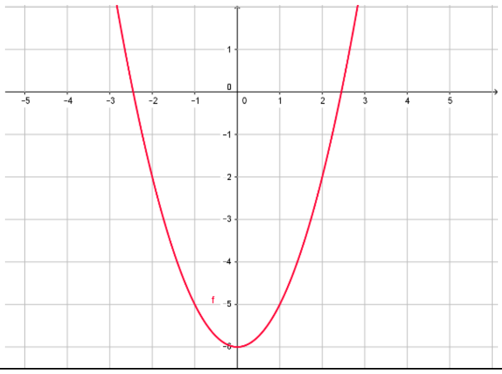
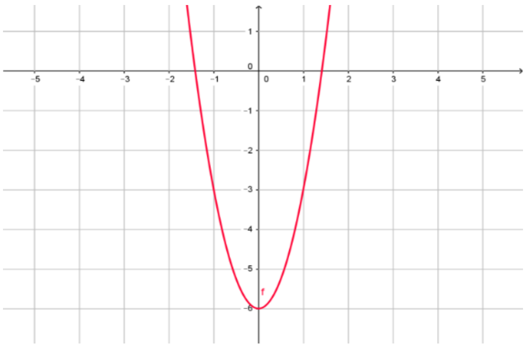
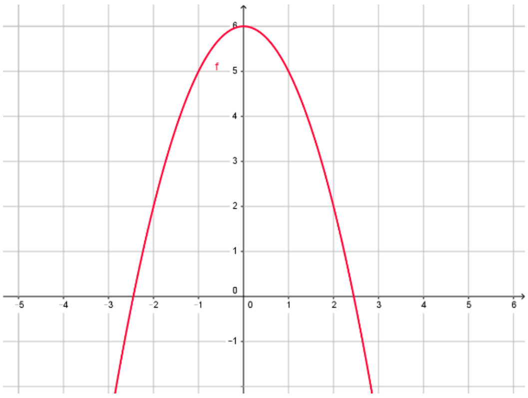
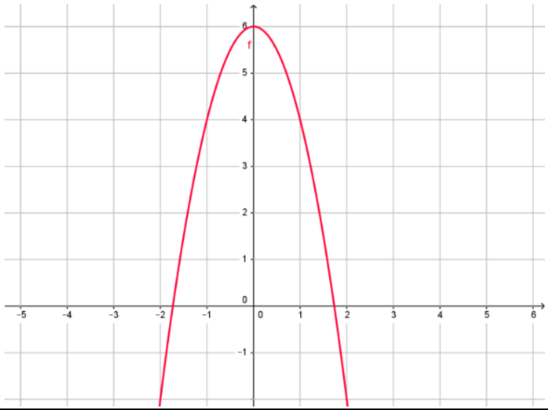
1. Which one of the following graphs defines the given equation: $y = \frac{3}{x} - 2$

A	B
	
C	**D**
	

A	B	C	D

2. Which one of the following graphs define the given equation: $f(x) = 2x - 4$	
A 	B 
C 	D 

A	B	C	D

3. Which one of the following graphs define the given equation: $f(x) = x^2 - 6$	
A 	B 
C 	D 

A	B	C	D

Section C : Algebraic function concepts: parameters a& q and x &y intercepts

Instructions

For each of the following questions select an option that best describes the question, make a cross on the correct letter in the box. Read the question carefully before you select the correct answer.

1. y intercept (q) is defined as:

A. a point where the graph cuts in the y axis.
B. a point where x is zero.
C. a point that the graph cuts in the y axis where x is zero.
D. None of the above options are correct

A	B	C	D

2. x intercept(s):

A. Is identified as a from linear equation $y = ax + q$.
B. Are point(s) on the x axis that the graph cuts, where y is zero and are called roots from quadratic function $y = ax^2 + q$.
C. Is identified as q from hyperbolic function $y = \frac{a}{x} + q$.
D. All the above descriptions about the x intercepts are correct.

A	B	C	D

3. If a quadratic function $f(x) = x^2 - 1$ shifts 3 units down , It's image will be:

A. $f'(x) = 3x^2 - 1$
B. $f'(x) = x^2 - 3$
C. $f'(x) = x^2 - 4$
D. $f'(x) = x^2 + 2$

A	B	C	D

4. What will be the image $g'(x)$ if $g(x) = \frac{4}{x} + 2$ is reflected about the x axis?

A. $g'(x) = -\frac{4}{x} + 2$
B. $g'(x) = -\frac{4}{x} - 2$
C. $g'(x) = \frac{2}{x} + 4$
D. $g'(x) = \frac{2}{x} - 4$

A	B	C	D

5. Which one of the following equations represents a linear function $y = ax + q$ with a slope/gradient given as 2, y intercept $(0,4)$ and x intercept$(-2,0)$?

A. $y = -2x + 2$

B. $y = 4x - 2$

C. $y = 2x - 2$

D. $y = 2x + 4$

A	B	C	D

6. The graphical representation of a quadratic function $y = x^2 - 9$ will have a:

A. maximum turning point $(0; 9)$

B. minimum turning point $(0, -9)$

C. maximum turning point $(0; -9)$

D. minimum turning point $(0; 9)$

A	B	C	D

7. Asymptotes from a hyperbolic function (i.e. $y = \frac{a}{x} + q; x \neq 0$ & $y \neq q$) are lines that the graph should never touch. Which one of the following lines represent hyperbolic asymptotes?

A. lines $x = 0$ & $y = 0$.

B. lines $x = 0$ & $y = q$

C. lines $x = a$ & $y = q$

D. both options A and B

A	B	C	D

APPENDIX 7: GeoGebra intervention worksheet

Intervention: Integrating GeoGebra software in algebraic functions.

Instructor:	Miss A Ngwabe	Name:	
Class:	Group	Date:	
Title:	Algebraic functions		

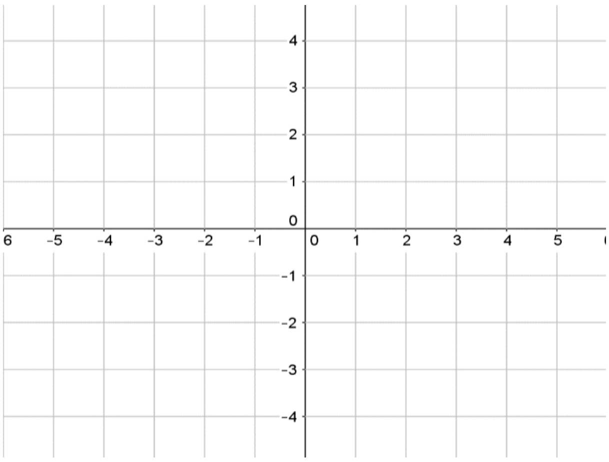
SECTION A: Using GeoGebra software to analyze the effect of q in algebraic functions.

Instructions: For each of the following functions given on GeoGebra applets, change the value of q by using slider q and draw new resulting graph for each q value. Thereafter, explain the effect of q on a space provided.

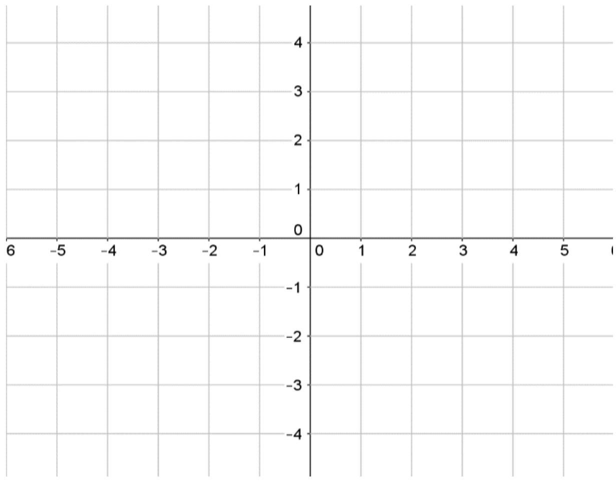
1. LINEAR FUNCTION GIVEN: $h(x) = -x + 4$

For each of the following q values plot the resulting graph on the Cartesian system given below, write down the new equation and identify the correct vertical shift.

