# Teaching NCS (CAPS) FET Mathematics: A comparison between an offline Techno-Blended Model and a traditional approach 

Dissertation

by

Noname Munemo<br>Submitted in fulfilment of the requirements for the<br>Degree of Master of Education: Mathematics Education (Research)<br>in<br>The Faculty of Education<br>at the<br>Nelson Mandela University (Port Elizabeth, RSA)<br>Supervisor: Prof W.A. Olivier<br>Co-supervisors: Prof S.E. van Rensburg \& Mr V. Matsha

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## DECLARATION BY STUDENT

I, the undersigned, declare that this dissertation, which I hereby submit for the degree Master of Education research in Mathematics Education at the Nelson Mandela University, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

Signature:


Date:
30 November 2018

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In mathematics I can report no deficiency, except it be that men do not sufficiently understand the excellent use of mathematics.

## Francis Bacon


#### Abstract

This research is a comparative research study that compares the way learning has taken place when selected mathematics topics were taught to two groups of Grade 11 Mathematics learners. An offline Techno-Blended Teaching and Learning Model was used for one group, while the other group was taught without the integration of technology. The cognitive and affective impact of the use of technology when an offline Techno-Blended T\&L Model was followed, was compared to a corresponding impact of an approach where no technology was integrated during the teaching of the topics. The research study involved teachers and selected learners in the Mathematics classes from four different high schools selected from two urban districts in the Eastern Cape Province in South Africa. The curriculum topics that the research focused on were Euclidean Geometry and Trigonometry.


## PRELUDE TO THE DISSERTATION

The following key words/concepts will be encountered in this research dissertation and their meanings follow in the next sub-section.

## KEY WORDS/CONCEPTS

Traditional face-to-face teaching approach or Traditional approach; blended learning; Offline Techno-Blended Model (TBM) approach; constructivism and constructivist learning environments; Vygotsky's Zone of Proximal Development (ZPD); scaffolding support; Self-Directed Learning (SDL); affective and cognitive learning experience.

## MEANINGS OF KEY WORDS/CONCEPTS

## 1. Traditional approach:

The meaning of the term 'traditional approach' in the title of this research dissertation simply implies the application of traditional face-to-face teaching techniques. In this approach, no technology is integrated in the teaching and learning of Mathematics and the teacher mostly uses the chalk board for instruction.

## 2. Blended learning:

According to Massoud, Iqbal, Stockley \& Noureldin (2011), blended learning is learning that results from employing a mixture of both face-to-face teaching strategies, and online or offline methods for instruction in the classroom. It is a type of learning that results
from an expansion of the traditional face-to-face teaching methods (synchronous) with electronic learning experiences (asynchronous).

## 3. Offline Techno-Blended Model (TBM) approach:

Offline Techno-Blended Model (TBM) approach, in short, is a teaching approach that integrates the use of off-line technology in the teaching and learning of Mathematics to create a blended teaching and learning environment in a Mathematics classroom. It is a teaching approach that utilizes various off-line technologies, in an integrated way, to create constructivist learning environments in a Mathematics classroom or elsewhere (Olivier, 2017).

## 4. Vygotsky's Zone of Proximal Development (ZPD):

The Zone of Proximal Development (ZPD) is the difference between what a learner can do without assistance and what a learner can do with assistance. Vygotsky (1978) defined ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers".

## 5. Constructivism and Constructivist learning environments:

The concept of constructivism emphasizes the student as being the active learner, playing a central role in mediating and controlling learning (Jonassen, 1999). The focus is on the
constructivist view of learning as an active process of participative constructing rather than acquiring knowledge, and instruction as a process that supports construction rather than communicating knowledge. According to Tynjala's article (1999) 'Constructivist learning environment' is a term used to describe teaching and learning situations which are explicitly based on constructivist epistemology and are designed to support learners' knowledge construction processes. Constructivist learning environments must be designed to engage the learner in complex thinking exercises that require reasoning and investigation of the problem to be undertaken.

## 6. Scaffolding support:

Scaffolding support is defined by Greenfield (1984) as the teacher's selective intervention that provides a supportive tool for the learner, which extends his or her skills, thereby allowing the learner to successfully accomplish a task not otherwise possible. This support occurs in every learner's zone of proximal development (ZPD) which is a theory on learning postulated by Lev Vygotsky in 1978. Scaffolding process results in the development of task competency by the learner at a pace that would far outstrip his unassisted efforts (Wood, Bruner \& Ross, 1976). Scaffolding can only be successful if the task at hand is meaningful and challenging (Lau, 1998).

## 7. Self-Directed Learning (SDL):

According to Knowles (2017) self-directed learning is a process in which individuals take the initiative, with or without the help of others, in diagnosing their learning needs, formulating learning goals, identifying human and material resources for learning, choosing and implementing appropriate learning strategies, and evaluating learning outcomes, that is, they take responsibility for, and control of, their own learning.

## 8. Affective and cognitive learning experience:

Affective and cognitive learning experience is a two-dimensional learning framework adapted from Li, Pow, Wong \& Fung's (2009) four-dimensional information literacy framework to examine students' learning which has the following dimensions: Affective dimension, Cognitive dimension, Meta-cognitive dimension, and Socio-cultural dimension. In this dissertation, I will give an analysis of the learning experience from the first two dimensions to give the affective learning experiences that result in change in motivation, attitude and participation as a result of the affective impact on learning, and the cognitive learning experiences resulting in conceptual change and skills improvement from the cognitive impact on learning.

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## CHAPTER 1

## ORIENTATION TO RESEARCH

### 1.1 Background

Globally there is a major concern about the poor performance of learners in mathematics (Siyepu, 2013). In particular, the learners' performance in mathematics at FET level in South Africa is quite alarming. The subject matter knowledge of the majority of learners in South Africa is very parlous and learners experience problems relating to their limited technical vocabulary of mathematics (Van der Walt, Maree \& Ellis, 2008). The South African Human Sciences Research Council (HSRC) conducted a study of the performance of learners at the Grade 8 level in mathematics and science in 2003. Among the 46 countries that participated in the Trends in International Mathematics and Science Study [TIMSS] (2003), South Africa was in position 45, and from the six African countries (Egypt, Tunisia, Morocco, Botswana, Ghana, and South Africa) that took part in the study, South Africa had the lowest score in mathematics and science.

An analysis of learners' performance in South African schools reflected that learners in the former white schools have higher scores than learners in the traditional African schools. Furthermore, private schools perform better than public schools. Reddy (2004) stated that "There is no single cause of South Africa's poor and diverse performance. Preliminary explanations could be linked to multiple, complex and connected sets of issues, including the following: issues of poverty, resources and infrastructure of schools,
low teacher qualification, and poor learning cultures in schools". This was supported by researchers such as Van der Walt et al. (2008) and Ndlovu (2011). Mji and Makgato (2006) assert that "outdated teaching practices and lack of basic content knowledge have resulted in poor teaching standards in mathematics". Thus, the continued poor performance in mathematics by learners in South Africa requires research to find finding new or innovative ways to enhance performance in the subject. It is therefore necessary to conduct research to find more effective teaching practices that will lead to better performance in mathematics.

### 1.2 Rationale

The educational requirement of all countries, and in particular South Africa, is that "everybody should be mathematically literate in this information era and be able to compete in the new world order" (Ahuja, 2006). Despite the government's effort to achieve this goal, many students still lack interest in mathematics, even when the learning of mathematics has been made accessible in all schools. This problem can be attributed to, among other factors, the present model of teaching, lack of qualified Mathematics educators in schools, learners' attitude, feeling of discouragement and boredom in doing mathematics, lack of resources such as textbooks, infrastructure shortage in vast rural areas, and disparities in schools created by the past political history (Olivier, 2017).

While technology is used globally in education, with great success (Isman \& Yaratan, 2006), in South Africa, various attempts at harnessing technology in education, especially in socio-economically deprived communities, have failed dismally. Reasons for this
include lack of ICT literacy amongst teachers, insufficient internet bandwidth, problems with security, and lack of technical support in schools (Olivier, 2017). Due to these unique challenges faced by South African schools, an innovative approach is needed in order to improve the teaching and learning of mathematics. This approach has to take into consideration the needs of the present generation of learners that finds technology to be very appealing. If this technology can appropriately be incorporated into mathematics education, it may enhance their learning of mathematics by making it more exciting and effective which may result in improved performance in the subject.

It is therefore a good idea to study successful education models of other countries. This will allow us, as a country to grow and develop a better education model that incorporates technology at lower costs, to cater for the large section of our population that lives in poverty with poor socio-economic conditions, mostly in rural areas as well as in some urban areas around the country.

### 1.3 Significance of the research

This research will provide a comparison of the learning that has taken place when two groups of Grade 11 Mathematics learners were taught selected mathematics topics from the CAPS curriculum using an offline Techno-Blended Model (TBM approach) for one group and a traditional approach for the other group. In this research I shall compare the cognitive and affective impact on learning when using an offline TBM approach and using an approach where no technology is applied (traditional approach).

The significance of this study is that it will establish if the use of certain types of offline technologies when teaching mathematics will bring about:

- better understanding,
- improvement/better results,
- solutions to various cognitive and affective challenges during the learning of mathematics.

Should it be established that the Techno-Blended Model leads to improvement; the research findings will be shared with administrators of schools for possible planning and designing of programmes aimed at improving the performance of students taking Mathematics as a subject. Furthermore, the information on the impact of using the digital teaching and learning material that accompanies the offline Techno-Blended Model could also be made available to some stakeholders including the Department of Education.

### 1.4 Theoretical framework

In any scientific research setting in the field of education, a researcher does not only need to conduct his or her research practically but also need to place it within some theoretical framework. Research in the area of educational technology has often been critiqued for a lack of theoretical grounding (Mishra \& Koehler, 2006). A challenge exists to develop frameworks within which to describe, theorize, and interpret the range of learning
activities that teachers and students engage in as they interact in technology enriched settings. Such frameworks serve at least two purposes: first, to capture and interpret characteristic forms of technology use exhibited by students and teachers in classroom learning episodes; and second, to theorize teaching actions as they are used to orchestrate the learning of students (Galbraith \& Goos, 2009). As Bednar, Cunningham, Duffy, and Perry (1995) argued, instructional strategies and tools must be based on some theory of learning and cognition.

Since this research is a comparative study that will compare the way learning has taken place when two groups of Grade 11 Mathematics learners are taught selected mathematics topics using two different approaches, the theoretical foundation or theoretical framework(s) that I will look at in this research are the frameworks of constructivism, Vygotsky's Zone of Proximal Development (ZPD), and scaffolding. These three theoretical learning frameworks, have been established by various researchers to have a complementary relationship with the teaching of mathematics using technology with the implementation of each one benefitting the other (see for example in Brush \& Saye, 2000; Duffy \& Cunningham, 1996; Nanjappa \& Grant, 2007). These researchers discovered that constructivist theoretical learning strategies, Vygotsky's Zone of Proximal Development (ZPD), and scaffolding strategies exploit technology for optimal impact in learning. Therefore, the integration of technology in the teaching and learning of mathematics articulates with the three theoretical learning frameworks that have been stated above.

### 1.5 What is offline Techno-Blended Teaching and Learning Model?

Offline Techno-Blended Teaching and Learning Model (TBM) is a teaching approach that was developed by the Govan Mbeki Mathematics Development Centre (GMMDC), at the Nelson Mandela University (NMU). This model has evolved through an extensive process of participative action research (PAR) over the period 2010 - 2016. This teaching approach integrates the use of various offline modern technologies in teaching mathematics inside and outside the school classroom. The basis for the TBM is the TouchTutor® digital resource package which consists of a comprehensive series of curriculum aligned video and Power Point content lessons and tutorials, examination revision material, CASIO Calculator emulator videos, mathematics assessments, and lesson aligned learner workbooks. During the development of the TBM the TouchTutor® material was installed on laptops for teachers and Android Tablet PC for individual learners, in the FET phase, in ten selected urban secondary schools in the Eastern Cape Province. This TBM model is independent of the internet and the Mathematics educator or learner can utilize the curriculum-aligned video or Power Point lessons on the laptop/Tablet whenever he/she chooses to do so. The material also serves as a curriculum reference and after hours scaffolding support platform for educators and learners at the project schools.

Research shows that dynamic visualization and multiple representations of mathematical concepts, relationships and results can contribute richly to conceptual understanding and learner interactivity during lesson presentations (Chan \& Tutkaluk, 2010; Roschelle, 2013). Hence, as part of the TBM, the dynamic graphic software (GeoGebra) was used to
create a series of animated demonstrations to present curriculum topics with the aim of optimal visualization for improved conceptual understanding. Another video resource component of the TBM was established by creating an examination revision video series that covers the compulsory final NCS national examinations (Papers 1 and 2) from 2008 to 2012. In addition, a comprehensive series of digital Mathematics learner workbooks with separate solutions was developed to assist learners. This was done for Grades 10-12 and the workbooks are aligned with content lessons of the NCS CAPS Mathematics video series. All the existing mathematics content and support resource material for Grades 10-12 were combined with exciting additional support functions for teachers and learners in an innovative way to form the TouchTutor® Mathematics support package which runs in an offline local browser environment. A flexible menu system within the software package allows for the easy use of the TouchTutor® Package with Android Tablets, Windows Desktop PCs and Laptops. Users of this package can freely navigate to any component of the TouchTutor ${ }^{\circledR}$ Mathematics resource material by clicking a mouse or touching a digital screen (Olivier, 2017). Figure 1 below shows some of the components of the offline Techno-Blended Mathematics Teaching and Learning Model:


Figure 1: Components of the offline TBM Teaching and Learning Model

### 1.6 Problem Statement

The problem to be investigated falls within the ambit of Mathematics Education. As mentioned earlier, many students in South Africa struggle with the learning of mathematics at FET level which they find to be challenging and difficult to understand (TIMSS, 2003; Van der Walt et al., 2008). The poor performance and inability to pass mathematics, which results in learners failing to progress to Grade 12 and beyond, poses a grave challenge to the economic growth of our country (Olivier, 2017). This problem can be attributed to, among other factors, the content deficit that had developed for most learners in the years prior to the FET phase. Also the present provision of teaching
capacity and teaching and learning resources provided by the Government is inadequate to address the challenges faced in mathematics education in most government schools. The educational needs of $21^{\text {st }}$ century learners call for different approaches to teaching and learning in schools. Most contemporary teaching strategies that are applied in today's mathematics classroom involve the "talk and chalk" model which is no longer adequate to cater for the needs of the present $21^{\text {st }}$ century generation of learners. Against the backdrop of the above, the problem that will be investigated can be stated as follows:

Will there be a significant difference in the impact on learning when a modern offline technology-assisted teaching and learning model of the TBM approach is integrated in the teaching and learning of mathematics in the classroom compared to where the traditional approach is applied?

### 1.7 Research question and sub-questions

Following the statement of the problem presented above, the main research question that will be investigated or addressed will be:

Does the use of the offline Techno-Blended Teaching and Learning Model (TBM approach) contribute to an improved affective and cognitive learning experience for mathematics learners at the FET level?

The sub-questions to be addressed will be:

1. Does the use of the TBM approach assist in the creation of a learning environment that will motivate and encourage learners to be more interested in learning Mathematics?

## [Affective impact]

2. Does the use of the TBM approach contribute to the improvement of results in Mathematics?
[Cognitive impact]
3. Which technological components of the Techno-Blended Model did the learners find to be most valuable during the learning of mathematics?
[Affective and cognitive impact]

### 1.8 Research aims and objectives

In this research project, I investigated the impact on learning when the two different pedagogical approaches, teaching using the TBM approach and teaching using the traditional approach, are applied to two groups of Grade 11 Mathematics learners. I presented a comparison of the impact on learning in each case by analyzing and comparing the learning outcomes of the two groups of learners. According to Paivi Tynjala's article (1999), learning outcomes can be examined from three different viewpoints; (1) as the students' subjective learning experiences, (2) as conceptual change, and (3) as measured by a traditional examination. Since the main focus of this research study is to investigate and compare the Affective and Cognitive impact on learning when the two different pedagogical approaches are applied, the researcher will examine the
students' learning outcomes from viewpoint (1) by analyzing the responses that learners will provide in the questionnaires and interviews conducted to get the learners' subjective learning experience for the affective impact on learning. The learning outcomes will also be examined from viewpoints (2) and (3) by giving the learners a pre-test followed by a post-test exercise and analyzing the responses the learners provide for the cognitive impact on learning. The research also aims to establish which approach will be the best in motivating students to:

- have a desire to learn mathematics [affective impact],
- change their behaviour or attitude towards the subject [affective impact],
- participate actively in the learning process of mathematics [affective impact],
- and improve performance in the subject [cognitive impact].

The objective of the research study is to compare the two different teaching and learning models and establish which one is more effective in improving students’ learning and foster them to acquire a deeper understanding of mathematics. The research study aimed to advance an understanding that different classroom practices may lead to different learning experiences if same knowledge areas are presented to learners in different classes.

In the study, the impact of implementing a comprehensive technology based blended teaching and learning model in the NCS Mathematics classrooms was measured and
compared with findings from a control group where no technology was used. In so doing, the researcher was in a position to be able to explore technology-assisted teaching practices that may be more effective in improving students' learning of mathematics and improve performance in the subject.

### 1.9 Literature review

The use of technology has a long history in mathematics education. Many societies, for example, introduce arithmetic with an abacus, for two reasons. First, the abacus supports computation and, second, the abacus presents a tangible image of mathematics, which helps students understand both the computation and representation of arithmetic (Centre for Technology Learning: Texas, 2007). Technology, such as calculators, standard software programmes and computers may be used effectively to enhance teaching and learning in a number of ways. These include allowing students to perform tedious calculations more quickly, organizing data for tables and graphs efficiently, and presenting processes and findings more clearly (Willard, 2005). Technology also creates flexible learning environments in which students can easily construct and learn new information, solve problems and enhance the stability and quality of learning in a coherent manner. Technology is not only electronic instruments; it also involves new teaching and learning methods that can be used in a beneficial way in education (Isman, 2003; Mishra \& Koehler, 2006).

Research results indicate that most mathematics teachers do not use educational technology to teach mathematics even though educational technology motivates students
to learn more (Isman \& Yaratan, 2006). Technology is an essential tool for learning mathematics in the $21^{\text {st }}$ century, and all schools should ensure that all their students have access to technology (NCTM Position, 2008). In technology-enhanced teaching and learning, the addition of computers and calculators to existing curricula and instruction, was done so primarily as an aid to computation (Behr \& Wheeler, 1981) or for the delivery of existing content (Kraus, 1982 in Kaput \& Thompson, 1994). The use of technology in mathematics education is not intended to replace conceptual understanding, computational fluency or problem-solving skills amongst learners, but it is used to provide scaffolding and support to gain access to mathematical content and problem-solving contexts, extend mathematical reasoning and sense making, and to enhance computational fluency. One aspect of electronic technologies is that it promotes interactivity in mathematics education. It has the power to change the experience of doing or learning mathematics, reshaping and expanding it from direct experience in a physical space to experience mediated by the computational medium.

To improve the low pass rate of Mathematics learners in South Africa, new approaches to teaching the subject are needed. This is because learners have changed radically and our learners today are no longer the same people our education system was designed to teach (Prensky, 2001). The paradigm shift from traditional teaching methods to contemporary constructivist teaching and learning methods should also be complemented with the integration of technology, in a blended teaching and learning environment, to cater for the needs of the present generation of learners. It does not make sense to try to prepare learners for the future by teaching them using exclusively methods that are historic and
outdated. The prevalence of digital technologies in society today has brought education to a point where teachers have an overpowering responsibility to incorporate these technologies effectively into their teaching (Way \& Beardon, 2009). When technology is used strategically, it can provide access to mathematics for all students. Teachers should maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics.