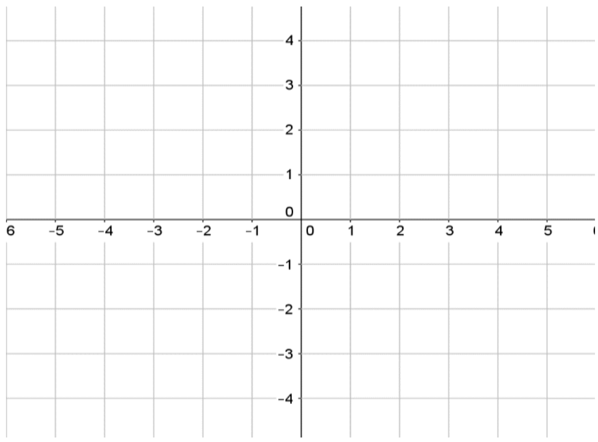
1.1. Use slider a to make $\boldsymbol{q = -4}$



1.2. Use slider a to make $\boldsymbol{q} = \mathbf{0}$



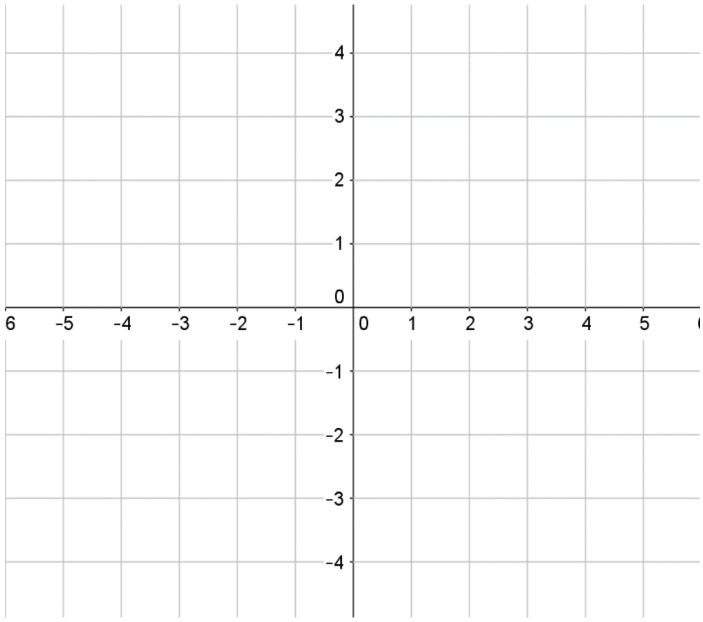
1.3. Use slider a to make $\boldsymbol{q} = \mathbf{2}$



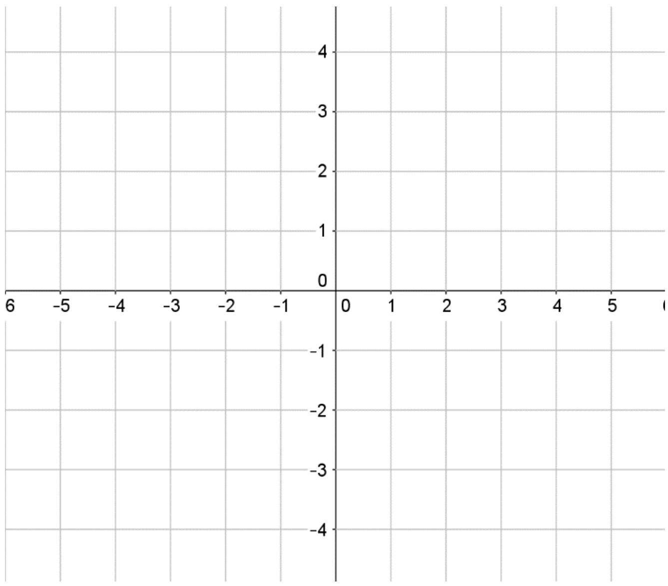
2. PARABOLA FUNCTION GIVEN: $f(x) = x^2 - 4$

For each of the following q values plot the resulting graph on the Cartesian system given below and explain the effect of variable q.

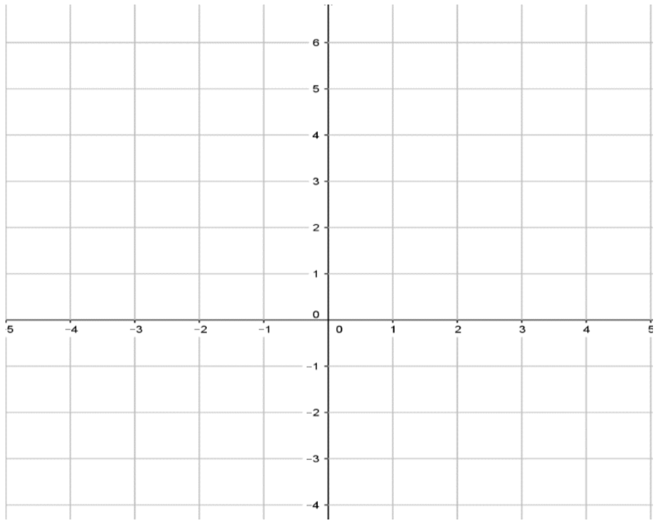
2.1. Use slider a to make $\boldsymbol{q = -2}$.



2.2. Use slider a to make $\boldsymbol{q = 0}$



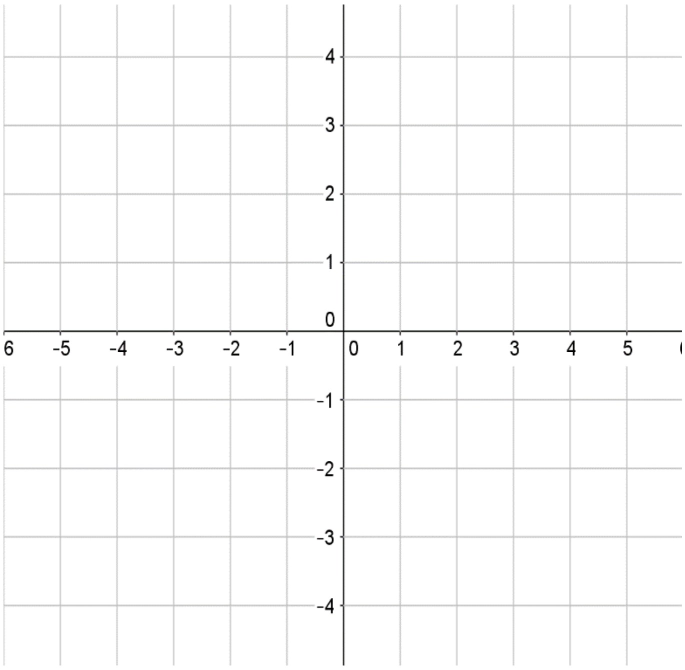
2.3. Use slider a to make $\boldsymbol{q = 4}$

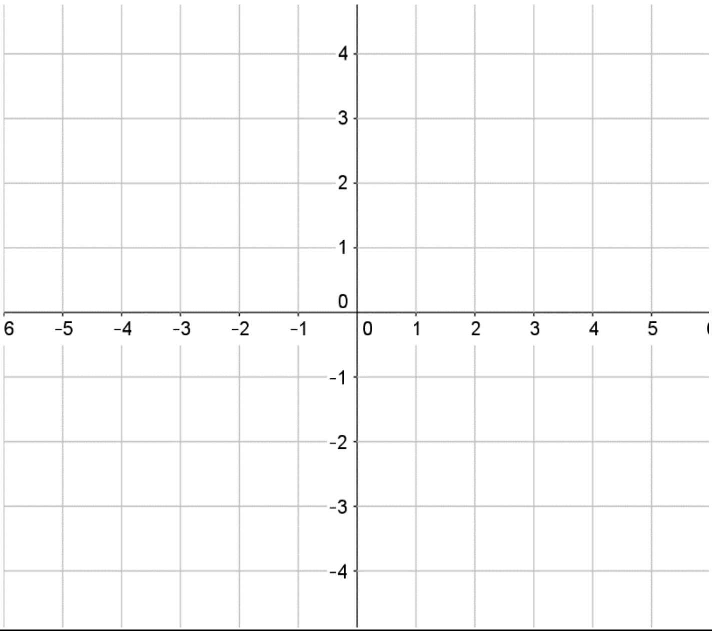
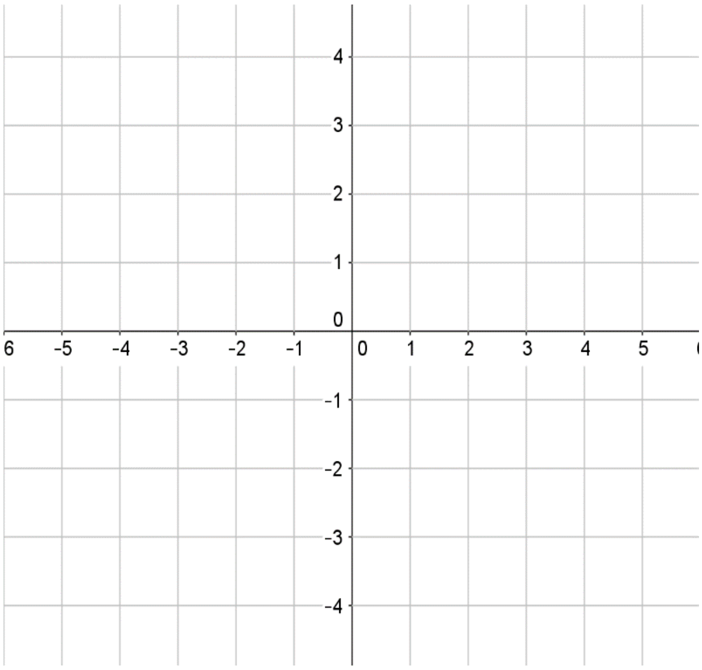


3. HYPERBOLA FUNCTION GIVEN: $\boldsymbol{g(x) = \frac{3}{x} + 1}$

For each of the following q values plot the resulting graph on the Cartesian system given below and explain the effect of variable q.

3.1. Use slider a to make $\boldsymbol{q = -1}$



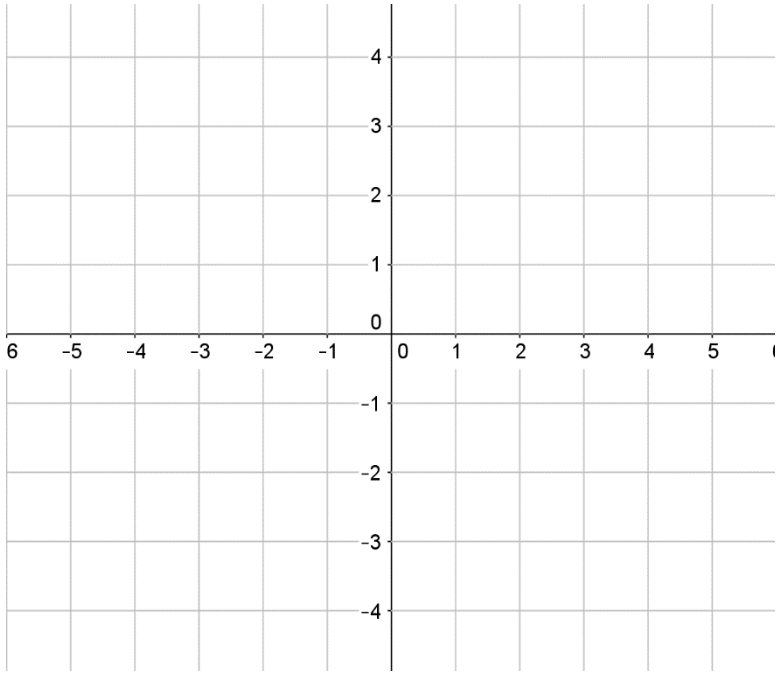
3.2. Use slider a to make $\boldsymbol{q = 2}$ 
3.3. Use slider a to make $\boldsymbol{q = 0}$ 

SECTION B: Using GeoGebra software to analyze the effect of a in algebraic functions.

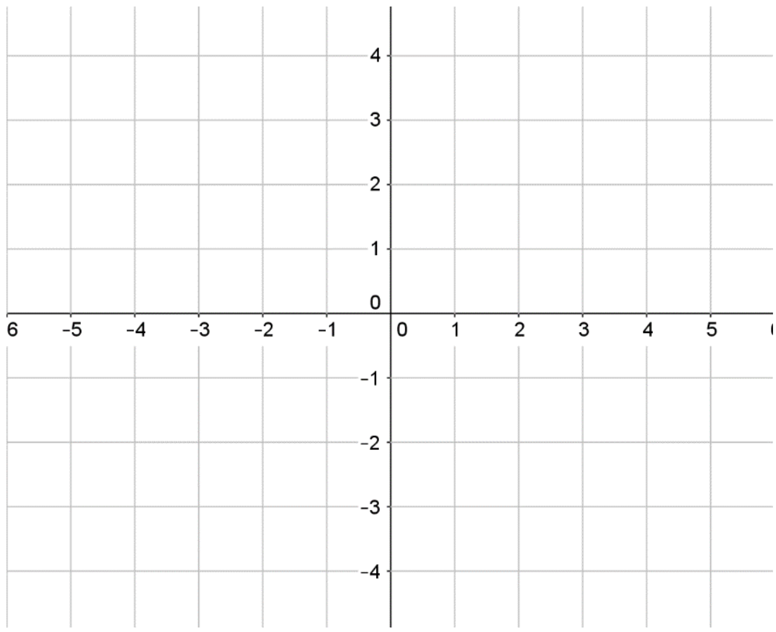
1. LINEAR FUNCTION GIVEN: $h(x) = -x + 4$

For each of the following a values plot the resulting graph on the Cartesian system given below and explain the effect of variable a.

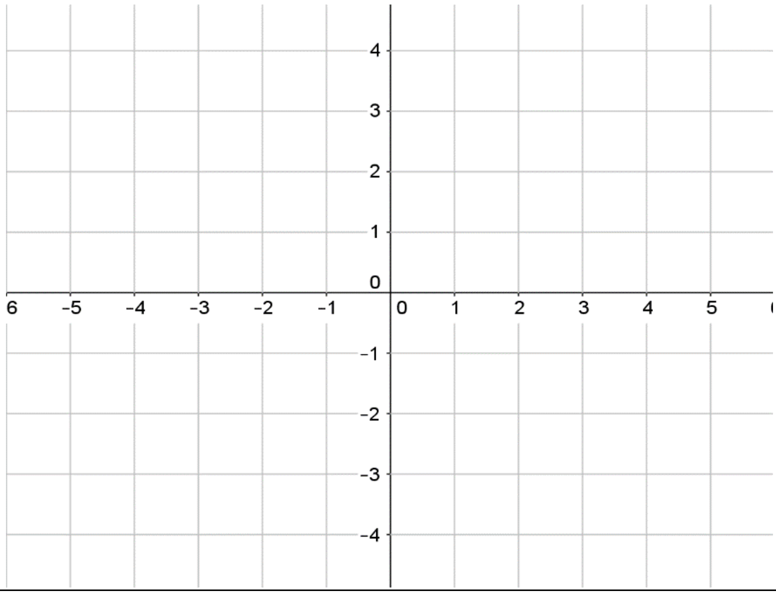
1.1. Use slider a to make $\boldsymbol{a = 1}$



1.2. Use slider a to make $\boldsymbol{a = -2}$



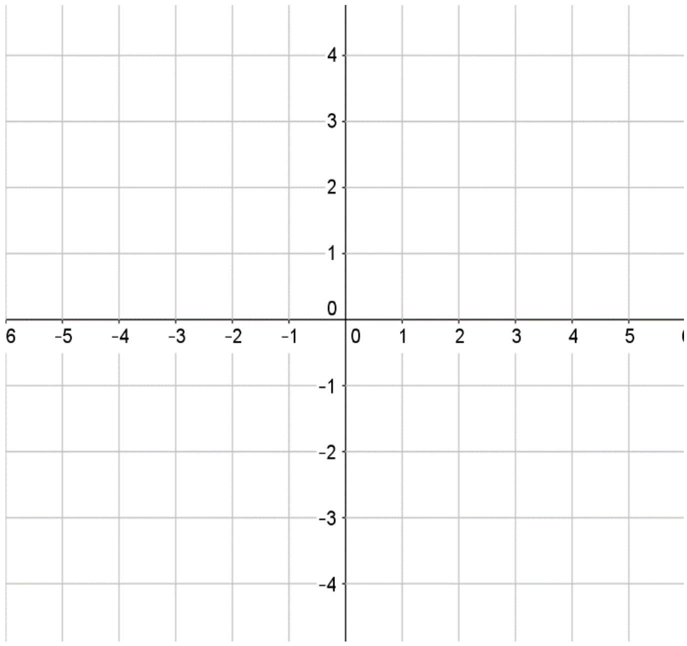
1.3. Use slider a to make $\boldsymbol{a = 0}$



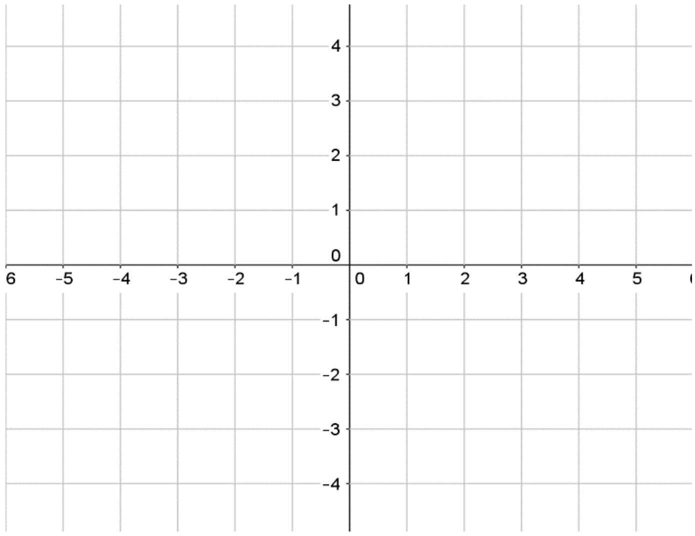
2. PARABOLA FUNCTION GIVEN: $\boldsymbol{f(x) = x^2 - 4}$

For each of the following a values plot the resulting graph on the Cartesian system given below and explain the effect of variable a.