Although most research shows that the integration of digital technologies could reinforce the pedagogical shift to learner-centered active learning, it is worth noting that the technology can be used for almost any purpose, including putting learners in a passive role or teaching the memorization of facts or rules. The integration of technology into mathematics education is not without controversy. Many educators fear that calculators and other technologies will eliminate opportunities for students to sharpen their computational abilities. However, teachers and learners alike should use calculators as a tool rather than a crutch, as a means of extending mathematics in new forms, exercising higher order thinking skills, and working more efficiently and accurately, without sacrificing an underlying comprehension of the concepts (NCTM, 2008). When properly applied to mathematics teaching and learning, technology has the potential to intensify and reinforce the integration of learners' analytical abilities, creative capacity, cooperative work skills, problem-solving strategies and communication skills with their information and technology skills (Powers \& Blubaugh, 2005; Niess, 2006).

Despite the many potential benefits of technology in mathematics education, issues and barriers exist that should be addressed. The greatest obstacles to technology in any field of education are funding and access. Teachers and learners need access to the hardware and software before any useful application of the technology can occur, and teachers require professional development and construction time to utilize technology effectively (NCTM, 2008). Despite these issues and challenges, the fact remains that technology has the potential to play an important role in enhancing the learning process of mathematics, which may lead to improvement of results in the subject. In this research study, the researcher is going to take a closer look at the potential role of offline technology in enhancing the learning of mathematics in a mathematics classroom - whether it is able to support learners to become independent self-directed learners (SDL) and increase their confidence in solving problems in mathematics education?

### 1.10 Research design and methodology

### 1.10.1 Research method

The research method that was used was a mixed methods approach which comprised both qualitative and quantitative research methods (Macmillan \& Schumacher, 2009). The research was a comparative research study that compared the way learning has taken place when two different teaching methods (teaching using the traditional approach and teaching using the offline TBM approach) were used to teach the topics Euclidean Geometry and Trigonometry in the CAPS curriculum to two groups of Grade 11 Mathematics learners.

### 1.10.2 Method of sample selection

The research study investigated the impact on learning from a two-dimensional framework view: (1) Cognitive impact - conceptual understanding and improvement in performance, and (2) Affective impact - motivation, attitude, and participation. One class from each of the four participating educators' normal classes of Grade 11 Mathematics learners was selected to take part in the study. So the method of sample selection that was used is the Cluster Sampling Method because the selection of participants for the research came from naturally occurring groups (i.e. the four already existing classes that were taught by the four educators selected from four different high schools from the two urban districts in the Eastern Cape Province in South Africa). The research study involved learners in the Grade 11 Mathematics classes taught by the four in-service educators. Two of the selected four classes were taught mathematics using an offline TBM approach. Learners from the two classes had Tablets with TouchTutor® package pre-loaded and their teachers were trained to use this model. The other two groups of learners were taught using the traditional approach and were selected from two other high schools that were not exposed to the TBM approach. So, Group A (control group) comprised learners from the two classes that were taught Geometry and Trigonometry using the traditional approach, while Group B (experiment group) comprised learners from the two classes that were taught the same topics using the offline TBM approach.

### 1.10.3 Data collection instruments, procedures and analysis strategies

The selected learners had to complete two structured questionnaires that aimed to establish how they experience each approach, in particular, how the approach influenced
their motivation, attitude, and active participation in the learning process of mathematics (under the affective domain). I also conducted interviews with the learners to get as much feedback as I could on how they found each teaching approach. Each group of learners also wrote a baseline and a summative assessment which comprised two sets of questions that the learners answered in class on two separate occasions or sessions that I met with them prior and after the topics were completed by their teachers. The two test exercises were used to examine the learners' conceptual understanding and improvement in performance (under the cognitive domain). The first exercise (pre-test) that was given to the learners was used as a diagnostic exercise that aimed to test their knowledge base prior to the topic chapters being taught in Grade 11. This pre-test comprised mostly Geometry and Trigonometry questions from the Grade 10 CAPS Mathematics syllabus content. The second exercise (post-test) was given after the project topics had been taught in Grade 11, and so comprised mostly Geometry and Trigonometry questions from Grade 11 CAPS Mathematics syllabus content covered after the two topics were completed during Grade 11. Both the pre-test and post-test results were analyzed to test changes in conceptual understanding and demonstration of relevant mathematical skills of all the learners in the two groups. In order to help evaluate the effectiveness of each teaching approach in terms of learners' conceptual understanding and improvement in performance, the researcher used descriptive and inferential statistical methods to do the analysis and evaluation of the responses provided by the learners in both test exercises. The researcher also provided qualitative interpretation to the statistical analysis of each set of data that was analyzed.

### 1.10.4 Research design

In the research, two overall parameters were controlled. These parameters were the two pedagogic instruction methods (which are the independent variables), while the dependent variable(s) affected by the interventions were the affective and cognitive impact on learning realized in the two different teaching and learning contexts. What were different in the two groups of learners were the two different pedagogic instruction approaches that the different educators applied in their different classrooms which I chose to label as Intervention $X_{1}$ (teaching using the traditional approach) and Intervention $X_{2}$ (teaching using the offline TBM approach) which were applied to Group A (control group) and Group B (experimental group) respectively. Learners in the control group continued using their traditional teaching and learning model for the content topics that were taught while those in the experimental group condition used the technology-enhanced teaching and learning support of the offline Techno-Blended Model to cover the same content topics. So the research design that was used was the Nonequivalent Groups Pretest-Posttest Comparison Design. A graphical representation of the design is given in Figure 2 below:


Figure 2: Structure of the Groups Comparison Research Design

Thus, the research design was an experimental research design because it was concerned with the phenomenon of cause and effect where the dependent variable (impact on learning) was measured when the independent variable (teaching strategy) was different for each group of learners to which it was applied. In the research, I gave a comparison of the impact on learning between the subjects/learners, from the two groups, that had experienced the different intervention teaching methods, and I did so by using the evidence or data collected from the research findings. Thus, the research is a scientific or an evidence-based research as its basis was to establish knowledge about which educational practices had the most positive impact. It is also an action or applied research in the field of qualitative and quantitative research in that it may help to
improve practice in the field of mathematics education by examining the impact of the two pedagogic instruction methods on learning.

### 1.11 Measures taken to avoid bias in the research

This research study involved learners in the Grade 11 Mathematics classes taught by four educators (two that taught mathematics using the offline TBM teaching and learning approach and the other two that used the traditional approach) selected from four different high schools. The reason why four educators were selected and not two was to eliminate bias where the teacher's teaching ability, and not the mode of teaching, might influence the outcome of the research. Further, the four schools from which the four educators were selected were schools where no disturbances in the teaching and learning process occurred for the period from Grade 10 to 11 as this would disadvantage learners in some classes in a group when compared to others due to content gaps created as a result of certain sections not having been properly taught to the learners thereby creating a bias. In order for the research results to be also unbiased, the four educators selected were all qualified to teach Mathematics at FET level with approximately the same number of years of experience and the same theoretical knowledge and understanding of the subject at the FET level.

### 1.12 Ethical considerations

The First Rand - DST National Chair in Mathematics education programme (2011-2015) has already received ethical clearance from the Ethical Clearance Committee of the NMU
to conduct research in projects in schools that are linked to the implementations of the TBM in Mathematics classrooms. This was reflected in the forms handed out to departmental officials, principals, teachers, parents and learners who participated in the research. In the forms that were handed out, educators involved in the study as well as the learners from the experimental and control groups were provided with an explanation of the purpose of the research and why it was necessary for them to take part in the research study. The forms also informed both the educators and learners involved that their privacy would be protected at all times during the research period and any information collected from them would be kept confidential. Their names as well as the school where they come from would not be revealed in the research project or during the presentation/publication of its results. All the teachers that were involved in the study were requested to provide confirmation of their willingness to participate by signing letters of consent as an indication that they had full knowledge about the purpose and nature of the study. The same process was applied to all the learners that were involved in the study. Furthermore, all the learners' parents or guardians were requested to sign in the space provided for parents on the forms that were provided to the learners to give consent for their children to participate in the research study.

### 1.13 Summary

In this chapter a general orientation to the research study was given, covered under the following headings: Background to the research, Rationale, Significance of the research, Theoretical framework, What is offline Techno-Blended Teaching and Learning Model?, Problem Statement, Research question and sub-questions, Research aims and objectives,

Literature review, Research design and methodology, Measures taken to avoid bias in the research, Ethical considerations, and lastly Summary of the chapter. In the next chapter the theoretical framework underpinning the investigation will be elucidated.

## CHAPTER 2

## THEORETICAL FRAMEWORK

### 2.1 Introduction

Research in the area of educational technology has often been critiqued for a lack of theoretical grounding (Mishra \& Koehler, 2006). A challenge exists to develop frameworks within which to describe, theorise, and interpret the range of learning activities engaged by teachers and students in technology enriched settings. Such frameworks serve at least two purposes: first, to capture and interpret characteristic forms of technology use exhibited by students and teachers in classroom learning episodes; and second, to theorise the teaching actions as they are used to orchestrate the learning of students (Galbraith \& Goos, 2009). As Bednar, Cunningham, Duffy, and Perry (1995) argued, instructional strategies and tools must be based on some theory of learning and cognition.

In a scientific research or many other forms of research in the field of education, a researcher does not merely need to conduct his or her research practically but needs to substantiate it with some theoretical framework. Since this research is a comparative research that will compare the way learning has taken place when two groups of Grade 11 Mathematics learners are taught selected mathematics topics using an offline Techno-Blended Model and a traditional approach, the theoretical foundation or theoretical framework(s) that the researcher will look at in this chapter are the
frameworks of constructivism, Vygotsky's Zone of Proximal Development (ZPD), and scaffolding. These three theoretical learning frameworks, have been found by various researchers to have a complementary relationship with the teaching of mathematics using technology with the implementation of each one benefitting the other (for example in Nanjappa \& Grant, 2007; Brush \& Saye, 2000; Duffy \& Cunningham, 1996). The researchers cited that constructivist theoretical learning strategies, Vygotsky's Zone of Proximal Development (ZPD), and scaffolding strategies exploit technology for the greatest impact in learning. The researcher have looked at the three theoretical learning frameworks in detail below, in particular how each one of them have enhanced the teaching and learning of mathematics using technology.

### 2.2 Constructivism as a theoretical learning framework

### 2.2.1 What is constructivism?

As mentioned earlier, a complementary relationship exists between technology and constructivism. Constructivism, derived mainly from the works of Piaget (1970); Bruner (1962, 1979); and Vygotsky (1962, 1978), is both a philosophical and psychological approach based on social cognitivism that assumes that persons, behaviours and environments interact in reciprocal fashion (Schunk, 2000). Constructivism is a school of thought which states that learning takes place in contexts, and that learners form or construct much of what they learn and understand as a function of their experiences in a situation (Schunk, 2000). The concept of constructivism emphasizes the student as being the active learner, playing a central role in mediating and controlling learning (Jonassen,
1999). The focus is on the constructivist view of learning as an active process of constructing rather than acquiring knowledge, and instruction as a process that supports construction rather than communicating knowledge. Constructivist learning environments are designed to engage the learner in more complex thinking exercises that require reasoning and investigation of the problem to be undertaken.

### 2.2.2 Types of constructivism

Cobb (1994) identified two variations of constructivism - cognitive constructivism and social constructivism, and there are undoubtedly more of these variations. Cognitive constructivism tends to draw insight from Piaget and focuses on individual constructions of knowledge discovered in interaction with the environment. Social constructivism relies more on Vygotsky (1978) and views learning as connection with an appropriation from sociocultural context within which we are all immersed. Collaborative learning tools (e.g. technology tools) can be used from both a cognitive constructivist and social constructivist perspective (Bonk \& Cunningham, 2002).

### 2.2.3 How the use of technology enhances constructivism

Many technology tools enable teachers to structure learning activities that address student misconceptions, seek student elaboration of their answers, and pose questions (Bonk \& Cunningham, 2002). Perhaps, even more importantly, some educators have come to recognize the importance of social constructivism for electronic learning because the potential for collaboration and negotiation embedded within it provides the learner
with the opportunity to obtain alternative perspectives on learning issues and offer personal insights to engage in meaning making and knowledge negotiation (Duffy \& Cunningham, 1996). Whereas cognitive constructivists focus on making learning more relevant, building on student prior knowledge, posing contradictions, and addressing misconceptions (Brooks, 1990), social constructivists emphasize human dialogue, interaction, negotiation, and collaboration (Bonk \& Cunningham, 2002).

Modern constructivist learning environments are technology-based in which learners are engaged in meaningful interactions. The emphasis is on learners who interpret and construct meaning based on their own experiences and interactions. Therefore, if educators are to adopt a constructivist approach of the $21^{\text {st }}$ century, they are now challenged to adapt and change instructional design strategies to integrate technology so as to engage learners in meaningful projects and activities that promote exploration, experimentation, construction, collaboration, and reflection of what these learners are studying.

Jonassen (1994) indicated that the growing demand and use of cognitive tools in education is placing students and technology at the center of educational practice, and that learners will increasingly demand that the technology in teaching relates to their real world needs. Technology, according to Jonassen, Peck and Wilson (1999) refers to the designs and environments that engage learners. The focus of both constructivism and technology is then on the creation of learning environments. Likewise, Hannfin and Hill (2002) depict these learning environments as "....contexts in which knowledge-building
tools and the means to create and manipulate artifacts are provided, not one in which concepts are explicitly taught but a place where learners work together and support each other as they use a variety of tools and learning resources in their pursuit of learning goals and problem-solving activities."

In early 2000, researchers started to re-focus and re-examine constructivism as a theoretical foundation in an age where learning was becoming increasingly impacted through technology. Technology in the 2000s has reorganized how we live, how we communicate, and how we learn. Learning needs and theories that describe learning principles and processes should be reflective of underlying social environments. Articles in the 2000s on constructivism mostly addressed the issues of collaborative learning, scaffolding, knowledge-building in multimedia or computer-based learning environments, interactivity (for example see Kang, Choi \& Chang, 2007). Likewise, educational technology in the 2000s also included a great deal of research on IT-related research and studies, while the aspect of 'high touch' (not to mention that of 'high tech') gradually gained the attention of researchers. The researchers on this trend mostly contend that the theoretical basis for their IT practices stem from 'social constructivism' or 'socio-cultural perspectives’ (Down, 2005; Lave \& Wenger, 1991; Vygotsky, 1978).

Constructivism, which first drew attention to itself as an alternative learning method and approach from the mid 90 s, is gradually preparing its own evolution and transformation for the onset of the $2^{\text {nd }}$ generation of constructivism which seeks to function as a theoretical basis of e-learning or IT-enhanced learning environments in the digital age
(Kang et al., 2007). The claim that constructivism is maintained as a theoretical basis for e-learning or IT-mediated learning environments, tends to be accepted more unanimously by most researchers (e.g. in Kang et al., 2007; Down, 2005; and Duffy, 2004). Thus, a more mature and deeper understanding of constructivism as a basis of IT-mediated learning is therefore absolutely necessary to build sound educational environments appropriate to the $21^{\text {st }}$ century.

### 2.2.4 Relationship between constructivism and technology

According to Nanjappa \& Grant (2007), the relationship between constructivism and technology can be viewed by looking at (a) technology as cognitive tools, and (b) technology as a constructive view of the thinking process.

### 2.2.4.1 Technology as cognitive tools

A central assumption of constructivism is that learning is mediated by tools and signs (Duffy \& Cunningham, 1996). The computer is an example of a mediational means that has aspects of both tool and sign. The computer's role in education has been largely viewed as an instructional tool and for providing a richer and more exciting learning environment (Duffy \& Cunningham, 1996). However, by focusing on the learner, the role of technology can support new understandings and capabilities, thus offering a cognitive tool to support cognitive and meta-cognitive processes. Thus the role of the computer is to make available new learning opportunities. Technologies, primarily computers, help build knowledge bases, which will engage the learners more and result in more
meaningful and transferable knowledge (Duffy \& Cunningham, 1996). Learners function as designers using the technology as tools for analyzing the world, accessing information, interpreting and organizing their personal knowledge, and representing what they know to others (Jonassen, 1994). Technological tools can be used by students to analyse subject matter, develop representative mental models, and then transcribe them into knowledge bases (Jonassen, 1994; Jonassen \& Carr, 2000). All of the above is aptly captured by Swain and Pearson (2001), namely that teachers and students must be educated to use the computer as a productivity tool, as well as a tool for learning, research, networking, collaboration, telecommunications, and problem-solving.

### 2.2.4.2 Technology as a constructive view of thinking

The process of thinking in constructivist paradigms requires higher-order skills delving deeper into content and context (Swain \& Pearson, 2001). Traditional schooling, according to Manzo (1998), actually discourages constructive thinking and has goals of transmitting existing knowledge that conflicts with any real attempt to generate new understanding. Constructivist thinking combines both the critical and creative intellectual processes. It can be practiced by encouraging critical analysis in activities. Cognitive tools, along with constructivist learning environments, guide and activate cognitive learning strategies and critical thinking (Jonassen, 1994). These (cognitive) tools assist in knowledge construction and not knowledge reproduction. The knowledge constructed by the learners reflects their comprehension and conception of the information (Nanjappa \& Grant, 2007). Reflective thinking, that requires careful deliberation, is also encouraged by constructivists (e.g. Walker, 2000; Swain \& Pearson, 2001). In addition,
meta-cognition, or the self-monitoring and self-control of the learning process is emphasized and the new knowledge which is composed is added to previous representations, modifying them in the process. This usually requires external scaffolding in the form of people, books, or technologies such as computers (Nanjappa \& Grant, 2007).

### 2.3 Vygotsky's ZPD as a theoretical learning framework

### 2.3.1 Zone of Proximal Development

The Zone of Proximal Development (ZPD) can briefly be explained as the difference between what a learner can do without help and what a learner can do with help. Vygotsky (1978) defined ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers". Teachers may make use of the Zone of Proximal Development (ZPD) to bridge the gap between what a learner can do without help and what a leaner can do with assistance (Siyepu, 2013). Vygotsky (1978) argues that learner's thinking and problem solving ability fall into three categories: those that can be performed independently, those that can be performed with assistance, and those that cannot be performed even with assistance. Those problem solving actions that cannot be performed even with assistance are those that lie beyond the ZPD.


Figure 3: A Model of the Zone of Proximal Development.

## (Adapted from InnovativeLearning.com)

The interpretation of Vygotsky's socio-cultural approach of ZPD on cognitive development is that one should understand the two main principles of Vygotsky's work: More Knowledgeable Other (MKO) and Zone of Proximal Development (ZPD). The MKO refers to someone who has a better understanding or a higher ability level than the learner with respect to a particular task, process, or concept (Galloway, 2001). In the early stages of development, this is likely to be a parent, but it can also be a teacher, peers, or a technology. Vygotsky (1978) highlights that "what is in the ZPD today will be the actual developmental level tomorrow, that is, what a learner can do with assistance today, she or he will be able to do alone tomorrow". Vygotsky believed that when a
learner is at the ZPD for a particular task, providing the appropriate assistance will give the learner advancement to achieve the task (Galloway, 2001).