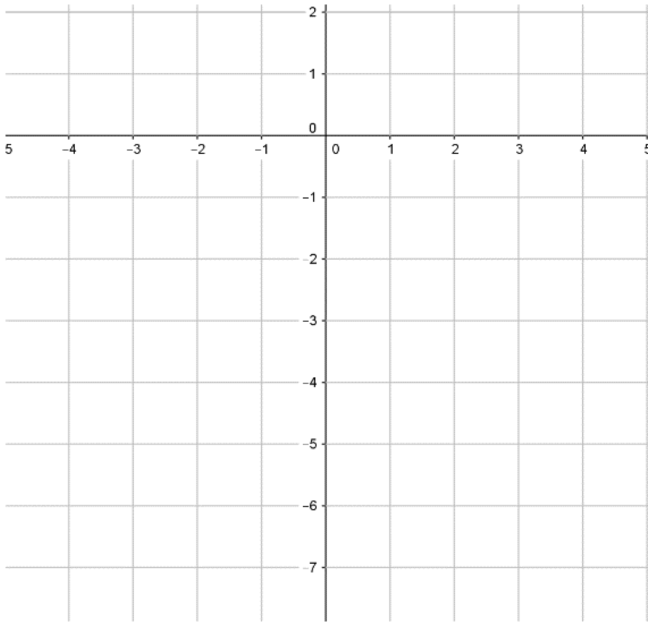
2.1. Use slider a to make $\boldsymbol{a = 3}$.



2.2. Use slider a to make $\boldsymbol{a} = \mathbf{0}$



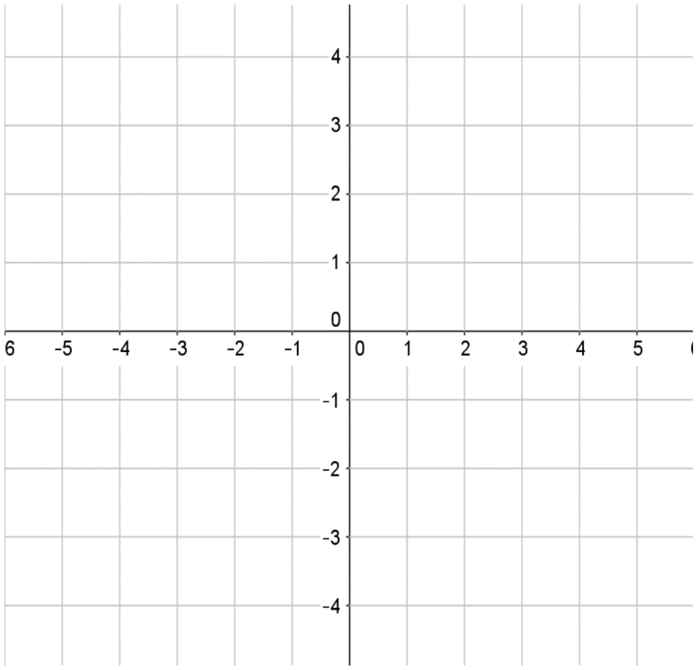
2.3. Use slider a to make $\boldsymbol{a} = -\mathbf{1}$



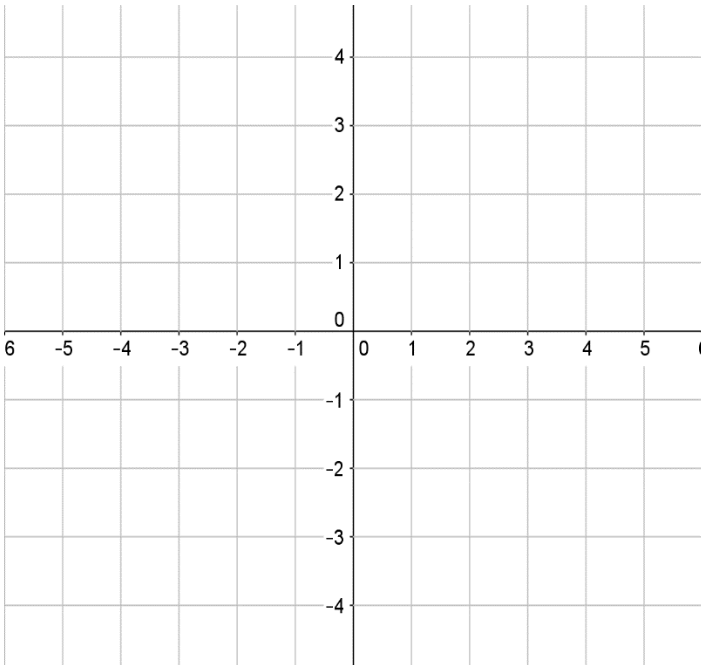
3. HYPERBOLA FUNCTION GIVEN: $g(x) = \frac{3}{x} + 1$

For each of the following a values plot the resulting graph on the Cartesian system given below and explain the effect of variable a.

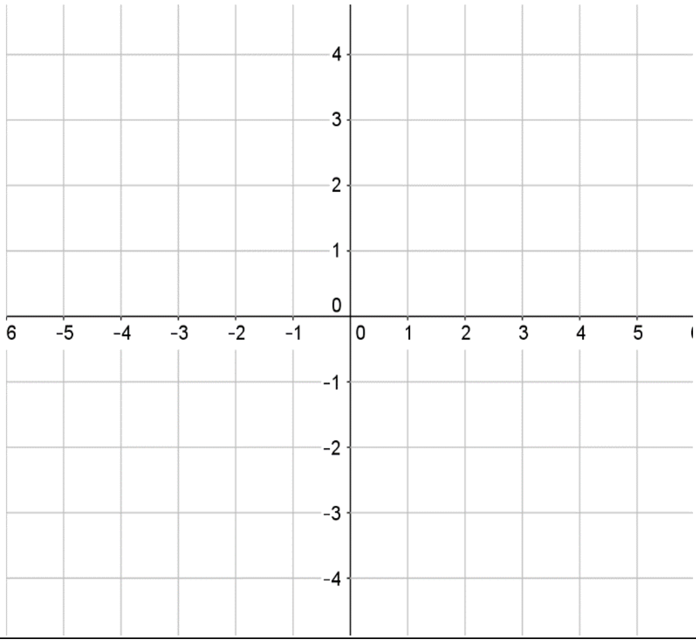
3.1. Use slider a to make $\boldsymbol{a = 1}$



3.2. Use slider a to make $\boldsymbol{a = -3}$



3.3. Use slider a to make $\boldsymbol{a = -1}$



Section C: Learners creating and reflecting algebraic functions using GeoGebra software

Instructions: make use of the GeoGebra applet with a & q sliders to answer the following questions.

1. Type $f(x) = ax + q$ on GeoGebra applet input space to create linear function.

   1.1. Move around sliders a &q and observe their effects on the graph. Write down your observations.

   1.2. Indicate how will the graph of $y = 0{,}3x + 10$ look like?

2. Type $h(x) = ax^2 + q$ on the GeoGebra applet input space to create a parabola function.
   2.1. Move around sliders a &q and observe their effects on the graph. Write down your observations.

   2.2. Indicate how will the graph of $y = -0{,}5x^2 + 8$ look like?

3. Type $g(x) = \frac{a}{x} + q$ on the GeoGebra applet input space to create a hyperbola function. Then type $y = q$ to create the line that indicates y asymptote.
   3.1. Move around sliders a &q and observe their effects on the graph. Write down your observations.

3.2. Indicate how will the graph of $y = \frac{-6}{x} - 25$ look like?

4. Does variable a have same effect in the algebraic functions you created? Explain the effect of variable a for each function.

Linear function:

Hyperbola Function:

Parabola function:

5. Does variable q have same effect on the algebraic functions you created? Explain the effect of variable q for each function.

Linear function:

Hyperbola function:

Parabola function:

APPENDIX 8: Post-test worksheet

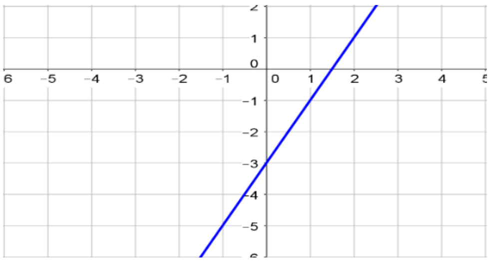
Instructor:	Miss A Ngwabe	Name:	Learner
Class:	Group	Date:	
Title:	Algebraic functions		

Section A: Converting graphs to equations.

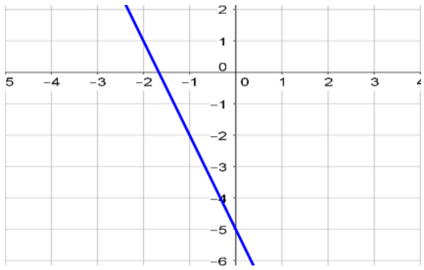
Instructions: For each of the following questions select an option that best describe the question, make a cross on the correct letter in the box at the end of each question.