### 2.3.2 How the use of technology enhances the ZPD

The use of technology in the teaching and learning of mathematics enhances ZPD in a number of ways in that technology is used as a tool that mediates learning. Mediation is central to Vygotsky's socio-cultural theory. He claimed that the secret of effective learning lies in the nature of the social interaction between two or more people with different levels of skills and knowledge. Furthermore, he regarded the process of learning through mediation in the ZPD as a process that uses "cultural tools". Wertsch (1990) suggested that these cultural tools consist of "technical tools" which are physical learning resources such as textbooks, teaching notes, calculators and classroom written activities, and "psychological tools" which are tools such as language, symbols, mnemonic techniques, counting systems, art, writing, diagrams, maps etc. Various discussions about the use of ZPD in the teaching and learning of mathematics indicate the importance of availing physical and psychological resources to both teachers and learners.

The integration of technology in mathematics education complements Vygotsky's ZPD. For example, the introduction of the computer was important since the computer acts as a cultural tool which is both a physical learning resource (technical tool) as well as a psychological tool with the mathematics software programs that it contains. With the advent of technology, Vygotsky's traditional role of the more knowledgeable other (MKO) has been transformed. This transformation shifts the power of a facilitator of
learning from the teacher or more capable peers to also include the technological tools and signs. Such transformation possesses significant value in the mathematics classroom where collaboration is essential for student learning (Cicconi, 2013).

Technology now transcends its previous isolative barriers and acts as a conduit for collaborative learning - simultaneously transforming typical students into their peers' more knowledgeable others (MKOs). In the pre-twenty-first century classroom the MKOs were most often teachers or advanced classmates. The advent of Web-Based Learning (WBL) has dramatically increased the opportunities for learning from a more knowledgeable other (Cicconi, 2013). In some cases today's MKO is a computer adaptive mathematics programme that creates an individualized tutoring series for students to help them to further understand mathematics, reduce content gap for behind syllabus students as well as make other learners understand how to solve challenging problems in mathematics by providing scaffolding assistance through each individual learner's Zone of Proximal Development (ZPD).

### 2.4 Scaffolding as a theoretical learning framework

### 2.4.1 What is scaffolding?

Scaffolding is another fundamental concept of the ZPD theory. Wood, Bruner, and Ross (1976) coined the term 'scaffolding' which they defined as assistance from experts that enables children to achieve what is beyond their ability to accomplish independently.

Wood et al.'s (1976) conceptualization of scaffolding was consistent with Vygotsky's model of instruction and emphasizes the teacher's role as a more knowledgeable other to help learners to solve problem-oriented tasks within their ZPDs (Vygotsky, 1978). The scaffolding support that is provided initially is gradually decreased as learners become more capable. So in this context, scaffolding is used to explain the social and participatory nature of teaching and learning which occurs in the ZPD (Denhere, Chinyoka \& Mambeu, 2013). Educators and researchers have used the concept of scaffolding as a metaphor to describe and explain the role of adults or more capable peers in guiding children's learning and development (Hammond, 2002; Daniels, 2001). In the teaching and learning of mathematics, scaffolding strategies include the use of combinations of several techniques such as modelling, offering explanations, clarifying students' responses, demonstration (e.g. modelling the problem solving process based on the performance of the problem solvers), motivating and challenging problem solvers, use of scaffolding questions, use of visual organizers, cooperative learning, guided practice, computer technology, etc. (Casem \& Oliva, 2013).

### 2.4.2 How the use of technology enhances scaffolding

Studies on the use of diverse scaffolding strategies have proved that scaffolding is difficult to implement in complex, everyday classrooms (Azevedo \& Jacobson, 2008; Ertmer, 2005; Hannanfin \& Kim, 2003). However, some of the studies further discovered that when technology is integrated, it has an effect of minimizing some of the difficulties by allowing students to individually access interactive materials and obtain just-in-time assistance by the use of these scaffolding technologies (Hannanfin \& Kim, 2003;

Jonassen, 2000). Recently, researchers have studied alternatives to designing and using technology to complement or enhance scaffolding learning strategies (e.g. Saye \& Brush, 2012; Hill \& Hannanfin, 2005). Consistently, research on meta-cognitive tools has underscored the significance of adaptive, human scaffolding in facilitating the learning of mathematics with technologies. Furthermore, computer-enhanced scaffolding can assist students in structuring complex tasks by "problematizing" content knowledge (Reiser, 2004). Scaffolding technologies can be applied to help students articulate and act upon problem-solving processes and learning activities (Kim \& Hannafin, 2011). Thus, technology enhances scaffolding in a number of ways.

### 2.4.3 Types of scaffolds

There are three types of scaffolds: teacher scaffolding, peer scaffolding, and technology-enhanced scaffolding. The three scaffolds are often used in complementary rather than isolated ways, and can either be procedural, conceptual, meta-cognitive or strategic (Kim \& Hannafin, 2011). Procedural scaffolds guide the student in addressing operational aspects of the learning environment. Conceptual scaffolds help students to identify essential knowledge gaps between what they already know and what they need to know. They guide students' understanding about the problem content, provide support to enhance students' understanding of the problem and related knowledge, and are gradually faded as students negotiate the knowledge and skills needed to solve the problem. Meta-cognitive scaffolds assist students in assessing their state of understanding, reflect on their thinking, and monitor their problem-solving processes. Strategic scaffolds help
students to consider alternative approaches to addressing problems (Kim \& Hannafin, 2011).

### 2.4.4 Scaffolding interactions

According to Kim \& Hannafin (2011), scaffolding interactions are either static or dynamic. Static scaffolding is provided in the form of fixed guidelines, procedures, or information that typically does not involve negotiation between the students and the scaffold source (e.g. text, tool, or technology). Dynamic scaffolds provide interactive methods to assess learners' progress and provide feedback in response to differential learners' needs (e.g. cures and prompts).

### 2.5 Summary

In this chapter, the researcher endeavoured to explain the theoretical framework underpinning this research study. An overview was given of Constructivism, Vygotsky's ZPD and Scaffolding in relation to $21^{\text {st }}$ Century technology-assisted learning. In the next chapter, the researcher will focus on an expanded Literature Overview relevant to the experimental research that was undertaken in this study.

## CHAPTER 3

## LITERATURE REVIEW

### 3.1 Introduction

The purpose of this chapter is to present an extended literature overview of the research topic being investigated in this study. By so doing, the researcher will create an overall academic backdrop of what other researchers have documented about in the same field where contemporary research has been undertaken. In the first section, the researcher looked at the common characteristic similarities of learners in Generation Y and the new Generation Z, and compare them with those from the previous generations - Generation X, Baby Boomers and Traditional generation (Tapscott, 1999; Kelan \& Lehnert, 2009; Brownlee, 2010; Rosen, 2011; Renfro, 2012). The purpose of this section is to show that the educational needs of different generations are different because their characteristic traits are not similar as proven by various research findings (see Appendix $F$ section). Because of the differences in characteristic traits of different generations, the teaching strategy or strategies used for learners in one generation will most likely not be effective for learners in another generation; and so the teaching strategies have to change to suit the learning style of each generation's learners. In the next section, the researcher looked at the use of technology in mathematics education, and this will be followed by the benefits of using technology in mathematics education, blended learning, references to online and offline blended learning, and lastly, the summary of the chapter.

### 3.2 Generations $Y$ and $Z$ and the advent of technology

Generation Y, whose members are presently in tertiary education institutions while others are working, were people born in the period between 1980 and 1999, while Generation Z, who are currently undergoing primary and high schooling, are young people born from year 2000 and after i.e. those born into the $21^{\text {st }}$ century. Although an exact starting point and stopping point are always unclear in generational labels (Renfro, 2012), these two generations have the distinction of living in a world that has always had the internet (see table B9 in Appendix $F$ for detailed explanation of each generation's characteristic similarities). From the table, you will find that for these two generations Y and Z , the internet, technology, and social media always shaped their lives to some extent. This is so because the two generations have grown up in a world that has been transformed by new technologies that make new ways of communicating, working and exchanging information and creating knowledge possible. These changes or transformation, however, became of age at a time when the two generations were still faced with institutions shaped by the old model but their way of behaving is more in line with new ways of behaving. While institutions change slowly, these generations have already lived the new lifestyle predicted by theorists of the information and knowledge society (Kelan \& Lehnert, 2009). In order to cater for the educational needs of the new generation Z that find technology to be very appealing to their lives, it is time for education in South Africa to look at new ways to transform its educational system by appropriately incorporating technology to the system. In particular, for mathematics education in South Africa, some of the possible reasons why learners continue to perform poorly in mathematics during high school could either be that (1) the technology is not incorporated at all in some
schools due to some reasons e.g. security concerns or it is very expensive to install, or (2) the technology is wrongly incorporated in the schools. The researcher believes that careful considerations need to be taken in the selection of which type of technology is appropriate to be incorporated in our public schools. The goal should be to use technologies that will be effective in improving performance in Mathematics in all the high schools in the country. In the next section, the researcher looked at the use of technology in mathematics education and how its appropriate integration into mathematics education has yielded positive results in some reported research findings.

### 3.3 Use of technology in mathematics education

The use of technology in mathematics education has the potential to increase productivity in educational activities and affect the quality of education in terms of meaningful learning and effective teaching (Isman, 2003). Eyyam and Yaratan (2014) states that in order to prepare learners for the future and help them learn how to think, learn, and gain different perspectives, technology has to be integrated into the classroom. Technology provides information to adopt new teaching and learning developments. It also creates flexible learning environments in which learners can easily construct and learn new information and knowledge (Behr \& Wheeler, 1981; Kraus, 1982; Kaput \& Thompson, 1994; Isman, 2003). The use of technology in mathematics education is not intended to replace conceptual understanding, computational fluency or problem-solving skills amongst learners, but it is used to provide scaffolding and support to gain access to mathematical content and problem-solving contexts, extend mathematical reasoning and sense making, and to enhance computational fluency. One aspect of digital technologies
is that it promotes interactivity in mathematics education. It has the power to change the experience of doing or learning mathematics, reshaping and expanding it from direct experience in a physical space to experience mediated by the computational medium. When technology is integrated into a mathematics classroom, it enhances teaching and helps students learn how to broaden their perspectives, and provide a better learning environment by bringing the world into the classroom (Ittigson \& Zewe, 2003; Niess, 2006).

### 3.4 The benefits of using technology in mathematics education

Many researchers carried out studies to evaluate the benefits of using technology in mathematics education. For example, an experimental research conducted by Hedren (1990) on learners aged 11-12 years found that mathematical computer software programmes integrated in the instruction of the subject did foster an active, exploring and investigative style of learning resulting in improved knowledge in arithmetic and geometry. The role of the teacher was also found to be critical for this. In a summary of the key benefits of the use of technology in mathematics education, Becta (2003) stated that: Technology promotes greater collaboration among learners and encourages communication and sharing of knowledge. Technology gives rapid and accurate feedbacks to learners and this contributes towards positive motivation. It also allows them to focus on strategies and interpretations of answers rather than spend time on tedious computational calculations. Technology also supports constructivist pedagogy, wherein learners use technology to explore and reach an understanding of mathematical concepts. This approach promotes higher order thinking and better problem solving
strategies which are in line with the recommendations forwarded by the National Council of Teachers of Mathematics (NCTM, 2008).

In most developed countries today technology is being used extensively in classrooms (Eyyam \& Yaratan, 2014). As Bitter and Pierson (2005) and Wiske, Franz, and Breit (2005) have pointed out, the use of instructional technology in class enhances learning so that students can learn more effectively. In technology-implemented classes, interactive student involvement in the learning process is fostered, and learning becomes more fun and more attractive for the students (Smaldino, Russell, Heinich \& Molenda, 2005). As stated by numerous researchers (e.g., Alessi \& Trollip, 2001; Ashburn \& Floden, 2006; Bitter \& Pierson, 2005; Egbert, 2009; Jonassen, Howland, Mara, \& Crismond, 2008; Kent, 2008), it is an inevitable fact that, in learning environments where educational technology is integrated into instruction, both students and teachers experience benefits from using it. As Smaldino et al. (2005) noted, the use of technology in instruction enhances not only the learning capabilities of students but also their motivation, making them more engaged in the learning process. Barron, Ivers, Lilavois, and Wells (2006) stated: "Technology provides an excellent avenue for learner's motivation, exploration, and instruction" (p. 17). It has become evident that teaching, learning, and technology work synergistically to provide effective and efficient knowledge transfer because educational technology helps teachers create learning contexts that were not previously possible with traditional teaching methods (Wiske et al., 2005). Bitter and Pierson (2005) stated: "A recent meta-analysis demonstrated that learners that used technology
had modest but positive gains in learning outcomes over those who use no technology" (p. 107).

The use of technology in education or teaching helps teachers provide immediate feedback to students and motivates active student learning, collaboration, and cooperation. It also helps teachers provide individualized learning opportunities and flexibility for their students (Eyyam \& Yaratan, 2014). Technology offers blended learning environments that are rich with dynamic visualization and multiple presentations of mathematical concepts, relationships and results which may contribute immensely to better conceptual understanding and greater learner interactivity during lesson presentations. These environments result from the blended learning process or strategy which is a teaching and learning strategy that combines multiple delivery media (which includes both face-to-face teaching and, offline and online forms of learning) that are designed to complement each other and promote learning.

In the next section, the researcher looked at this blended learning strategy which is a teaching and learning strategy that is recommended for application in today's classroom learning situations and is supported by various research studies (e.g. Singh, 2003; Precel, Eshet-Alkalai \& Alberton, 2009; Massoud, Iqbal, Stockley \& Noureldin, 2011; Padayachee, Boshoff, Olivier \& Harding, 2011; Wolpert-Gawron, 2011).

### 3.5 Blended learning

According to Massoud, Iqbal, Stockley \& Noureldin (2011), blended learning is learning that results from employing a mixture of both face-to-face teaching strategies and, online or offline methods for instruction in the classroom. It is a type of learning that results from an expansion of the traditional face-to-face teaching methods (synchronous) with electronic learning experiences (asynchronous). In the late 1990s, electronic learning (e-learning) technology was applied in many educational institutions. E-learning transformed the traditional classrooms into virtual classrooms using local area networks or the World Wide Web. Experiments and scientific research has since demonstrated drawbacks of e-learning including the fact that it is very expensive to implement and it reduces face-to-face interaction between the student and the teacher. Moreover, a person needs to be trained for discussion and exchanges within a hypothetical environment. To conquer the limitations of e-learning, a combination of traditional technique augmented by innovating technologies is suggested, resulting in a blended learning format (Massoud et al., 2011). Typically, blended learning makes extensive use of learning technologies through the "blend" of physical and virtual environments in order to supplement traditional face-to-face classroom interaction by adding the domain of modern technology (Singh, 2003; Bersin, 2004; Bonk, 2004; Rovai \& Jordan, 2004). So blended learning has greater advantage over traditional face-to-face teaching and e-learning apart because it combines the two instructional strategies which complement one another to improve students' learning and knowledge production. Blended learning is most likely to make learning more effective as the integration of technology to augment student-teacher classroom interactions will not replace the traditional face-to-face
interactions which are a vital component of the blended learning process. This was reiterated by Massoud et al. (2011) when they stated: "Students value a strong, active presence by the instructor during the course, especially in discussion groups. Students highly appreciate the added wisdom that the instructor's perspective brings to their discussions with other students, while monitoring and channeling the discussions onward in a blended learning environment" (p. 5).

In the 2009 study, Olive and Makar along with four members of their working group focused on the mathematical knowledge and practices that may result from access to digital technologies. They put forward a new tetrahedral model derived from Steinbring's (2005) didactic triangle and this new model illustrates how interactions among the didactical variables: student, teacher, task and technology (that form the vertices of the tetrahedron) create a space in which new mathematical knowledge and practices may merge in a blended learning classroom situation (see Figure 4 below).


Figure 4: The Didactical Tetrahedron for Blended Learning.
(from Olive \& Makar, 2009, p. 169)

Olive and Makar stated that: "It is not arbitrary that we place the student at the top of this tetrahedron as, from a constructivist point of view, the student is the one who has to construct the new knowledge and develop the new practices, supported by the teacher, task and technology" (p. 168). In the model, the learner assumes a new position of being in charge of his/her own learning process while the role of the teacher becomes more of a facilitator according to the constructivist learning theory. The use of technology in blended learning had been contextualized by various researches as one process which facilitates a paradigm shift from teacher centeredness to learner centeredness in mathematics education (e.g. in Massoud et al., 2011; Wolpert-Gawron, 2011). In the next section, the researcher focused on online and offline blended learning - which are two learning approaches of blended learning that can be used to enhance learning in
mathematics education in the $21^{\text {st }}$ century. The advantages of using one approach over the other, or using both approaches, in South African educational context will also be discussed in this section.

### 3.6 Online and offline blended learning

A blended learning approach combines multiple delivery media that are designed to complement each other and promote learning by mixing various event-based activities, including face-to-face classrooms, offline and online forms of learning where the online learning usually means "over the Internet", self-paced learning, and performance support tools to support the appropriate execution of tasks (Singh, 2003). Online and offline blended learning are two components of blended learning that can be used separately or together to enhance and improve learning in mathematics education. The advantage of offline blended learning model over the online model is that the offline model is independent of data usage via the internet and is therefore less expensive to operate. The expensiveness of producing and operating an online model was reiterated by Singh (2003) when he stated: "A totally online, self-paced, media-rich, Web-based training content may be too expensive to produce but combining virtual collaborative and coaching sessions with simpler self-paced materials such custom content, documents, recorded e-learning events, texts assignments, and PowerPoint presentations may be just as effective or even more effective".

An offline blended learning model is a teaching and learning approach that integrates the use of various offline modern technologies for mathematics instruction and the
programme package of the model can be installed on a Tablet, Laptop or PC and may be used anywhere, including in very remote areas where there is no internet connection. The program package also contains learning material that can be installed on tablet PCs for individual learners which they can use for revision during and after school. An example of an offline model is the offline Techno-Blended Teaching and Learning Model (TBM approach) which is a teaching approach that was developed by the Govan Mbeki Mathematics Development Centre (GMMDC) at the Nelson Mandela University (NMU). This model was inspired by the "blended learning" teaching and learning approach which is a teaching approach in which "technology is blended along with the traditional teaching and learning model", therefore the name offline Techno-Blended Teaching and Learning Model (TBM) was derived from "Offline Technology Blended along with the traditional Teaching and Learning Model". The programme package of the TBM approach comprises a TouchTutor® Package which consists of a comprehensive series of curriculum aligned video and Power Point content lessons and tutorials, examination revision material, CASIO Calculator Emulator, GeoGebra support material, and lesson aligned learner workbooks that were installed on laptops for teachers and Tablets for selected learners in the FET phase (see Section 1.5 for detailed explanation of the TBM approach). The other advantage of the offline Techno-Blended Model is that it is easy to operate because of its TouchTutor® software package programme which only requires a mouse click to access any component of the package. Hence the TouchTutor® package is user-friendly and does not require much training for teachers or learners to be able to utilize it fully.

This teaching approach is ideal to apply in South African public educational system because, geographically, the country has very large rural areas that lack infrastructure, which may create problems with technology such as internet connection. Moreover, the majority of the country's population lives in socio-economically challenged areas where internet connections are not stable or affordable. This model is also suitable for application in any type of school in the country, whether the school is in an urban or rural area. The question that was asked was whether the model could improve performance in mathematics if it is applied in public high schools in South Africa? To help give answers to this question, the model was piloted in ten selected high schools in one district of the Eastern Cape Province over the period 2010-2015. The maturity of the TBM after 2015 and the attempts that were made by the GMMDC to include technology-assisted educational elements that are in harmony with the socio-economic and academic challenges in public schools made this offline model an attractive choice for this study.