1. Which one of the following graphs defines the given equation: $f(x) = -3x - 5$

A



B



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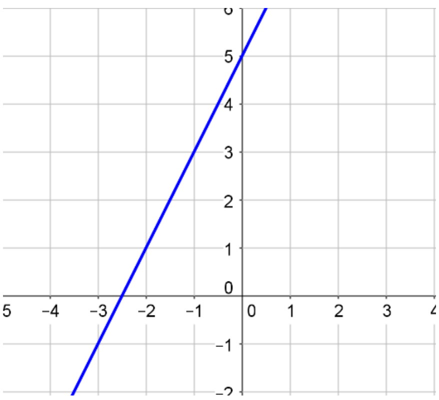

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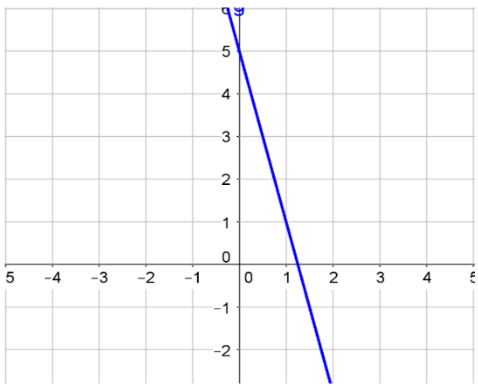
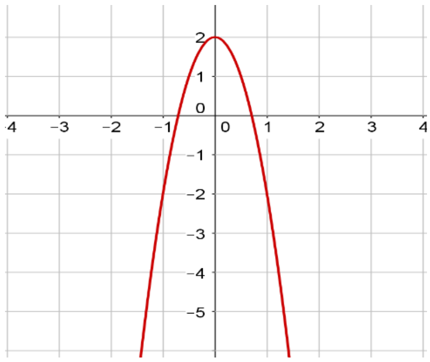


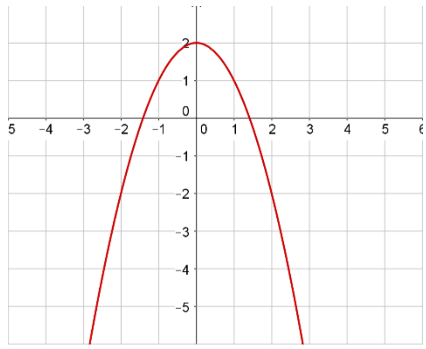
A	B	C	D

2. Which one of the following graphs defines the given equation: $f(x) = 4x^2 + 2$

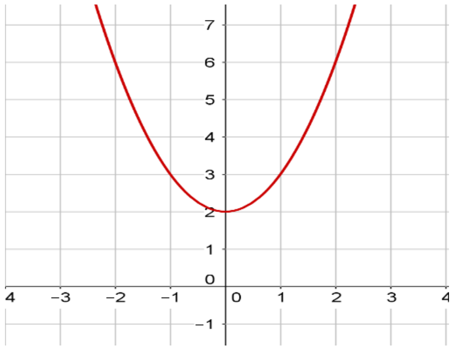
A



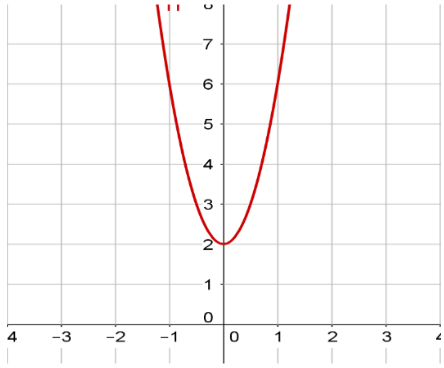
B



C



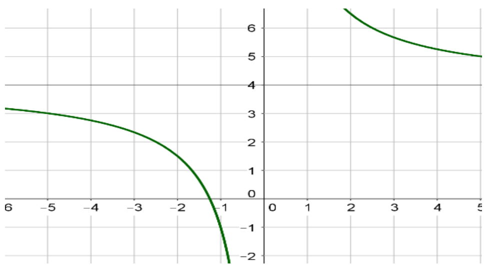
D



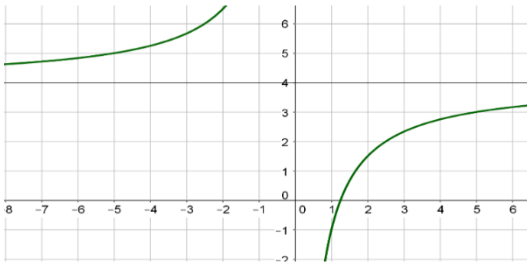
A	B	C	D

3. Which one of the following graphs defines the given equation: $y = \frac{4}{x} - 3$

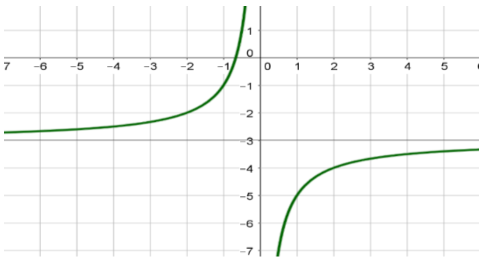
A



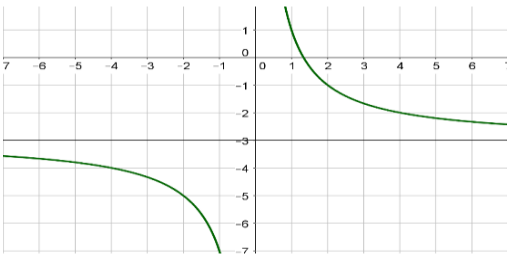
B



C



D



A	B	C	D

Section B: Converting equations to graphs

Instructions

For each of the following questions select an option that best describes the question, make a cross on the correct letter in the box. Read the question carefully before you select the correct answer.

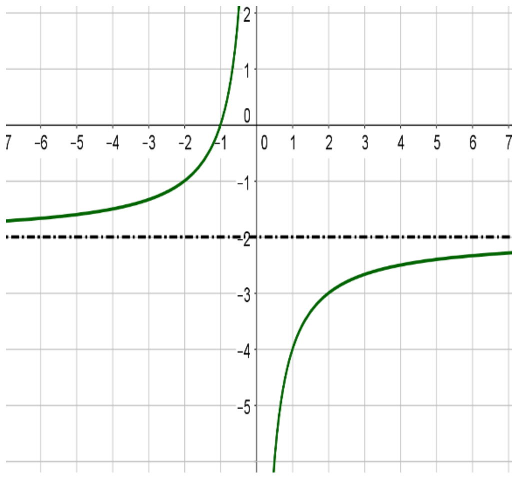
1. Which of the following equations defines the given graph?

A. $f(x) = \frac{-2}{x} - 2$

B. $f(x) = \frac{2}{x} - 2$

C. $f(x) = \frac{1}{x} - 2$

D. $f(x) = \frac{-1}{x} - 2$



A	B	C	D

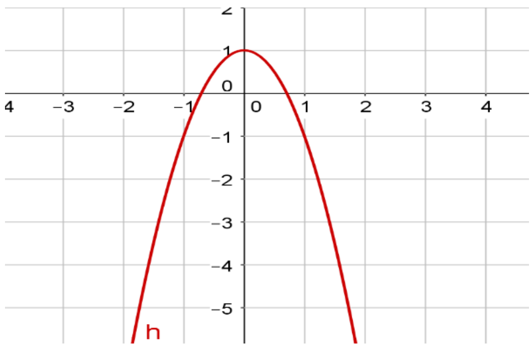
2. Which of the following equations defines the given graph?

A. $f(x) = -1x^2 + 1$

B. $f(x) = 2x^2 + 1$

C. $f(x) = -2x^2 + 1$

D. $f(x) = x^2 + 1$



A	B	C	D

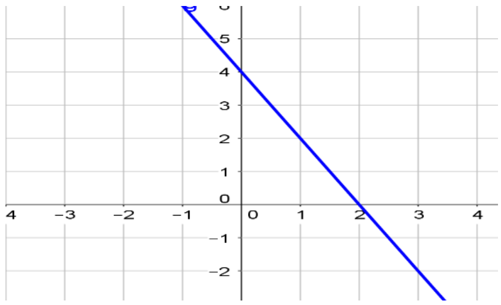
3. Which of the following equations defines the given graph?

A. $f(x) = -2x + 4$

B. $f(x) = x + 4$

C. $f(x) = -x + 4$

D. $f(x) = 2x + 4$



A	B	C	D

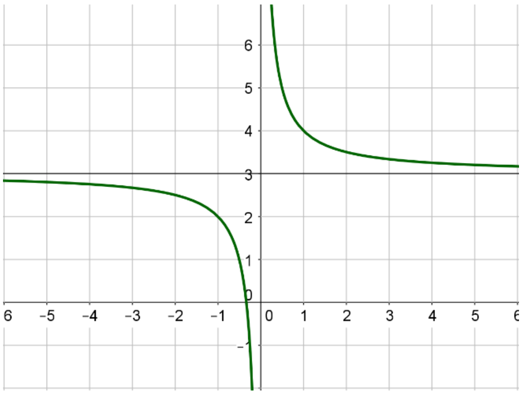
4. Which of the following equations defines the given graph?

A. $f(x) = \frac{2}{x} - 2$

B. $f(x) = \frac{2}{x} + 3$

C. $f(x) = \frac{2}{x} - 3$

D. $f(x) = \frac{2}{x} + 2$



A	B	C	D

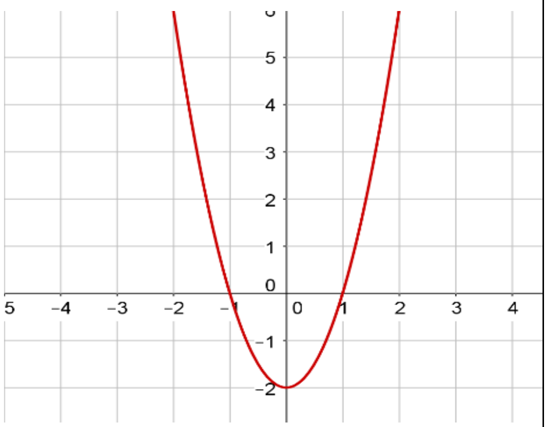
5. Which of the following equations defines the given graph?

A. $f(x) = 2x^2 - 1$

B. $f(x) = 2x^2 + 1$

C. $f(x) = 2x^2 + 2$

D. $f(x) = 2x^2 - 2$



A	B	C	D

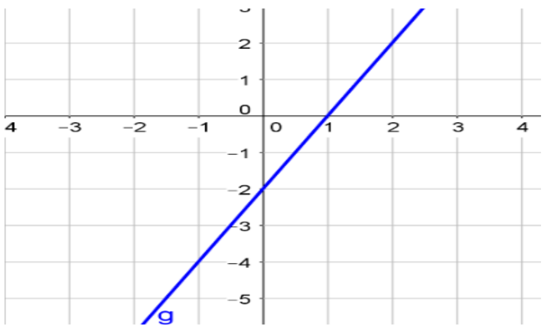
6. Which of the following equations defines the given graph?

A. $f(x) = 2x + 1$

B. $f(x) = 2x - 2$

C. $f(x) = 2x - 4$

D. $f(x) = 2x + 2$



A	B	C	D

Section C: Algebraic function concepts: parameters $\boldsymbol{a}$ & $\boldsymbol{q}$

Instructions

For each of the following questions select an option that best describes the question, make a cross on the correct letter in the box. Read the question carefully before you select the correct answer.

1. The following defines the graphical representation of a parabola equation given as $y = -2x^2 - 8$:

A. Decreasing graph turning at $y = -8$
B. Increasing graph turning at $y = 8$
C. Decreasing graph turning $y = 8$
D. Increasing graph turning at $y = -8$

A	B	C	D

2. The following defines the graphical representation of a linear equation given as $y = -3x + 5$:

A. Increasing line cutting at $y = 5$
B. Decreasing line cutting $y = -5$
C. Decreasing line cutting $y = 5$
D. Increasing line cutting $y = -5$

A	B	C	D

3. The following defines the graphical representation of a hyperbola equation given as $y = -\frac{1}{x} - 4$:

A. The graph occupies second and fourth quadrants with horizontal asymptote cutting $y = 4$
B. The graph occupies first and third quadrants with horizontal asymptote cutting $y = -4$
C. The graph occupies first and third quadrants with horizontal asymptote cutting $y = 4$
D. The graph occupies second and fourth quadrants with horizontal asymptote cutting $y = -4$

A	B	C	D

APPENDIX 9: Schedule for GeoGebra intervention

GROUP NAME	SESSION 1	SESSION 2	SESSION 3	SESSION 4
A	11/08/17 8h00-8h55	18/08/17 8h00-8h55	25/08/17 8h00-8h55	01/09/17 8h00-8h55
B	11/08/17 9h00-10h55	18/08/17 9h00-10h55	25/08/217 9h00-10h55	01/09/17 9h00-10h55
C	11/08/17 11h00-11h55	18/08/17 11h00-11h55	25/08/217 11h00-11h55	01/09/17 11h00-11h55
D	08/08/17 13h00-13h55	15/08/17 13h00-13h55	22/08/17 13h00-13h55	29/08/17 13h00-13h55

APPENDIX 10: NCV level two Mathematics Subject Guidelines

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATES (VOCATIONAL)

SUBJECT GUIDELINES

MATHEMATICS

NQF LEVEL 2

June 2010

INTRODUCTION

A. What is *Mathematics?*

Reader's Digest Oxford Complete Wordfinder defines Mathematics as *"the abstract science of number, quantity and space studied in its own right."*

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. Through mathematical problem solving, students develop an understanding of the world and can use that understanding to great effect in their daily lives.

Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. The Subject Outcomes and Assessment Standards for Mathematics are designed to allow all students to become citizens who will be able to confidently deal with Mathematics as and when it impinges on their daily lives, their community and the world in general.

B. Why is *Mathematics* Important as a Fundamental subject?

The subject Mathematics (NQF Level 2 – 4) empowers students to:

- Communicate appropriately using numbers, verbal descriptions, graphs, symbols, tables and diagrams.
- Use mathematical process skills to identify, pose and solve problems creatively and critically.
- Organise, interpret and manage mathematical information which demonstrates responsibility and sensitivity to personal and broader societal concerns.
- Work collaboratively in teams and groups to promote understanding in general.
- Collect, analyse and organise quantitative data to evaluate and comment on conclusions.
- Engage responsibly with quantitative arguments relating to local, national and global concerns.

C. How do the Learning Outcomes link with the Critical and Developmental Outcomes?

The Learning Outcomes provide a platform for students to achieve the following Critical Cross field Outcomes and Developmental Outcomes:

- Identify and solve problems and make decisions using critical and creative thinking.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and/or language skills in various modes.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.
- Reflect on and explore a variety of strategies to learn more effectively.

D. Which factors contribute to achieving the Learning Outcomes?

A learning enabling environment for Mathematics is created by:

- Encouraging an attitude of *"I can do Mathematics"* in students.
- Using different media and learning approaches to accommodate different learning styles.
- Applying different strategies to develop and encourage creativity and problem solving capabilities.
- Focusing on strategies that develop higher level cognitive skills such as analytical and logical thinking and reasoning.
- Adopting a learning pace that will instil a sense of achievement rather than one of constant failure.
- Practical and relevant examples so that students can apply abstract concepts in real everyday life situations.
- Providing remedial and support interventions for those students that struggle to grasp fundamental outcomes.
- Encouraging continuous work and exercise for students to develop a sense of achievement and success.

MATHEMATICS – LEVEL 2

CONTENTS

1. DURATION AND TUITION TIME

This is a one year instructional programme comprising 200 teaching and learning hours. The subject may be offered on a part-time basis provided all the assessment requirements are adhered to.

Students with special education needs (LSEN) must be catered for in a way that eliminates barriers to learning.

2. SUBJECT LEVEL OUTCOMES AND FOCUS

Students will be able to:

- Recognise and work with numbers and their relationships to estimate, calculate and check solutions.
- Investigate and represent a wide range of algebraic expressions and functions and solve related problems.
- Describe, represent, analyse and explain properties of shape in two- and three-dimensional space with justification.
- Analyse data to establish statistical models to solve related problems.
- Plan personal finances in the context of income and expenditure, basic budgets and the impact of interest rates.

3. ASSESSMENT REQUIREMENTS

3.1. Internal assessment (25 percent)

All internal assessments must be finalised by a competent assessor. Refer to the *Assessment Guideline for Mathematics Level 2* for specific mark allocation on internal assessment.

Three formal written tests & one internal examination	70% of ICASS
Two assignments & one practical assessment	30% of ICASS

Possible distribution of tests, practical assignments and internal examination

Term 1	Term 2	Term 3	Term 4	Total
2	2	*2	1	7

*One of these must be an internal examination

- **Some examples of practical assessments include, but are not limited to:**

  A. Practical exercise work and applications to contextual problems
  B. Presentations (lectures, demonstrations, group discussions and activities, practical work, observation, role-play, self activity, judging and evaluation)
  C. Use of aids
  D. Exhibitions
  E. Visits

- **Evidence in practical assessments**

All evidence pertaining to evaluation of practical work must be reflected in the students' Portfolios of Evidence (PoE).

3.1.1 Processing of internal assessment mark for the year

A year mark out of 100 is calculated from marks obtained during the internal continuous assessment (ICASS).

3.1.2 Moderation of internal assessment mark

Internal assessment is subjected to both internal and external moderation procedures as set out in the *National Examination Policy for FET College Programmes.*

3.2 External assessment (75 percent)

A National Examination is conducted annually in October or November by means of a paper/s set and moderated externally.

External assessment details and procedures are set out in *Assessment Guidelines: Mathematics* (Level 2).

4. WEIGHTED VALUES OF TOPICS

TOPICS	WEIGHTED VALUE
1. Numbers	15
2. Functions and Algebra	25
3. Space, Shape and Measurement	30
4. Data Handling	20
5. Financial Mathematics	10
TOTAL	**100**

5. CALCULATION OF FINAL MARK

Continuous assessment: X/100 x 25/1 = a mark out of 25 (a)
Examination mark: X/100 x 75/1= a mark out of 75 (b)
Final mark: (a) + (b) = a mark out of 100

All marks are systematically processed and accurately recorded to be available as hard copy evidence for, amongst others, moderation and verification purposes.

6. PASS REQUIREMENTS

The student must obtain a minimum of 30 percent in Mathematics. A pass will be condoned at 25 percent if it is the only subject stopping the student from progressing to Level 3.