### 3.7 Conclusion

The learning challenges faced by today's students of generations Z require us to look for new approaches to the way mathematics is taught in schools. Generation Z has grown up in a world that has been transformed by new technologies that make new ways of communicating, working and exchanging information and creating knowledge possible (Kelan \& Lehnert, 2009). Because of this, new pedagogical approaches need to have technology integrated appropriately in order to create teaching strategies that will be able to meet the learning expectations of modern learners of this generation.

One powerful reason why most of the learners in this country perform poorly in mathematics in high school while others "drop-out" before completing their high school is that education in many schools is presented in the same way as it was in the $19^{\text {th }}$ and $20^{\text {th }}$ centuries. This is so because technology is mostly not incorporated at all in mathematics education in public schools. Some reasons for this may include costs, security, and lack of infrastructure. In some schools where attempts were made to incorporate technology, it was not used correctly due to lack of teachers training or support infrastructure. While technology is used globally in education, with great success (Isman \& Yaratan, 2006), in South Africa, various attempts at harnessing technology in education have failed dismally. In some cases, the adoption of sophisticated models of blended learning that relies on internet connection, mostly adopted from first world countries, that were not designed with our unique challenges in mind, have immensely contributed to this dismal failure. Our country contains large rural areas with inadequate infrastructure to support teaching with technologies that relies on internet connection. For these reasons, a teaching approach that makes use of offline technology could be much more effective.

## CHAPTER 4

## RESEARCH DESIGN AND METHODOLOGY

### 4.1 Introduction

The purpose of this chapter is to specify the plan that was used to generate empirical evidence which was used to answer the research questions. In this chapter, the researcher will explain how the research was set up, which methods were used during the selection of the subjects and which methods of data collection were used.

### 4.2 Methods of sample selection

The research study involved teachers and selected learners in the Mathematics classes from four different high schools selected from two urban districts in the Eastern Cape Province of South Africa. The schools were selected to be similar in terms of academic profile and quintile status as they were all schools located in townships of the two urban districts. So, most students enrolled at the four schools, labelled school W, X, Y and Z, came from homes in the surrounding township areas with more or less the same socio-economic profiles. Thus, the socio-demographic and academic profiles of the selected pupils were similar in that the learners came from the townships surrounding the schools. Hence no school had an added advantage in terms of it having more resources than other schools. Also from the selected schools, the four educators who were selected that were teaching the four Grade 11 Mathematics classes were qualified to teach the
subject at FET level with more or less the same number of years of experience. Furthermore, the four schools labelled $\mathrm{W}, \mathrm{X}, \mathrm{Y}$ and Z were selected on the basis of being identified as functional schools were teaching and learning process occurred without interruption for the period from Grade 10 to Grade 11. Hence the selected learners from any of these schools were not disadvantaged as a result of content gaps that may have happened because of certain sections not having been taught in the previous Grade. The above measures were taken by the researcher in order to minimize the potential for bias in the research outcomes or research findings.

Schools W and X were selected from the first urban district, and schools Y and Z were selected from the second urban district. Mathematics educators from schools X and Y were trained to teach mathematics using the offline Techno-Blended Model while those from the other schools W and Z did not receive any training and therefore taught the two content chapters using the traditional approach. One Grade 11 class that was assigned to each educator was selected to take part in the study. So the method of sample selection that the researcher used was a Cluster Sampling Method because the selection of participants for the research came from naturally occurring groups (i.e. the four already existing classes taught by the four educators). Furthermore, a total of 60 learners were selected at random from the four selected classes of the four educators to give the final number for analysis and the learners were selected using the Random Sampling Method. There were 30 selected learners in Group $A$ which comprised learners from the two classes that were taught Geometry and Trigonometry using the traditional approach, and the other 30 selected learners were in Group $B$ which comprised learners from the other
two classes that were taught the same content chapters using the offline TBM approach. So Group $A$ was the control group while Group $B$ was the experiment group, and a Cluster Sampling Method followed by Random Sampling Method were the two methods of sample selection used for the selection of learners in each group (see the two tables below).

Table 1: Learners selected for the control group
$\begin{array}{|l|c|c|c|}$\cline { 2 - 4 } \& Teacher$\left.W & \text { Teacher } Z & \begin{array}{l}\text { Total number of learners } \\ \text { for the traditional approach }\end{array} \\ \text { (Group A) }\end{array}\right\}$

Table 2: Learners selected for the experiment group

|  | Teacher X | Teacher Y | Total number of learners for the TBM approach <br> (Group B) |
| :---: | :---: | :---: | :---: |
| Number of learners in class | 21 | 26 | 47 |
| Number of learners randomly selected from the class | 11 | 19 | 30 |

### 4.3 Research method

A mixed-model approach similar to the method prescribed by Macmillan and Schumacher (2009) was used in this study. This approach, which comprises both qualitative and quantitative research methods, was used to establish and give a comparison of the affective and cognitive impact on learning when the topics Euclidean Geometry and Trigonometry in the CAPS curriculum were taught to two groups of Grade 11 Mathematics learners using an offline Techno-Blended Teaching and Learning Model for one group and a traditional approach for the other group. So the research was a comparative research study. In this study, the impact on learning was investigated from a two-dimensional framework adapted from Li et al. (2009) which described a four-dimensional information literacy framework used to examine students'
learning which has the following dimensions: Affective dimension, Cognitive dimension, Meta-cognitive dimension and Socio-cultural dimension. In the study, an analysis and comparison of the learning experience from the first two dimensions was provided to give the affective learning experiences which resulted in change in motivation, attitude and participation towards the learning of mathematics under the affective impact on learning, and the cognitive learning experiences which resulted in conceptual understanding and improvement in performances under the cognitive impact on learning.

### 4.4 Data collection instruments, procedures and analysis strategies

Two questionnaires (in Appendix $C$ section) were used as instruments to measure the affective impact on learning. Responses provided by the students in the two questionnaires were used to examine the students' motivation, attitude, and participation towards the learning of mathematics in the affective domain. The first questionnaire comprised 24 questions and was given to all the selected learners for them to respond to before the two chapters were taught to them during Grade 11. The second questionnaire was given to the learners after the chapters were completed in Grade 11.

The first questionnaire entitled: Anticipated affective impact of offline technology on learning, was given to learners from both the control and experiment groups before the intervention of the two teaching approaches which are (1) teaching using the traditional approach which was applied to the control group A, and (2) teaching using an offline Techno-Blended Model (TBM) which was applied to the experiment group B. In the first
questionnaire, learners were asked to imagine that they had offline technology support and provide responses to questions asking what effect such support could have on improving their emotional motivation towards the learning of mathematics as a subject. The responses learners provided on this instrument were based on a series five-point Likert-scale $(1=$ Strongly disagree, $2=$ Disagree, $3=$ Neutral, $4=$ Agree, $5=$ Strongly agree). The learners also provided responses to questions in the second questionnaire that were also based on a series five-point Likert-scale which was similar to the first questionnaire. In the second questionnaire, learners gave feedback on their educator's teaching approach as well as their learning experiences during the teaching of the two content chapters in Grade 11 when the two different pedagogical teaching approaches were applied by their educators in the two groups. Individual and group interviews were also conducted with the learners to get more feedback on how they have experienced their educator's teaching approach.

To measure the cognitive impact on learning, a paper-based pre-test followed by a post-test were written by all the selected learners which they wrote prior to and after the topics were presented in Grade 11. The responses provided by the learners in both tests were used to examine the students' conceptual understanding and improvement in performance in the cognitive domain. The pre-test which comprised mostly Geometry and Trigonometry questions from the Grade 10 Mathematics curriculum was used as a diagnostic exercise to test the students' knowledge base prior to the chapters being taught in Grade 11. And the post-test which comprised mostly Geometry and Trigonometry questions from the Grade 11 Mathematics curriculum was given after the topics were
completed in Grade 11. Both test exercises had the same format and during the responses to each test exercise, students were asked to read each question carefully and write their solution indicating all the steps of their working in the space provided below the question. Sections B and C of the tests comprised of long questions while Section A comprised multiple choice questions. In both tests, there were 10 multiple choice questions each from the two content chapters (see Appendix $D$ for the design structure of the tests). Both tests written by the selected learners were both aimed at testing conceptual understanding and demonstration of skills by the learners in the two groups. In order to help evaluate the effectiveness of each teaching approach in terms of students' conceptual understanding and improvement in performance, statistical methods were used to analyze responses provided by the learners in both test exercises.

So far, the researcher has looked at the methods of sample selection, research method and instruments that were used for data collection in this research. In the next section, the researcher focused on the research design that was used in the study.

### 4.5 Research design

In the research, the interventions that were controlled were the two pedagogical instructional methods (which were the independent variables), while the dependent variable(s) affected by the interventions were the different impact on learning realized in the two different contexts. What was different in the two groups of learners was the two different pedagogical instructional approaches that the different educators applied which
the researcher chose to label as Intervention $X_{1}$ (teaching using the traditional approach) and Intervention $X_{2}$ (teaching using the offline TBM approach) applied to Group A [control condition] and Group B [treatment condition] respectively. During the research period, learners in the control condition continued using their existing textbooks on the same topics while those in the test/experiment condition used the technology-enhanced teaching and learning support provided by the offline Techno-Blended Model to cover the same content topics in Grade 11. So the research design that the researcher used was the Nonequivalent Groups Pretest-Posttest Comparison Design. A graphical representation of this design is given in Figure 5 below:


Figure 5: Research Design Model

The research design was an experimental research design because it was concerned with the phenomenon of cause and effect where the dependent variable (impact on learning) is measured when the independent variable (teaching strategy) changes for each group of learners to which it is applied. In the research analysis, I gave a comparison of the impact on learning between the subjects/learners, from the two groups, that have experienced the different intervention teaching methods, and I did so by using the evidence or data collected from the research findings. Thus, the research was also a scientific or an evidence-based research exercise as its basis was to establish knowledge about which educational practices have the most positive impact on learning. The research could also be viewed as an action or applied research in the field of qualitative and quantitative research in that it could help to improve practice in the field of mathematics education by examining the impact of the two pedagogic instruction methods on learning.

### 4.6 Summary

In this chapter, the researcher provided the methods of sample selection that were used in this research. He also provided an explanation of the research method and the data collection instruments that were used to collect the research data for analysis as well as the research design that was used in the research. In the next chapter the researcher will look at the pre-test and post-test data analysis and interpretation.

## CHAPTER 5

## ANALYSIS OF QUESTIONNAIRES FOR THE

## AFFECTIVE IMPACT ON LEARNING

### 5.1 Introduction

In this chapter, the researcher provides an analysis of the responses provided by the selected groups of learners who completed the two questionnaires about the affective impact on their learning. A qualitative interpretation of the data collected from the questionnaires is also provided by the researcher in the sequel.

### 5.2 Analysis of first questionnaires for the control and experiment

## groups' anticipated impact of technology

It is clear from the two histograms A5 and A6 of the students' responses to the first questionnaire for both the control and experiment group respectively (in Appendix C) that most of the learners from the two groups overwhelmingly reported Agree or Strongly Agree to the 24 questions in the first questionnaire, that were based on a five-point Likert-scale (Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree). These questions sought to get feedback on the anticipated affective impact on learning across the three affective domains: motivation, attitude, and participation.

Thus, judging from the predominantly positive responses to the first questionnaire, we can conclude that there were strong pre-conceived perceptions among the students from both groups that, if technology was integrated in the teaching and learning of mathematics, it would enhance the learning of mathematics, amongst others, by changing the learners' attitude towards the subject. Perceptions (emanating from the responses to the first questionnaire) that the use of technology will also make the learners to be more interested in learning mathematics will also result in an improvement in participation in the learning process of mathematics, and may lead to an improvement of performance in the subject under the cognitive impact on learning.

Next in this section, the researcher further investigated the anticipated affective impact on learning in the three categories (motivation, attitude, and participation) previously mentioned. This was be done by splitting the 24 questions in the first questionnaire into the three affective domain categories, and use the data from the students' responses to questions in each category to represent histograms. Analysis of the data representations from the three histograms will be provided to interpret the students' self-reflection responses to the anticipated affective impact questions on learning with technology.

### 5.2.1 Motivation as an affective impact domain

From the first questionnaire, Questions 3, 11, 13, 15, 18 and 23 were identified by the researcher as falling under the affective impact domain of motivation. Under this domain, students from both the control and experiment groups overwhelmingly reported that their motivation to learn mathematics will increase if technology was to be used to
support the learning process in the subject. This is so because most students from both groups reported Agree or Strongly Agree to the six questions under this category (see histograms of Figures 6 and 7 below).


Figure 6: Histogram for control group's anticipated affective impact of technology (Motivation Domain)


Figure 7: Histogram for experiment group's anticipated affective impact of technology (Motivation Domain)

### 5.2.2 Attitude as an affective impact domain

A total of 14 questions (Questions 1, 4, $5-10,12,14,17,21$ and 24) from the first questionnaire were identified by the researcher as falling under the affective impact on learning domain of attitude (i.e. the attitude towards the learning of mathematics). Again most learners from all the groups overwhelmingly reported that they Agree or Strongly Agree to all the 14 questions under this category which was an indication that most of the learners agreed that if technology was integrated into mathematics education teaching and learning, it was going to change their attitude towards the subject (see histograms of Figures 8 and 9 below).


Figure 8: Histogram for control group's anticipated affective impact of technology (Attitude Domain)


Figure 9: Histogram for experiment group's anticipated affective impact

## of technology (Attitude Domain)

### 5.2.3 Participation as an affective impact domain

Questions 2, 16, 19 and 22 were identified by the researcher as falling under the affective impact on learning domain of participation in the learning process of mathematics. For the students' responses to these four questions, similar trends were again observed by the researcher from both groups with most learners overwhelmingly reporting that they Agree or Strongly Agree to all the questions under this category. This confirms that the majority of the learners agreed that if technology was integrated during the teaching and learning process of mathematics, it will make the learners to be
interested in the subject and will result in them actively participating in the learning process (see histograms of Figures 10 and 11 below).


Figure 10: Histogram for control group's anticipated affective impact of technology (Participation Domain)


Figure 11: Histogram for experiment group's anticipated affective impact of technology (Participation Domain)

### 5.2.4 Summary of the anticipated affective impact on learning domains

From an analysis of histograms of the students' responses to questions under the three categories, it can be concluded that, most of the learners from both groups agree that if technology is integrated into mathematics education teaching and learning, it will make them to:

- become more motivated to learn mathematics (Figures 6 and 7),
- have a more positive attitude towards the subject (Figures 8 and 9), and
- be more active in participating in the learning processes provided to them by their mathematics educators (Figures 10 and 11).


### 5.3 Analysis of second questionnaires for the control and experiment

## groups

In this section, the researcher will give an analysis of the responses that were provided by the learners when they completed the second questionnaire. This questionnaire intended to gather the learners' views on their experience of each teaching approach when the two Grade 11 topics Geometry and Trigonometry were taught by their mathematics educator. Interpretation of the observable trends as it relates to how the different approaches affectively impacted on their learning of the two areas of the mathematics curriculum will also be provided. This interpretation will be carried out from the analysis of the data compiled from the second questionnaires' responses for the two groups (see Tables B3 and B4 in Appendix C).

### 5.3.1 Analysis of learners' views of the educator's teaching approach

For their views on the educator's teaching approach, learners were given questions based on a similar five-point Likert-scale as for the pre-test questionnaire with options (Not effective at all, Less effective, Somewhat effective, Effective, and Highly effective) that they had to respond to. The percentage of the students that reported Effective or Highly effective teaching was used by the researcher as a measuring instrument to indicate what proportion of learners in each group reported that their educator's teaching approach was effective. From the compiled data respectively, $33 \%$ of the control group and $80 \%$ of the experiment group reported that they experienced Effective or Highly effective teaching of Geometry. On the effectiveness of the teaching of trigonometry
experienced respectively, $37 \%$ of the control group and $73 \%$ of the experiment group reported that they experienced Effective or Highly effective teaching. Thus, a larger number of learners from the experiment group reported that the Techno-Blended Teaching and Learning Model (TBM approach) was more effective when compared to the control group where fewer learners reported the traditional approach to be effective. Therefore, in both cases of Geometry and Trigonometry teaching, majority of learners in the control group reported a less than effective teaching experience. Hence the overall teaching measure indicates a significant difference in the experience of the learners for both content areas in favour of the TBM approach that was used. These trends are shown using histograms in Figure 12 and Figure 13 below.


Figure 12: Control group learners' views on the effectiveness of teaching


Figure 13: Experiment group learners' views on the effectiveness of teaching

### 5.3.2 Analysis of the learners' self-reflection on the conceptual understanding of the sections of the content topics that were taught

For the self-reflection on the conceptual understanding of sections that were taught in the two topic sections Geometry and Trigonometry in the post-test questionnaire, learners from the two groups provided responses to questions that were based on a five-point Likert-scale (Very little understanding, Little understanding, Moderate understanding, Good understanding, Complete understanding). The percentage of learners that reported Good or Comprehensive understanding after teaching, was used by the researcher as a measuring instrument to show what proportion of learners in each group believed that they clearly understood the sections of the topics that were taught to them. The
percentages were found to be lower in the control group compared to the experiment group for all the sections under each topic. This was a clear indication that the majority of learners from the experiment group who were taught the two topics using the TBM approach understood the topics better than their counterparts from the control group that were taught the same topics using the traditional approach. This is so because for the first topic Geometry (see Figures 14 and 15), 20\% of the control group learners reported Good or Comprehensive understanding after teaching for the section Circle geometry while $57 \%$ of the experiment group learners reported Good or Comprehensive understanding for the same section. Also under geometry, for the two groups respectively, $13 \%$ and $67 \%$ reported Good or Comprehensive understanding of the section Cyclic quadrilaterals, and $20 \%$ and $33 \%$ reported Good or Comprehensive understanding of the section Tangents to a circle. These observations clearly indicated that the TBM teaching approach was more effective in fostering understanding when compared to the traditional approach during the teaching of the Geometry topic.

Similar trends or patterns were also observed when percentages of learners that reported Good or Comprehensive understanding after the second topic Trigonometry was taught were analysed with $17 \%$ and $80 \%$ reporting Good or Comprehensive understanding of The Area Rule section, 23\% and $83 \%$ for The Sine Rule section, $30 \%$ and $80 \%$ for The Cosine Rule section, and $23 \%$ and $57 \%$ for The Mixed Problems section for both the control and experiment groups respectively (see Figures 14 and 15). The analysis of the above results clearly show, to a greater extent, that the application of the TBM
approach in the teaching and learning of mathematics has led to improved affective learning experience for mathematics learners at the FET level.


Figure 14: Control group learners' self-reflection on understanding
of sections that were taught


Figure 15: Experiment group learners' self-reflection on understanding

## of sections that were taught

The histogram in Figure 16 gives a comparison of the percentage of learners from the experiment group to the percentage of learners from the control group that reported Good or Comprehensive Understanding of topics' sections that were taught to both groups. From the graph, trend clearly shows that in all sections of the topics that were taught, a greater percentage of learners from the experiment group (as compared to the control group) indicated that they understood the sections from the two topics that were taught more clearly. This seem to suggest that the TBM teaching approach had a stronger positive impact on the cognitive understanding of sections that were taught when compared to the teaching using the traditional approach.