7. SUBJECT AND LEARNING OUTCOMES

On completion of Mathematics Level 2, the student should have covered the following topics:

Topic 1: Numbers
Topic 2: Functions and Algebra
Topic 3: Space, Shape and Measurement
Topic 4: Data Handling
Topic 5: Financial Mathematics

Topic 1: Numbers

Subject Outcome 1.1: Use computational tools and strategies and make estimates and approximations.
Learning Outcomes
Students are able to:

- Use a scientific calculator correctly to solve expressions involving addition, subtraction, multiplication, division, squares, cubes, square roots and cube roots.
- Estimate and approximate physical quantities to solve problems in practical situations. Quantities include length, time, mass and temperature

Subject Outcome 1.2: Demonstrate an understanding of numbers, relationships among numbers and number systems and represent numbers in different ways.
Learning Outcomes

Students are able to:

- Identify rational and irrational numbers.
- Round off rational and irrational numbers to an appropriate degree of accuracy
- Convert rational numbers between terminating and recurring decimals to the form $\frac{a}{b}$; $a,b \in Z$; $b \neq 0$.
- Apply the following laws of exponents.

$a^m \times a^n = a^{m+n}$ $\quad$ $a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{m \times n}$ $\quad$ $(ab)^m = a^m b^m$

$(a^m b^n)^p = a^{mp} b^{np}$ $\quad$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$ $\quad$ $a^{-n} = \frac{1}{a^n}$

$\frac{1}{a^{-n}} = a^n$ $\quad$ $a^0 = 1$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

- Rationalise fractions with surd denominators (binomial and monomial denominators) without using a calculator.
- Add, subtract, multiply and divide simple surds.
- Manipulate simple technical and non technical formulae.
- Solve an unknown variable in simple technical and non technical formulae.
- Identify and work with arithmetic sequences and series.

Topic 2: Functions and Algebra.

Subject Outcome 2.1: Use a variety of techniques to sketch and interpret information from graphs of algebraic and transcendental functions.

Learning Outcomes

Students are able to:

- Generate graphs by means of point-by-point plotting supported by available technology.
- Use the generated graphs to make and test conjectures.
- Generalise the effects of the parameters *a* and *q* on the generated graphs of functions including the following:

$y = ax + q$

$y = ax^2 + q$

$y = \frac{a}{x} + q$

$y = ab^x + q$, $b > 0$

$y = a \sin x + q$

$y = a \cos x + q$

$y = a \tan x + q$

- Define functions

- Identify the following characteristics with of functions:
  - Domain and range.
  - Intercepts with axes.
  - Turning points, minima and maxima.
  - Asymptotes
  - Shape and symmetry.
  - Periodicity and amplitude
  - Functions or non functions.
  - Continuous or discontinuous.
  - Sketch graphs and find equations of graphs for the following functions:

    $y=ax+q$

    $y=ax^2+q$

    $y=\frac{a}{x}+q$

    $y=ab^x+q \ , b>0$

    $y=a\sin x+q$

    $y=a\cos x+q$

    $y=a\tan x+q$

Subject Outcome 2.2: Manipulate and simplify algebraic expressions.

Learning Outcomes

Students are able to:

- Find products of two binomials
- Find products of binomials with trinomials
- Factorise by identifying/ taking out of common factor.
- Factorise by grouping in pairs
- Factorise the difference of two squares.
- Factorise trinomials.
- Simplify algebraic fractions with monomial denominators.

Subject Outcome 2.3: Solve algebraic equations and inequalities.

Learning Outcomes

Students are able to:

- Solve linear equations.
- Solve quadratic equations by factorization.
- Solve exponential equations in the form $ka^x = m$ (where x is an integer) by using the laws of exponents.
- Solve inequalities in one variable and represent the solution in set builder notation, interval notation and on the number line.
- Solve simultaneous equations with two unknowns algebraically and graphically, where both equations are linear.

Topic 3: Space, Shape and Measurement.

Subject Outcome 3.1: Measure and calculate physical quantities.

Learning Outcomes

Students are able to:

- Read scales on measuring instruments correctly. Instruments to include the ruler and protractor.
- Use symbols and Systeme Internationale (SI) units as appropriate to the situation.

Subject Outcome 3.2: Calculate perimeter, surface area and volume in two and three dimensional geometrical shapes.

Learning Outcomes

Students are able to:

- Calculate the perimeter and surface area of the following laminas:
  - Square
  - Rectangle
  - Circle
  - Triangle
  - Parallelogram
  - Trapezium
  - Hexagons

- Calculate the volume of the following geometric objects:
  - Cubes
  - Rectangular prisms
  - Cylinders
  - Triangular prisms
  - Hexagonal prisms

- Investigate the effect on area of laminas where one or more dimensions are multiplied by a constant factor k is investigated

- Investigate the effect on the volume and surface area of right prisms and cylinders , where one or more dimensions are multiplied by a constant factor k

Subject Outcome 3.3: Use the Cartesian co-ordinate system to derive and apply equations.

Learning Outcomes

Students are able to:

- Use the Cartesian co-ordinate system to plot points, lines and polygons.
- Use the Cartesian co-ordinate system to calculate the distance between two points.
- Use the Cartesian co-ordinate system to find the gradient of the line joining two points.
- Use the Cartesian co-ordinate system to find the co-ordinates of the midpoint of a line segment joining two points

Subject Outcome 3.4: Use and apply transformations to plot co-ordinates.

Learning Outcomes

Students are able to:

- Find the co-ordinates of the point $(x; y)$ after it is translated p units horizontally and q units vertically.
- Find the co-ordinates of the point $(x; y)$ after it is reflected about the x-axis, the y-axis, and the line $y = -x$ and the line $y = x$.

Subject Outcome 3.5: Solve problems by constructing and interpreting geometrical models.

Learning Outcomes

Students are able to:

- Investigate the relationship between the sides of a right-angled triangle to develop the Theorem of Pythagoras.
- Use the Theorem of Pythagoras to calculate a missing length in a right-angled triangle leaving answers in the most appropriate form.

Subject Outcome 3.6: Solve problems by constructing and interpreting trigonometric models.

Learning Outcomes

Students are able to:

- Define and use the following trigonometric functions: $\sin\theta$; $\cos\theta$; $\tan\theta$;
- Calculate trigonometric ratios in each of the quadrants where one ratio in that quadrant is given

  $\sin\theta = \dfrac{3}{5}$ and $90° \le \theta \le 180°$

- Solve problems in two dimensions by using the trigonometric functions ($\sin\theta$; $\cos\theta$; $\tan\theta$;)
- Express an appreciation of the contribution to the history of the development and the use of geometry and trigonometry by various cultures.
  (Not to be examined, delivered by means of a presentation by the lecturer or completed as a project)

Topic 4: Data Handling

Subject Outcome 4.1: Calculate central tendencies and dispersion of data.
Learning Outcomes
Students are able to:

- Calculate central tendency of ungrouped data, namely the mean, median and mode.
- Calculate measures of dispersion including range, percentiles, quartiles, inter-quartile and semi-inter-quartile range.

Subject Outcome 4.2: Represent data effectively.
Learning Outcomes
Students are able to:

- Represent data effectively, choosing appropriately from:
  - Construction of Frequency Distribution/Tally Chart
  - Bar and compound bar graphs;
  - Construction of the stem and leaf plot;
  - Histograms (grouped data);
  - Frequency polygons;
  - Pie charts;
  - Line and broken line graphs.

Topic 5: Financial Mathematics

Subject Outcome 5.1: Plan and manage personal and household finances.
Learning Outcomes
Students are able to:

- Describe financial concepts related to personal finances, methods of financing and financial control.

  Range: Needs, wants, salary, wage, income, expense, budget, fixed expense, variable expense, savings account, cheque account, short term investment, medium term investment, fixed deposit, endowment, long term investment, credit card account, debit card, credit card, bank fees , variance, cheque book, unit trusts, retirement annuities, pension fund, interest, interest rate, simple interest, compound interest, principle amount, hire purchase agreement, stokvels, mashonisa

- Draw up a projected personal and household monthly budget.
- Record actual income and expenditure over a period (one month, six months or twelve months) and compare to the projected budget.
- Identify and explain variances between actual and projected figures
- Provide possible corrective methods of financial control.

Subject Outcome 5.2: Use simple and compound interest to explain and define a variety of situations.
Learning Outcomes
Students are able to:

- Differentiate between simple and compound interest.
- Explain the advantages and disadvantages of using simple and compound interest in specific situations.
- Use and manipulate the simple growth formula $A=P(1+in)$ to solve problems.
- Use and manipulate the compound growth formula $A=P(1+i)^n$ to solve problems subject to only annual compounding being made.
  (Range: Manipulation of only A; P; i)

8. RESOURCE NEEDS FOR THE TEACHING OF MATHEMATICS – LEVEL 2

- *Physical resources*
  - Files for portfolios
  - Scientific calculators
  - Graph paper
  - Textbook or workbook
  - Computer stiffy or memory stick
  - Computer and printing facilities
  - Data Projector
  - Applicable graphing software
  - Geometric sets
  - Chalk and chalkboards
  - Paper
  - Overhead Projectors
  - Current newspapers and information about financial packages from banks and investment companies.
  - Internet access or access to a good library or resource centre.
  - Models

- *Human resources*

  A lecturer must have NQF Level 5 Mathematics or an equivalent with an appropriate teaching qualification to teach Level 2 Mathematics.