Figure 16: Percentage of learners that reported Good or Comprehensive Understanding of sections that were taught
5.3.3 Analysis of the learners' experience of the teacher's pedagogical approach when geometry was taught

A five-point Likert-scale (Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree) was used to gather data for questions on the learners' experience when geometry was taught. The percentage of learners that reported Agree or Strongly Agree was used by the researcher as a measuring instrument to show what proportion of learners in each group at least agreed with the statement of each question. From the two groups, and for most questions, the percentage of the learners who Agreed or Strongly Agreed to responses to the questions were again found to be greater for the experiment group than for the control
group. In the case of a question where the design of the Likert instrument was set up for contra-indication, the outcome was different. One such instance was the response to the question/statement "I feel my teacher needs to consider using other teaching approaches or styles", where $70 \%$ of the control group reported that they Agree or Strongly Agree with the statement as opposed to $47 \%$ of the experiment group. From the responses provided for this question, it was clear that a majority of learners from the control group felt that the teaching approach applied by their mathematics teacher(s) was inadequate and did not meet their learning requirements or needs while the experiment group's responses showed that a majority were by and large satisfied by the teaching approach (TBM approach) applied by their mathematics educator(s).

For question 2.8, the percentages of respondents that reported Agree or Strongly Agree to the question's statement "The teacher did not interact with us" were $7 \%$ for the control group and $0 \%$ for the experiment group and since the two percentages are close to one another and nearer to zero, this means that the learners from both groups mostly disagreed with the statement or did agree that interaction with their mathematics teachers took place during the teaching and learning process of the content of the geometry chapter. The rest of the other responses to questions' percentage Agree or Strongly Agree responses may be summarised in Table 3 for both the control group (A) and experiment group (B) as:

Table 3: Percentage responses of learners' experience of teacher's pedagogy when geometry was taught

| QUESTION/STATEMENT ON GEOMETRY TEACHING | Agree or <br> Strongly Agree |  |
| :--- | :---: | :---: |
|  | (A) | (B) |
| 2.1 The teacher's approach enabled me to solve problems independently. | $30 \%$ | $57 \%$ |
| 2.3 The teacher's presentation of the chapter was very clear and understandable. | $37 \%$ | $67 \%$ |
| 2.4 The teacher assumed we had sufficient pre-knowledge to follow the lesson. | $27 \%$ | $47 \%$ |
| 2.5 The teaching gave me a very strong reason to like the topic. | $27 \%$ | $73 \%$ |
| 2.6 The teaching stimulated my interest to participate in classwork. | $40 \%$ | $73 \%$ |
| 2.7 The teacher raced against time to complete this syllabus topic. | $40 \%$ | $60 \%$ |
| 2.9 The teaching stimulated me to do more maths problems. | $43 \%$ | $80 \%$ |
| 2.10 Using more technology could have made the teaching more interesting. | $77 \%$ | $90 \%$ |

From Table 3, it can be observed that a larger percentage of learners from the experiment group reported Agree or Strongly Agree to statements of the questions 2.1, 2.3, 2.5, 2.6 and 2.9 when compared to the control group. The qualitative interpretation of what this means is that, the teacher's presentations of the Geometry sections were found by the experiment group to be more clear and understandable as compared to the control group.

The same trend can be observed in respect of the presentations making the group to have a very strong reason to like the topic and participate in classwork exercises given to them by their mathematics teacher. This observation is aligned to many similar research studies on the use of technology in mathematics education. In many such studies it were shown that dynamic visualization and multiple representations of mathematical concepts, relationships and results which were supported by technology-rich teaching and learning environments, contributed to conceptual understanding and learner interactivity among learners during mathematics lesson presentations (for instance in Chan \& Tutkaluk, 2010; Roschelle, 2013).

The percentage responses to question 2.4 were less than $50 \%$ for both groups which is an indication that majority of learners disagreed with the statement, i.e. the learners' perception indicates that they had an impression their teacher(s) conducted some form of diagnostic testing to establish what the learners already new prior to the beginning of each chapter. A larger percentage of learners furthermore reported Agree or Strongly Agree to statements of the questions 2.7 and 2.10. In question 2.7, majority of students from the experiment group reported that the pace at which the chapter was completed was faster than the control group. This, as was expected, could be the case because the use of technology in the TBM approach allows mathematics educators to present diagrams and solution methods at a faster pace than when only the chalkboard is used in the traditional approach. This would have further provided each educator who have used the TBM approach more time to explain each solution method to the learners and hence provided them with more chances to understand the content material better. The percentage
responses to question 2.10 were $77 \%$ and $90 \%$ for both the control and experiment groups respectively, which is an indication that the learners reported that their teachers could have utilized or blended various technologies during the Geometry lesson presentations better to make each lesson more interesting. Figures 17 and 18 give a section by section overview or summary of the learners' experiences of Geometry teaching for the two groups involved in this study.


Figure 17: Control group's experience of Geometry teaching


Figure 18: Experiment group's experience of Geometry teaching

### 5.3.4 Analysis of the learners' experience of the teacher's pedagogical approach

## when trigonometry was taught

A similar pattern was also observed for responses to questions on the learner experiences of both groups when sections of trigonometry were taught. The questions given to the learners in the case of the teaching of Trigonometry were similar to those that were discussed in section 5.3.3. The questions were again based on the same five-point Likert-scale options as in the previous section. Figures 19 and 20 show respectively for the control and experiment groups, a summary of the responses provided by the learners about their experience when sections of the trigonometry syllabus were taught. Again, for these two diagrams, the percentage responses to questions that indicated Agree or

Strongly Agree was used as a measuring instrument to determine what proportion of the learners in each group at least agreed with the statement of the question.


Figure 19: Control group's experience of Trigonometry teaching


Figure 20: Experiment group's experience of Trigonometry teaching

For the response to the question/statement "I feel my teacher needs to consider using other teaching approaches or styles", $70 \%$ of the control group reported that they Agree or Strongly Agree with the statement as opposed to $40 \%$ of the experiment group, an indication that the traditional teaching approach was less appealing to the majority of learners in the control group while for the majority of learners in the experiment group, the TBM approach was more appealing and exciting to these learners. The responses to the question/statement "The teacher did not interact with us" were $7 \%$ for the control group and $13 \%$ for the experiment group, which again is an indication that most learners from both the two groups disagreed with the statement or did agree that interaction with their mathematics teachers took place during the teaching and learning process of the
trigonometry chapter. For the rest of the other of questions 3.1, 3.3, 3.5, 3.6 and 3.9 as well as questions 2.4, 2.7 and 2.10 , the percentage of the responses to questions that indicated Agree or Strongly Agree follow a similar pattern as those of the previous section with a larger percentage of learners from the experiment group (B) reporting Agree or Strongly Agree to the statements of the questions when compared to the control group (A). These trends are summarised in Table 4 below.

Table 4: Percentage responses of learners' experience on trigonometry teaching

| QUESTION/STATEMENT ON TRIGONOMETRY TEACHING | Agree or <br> Strongly Agree |  |
| :--- | :---: | :---: |
|  | (A) | (B) |
| 3.1 The teacher's approach enabled me to solve problems independently. | $30 \%$ | $67 \%$ |
| 3.3 The teacher's presentation of the chapter was very clear and understandable. | $37 \%$ | $73 \%$ |
| 3.4 The teacher assumed we had sufficient pre-knowledge to follow the lesson. | $27 \%$ | $47 \%$ |
| 3.5 The teaching gave me a very strong reason to like the topic. | $27 \%$ | $77 \%$ |
| 3.6 The teaching stimulated my interest to participate in classwork. | $40 \%$ | $80 \%$ |
| 3.7 The teacher raced against time to complete this syllabus topic. | $40 \%$ | $47 \%$ |
| 3.9 The teaching stimulated me to do more maths problems. | $43 \%$ | $87 \%$ |
| 3.10 Using more technology could have made the teaching more interesting. | $77 \%$ | $90 \%$ |

An analysis of the results from the above table, again show that the teacher's presentation of the Trigonometry chapter was found by the experiment group to be very clear and understandable and gave them a very strong reason to like the topic and participate in classwork exercises given to them by their mathematics teacher. From the above analysis, it can therefore be concluded that the affective impact on learning was greater for the experiment group than the control group. The observations made in the above Sections 5.3.2, 5.3.3 and 5.3.4 all shows evidence that for the two chapters that were taught, the application of the TBM approach led to an improvement in the affective impact on learning for learners in the experiment group.

### 5.4 Conclusion

The analysis of data linked to the pre-test and post-test questionnaires on the affective impact on learning when, respectively, the TBM and the traditional approaches were used to teach two content areas of the mathematics curriculum at FET level has led to a noticeable difference in the trends that were observed. A strong perception exist among students who were taught Geometry and Trigonometry sections with the integrated use of technology, that when technology is integrated in the teaching and learning of mathematics, it will enhance the learning of mathematics by making them to become motivated to want to learn the subject. They are also likely have a positive attitude towards the subject resulting in more active participation in classwork and homework activities that are provided to them by their mathematics educators. This is because the experiment group reported more positively about their experiences in all three affective learning domains that were investigated when compared to the control group. It can
therefore be concluded that the use of offline Techno-Blended Teaching and Learning Model does contribute to an improved affective learning experience for mathematics learners at the FET level. This statement provides strong evidence to answer part of the main research question which was stated in section 1.7 of this research project. In the next chapter, the researcher investigated whether this positive affective impact on learning when technology is integrated will result in an improvement in the performance in mathematics for the experiment group when compared to the control group.

## CHAPTER 6

## ANALYSIS OF PRE-TESTS AND POST-TESTS FOR THE COGNITIVE IMPACT ON LEARNING

### 6.1 Introduction

In this chapter, the researcher conducted an analysis of the pre-test and post-test results for the cognitive impact on learning. This analysis was done using the final test scores data obtained from the two paper-based content tests that were written by learners in both the control and experiment groups (please refer to Appendix D for access to the two content tests that were conducted). The test scores data will be ordered by means of standard statistical methods. The two types of statistical methods that will be used for the data analysis are: descriptive statistics and inferential statistics (for more detailed explanation of these methods, please refer to Appendix G). After that, the researcher looked at the analysis of learner responses to interview questions and investigated if evidence from this analysis agrees with findings from the analysis of questionnaires for the affective impact on learning as well as the analysis of the pre-tests and post-tests for the cognitive impact on learning.

### 6.2 Descriptive statistical analysis and interpretation

The following tables give a summary of the descriptive statistics calculations that were carried out in Appendix G on the test scores data obtained from the two groups of learners which were involved in this research study.

Table 5: Summary of descriptive statistics calculations on the pre-test scores

| Pre-test analysis | Control group (A) | Experiment group (B) |
| :--- | :---: | :---: |
| Mean score $(\bar{x})$ | $31.6 \%$ | $35.5 \%$ |
| Median score $\left(Q_{2}\right)$ | $29 \%$ | $34 \%$ |
| Lower quartile $\left(Q_{1}\right)$ | $22 \%$ | $24 \%$ |
| Upper quartile $\left(Q_{3}\right)$ | $43 \%$ | $42 \%$ |
| Minimum score $\left(x_{\min }\right)$ | $07 \%$ | $11 \%$ |
| Maximum score $\left(x_{\max }\right)$ | $55 \%$ | $77 \%$ |
| Range $(R)$ | 48 | 66 |
| Standard deviation $(\sigma)$ | 12.88 | 13.62 |

Table 6: Summary of descriptive statistics calculations on the post-test scores

| Post-test analysis | Control group (A) | Experiment group (B) |
| :--- | :---: | :---: |
| Mean score $(\bar{x})$ | $25.7 \%$ | $40.6 \%$ |
| Median score $\left(Q_{2}\right)$ | $24.5 \%$ | $44.5 \%$ |
| Lower quartile $\left(Q_{1}\right)$ | $19 \%$ | $19 \%$ |
| Upper quartile $\left(Q_{3}\right)$ | $28 \%$ | $57 \%$ |
| Minimum score $\left(x_{\min }\right)$ | $76 \%$ | $10 \%$ |
| Maximum score $\left(x_{\max }\right)$ | 74 | $86 \%$ |
| Range $(R)$ |  | 76 |
| Standard deviation $(\sigma)$ |  |  |

### 6.2.1 Comparison of the test scores data using box and whisker diagrams

Using the compiled data from the descriptive statistics calculations on the pre-test and post-test scores in Tables 5 and 6, we can draw box and whisker diagram representations of the data. We will place the diagrams side by side in order to assist us to compare the performance by the two groups in both the pre-tests (Figure 21) and post-tests (Figure 22).

## Pre-Test Control Group



## Pre-Test Experiment Group



Figure 21: Pre-test box plots for control and experiment groups

From Table 5 or the two box plots in Figure 21, we can see that the majority of the experiment group's pre-test scores are concentrated between $11 \%$ and $55 \%$ with the maximum score $77 \%$ falling out as an outlier as it is a distance away from the cluster under which the rest of the scores are concentrated. The pre-test scores for the control group lie between $7 \%$ and $55 \%$ which compares closely with those of the experiment group. The lower quartiles, medians, upper quartiles, and mean scores also compare very closely for both groups. Furthermore, the standard deviations for the two groups are very close to one another which means that the dispersion of scores around the mean for the two groups are similar and very closely comparable. So we can conclude, from

Table 5 or Figure 21, that there was no significant difference in performance in the pre-tests for the two groups of learners that were involved in this research study.

## Post-Test Control Group



## Post-Test Experiment Group



Figure 22: Post-test box plots for control and experiment groups

In the second box and whisker diagrams (Figure 22), the control group's post-test scores data are more concentrated between $6 \%$ and $40 \%$ with the four scores $2 \%, 4 \%, 68 \%$ and $76 \%$ standing out as outliers. This is not the same case with the post-test scores data of the experiment group which is more spread out between $10 \%$ and $86 \%$. From Table 6 or the two box plots in Figure 22, about 50 percent of the post-test scores are concentrated between the lower and upper quartiles of $19 \%$ and $28 \%$ for the control group while for
the experiment group, the 50 percent of the post-test scores are spread-out (or dispersed) between the lower and upper quartiles of $19 \%$ and $57 \%$ respectively. This is supported by the standard deviation values for the two groups of 15.09 for the control group and 20.78 for the experiment group, which is an indication that the dispersion of scores are more spread out around the mean score of $40.6 \%$ for the experiment group than in the control group where they are more concentrated around the mean score of $25.7 \%$. There was no improvement in the mean score of the control group from the pre-test to the post-test while for the experiment group, a significant improvement was realized and this resulted in the box plot for the experiment group being skewed more to the right than that of the control group, an indication that the experiment group performed better than the control group in the post-test exercise.

### 6.2.2 Comparison of the pre-test and post-test using histograms

The frequency distribution table for the two groups for both the pre-test and post-test scores is given below (see also Table B16 in Appendix G).

Table 7: Frequency distribution table for the pre-test and post-test scores

| Mark Range | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ | $70-79$ | $80-89$ | $90-100$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency |  |  |  |  |  |  |  |  |  |  |
| $f_{A(\text { Pretest })}$ | 1 | 5 | 10 | 4 | 8 | 2 | 0 | 0 | 0 | 0 |


| Frequency |  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{B(\text { Pretest })}$ | 0 | 2 | 8 | 10 | 5 | 4 | 0 | 1 | 0 | 0 |
| Frequency |  |  |  |  |  |  |  |  |  |  |
| $f_{A(\text { Postest })}$ | 3 | 5 | 17 | 1 | 2 | 0 | 1 | 1 | 0 | 0 |
| Frequency |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| $f_{B(\text { Postest })}$ | 0 | 8 | 4 | 2 | 5 | 6 | 3 | 1 | 1 | 0 |

From Table 7, we can represent a histogram of the pre-test results of both groups for comparison purposes (see Figure 23 below).


Figure 23: Comparison of pre-test histograms

From Figure 23, it can be observed that the frequencies of the two histograms compare very closely. This is so because the majority of the test scores from both groups are concentrated in class intervals between $10 \%$ and $60 \%$ with the test score of $77 \%$ from the experiment group and $7 \%$ from the control group lying out as outliers as they are outside the above mentioned interval. Thus, we can conclude that, there was no significant difference in performance between the two groups of learners in the pre-test i.e. the two groups performed the same during the pre-test exercise.


Figure 24: Comparison of post-test histograms

In the histograms for the post-tests (Figure 24), the frequencies of scores for the experiment group are clearly more skewed to the right. This is an indication that the experiment group performed much better than the control group in the post-test exercise
of this research project. This observation, again, reaffirms the conclusion that was made from the box and whisker diagrams from the previous section.

### 6.2.3 Discussion of each group's performance from the pre-test to the post-test

The pre-test mean score of the control group was $31.6 \%$ while its post-test mean score was $25.7 \%$. This is an indication that the group did not seem to have benefitted from the traditional approach used to teach Geometry and Trigonometry as there was no improvement in performance from the pre-test to the post-test. This observation is supported by the frequencies of scores in the pre-test histogram of this group which are closely comparable to the frequencies of scores of the same group in the post-test histogram (see Figure 25).

On the other hand, the mean score of the experiment group which was $35.5 \%$ in the pre-test improved to $40.6 \%$ in the post-test. This was an improvement by more than $14 \%$ from the pre-test to the post-test. This evidence of improvement is also seen in Figure 26 where it can be observed that the frequencies of scores in the post-test histogram for this group are higher than the frequencies of scores in the pre-test histogram for the same group as you move to the right. It can therefore be concluded from above that the use of the TBM approach in the teaching and learning of mathematics did contribute to an improvement of results of the experiment group. This shows strong evidence of a positive answer to the research sub-question number 2 which was stated in section 1.7 of this dissertation.


Figure 25: Control group pre-test-post-test histograms comparison


Figure 26: Experiment group pre-test-post-test histograms comparison

### 6.3 Inferential statistical analysis of data and interpretation

In this section we will determine the probability level of rejecting the null hypothesis which states that there is no significant difference between the two means (or scores averages) from the two groups of learners in both the pre-test and post-test scores respectively. The calculated $t$-values for the pre-test and post-test scores in Appendix G were used to determine the probability level of rejecting the null hypothesis for the two cases. Because the test scores data are from two groups or samples that are not related (control group and experiment group) which were selected by means of random sampling, we use the independent samples $t$-test. This t -test (which is a parametric test) is a statistical procedure for determining the probability level of rejecting the null hypothesis which was stated above.

## Case I: t -test for the pre-test scores data

Following the six steps that need to be followed when using a t-test we have:
(1) Formulation of the null hypothesis $\left(H_{0}\right)$
$H_{0}$ : There is no significant difference between the pre-test mean score of the control group and the pre-test mean score of the experiment group.

## (2) Which t-test method is to be used?

The independent samples t-test method because the test scores data are from two
groups or samples that are not related which were selected by means of random sampling.

## (3) Calculation of the t value

From Appendix G, the calculated $t$-value for the pre-test scores was equal to 0.41
(4) Degrees of freedom

$$
d f=\left(n_{A}-1\right)+\left(n_{B}-1\right)=(30-1)+(30-1)=29+29=58
$$

## (5) Results

From table B19 in Appendix G, the critical values are 1.296 (at $10 \%$ level), 1.672 (at $5 \%$ level) and 2.392 (at $1 \%$ level).

## (6) Interpretation

Since the calculated $t$ value is less than all the critical values at $10 \%$ level, $5 \%$ level and $1 \%$ level, the null hypothesis may not be rejected. This means that there is no significant difference between the two pre-test mean scores i.e. the two groups' performance was similar in the pre-test exercise.

## Case II: t-test for the post-test scores data

For the post-test exercise, we also follow the six steps needed to be followed when applying a t-test which are:
(1) Formulation of the null hypothesis $\left(H_{0}\right)$
$H_{0}$ : There is no significant difference between the post-test mean score of the control group and the post-test mean score of the experiment group.

## (2) Which t-test method is to be used?

The independent samples t-test method because the test scores data are from two groups or samples that are not related which were selected by means of random sampling.

## (3) Calculation of the t value

From Appendix G, the calculated $t$-value for the post-test scores was equal to 1.47

## (4) Degrees of freedom

$$
d f=\left(n_{A}-1\right)+\left(n_{B}-1\right)=(30-1)+(30-1)=29+29=58
$$

## (5) Results

From table B19 in Appendix G, the critical values are 1.296 (at $10 \%$ level), 1.672 (at $5 \%$ level) and 2.392 (at $1 \%$ level).

## (6) Interpretation

Since the calculated $t$ value is greater than the critical $t$ value at the $10 \%$ level but not at the $5 \%$ level and $1 \%$ level, the null hypothesis is rejected at the $10 \%$ level only. This means that there is a $90 \%$ confidence that a statistically significant difference does exist between the two mean scores. The experiment group performed significantly better in the post-test (mean of $40.6 \%$ ) compared to the control group (mean of $25.7 \%$ ).

Thus based on the inferential statistics analysis, the learners from the experiment group performed better than their counterparts from the control group in the post-test exercise. This improvement in performance is a further indication that the TBM approach enhanced the learning of mathematics by helping the learners to understand mathematics and make them to be able to solve mathematical problems given to them by their mathematics teacher. Therefore, the use of offline Techno-Blended Teaching and Learning Model also contribute to an improvement in cognitive learning experience for mathematics learners at the FET level, and this again confirms a positive response
to the cognitive part of the main research question which was stated in this research dissertation.

### 6.4 Analysis of responses to interview questions in the sections of the

## survey

In this section, the researcher will give an analysis of the students' responses that were provided to interview questions (refer to Appendix E for the questions). This analysis will be done in order to help the researcher to address some of the research sub-questions as well as to examine if similar trends to the previous observations on affective and cognitive learning experience can be drawn from the learners' responses to the interview questions that were asked. The interview questions comprised of nine questions from which, six were to be responded to by the control group and eight were to be responded to by the experiment group. Both the two groups responded to interview questions from Section A with five questions, while Question 6 from Section B was further responded to by the control group only, and Question 7 from Section C and Questions 8 and 9 from Section D were furthermore responded to by the experiment group only.

During the interview process, all the learners in each of the two groups were individually provided with a copy of the interview questions and after explaining the instructions, the researcher read out each question verbally to the learners in the group. Thereafter, the learners were given time to individually respond to the question before moving on to the next one. Therefore, the type of interviewing conducted by the
researcher was group interview questioning. Questions from Section A and Section D in the interview were structured/closed questions that were based on a five-point Likert-scale (Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree) while Section B and Section C questions were open-ended questions. All the questions asked during the interview process were intended to elicit more information from the students on their affective and cognitive learning experiences as well as to confirm whether parallel findings similar to the previous analyses on both the affective and cognitive impact on learning may be drawn.

### 6.4.1 Comparison of the learners' responses to Section A

For the structured questions of Section A which were based on the five-point Likert-scale mentioned above, the percentage of the learners that reported Agree or Strongly Agree were used by the researcher as a measuring instrument to show what proportion of learners in each group agree with the statement of each question. Tables 8 and 9 show the percentage Agree or Strongly Agree responses to Section A questions for the two topics Geometry and Trigonometry respectively, and Figures 27 and 28 are the line graphs of the percentage responses for questions on the understanding of the two content areas.

Table 8: Percentage responses of learners to Section A questions on geometry understanding

|  | \% Agree or Strongly Agree |  |
| :---: | :---: | :---: |
| Interview question asked for Geometry topic | Group (A) | Group (B) |
| 1. The teacher's teaching approach was understandable during the teaching of the Geometry topic in Grade 11. | 37\% | 43\% |
| 2. The teaching of Geometry in Grade 11 gave me a very strong reason to like this topic and want to practice more problems on my own. | 37\% | 57\% |
| 3. The teaching approach to Geometry increased my interest to participate in the classwork activities given to us by our teacher. | 63\% | 67\% |
| 4. The teaching approach to Geometry increased my interest to participate in the homework activities given to us by our teacher. | 43\% | 67\% |
| 5. The teacher's teaching approach motivated me to want to learn mathematics. | 43\% | 80\% |



Figure 27: Response to interview questions on Geometry understanding

Table 9: Percentage responses of learners to Section A questions on trigonometry understanding

|  | $\%$ Agree or Strongly Agree |  |
| :--- | :---: | :---: |
| Interview question asked for Trigonometry topic | Group (A) | Group (B) |
| 1. The teacher's teaching approach was understandable during the | $40 \%$ | $63 \%$ |
| teaching of the Trigonometry topic in Grade 11. |  |  |
| 2. The teaching of Trigonometry in Grade 11 gave me a very strong <br> reason to like this topic and want to practice more problems on my <br> own. | $23 \%$ | $67 \%$ |


| 3. The teaching approach to Trigonometry increased my interest to <br> participate in the classwork activities given to us by our teacher. | $57 \%$ | $70 \%$ |
| :--- | :---: | :---: |
| 4. The teaching approach to Trigonometry increased my interest to <br> participate in the homework activities given to us by our teacher. | $50 \%$ | $70 \%$ |
| 5. The teacher's teaching approach motivated me to want to learn <br> mathematics. | $43 \%$ | $80 \%$ |



Figure 28: Response to interview questions on Trigonometry understanding

As expected, the percentage Agree or Strongly Agree is higher for the experiment group (B) when compared to the control group (A) for questions on the experiences during the teaching of both content areas. This observation agrees with the findings obtained from analysis of the two questionnaires for the affective impact on learning. From Figures 27 and 28, it can therefore be concluded that a majority of learners from the experiment group reported that the TBM approach was more understandable and motivated them more to want to learn mathematics and participate in the classroom activities provided to them by their mathematics educator(s). This was not the same case with the control group where fewer learners reported likewise for the same questions under Section A. From the above observations or findings, it can therefore be concluded that the TBM approach assisted in the creation of a learning environment that motivated and encouraged learners to be more interested in learning Mathematics. This provides evidence that answers the research sub-question 1 which was stated in Section 1.7 of this research dissertation.

### 6.4.2 Control group's responses to Section B

The following are some responses provided by some of the learners from the control group in response to Section B , interview question 6 "How could the use of technology during the teaching of Geometry and Trigonometry have helped my learning?"

A1: The use of PowerPoint and video lessons will make our teacher have less time for writing on the chalkboard and more time for him/her to explain solutions of
mathematical problems.

A6: The use of technology would create different teaching styles that would make mathematics to be more interesting to learn and I would use the tablet for self-assessment and getting feedback during my own study time after school.

A12: When mathematical solution methods are presented using PowerPoint and video lesson, they are easier to remember and will enhance the learning of mathematics by making the lessons presented to be more interesting and will motivate me to want to study the subject more.

A21: Some of the graphical solution methods presented by our teacher using the chalkboard were not very clear and I believe that if the teacher had used technology, it would improve our learning of mathematics by making the presentation of these solutions to be clearer and easier to understand.

From the responses provided above, it can be observed that the learners from this group anticipated that the use of technology could bring with it different teaching approaches that may make the learning of mathematics to be more interesting. The learners also indicated that the use of technology could enhance learning as graphical mathematical
solutions will be presented with clear visual images that are easier to follow and understand. This would avail more time to the mathematics teacher to explain these solutions as he/she would spend less time writing the solutions on the chalkboard. The above responses also agree with findings from the analysis of the two questionnaires for the affective impact on learning in Chapter 5 and the analysis of the pre-tests and post-tests for the cognitive impact on learning from the beginning of this chapter.

### 6.4.3 Experiment group's responses to Section C

For responses to Section C , interview question 7 "What are your views about the use of technology (Tablet and TouchTutor® package) to assist with the learning of Geometry and Trigonometry", learners from the experiment group indicated that the watching of Power Point and video lesson presentations on the tablets helped them to further understand the two content chapters by exposing them to different ways of approaching and solving problems. The video lessons helped the learners to catch up on sections that they did not fully understand in class as well as address any content-gaps that could have been created by disturbances resulting from some unforeseen circumstances occurring during the earlier course of their study at school. They also indicated that the graphs presented in graphical mathematical solutions were very clear and the solution methods presented were easy to follow and understand which made the learning of mathematics to be more interesting. The learners indicated that discussions of Geometry and Trigonometry questions from Past Examination Papers with memos also helped them to consolidate their understanding of the two content chapters during revision with their mathematics educator(s).

Shown below are some of the responses provided by learners in this group in response to the above interview question:

B2: I found the tablet to be quite resourceful in that I was able to use the video and PowerPoint lesson presentations to catch up on sections that I did not understand in class as well as read new sections on my own before they are taught to us by our mathematics teacher.

B13: During the video and PowerPoint lesson presentations, graphs are clearly presented in examples that expose you to different ways of approaching and solving problems in the two chapters and when you do self-assessment, you get feedback so that you can see where you made a mistake and correct yourself.

B18: The video lessons on the tablet helped me a lot during my free time as I would repeat watching some lesson presentations of the content that I did not fully understand in class; it felt like I had a teacher in front of me every time I watched these lessons.

B19: I found TouchTutor ${ }^{\circledR}$ package program on the tablet very easy to use and the worked examples presented through video and PowerPoint presentations made me
to further understand the two chapters on my own as the examples presented in the tablet were different from those found in our mathematics textbooks.

B20: The tablet contained worked examples and extra exercises that we were able to solve with my colleagues and get feedback for the problems in these exercises which made our learning of mathematics very interesting.

B24: Watching the video lesson presentations made me to further understand what we had learnt in class and when we completed the chapters, the revision of Geometry and Trigonometry questions from Past Examination Papers with memos (in PDF format) furthermore helped me to understand the two chapters very well.

Again, it can be observed from the responses provided by learners from this group, that there is a high degree of agreement with findings from the analysis of the two questionnaires for the affective impact on learning in Chapter 5 and the analysis of the pre-tests and post-tests for the cognitive impact on learning from the beginning of this chapter.

### 6.4.4 Experiment group's responses to Section D

For the feedback on the experiment group learners' experience with the Tablet and TouchTutor® package, a five-point Likert-scale (Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree) was used to obtain responses for interview question 8 and the histogram in Figure 29 shows the number of respondents to each statement in the question (obtained from the compiled data of the responses to the question in table B7 in Appendix E)..


Figure 29: Experiment group learners' experience with Tablet and TouchTutor ${ }^{\circledR}$ package

From Figure 29, it can be observed that a large number of learners from this group indicated that they Agree or Strongly Agree with all the statements in question 8. Hence, we can conclude that a majority of learners from the experiment group found most components of the Tablet and TouchTutor® package to be very useful and valuable during the leaning of mathematics. In order to help the researcher to be able to fully address or answer the research sub-question 3 which was stated in Chapter 1 Section 1.7, the experiment group learners were also asked to individually respond to interview question 9 whose question was "Which component(s) of the Tablet and TouchTutor ${ }^{\circledR}$ support model did you find to be very helpful during your learning of Geometry and Trigonometry", and the histogram in figure 30 shows the frequency or the number of respondents for each component that was indicated in the question. These frequencies were obtained from the compiled data of the responses to the question in table B8 in Appendix E section.


Figure 30: Preferred Tablet and TouchTutor® components selected by the experiment group learners

From Figure 30, it can be observed that most of the learners in this group indicated that they found the Past Examination Papers and Solutions to be very useful while a least number indicated that they found the CASO Videos Emulator to be useful. Therefore, in descending order, the technological components of the TBM approach which the learners found to be more valuable are: Past Examination Papers and Solutions, Video Lessons, PowerPoint Lessons in PDF format, Multiple Choice Questions and Feedback, and CASIO Videos Emulator.

### 6.5 Conclusion

Learners from the experiment group performed better than their counterparts from the control group in the post-test exercise. This was confirmed by the descriptive and inferential statistics analysis of the results from the two tests that the learners wrote as part of this research investigation. This improvement in performance by learners in the experiment group occurred against the backdrop of this group also reporting a positive affective impact on their learning as a result of being taught using the TBM approach. So, a complementary relationship emerged linking the (1) teaching of mathematics using the TBM approach, (2) improvement in the affective impact on learning of mathematics, and (3) improvement in performance in the subject. Furthermore, the responses provided by learners in the interviews conducted by the researcher in the last part of this chapter, also confirmed or agreed with the findings from the analysis of the two questionnaires for the affective impact on learning in chapter 5 and the analysis of the pre-tests and posttests for the cognitive impact on learning at the beginning of this chapter 6. From above, we can therefore conclude that, the use of offline Techno-Blended Teaching and Learning Model contributed positively to an improved affective and cognitive learning experience for mathematics learners at the FET level.

## CHAPTER 7

## CONCLUSION AND RECOMMENDATIONS

### 7.1 Introduction

In this chapter, conclusions will be made based on the findings discussed in the previous chapters about the cognitive and affective impacts of the use of technology when an offline Techno-Blended Teaching and Learning Model was followed, compared to corresponding impacts after an approach where no technology was integrated. The researcher will also offer recommendations for schools and education districts to consider, make suggestions for further research, and lastly state a final word to conclude this research dissertation.

This research was a comparative research study that compared the way learning took place when selected mathematics topics from the CAPS curriculum were taught to two groups of Grade 11 Mathematics learners. An offline Techno-Blended T \& L Model (TBM approach) was used for one group, while the other group was taught without the integration of technology.

### 7.2 Findings and conclusions

The following is a summary of the research findings obtained from the previous chapters:

- The TBM approach contributed to an improvement in the affective learning experience for mathematics learners at the FET level

From chapter 5, the findings obtained from the analysis conducted in this chapter clearly showed that there was a significantly improved affective impact on learning when modern offline technology-assisted teaching and learning components of the TBM approach were integrated in the teaching and learning of selected mathematics topics in the classroom. This was in contrast to where the traditional approach was applied to a control group of learners. In this case no positive affective impact on learning was observed.

- The TBM approach contributed to an improvement in the cognitive learning experience for mathematics learners at the FET level

From chapter 6, the findings obtained confirm that there was no significant difference in performance in the pre-test exercise for the two groups prior to the intervention of the two teaching approaches (Techno-Blended Model and the traditional approach). However, a difference in performance was realized in the post-test exercise where the
experimental group performed significantly better than the control group after the two content chapters were taught to the two groups. This improvement in performance or positive quantitative impact was a clear indication that - the TBM approach enhances the learning of mathematics by helping the learners to understand the subject better and assisted them to solve problems which they encounter in the sections of the CAPS syllabus that were taught.

The following conclusions can be made from findings obtained in Chapters 5 and 6:

- The use of offline Techno-Blended Teaching and Learning Model (TBM approach)
did contribute to an improvement in the affective and cognitive learning experience for mathematics learners at the FET level.
- The TBM approach assisted in the creation of a learning environment that motivated and encouraged learners to be more interested in learning Mathematics.
- The TBM approach did contribute to an improvement of results in the Mathematics that were taught.
- The technological components of the TBM approach that were found to be more valuable by the experiment group were, in descending order: Past Examination

Papers and Solutions, Video Lessons, PowerPoint Lessons in PDF format,

Multiple Choice Questions and Feedback, and the CASIO Videos Emulator.

### 7.3 Recommendations

Based on the results and findings obtained from this study, it is recommended that Provincial Education Departments should be encouraged to promote the use of offline digital teaching and learning materials in schools that are curriculum aligned. More initiatives to ensure that learners are exposed to technology-based support models similar to the TBM are needed to stimulate interest and motivation amongst learners to interact with their mathematics studies.

The TBM approach which does not require costly installations and security arrangements at schools, and which can flexibly be accessed by learners anytime anywhere, holds great promise to overcome some of the challenges that learners experience in the learning environment at under-resourced schools and elsewhere. Furthermore, the TBM model also affords teachers new ways to enhance explanations of concepts during instruction which contribute to richer constructivist classrooms. The TBM model can also contribute to bridging the gaps between under-resourced schools whose learners perform poorly in mathematics and former Model C schools whose learners generally benefit more from their schools being well-resourced with technological support, better teachers and good infrastructure.

In order to meet the changing needs of today's learners in South African schools, teachers need to adapt to new instruction methods. Today's students are expected to learn about and use digital-technology in mathematics to prepare them for their future, the work force and challenges of everyday life (Hudson \& Porter, 2010). Hence a further recommendation is that in-service educators be exposed to more professional development programmes that lead towards greater awareness of the importance of the integration of technology, pedagogy and content knowledge (TPACK). This is an important body of knowledge for teaching mathematics which integrates technology in the teaching of mathematics to enhance learning of the subject (Mishra \& Koehler, 2006).

### 7.4 Suggestions for further research

The following are suggestions for further research:

- Similar research to investigative innovative teaching strategies which are effective in helping learners perform better in Mathematics should be conducted at more South African schools, especially for primary school learners as these strategies could help in making these learners understand the subject better thereby closing the knowledge gaps for supporting their transition from primary to high school in Mathematics.
- This research study investigated the cognitive and affective learning experiences
when selected mathematics topics were taught to two groups of Grade 11 Mathematics learners using the TBM approach for one group, and the traditional teaching approach for the other. An extension of this study to include more topics of the CAPS syllabus and also other Grades at the FET level could add value to the quest to measure the impact of using an offline techno-blended approach to support the teaching and learning of mathematics in secondary schools.
- The researcher also recommends further research to be conducted to further investigate the main reason why the TBM approach had such a positive impact on learning of mathematics when it was used with learners. In this respect, the following framework is recommended:
* Identify which component of the TBM approach was most popular amongst learners and why?
* Was a specific technology component key to the successful implementation of the TBM or was the impact a result of the integral use of a combination of technology support components?
* Can similar result be obtained by relying on exclusively web-based online support components?


### 7.5 Final word from the researcher

The multiple challenges (some historical and some contemporary) that South Africa is facing in terms of ensuring quality Mathematics Education for its millions of learners require innovative, $21^{\text {st }}$ Century solutions with high levels of customization. Only through modern learner-centered approaches can we ever have hope that knowledge gaps and deficits in conceptual understanding can be overcome. From findings obtained in this research, the TBM approach represent one such model that holds great promise to assist with addressing such challenges. In this research, the model was found to be very effective in allowing learners to have a chance to improve performance in Mathematics in South African high schools. The results from this current study shows a positive impact on mathematics learning as a result of the integrated and customized use of offline technology and curriculum aligned digital resources in classrooms of public schools. The outcomes from this study, furthermore, represents a unique contribution to a critically under-represented area of the mathematics education research literature in South Africa. Very few local studies have been conducted to measure the potential impact of integrating ICT in the classroom to assist with modern learner-centered delivery of the CAPS curriculum. As a result, Provincial Education Departments are encouraged to promote initiatives to ensure that learners are exposed to digital materials and technology-based models that are appropriately designed to stimulate interest and interactivity of learners.

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## APPENDICES SECTION

## APPENDIX A: CONSENT LETTERS

The following were the consent letters that were given to the principals, educators, and selected learners at the four schools that were chosen to participate in this research study.

## Consent Letters

## FRF/GMMDC NCS Maths Research and Development Programme School Principal Consent Form

I give consent for you to approach FET Mathematics learners and educators to participate in the research projects linked to the FRF/GMMDC NCS Maths Research and Development Programme.

I have read the Project Information Statement explaining the purpose of the research project and understand that:

- The role of the school in the research programme is voluntary
- I may decide to withdraw the school's participation at any time without penalty
- FET Mathematics learners will be invited on an ad hoc basis to participate and that permission will be sought from them and also from their parents.
- Only learners who consent and whose parents consent will participate in the project
- All information obtained will be treated in strictest confidence.
- The learners' names will not be used and individual learners will not be identifiable in any written reports about the study.
- The school will not be identifiable in any written reports about the study.
- Participants may withdraw from the study at any time without penalty.
- A report of the findings will be made available to the school.
- I may seek further information on the project from Prof WA Olivier on 0845102582.