APPENDIX 11: First focus group interview questions & transcripts before GeoGebra intervention

IDENTIFYING ALGEBRAIC MISCONCEPTIONS

FOCUS GROUP INTRODUCTION: By the researcher

1. WELCOME

Good day everyone and welcome to the session. Thanks for taking the time to join me to talk about algebraic functions. I appreciate your willingness to participate.

2. PURPOSE OF FOCUS GROUPS

The session is basically about your experience on the test you completed which was based on converting functions from their algebraic form to their graphical presentation and vice versa. I need your input and want you to share your honest and open thoughts and experiences with me.

GROUND RULES

- I NEED YOU TO DO THE TALKING.
  I would like everyone to participate. I may call on you if I haven't heard from you in a while.

- THERE ARE NO RIGHT OR WRONG ANSWERS
  Every person's experiences and opinions are important. Speak up whether you agree or disagree. I want to hear a wide range of views.

- WHAT IS SAID IN THIS ROOM STAYS HERE
  I want every learner to feel comfortable sharing when sensitive issues come up.

- I WILL BE TAPE RECORDING THE GROUP
  I want to capture everything you have to say. I will not identify anyone by name in my report. You will remain anonymous.

SESSION 1: Group of students showing misconceptions related to the y intercept q.

- *General questions about the y intercept q:*
  a) How can you identify the y intercept from the equation?
  b) How do you know if this is the y intercept from the graph?

- *Linear function $y = ax + q$ based questions*

a) What effect does a negative y intercept value have on the graph?
b) What is the role of the y intercept in a linear graph?

- *Parabola function $y = ax^2 + q$ based questions.*

a) Does the sign of the y intercept have any effect on the shape of the graph?
b) What is the role of the y intercept in a parabola graph?

- *Hyperbola function $y = \frac{a}{x} + q$ based questions.*

a) What is the role of the y intercept in a hyperbola function?
b) The hyperbola graph should not touch both x and y intercepts. Therefore, how do identify the y intercept graphically?

SESSION 2: Group of learners showed misconceptions related to the effect of parameter a in algebraic functions

- *Linear function $y = ax + q$ based questions.*

1. What effects do parameter a have in a parabola graph?
2. What effect does a negative a value have on the graph?
3. What effect does a positive a value have on a graph?

- *Parabola function $y = ax^2 + q$ based questions.*

1. What effects do parameter a have in a parabola graph?
2. What happens to the graph if the value of a is negative?
3. What happens to the graph if the value of a is positive?

- *Hyperbola function $y = \frac{a}{x} + q$ based questions*

1. Between the equation and the graph, which one can you identify the exact value of a?
2. What happens to the graph if the value of a is negative?
3. What happens to the graph if the value of a is positive?

➢ **PARAMETER q TRANSCRIPTS**

- **General questions about the y intercept q:**

c) How can you identify the y intercept from the equation?

B9: "I don't know…"

A8: "q represents a turning point"

C15: "I think q represents axis of symmetry"

d) How do you know if this is the y intercept from the graph?

B3: "that is where the graph turns"

D9: "q is always zero in the graph"

- **Linear function $y = ax + q$ based questions**

c) What effect does a negative y intercept value have on the graph?

A8: "a negative q value makes the straight line smaller"

B3: "mmmmm I don't know…"

C15: "I think if q is negative the graph decreases"

d) What is the role of the y intercept in a linear graph?

C15: "the y intercept is in the vertical line (y-axis), the graph cuts there" **D9:** "that is where the graph cuts"

- **Parabola function $\mathbf{y = ax^2 + q}$ based questions.**

c) Does the sign of the y intercept have any effect on the shape of the graph?

B3: "yes it will affect the shape, if q is negative the graph become a sad face, if it's positive it will be a happy face" **D9:** "it's going to be happy and sad face shape".

d) What is the role of the y intercept in a parabola graph?

A8: "if $y = -1$, the graph will cut the vertical line at -1" **C15:** "(referring to the worksheet) q in section A question 5 is $+5$"

- **Hyperbola function $\mathbf{y = \frac{a}{x} + q}$ based questions.**

c) What is the role of the y intercept in a hyperbola function?

D9: "I think, it is where the graph cuts the y-axis"

d) The hyperbola graph should not touch both x and y intercepts. Therefore, how do identify the y intercept graphically?

Researcher: "Identify the value of q from section A question 4 and explain why do you think the value represents q"

A3: "I saw the line $y = 2$, I think it represent parameter q". **B9:** "the asymptote line represent the value of q" **C15:** "the line is cutting $y = -2$ which means $q = -2$"

- **PARAMETER $\boldsymbol{a}$ TRANSCRIPTS**

- **Linear function $\boldsymbol{y = ax + q}$ based questions.**

4. What effects do parameter a have in a parabola graph?

C10: " it decreases and increases the graph"

5. What effect does a negative a value have on the graph?

B20: " the line decrease if the value of a is negative…"

D25: “the graph become smaller”

6. What effect does a positive a value have on a graph?

C10: “the graph increases”
D2: “the graph will be positioned from third quadrant to first quadrant…..”
B20: “the graph will increase”

- **Parabola function $y = ax^2 + q$ based questions.**

4. What effects do parameter a have in a parabola graph?

B20: “when a is negative the curve faces up and when $a$is positive the curve faces down”.
A7:“when a is negative is a smile face and when a is positive is a sad face”

- **Hyperbola function $y = \frac{a}{x} + q$ based questions**

4. Between the equation and the graph, which one can you identify the exact value of a?

A7: “mmm I think in the graph…?”
D2: “in the graph”
C10: “it’s easy to identify the value of a in the equation...”

5. What happens to the graph if the value of a is negative?

B20: “the graph will cut the negative y value…”
D25: “the graph will move away from the asymptotes”
D2: “the graph will be in the second and fourth quadrants”

6. What happens to the graph if the value of a is positive?

C10: “the graph will be allocated in the first and third quadrants”

APPENDIX 12: Second focus group interview questions & transcripts after GeoGebra intervention

- **QUESTIONS**

Learners' views and experiences of GeoGebra software

FOCUS GROUP INTRODUCTION: By the facilitator

3. PURPOSE OF FOCUS GROUP

The session is basically about your experience on using GeoGebra software to solve questions based on algebraic functions. I need your input and want you to share your honest and open thoughts and experiences with me.

4. FOCUS GROUP QUESTIONS

a) Do you think integration of GeoGebra when solving algebraic functions was helpful? Elaborate your answer.
b) Can you regard GeoGebra software as problem solving tool in algebraic functions.
c) GeoGebra software is defined as a dynamical, explorative and active tool that can be used to drag objects or graphs without spending time as compared to drawing with a pencil (Dejene, 2014). According to your own experience on GeoGebra, do you agree with the above statement?
d) How did GeoGebra software help you to visualize the change of algebraic graphs?
e) GeoGebra is also said to be a constructional tool where graphs can be developed. Would you therefore prefer to construct algebraic functions using GeoGebra software or the traditional way (using pencil and paper)? Justify your answer.
f) Are there any challenges you came across when utilizing GeoGebra software?

- **TRANSCRIPTS**

a) Do you think integration of GeoGebra when solving algebraic functions was helpful? Elaborate your answer.

A8: "yes it was helpful to me, it was easy to see the effect of a and q **D25:** "it was very helpful, it saves time and its easy working with it" **B9: "**GeoGebra helped me develop deeper understanding of the effect of parameter a in each function. For example, it was difficult for me to differentiate between a hyperbola graphical representations with $\mathrm{a} = 1$ and $\mathrm{a} = 3$".

b) Can you regard GeoGebra software as problem solving tool in algebraic functions.

B9: "I used applets to draw graphs and move them to see what will happen to the equation" **C15**: "it does solve problems, I managed to solve all the problems asked in the worksheet by using GeoGebra"

c) GeoGebra software is defined as a dynamical, explorative and active tool that can be used to drag objects or graphs without spending time as compared to drawing with a pencil (Dejene, 2014). According to your own experience on GeoGebra, do you agree with the above statement?

A7: “yes, that’s true. It was very easy to change values of a and q using sliders”
D2: “yes, it doesn’t take longer to draw a graph and also to change the equation. You just move the sliders, then the graph and its equation will change”

d) How did GeoGebra software help you to visualize the change of algebraic graphs?

D9: “when I drag the sliders then together at the same time the graph moves and its equation changes”
C15: it was easy to see the changes in a short period of time, to change the values doesn’t take longer. I was observing all the changes as I dragged the sliders”
A7: “it helped me to also know and see the effects of different a values in each graph. Before using applet it was difficult for me to differentiate if what happens to the graph if maybe $a = +1$ and when $a + 3$”

e) GeoGebra is also said to be a constructional tool where graphs can be developed. Would you therefore prefer to construct algebraic functions using GeoGebra software or the traditional way (using pencil and paper)? Justify your answer.

A8: “yes, constructing graphs using applets is much better than drawing in the paper”
B20: “yes, I prefer to draw graphs using GeoGebra”
A7: “I also found GeoGebra much better to work with graphs”

f) Are there any challenges you came across when utilizing GeoGebra software?

C10: “at the beginning I did not know how to drag a slider until my friend helped me out”