Principal $\qquad$ Signature $\qquad$ Date $\qquad$
Please return to: Mr N. Munemo (Researcher, MEd in Mathematics Education, NMU).

Figure A1: School Principal Consent Form

## ASSENT FORM for learners and parents to participate in Mathematics Education Research Project

## FRF/GMMDC Research and Development Programme (Learner Component)

## Explanation of the Study (What will happen to me in this study?)

The aim of the study is to improve learner performance in mathematics.
You will be asked to complete/participate in a questionnaire/interview about your experiences with the teaching approach applied by your teacher during his/her teaching of selected topics in your Mathematics syllabus. Questions are aimed at finding out which aspects of your teacher's teaching approach assisted you most and to what extent did those aspects contributed to support your studies in Mathematics. Your results in tests and exams subsequent to this assent may be used to determine the extent to which the teaching approach applied has had an impact on your performance in mathematics. Completing the form will only take about 10 minutes.

Risks or Discomforts of Participating in the Study (Can anything bad happen to me ?)

There are no known risks to participating in the study.

## Benefits of Participating in the Study (Can anything good happen to me?)

The support and resources provided in the programme may assist you to achieve better results in Mathematics. Information about the reasons for such improvement may also assist many other learners.

## Confidentiality (Will anyone know I am in the study?)

No one will know that you have participated in the study other than the researchers. Any information that is published will only be in the form of summaries. You will never be able to be identified in any publication. Your feedback will be anonymous or you will be identified by a random code so that no one will be able to link the code with you as an individual person.

## Contact Information (Who can I talk to about the study?)

You can contact Prof Werner Olivier werner.aolivier2@mandela.ac.za
Tel: 0415044743

Voluntary Participation (What if I do not want to do this?)
You have the right to stop completing the questionnaire at any time. You will not get into any trouble for not completing the questionnaire.

Do you understand this study and are you willing to participate?


Signature of Minor $\qquad$ Date $\qquad$

Signature of Parent/Guardian $\qquad$ Date $\qquad$

Please return to: Mr N. Munemo (Researcher, MEd in Mathematics Education, NMU).

Figure A2: Assent Form for Learners to Participate in Mathematics Education Research Project

## APPENDIX B: ETHICAL CLEARANCE

## NELSON MANDELA

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mandela.ac.za

30-11-2018
The First Rand - DST National Chair in Mathematics education programme (2011-2015) has already received ethical clearance from the Ethical Clearance Committee of the NMU to conduct research in projects in schools that are linked to the implementations of the TBM in Mathematics classrooms. This was reflected in the forms handed out to departmental officials, principals, teachers, parents and learners who participated in the research. In the forms that were handed out, educators involved in the study as well as the learners from the experimental and control groups were provided with an explanation of the purpose of the research and why it was necessary for them to take part in the research study. The forms also informed both the educators and learners involved that their privacy would be protected at all times during the research period and any information collected from them would be kept confidential. Their names as well as the school where they come from would not be revealed in the research project or during the presentation/publication of its results. All the teachers that were involved in the study were requested to provide confirmation of their willingness to participate by signing letters of consent as an indication that they had full knowledge about the purpose and nature of the study. The same process was applied to all the learners that were involved in the study. Furthermore, all the learners' parents or guardians were requested to sign in the space provided for parents on the forms that were provided to the learners to give consent for their children to participate in the research study.

Figure A3: Ethical Clearance

## APPENDIX C: PRE-TEST AND POST-TEST QUESTIONNAIRES

| NELSON MANDELA <br> Change the World <br> UNIVERSITY |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M Ed RESEARCH QUESTIONNAIRE FOR GRADE 11(NCS CAPS) MATHEMATICS LEARNERS |  |  |  |  |  |  |
| By N. Munemo (Student Number: 215133862) |  | Ethics Clearance: H13-SCI-MAT-006 |  |  |  |  |
| INSTRUCTIONS: Indicate your response with a cross (X) for each question. Please answer all the questions |  |  |  |  |  |  |
| FEEDBACK ON THE AFFECTIVE IMPACT ON LEARNING (1st QUESTIONNAIRE) |  |  |  |  |  |  |
| Dear Student <br> You are a learner who was not assisted by a tablet and pre-installed TouchTutor resource material for your learning. A tablet is a handheld device that is similar to a computer and the TouchTutor package has video and power point lessons installed in it that covers Grade 10 to 12 mathematics content in the CAPS syllabus. It also contains examination revision material, CASIO Calculator support videos and learner workbooks that are aligned with the content lessons. <br> At your own leisure or after hours at school or home, you can practice the mathematics on the tablet for further understanding so that you can improve your marks. The questions below want you to imagine you have such support and what effect it will have on empowering your emotions towards the subject so that your results in mathematics may improve. Think yourself into such a possibility and then complete this questionnaire. Indicate with a cross ( $X$ ) in each column under your response. Please answer all questions. |  |  |  |  |  |  |
| Meaning of terms used: |  |  |  |  |  |  |
| a.) Nurture my attitude :- help my attitude get more positive. |  |  |  |  |  |  |
| b.) Attitude :- way of how I generally feel towards the subject of Mathematics. |  |  |  |  |  |  |
| c.) Affect :- produce an effect on, move emotionally. |  |  |  |  |  |  |
| d.) Self-esteem :- my feelings about myself and my ability. |  |  |  |  |  |  |
| e.) Motivate :- something that encourages me to do mathematics. |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| FEEDBACK ON THE ANTICIPATED AFFECTIVE IMPACT ON LEARNING WITH USE OF TABLETs |  | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|  |  |  |  |  |  |  |
| 2. | It will make the subject easier to understand. |  |  |  |  |  |
| 3.1 It will encourage me to want to study. |  |  |  |  |  |  |
| 4. It will build my self-esteem that I can succeed. |  |  |  |  |  |  |
| 5. It will take my fear 'blockage' for the subject away. |  |  |  |  |  |  |
| 6.1 will not feel so helpless because I cannot cope on my own. |  |  |  |  |  |  |
| 7. I will not feel so frustrated about my ability. |  |  |  |  |  |  |
| 8. I will feel that I 'am not a lost case'. |  |  |  |  |  |  |
| 9. I will feel optimistic because I have support. |  |  |  |  |  |  |
| 10. | I will feel joy because I can help myself to learn. |  |  |  |  |  |
| 11. | It will be a pleasure to study because I can overcome most Maths learning stumbling blocks. |  |  |  |  |  |
| 12. | I will not be blocked by frustration. |  |  |  |  |  |
| 13. | Watching the lessons will motivate me to keep on trying. |  |  |  |  |  |
| 14. | I will not feel depressed that it is too difficult. |  |  |  |  |  |
| 15. | I will develop a keen interest in the subject. |  |  |  |  |  |
| 16. | It will make me participate actively in the learning process of mathematics. |  |  |  |  |  |
| 17. | Feelings of sadness will become less. |  |  |  |  |  |
| 18. | I will become motivated as I see success growing. |  |  |  |  |  |
| 19. | I will feel empowered to actively participate in learning the subject. |  |  |  |  |  |
| 20. | It will keep me positive about the subject. |  |  |  |  |  |
| 21. | I will feel I am the 'captain of my ship' I am in control of my mathematics achievement. |  |  |  |  |  |
| 22. | I will be free to use Maths material when and where I choose. |  |  |  |  |  |
| 23. | Knowing how and where to get help will boost my confidence. |  |  |  |  |  |
| 24. | It will feel as if I have a personal tutor. |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Thanks for completing the questionnaire!! |  |  |  |  |  |  |

Figure A4: First Questionnaire for the Affective Impact on Learning

Figure A4 is the first questionnaire to which both the control and experiment groups responded to at the beginning of the research study, and tables B 1 and B 2 show the compiled data of the first questionnaires' responses for the two groups respectively.

Table B1: Compiled data of the control group's responses to the first questionnaire

| Question <br> number | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1. | 0 | 0 | 0 | 15 | 15 |
| 2. | 0 | 0 | 0 | 8 | 22 |
| 3. | 0 | 0 | 0 | 11 | 19 |
| 4. | 0 | 0 | 1 | 13 | 19 |
| 5. | 0 | 0 | 7 | 14 | 9 |
| 6. | 0 | 0 | 7 | 15 | 8 |
| 7. | 0 | 0 | 3 | 17 | 10 |
| 8. | 0 | 0 | 2 | 18 | 10 |
| 9. | 0 | 0 | 2 | 10 | 18 |
| 10. | 0 | 0 | 0 | 6 | 24 |
| 11. | 0 | 0 | 0 | 14 | 16 |
| 12. | 0 | 0 | 0 | 18 | 12 |
| 13. | 0 | 0 | 0 | 11 | 19 |
| 14. | 0 | 0 | 1 | 23 | 6 |
| 15. | 0 | 0 | 1 | 7 | 22 |
| 16. | 0 | 0 | 0 | 13 | 17 |
| 17. | 0 | 0 | 1 | 13 | 16 |
| 18. | 0 | 0 | 0 | 12 | 18 |
| 19. | 0 | 0 | 0 | 12 | 18 |
| 20. | 0 | 0 | 0 | 11 | 19 |
| 21. | 0 | 0 | 0 | 12 | 18 |
| 22. | 0 | 0 | 0 | 12 | 18 |
| 23. | 0 | 0 | 0 | 8 | 22 |
| 24. | 0 | 0 | 0 | 9 | 21 |

Table B2: Compiled data of the experiment group's responses to the first questionnaire

| Question <br> number | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1. | 0 | 0 | 1 | 15 | 14 |
| 2. | 0 | 0 | 0 | 16 | 14 |
| 3. | 0 | 0 | 2 | 10 | 18 |
| 4. | 0 | 0 | 1 | 19 | 10 |
| 5. | 0 | 0 | 3 | 16 | 11 |
| 6. | 0 | 0 | 5 | 17 | 8 |
| 7. | 0 | 0 | 3 | 19 | 8 |


| 8. | 0 | 0 | 1 | 18 | 11 |
| :--- | :--- | :--- | :--- | :---: | :---: |
| 9. | 0 | 0 | 1 | 15 | 14 |
| 10. | 0 | 0 | 0 | 12 | 18 |
| 11. | 0 | 0 | 0 | 20 | 10 |
| 12. | 0 | 0 | 4 | 17 | 9 |
| 13. | 0 | 0 | 0 | 10 | 20 |
| 14. | 0 | 0 | 3 | 19 | 8 |
| 15. | 0 | 0 | 0 | 13 | 17 |
| 16. | 0 | 0 | 2 | 15 | 13 |
| 17. | 0 | 0 | 0 | 16 | 12 |
| 18. | 0 | 0 | 0 | 8 | 22 |
| 19. | 0 | 0 | 1 | 18 | 11 |
| 20. | 0 | 0 | 0 | 17 | 13 |
| 21. | 0 | 0 | 13 | 16 |  |
| 22. | 0 | 0 | 11 | 14 |  |
| 23. | 0 | 0 | 0 | 19 |  |
| 24. | 0 | 0 | 0 | 25 |  |

And figures A5 and A6 are the histograms of the data compiled in tables B1 and B2 which were compiled from the responses to questions of the first questionnaires for both the control and experiment groups respectively.


Figure A5: Histogram of the anticipated impact of technology for the control group


Figure A6: Histogram of the anticipated impact of technology for the experiment group

Figure A7 shows the second questionnaire that the selected learners from both groups provided responses to after the two chapters were taught to them using the traditional approach for the control group and the TBM approach for the experiment group.

| NELSON MANDELA |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M Ed RESEARCH QUESTIONNAIRE FOR GRADE 11(NCS CAPS) MATHEMATICS LEARNERS |  |  |  |  |  |  |
| By N. Munemo (Student Number: 215133862) Ethics Clearance: H13-SCI-MAT-006 |  |  |  |  |  |  |
| TIONS: Indicate your response with a cross (X) for each question, and answer all the questions. |  |  |  |  |  |  |
| 1. FEEDBACK ON EDUCATOR'S TEACHING APPROACH (2nd QUESTIONNAIRE) |  | Not effecive at all | Less effective | Somewhat effectve | Effective | Highly effective |
| 1.1 | After your teacher completed teaching the topic Geometry to your class, |  |  |  |  |  |
|  | what can you say about his/her teaching approach. |  |  |  |  |  |
| 1.2 | After your teacher completed teaching the topic Trigonometry to your |  |  |  |  |  |
|  | class, what can you say about his/her teaching approach. |  |  |  |  |  |
| Indicate how much you understand about the following sections after your teacher had taught you. |  | Very little <br> under <br> standing | Little under standing | Moderate under standing | Good <br> under <br> standing | Complete under standing |
| 1.3 | (a) Geometry - Circle theorems. |  |  |  |  |  |
|  | (b) Geometry - Cyclic quadrilaterals. |  |  |  |  |  |
|  | (c) Geometry - Tangents to a circle. |  |  |  |  |  |
|  | (a) Trigonometry - The area rule. |  |  |  |  |  |
|  | (b) Trigonometry - The sine rule. |  |  |  |  |  |
|  | (c) Trigonometry - The cosine rule. |  |  |  |  |  |
|  | (d) Trigonometry - Mixed problems: Sine/cosine/area rules. |  |  |  |  |  |
| 2. EXPERIENCE OF THE TEACHING OF GEOMETRY |  | Strongly Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| 2.1 | The teacher's approach enabled me to solve problems indipendently. |  |  |  |  |  |
| 2.2 I | I feel my teacher needs to consider using other teaching approaches/styles. |  |  |  |  |  |
| 2.3 | The teacher's presentation of the chapter was very clear and understanda |  |  |  |  |  |
| 2.4 | The teacher assumed we had sufficient pre-knowledge to follow the lesson. |  |  |  |  |  |
| 2.5 | The teaching gave me a very strong reason to like the topic. |  |  |  |  |  |
| 2.6 | The teaching stimulated my interest to participate in classwork. |  |  |  |  |  |
| 2.7 | The teacher raced against time to complete this syllabus topic. |  |  |  |  |  |
| 2.8 | The teacher did not interact with us. |  |  |  |  |  |
| 2.9 T | The teaching stimulated me to do more maths problems. |  |  |  |  |  |
| 2.10 U | Using more technology could have made the teaching more interesting. |  |  |  |  |  |
| 3. EXPERIENCE OF THE TEACHING OF TRIGONOMETRY |  | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| 3.1 | The teacher's approach enabled me to solve problems indipendently. |  |  |  |  |  |
| 3.2 I | I feel my teacher needs to consider using other teaching approaches/styles. |  |  |  |  |  |
| 3.3 | The teacher's presentation of the chapter was very clear and understanda | ble. |  |  |  |  |
| 3.4 | The teacher assumed we had sufficient pre-knowledge to follow the lesson. |  |  |  |  |  |
| 3.5 | The teaching gave me a very strong reason to like the topic. |  |  |  |  |  |
| 3.6 | The teaching stimulated my interest to participate in classwork. |  |  |  |  |  |
| 3.7 T | The teacher raced against time to complete this syllabus topic. |  |  |  |  |  |
| 3.8 T | The teacher did not interact with us. |  |  |  |  |  |
| 3.9 | The teaching stimulated me to do more maths problems. |  |  |  |  |  |
| 3.10 | Using more technology could have made the teaching more interesting. |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | Thanks for completing the questionnaire!! |  |  |  |  |  |

Figure A7: Second Questionnaire for the Affective Impact on Learning

And tables B3 and B4 show the compiled data of the responses to the second questionnaires provided by both the control and experiment groups respectively.

Table B3: Compiled data of the control group's responses to the second questionnaire

| FEEDBACK ON EDUCATOR'S TEACHING APPROACH |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Question number | Not effective at all | Less effective | Somewhat effective | Effective | Highly effective |
| 1.1 | 1 | 3 | 16 | 10 | 0 |
| 1.2 | 0 | 2 | 17 | 11 | 0 |
| FEEDBACK ON SECTIONS LEARNERS UNDERSTOOD IN BOTH GEOMETRY AND TRIGONOMETRY |  |  |  |  |  |
| Question number | Very little understanding | Little understanding | Moderate understanding | Good understanding | Complete understanding |
| 1.3 (a) | 1 | 13 | 10 | 6 | 0 |
| 1.3 (b) | 2 | 14 | 10 | 4 | 0 |
| 1.3 (c) | 0 | 15 | 9 | 5 | 1 |
| 1.4 (a) | 3 | 14 | 8 | 2 | 3 |
| 1.4 (b) | 3 | 13 | 7 | 3 | 4 |
| 1.4 (c) | 3 | 13 | 5 | 4 | 5 |
| 1.4 (d) | 1 | 17 | 5 | 7 | 0 |
| EXPERIENCE ON THE TEACHING OF GEOMETRY |  |  |  |  |  |
| Question number | Strongly Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| 2.1 | 1 | 12 | 8 | 8 | 1 |
| 2.2 | 2 | 5 | 2 | 20 | 1 |
| 2.3 | 0 | 12 | 7 | 10 | 1 |
| 2.4 | 3 | 5 | 14 | 8 | 0 |
| 2.5 | 2 | 13 | 7 | 4 | 4 |
| 2.6 | 1 | 10 | 7 | 11 | 1 |
| 2.7 | 2 | 2 | 14 | 9 | 3 |
| 2.8 | 4 | 12 | 12 | 1 | 1 |
| 2.9 | 1 | 12 | 4 | 8 | 5 |
| 2.10 | 1 | 3 | 3 | 14 | 9 |
| EXPERIENCE ON THE TEACHING OF TRIGONOMETRY |  |  |  |  |  |
| Question number | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 3.1 | 3 | 13 | 7 | 7 | 0 |
| 3.2 | 1 | 3 | 5 | 18 | 3 |
| 3.3 | 0 | 12 | 5 | 9 | 4 |


| 3.4 | 3 | 10 | 12 | 5 | 0 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 3.5 | 1 | 12 | 3 | 10 | 4 |
| 3.6 | 1 | 14 | 8 | 4 | 3 |
| 3.7 | 2 | 6 | 7 | 9 | 6 |
| 3.8 | 3 | 9 | 9 | 8 | 1 |
| 3.9 | 0 | 11 | 8 | 9 | 2 |
| 3.10 | 0 | 2 | 3 | 13 | 12 |

Table B4: Compiled data of the experiment group's responses to the second questionnaire

| FEEDBACK ON EDUCATOR'S TEACHING APPROACH |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :--- | :--- | :--- |
| Question <br> number | Not effective <br> at all | Less effective | Somewhat <br> effective | Effective | Highly <br> effective |  |
| 1.1 | 0 | 2 | 4 | 21 | 3 |  |
| 1.2 | 0 | 1 | 7 | 17 | 5 |  |
| FEEDBACK ON SECTIONS LEARNERS UNDERSTOOD IN BOTH GEOMETRY AND TRIGONOMETRY |  |  |  |  |  |  |


| EXPERIENCE ON THE TEACHING OF TRIGONOMETRY |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Question number | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 3.1 | 1 | 1 | 8 | 19 | 1 |
| 3.2 | 1 | 7 | 10 | 9 | 3 |
| 3.3 | 0 | 0 | 8 | 19 | 3 |
| 3.4 | 1 | 2 | 13 | 10 | 4 |
| 3.5 | 0 | 1 | 6 | 19 | 4 |
| 3.6 | 0 | 3 | 3 | 20 | 4 |
| 3.7 | 0 | 12 | 4 | 8 | 6 |
| 3.8 | 7 | 14 | 5 | 3 | 1 |
| 3.9 | 0 | 2 | 2 | 22 | 4 |
| 3.10 | 0 | 1 | 2 | 10 | 17 |

The above responses provide by both the control and experiment groups for both the first and second questionnaires are going to be used in chapter 5 when the researcher will look at the analysis of the questionnaires for the affective impact on learning.

# FIRST RAND FOUNDATION MATHEMATICS EDUCATION CHAIR RESEARCH SCHOOLS 

## GRADE 11 MATHEMATICS PRE-TEST EXERCISE EUCLIDEAN GEOMETRY \& TRIGONOMETRY

RESEARCHER'S NAME
SUPERVISOR
CO-SUPERVISORS

DATE

## MR MUNEMO

PROF W.A. OLIVIER
PROF VAN RENSBURG MR V. MATSHA

02 MARCH 2015

## INSTRUCTIONS AND INFORMATION

1. Answer all questions.
2. Show all the steps of your working of the solutions.
3. You may use an approved scientific calculator unless stated otherwise.
4. If necessary, round off answers to two decimal places, unless stated otherwise.
5. Write neatly and legible.

## SECTION A (Geometry - Multiple Choice)

(Do not guess!!! - try to work out the solution to each problem to the best of your ability)

## QUESTION 1

Use the following diagram to answer questions 1.1, 1.2 and 1.3
$\operatorname{In} \triangle A B C, \hat{C}=90^{\circ} . A C=3.2 \mathrm{~cm}$ and $\hat{B}=33.45^{\circ}$.

1.1 The length of $A B$ is equal to:
A. 1.76 cm
B. 5.81 cm
C. 3.20 cm
D. 2.67 cm
(2)
1.2 The size of $\hat{A}$ is equal to:
A. $56.55^{0}$
B. $123.45^{0}$
C. $30.25^{0}$
D. $146.55^{0}$
1.3 The length of $B C$ is equal to:
A. 1.76 cm
B. 2.67 cm
C. 2.11 cm
D. 4.84 cm
$1.4 \triangle P Q R$ is a right-angled triangle with $R=90^{\circ}, P R$ is 4.3 units and $P Q$ is 6.5 units.


The size of $\hat{Q}$, correct to one decimal digit, is:
A. $33.5^{0}$
B. $48.6^{0}$
C. $41.4^{0}$
D. $56.5^{0}$

Use the following diagram to answer questions 1.5 and 1.6

1.5 The value of angle $x$ is:
A. $30^{0}$
B. $90^{\circ}$
C. $120^{\circ}$
D. $60^{\circ}$
(2)
1.6 The value of angle $y$ is:
A. $120^{0}$
B. $100^{0}$
C. $150^{\circ}$
D. $60^{\circ}$

Use the following diagram to answer questions 1.7 and 1.8

1.7 The value of angle $x$ is:
A. $60^{\circ}$
B. $20^{\circ}$
C. $80^{\circ}$
D. $40^{\circ}$
(2)
1.8 The value of angle $y$ is:
A. $20^{\circ}$
B. $80^{\circ}$
C. $40^{\circ}$
D. $60^{\circ}$
(2)

Use the following diagram to answer questions 1.9 and 1.10

1.9 The value of angle $x$ is:
A. $60^{\circ}$
B. $50^{0}$
C. $95^{\circ}$
D. $35^{0}$
(2)
1.10 The value of angle $y$ is:
A. $120^{\circ}$
B. $130^{\circ}$
C. $100^{\circ}$
D. $95^{\circ}$
(2)

$$
(10 \times 2)=[20]
$$

## SECTION B (Geometry)

QUESTION 2
[9]
Determine the angles $x, y$ and $z$. Give reasons for all your steps.


## QUESTION 3

[15]


Giving reasons in each case, calculate
3.1 angles $a$ and $b$

## QUESTION 4

In the diagram, $\hat{D}_{1}=\hat{D}_{2}$ and $D E / / A B$.
Determine the length of $D C$. Show your workings.


## QUESTION 5

In the diagram, $B E=18 \mathrm{~cm}, E D=6 \mathrm{~cm}, E C=4 \mathrm{~cm}$; and $B A / / E F / / C D$.


Use the diagram to calculate:
5.1 the length $x$
(2)
5.2 the ratio $\frac{y}{z}$
(2)

## QUESTION 6

$A B C D$ is a parallelogram with $E$ a point on $B C$ so that $D E=D C$. Prove that
$6.1 \quad \hat{D}_{1}=\hat{A}$.
6.2 $A B=D E$.


## QUESTION 7

[13]

In the diagram, $B C D E$ and $A O D E$ are parallelograms.

7.1 Prove that $O F I I A B$.
(4)
7.2 Prove that $A B O E$ is a parallelogram.
(4)
7.3 Prove that $\triangle A B O \equiv \triangle E O D$.
(5)

## SECTION C (Trigonometry)

## QUESTION 8

[8]

In the diagram, $A C=9 \mathrm{~cm}, A \hat{C} B=35^{\circ}$ and $C \hat{A} D=15^{\circ}$.


Find the value of
8.1 angle $C \hat{A} B$
8.2 the length $A B$
8.3 the length $B C$
8.4 the length $C D$.

## QUESTION 9

Calculate the length of a vertical pole if the shadow of the pole is 4 m long when the angle of elevation of the sun is $43^{\circ}$.


QUESTION 10
[6]
A painter is standing on a ladder, 10 metres in length, which is leaning against a wall. The angle between the ladder and the ground is $65^{\circ}$.

10.1 At what height is the top of the ladder above the ground?
10.2 If the painter lowers the ladder by 2 metres, what will be the size of the angle between the ladder and the ground?

TOTAL MARKS : 100

Figure A8: Pre-test Exercise on Euclidean Geometry and Trigonometry

## FIRST RAND FOUNDATION MATHEMATICS EDUCATION CHAIR RESEARCH SCHOOLS

## GRADE 11 MATHEMATICS POST-TEST EXERCISE EUCLIDEAN GEOMETRY \& TRIGONOMETRY

RESEARCHER'S NAME

SUPERVISOR

CO-SUPERVISORS

DATE

MR MUNEMO

PROF W.A. OLIVIER

PROF VAN RENSBURG MR V. MATSHA

11 SEPTEMBER 2015

## INSTRUCTIONS AND INFORMATION

1. Answer all questions.
2. Show all the steps of your working of the solutions.
3. You may use an approved scientific calculator unless stated otherwise.
4. If necessary, round off answers to two decimal places, unless stated otherwise.
5. Write neatly and legible.

## SECTION A (Geometry - Multiple Choice)

(Do not guess!!! - try to work out the solution to each problem to the best of your ability)

## QUESTION 1

In the diagram $O$ is the centre of the circle. Use the diagram to answer questions 1.1 and 1.2

1.1 The value of the angle $\hat{a}$ is equal to:
A. $60^{0}$
B. $75^{0}$
C. $30^{\circ}$
D. $150^{\circ}$
(2)
1.2 The value of the angle $\hat{b}$ is equal to:
A. $75^{0}$
B. $300^{\circ}$
C. $150^{\circ}$
D. $105^{0}$
(2)

Use the following diagram to answer questions 1.3 and 1.4

1.3 The value of the angle $\hat{c}$ is equal to:
A. $34^{0}$
B. $94^{0}$
C. $43^{0}$
D. $68^{0}$
1.4 The value of the angle $\hat{d}$ is equal to:
A. $94^{0}$
B. $34^{0}$
C. $52^{0}$
D. $47^{0}$

In the diagram $O$ is the centre of the circle. Use the diagram to answer questions 1.5, 1.6 and 1.7

1.5 The value of the angle $\hat{e}$ is equal to:
A. $50^{0}$
B. $40^{\circ}$
C. $90^{\circ}$
D. $100^{\circ}$
1.6 The value of the angle $\hat{f}$ is equal to:
A. $40^{0}$
B. $50^{0}$
C. $100^{\circ}$
D. $130^{\circ}$
1.7 The value of the angle $\hat{g}$ is equal to:
A. $40^{\circ}$
B. $50^{\circ}$
C. $30^{\circ}$
D. $25^{0}$

In the diagram $O$ is the centre of a circle. Use the diagram to answer questions 1.8 and 1.9

1.8 The value of the angle $\hat{j}$ is equal to:
A. $30^{0}$
B. $22^{0}$
C. $45^{0}$
D. $68^{0}$
1.9 The value of the angle $\hat{k}$ is equal to:
A. $45^{0}$
B. $68^{\circ}$
C. $60^{\circ}$
D. $46^{0}$
1.10 In the diagram $O$ is the centre of the circle and $A \hat{O C}=110^{\circ}$.


The value of angle $\hat{x}$ is:
A. $110^{0}$
B. $55^{\circ}$
C. $70^{\circ}$
D. $140^{\circ}$
$(10 \times 2)=[20]$

## SECTION B (Geometry)

## QUESTION 2

From the diagram below, calculate the lengths $x$ and $y$.


QUESTION 3


From the above diagram determine
3.1 the size of the angle $\hat{x}$
3.2 the size of the angle $\hat{y}$ if $F G$ is a tangent to the circle $A B D C$.

## QUESTION 4

In the diagram $O$ is the centre of a circle, $\hat{C}=38^{\circ}$ and $\hat{E}=25^{\circ}$.


Determine the size of each of the following:
4.1 angle $\hat{A}$
4.2 angle $\hat{B_{2}}$
4.3 angle $\hat{F}_{3}$
4.4 angle $\hat{x}$

## QUESTION 5

In the diagram, $A B$ is the diameter and $A E$ is a tangent to the circle with centre $O$.

5.1 Determine two angles equal to $\hat{y}$.
(4)
5.2 Prove that $A D^{2}=B D \cdot D E$

Given that $A B C D$ is a cyclic quadrilateral.
Prove the theorem: $\hat{A}+\hat{C}=180^{\circ}$


QUESTION 7

In the diagram, $O$ is the centre of the circle. Points $A, B, C, K$ and $T$ lie on the circle. $A T$ produced and $C K$ produced meet at $N$. Also $N A=N C$ and $B=38^{\circ}$.

7.1 Calculate, with reasons, the size of
a) angle $K \hat{O} A$
b) angle $\hat{T}_{2}$
c) angle $\hat{C}$
d) angle $\hat{K}_{4}$
7.2 Show that $N K=N T$.

## SECTION C (Trigonometry)

## QUESTION 8

In the diagram, $P Q=50 \mathrm{~cm}, Q R=70 \mathrm{~cm}, P R=80 \mathrm{~cm}$ and $Q \hat{P} R=60^{\circ}$.
Calculate the length of $P S$.


## QUESTION 9

In the diagram below, $A B=7 \mathrm{~cm}, A C=9 \mathrm{~cm}$ and $\hat{B}=38^{\circ}$.
Calculate correct to two decimal places, the area of $\triangle A B C$.


## QUESTION 10

The perimeter of a farm is in the form of a triangle. Vertex $A$ is directly north of vertex $B$. $A B=7.8 \mathrm{~km}, B C=8 \mathrm{~km}$ and angle $A \hat{B} C=122.07^{\circ}$. Calculate the perimeter of the farm correct to one decimal place.


TOTAL MARKS : 100

Figure A9: Post-test Exercise on Euclidean Geometry and Trigonometry

## INTERVIEW QUESTIONS FOR PROJECT LEARNERS FROM RESEARCH SCHOOLS

## GRADE 11 MATHEMATICS EUCLIDEAN GEOMETRY \& TRIGONOMETRY

```
RESEARCHER'S NAME
SUPERVISOR
CO-SUPERVISORS
DATE
```

MR N MUNEMO

PROF W.A. OLIVIER

PROF S. VAN RENSBURG MR V. MATSHA

OCTOBER 2016

## INSTRUCTIONS AND INFORMATION

1. Give responses to all questions with a cross $(\mathbf{X})$ where necessary.
2. Sections to be completed:

- Control Group - Sections A and B.
- Experiment Group - Sections A, C and D.

SECTION A [Control group and Experiment group]

Indicate your responses with a $\operatorname{cross}(X)$ for each question when required.

1. The teacher's teaching approach was understandable during the teaching of the topics Geometry and Trigonometry in Grade 11.

| Geometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |


| Trigonometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |

2. The teaching of Geometry and Trigonometry in Grade 11 gave me a very strong reason to like these topics and want to practise more problems on my own.

| Geometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |


| Trigonometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |

3. The teaching approach to Geometry and Trigonometry increased my interest to participate in the classwork activities given to us by our teacher.

| Geometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |


| Trigonometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |

4. The teaching approach to Geometry and Trigonometry increased my interest to participate in homework activities given to us by our teacher.

| Geometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |


| Trigonometry |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |

5. The teacher's teaching approach motivated me to want to learn mathematics.

| Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly Agree |
| :--- | :--- | :--- | :--- | :--- |

SECTION B [Control group only]
6. How could the use of technology (*) during the teaching of Geometry and Trigonometry have helped my learning?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
*( for example video or PowerPoint lessons, demonstrations of calculator use, Tablets with self-assessment and feedback, digital access to past exam papers and solutions)

## SECTION C [Experiment group only]

7. What are your view(s) about the use of technology (Tablet and TouchTutor ${ }^{\text {TM }}$ package) to assist with the learning of Geometry and Trigonometry?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

SECTION D [Experiment group only]
8. Feedback on your experience with the Tablet and TouchTutor ${ }^{T M}$ package.

|  | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| a. | The Maths video lessons <br> on the tablet helped me to <br> understand the subject <br> content. |  |  |  |  |  |
| b. | The CASIO calculator <br> video with visual emulator <br> on the Tablet screen <br> assisted me to use my <br> calculator to solve <br> problems. |  |  |  |  |  |
| c. | The Maths video lessons <br> assisted me to prepare for <br> tests and exams. |  |  |  |  |  |
| d.The Maths lesson <br> workbooks helped to <br> practise my mathematics. |  |  |  |  |  |  |
| e.It was easy to find the <br> Maths content topics that I <br> wanted to view on the <br> Tablet. |  |  |  |  |  |  |


| f. | It was easy to use the <br> Maths menu on the tablet <br> to go to the support <br> section that I needed. |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| g. | Sharing the Maths material <br> on the tablet with peers <br> was easy. |  |  |  |  |  |
| h. | Sharing the Maths material <br> on the tablet with peers <br> helped me to learn. |  |  |  |  |  |

9. Please select which of the following components of the Tablet \& TouchTutor ${ }^{\text {TM }}$ support model helped you to learn Geometry and Trigonometry:
(You are free to select more than one component from the options provided)

| TouchTutor ${ }^{\text {TM }}$ Digital Components on the Tablet | Mark <br> with X |
| :---: | :---: |
| a. Video Lessons |  |
| b. CASIO Emulator and Videos |  |
| c. PowerPoint Lessons in PDF format |  |
| d. Multiple Choice Questions and Feedback |  |
| e. Past Exam Papers and Solutions |  |

## End of learner survey Interview Questions Thank you for your time.

Figure A10: Interview Questions for Project Learners from Research Schools

The following tables give the statistics summary of responses to the interview questions by both the control and experiment groups respectively

## 1. RESPONSE TO INTERVIEW QUESTIONS BY THE CONTROL GROUP

Table B5: Compiled data for the control group's responses to Section A interview questions

| Questions on Geometry topic (Section A) |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Question <br> Number | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| 1. | 2 | 8 | 9 | 11 | 0 |
| 2. | 2 | 11 | 6 | 7 | 4 |
| 3. | 1 | 7 | 3 | 14 | 5 |
| 4. | 2 | 10 | 5 | 7 | 6 |
| TOTAL | $\mathbf{7}$ | $\mathbf{3 6}$ | $\mathbf{2 3}$ | $\mathbf{3 9}$ | $\mathbf{1 5}$ |


| Questions on Trigonometry topic (Section A) |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Question <br> Number | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |  |
| 1. | 0 | 9 | 9 | 11 | 1 |  |
| 2. | 2 | 6 | 15 | 6 | 1 |  |
| 3. | 1 | 4 | 8 | 13 | 4 |  |
| 4. | 3 | 8 | 4 | 11 | 4 |  |
| TOTAL | $\mathbf{6}$ | $\mathbf{2 7}$ | $\mathbf{3 6}$ | $\mathbf{4 1}$ | $\mathbf{1 0}$ |  |

## Response to Question 5 (Section A)

| 5. | 1 | 11 | 5 | 7 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## 2. RESPONSE TO INTERVIEW QUESTIONS BY THE EXPERIMENT GROUP

Table B6: Compiled data for the experiment group's responses to Section A interview questions

| Geometry topic questions (Section A) |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Question <br> Number | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |  |
| 1. | 0 | 1 | 16 | 13 | 0 |  |
| 2. | 0 | 5 | 8 | 13 | 4 |  |
| 3. | 0 | 2 | 8 | 19 | 1 |  |
| 4. | 0 | 1 | 9 | 14 | 6 |  |
| TOTAL | $\mathbf{0}$ | $\mathbf{9}$ | $\mathbf{4 1}$ | $\mathbf{5 9}$ | $\mathbf{1 1}$ |  |


| Trigonometry topic questions (Section A) |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Question <br> Number | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |
| 1. | 1 | 2 | 8 | 18 | 1 |
| 2. | 0 | 4 | 6 | 14 | 6 |
| 3. | 0 | 2 | 7 | 18 | 3 |
| 4. | 0 | 0 | 9 | 12 | 9 |
| TOTAL | $\mathbf{1}$ | $\mathbf{8}$ | $\mathbf{3 0}$ | $\mathbf{6 2}$ | $\mathbf{1 9}$ |
|      <br> 5. Response to Question 5 (Section A)    |  |  |  |  |  |

Table B7: Compiled data for the experiment group's responses to Section D question 8

|  | Strongly <br> Disagree | Disagree | Neutral | Agree | Strongly <br> Agree |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| a. | The Maths video lessons on the <br> tablet helped me to understand the <br> subject content. | 0 | 0 | 4 | 14 | 12 |
| b. | The CASIO calculator video with <br> visual emulator on the Tablet <br> screen assisted me to use my <br> calculator to solve problems. | 1 | 2 | 10 | 11 | 6 |
| c. | The Maths video lessons assisted <br> me to prepare for tests and exams. | 0 | 0 | 4 | 14 | 12 |
| d. | The Maths lesson workbooks <br> helped to practise my mathematics. | 0 | 0 | 3 | 17 | 10 |
| e. | It was easy to find the Maths <br> content topics that I wanted to view <br> on the Tablet. | 0 | 3 | 3 | 10 | 14 |
| f. | It was easy to use the Maths menu <br> on the tablet to go to the support <br> section that I needed. | 0 | 1 | 3 | 18 | 8 |
| g.Sharing the Maths material on the <br> tablet with peers was easy. | 1 | 1 | 5 | 13 | 10 |  |
| h. | Sharing the Maths material on the <br> tablet with peers helped me to <br> learn. | 0 | 4 | 4 | 13 | 9 |

Table B8: Compiled data for the experiment group's responses to Section D question 9

| Response to Question 9 (Section D) |  |
| :--- | :---: |
| TouchTutor ${ }^{\text {TM }}$ Digital Component selected | Number of responses |
| a. Video Lessons | 24 |
| b. CASIO Emulator and Videos | 3 |
| c. PowerPoint Lessons in PDF format | 17 |
| d. Multiple Choice Questions and Feedback | 10 |
| e. Past Examination Papers and Solutions | 27 |

## APPENDIX F: GENERATIONAL CHARACTERISTIC SIMILARITIES

Various research studies suggest that the majority of people born between a rough set of dates actually do share many characteristics (e.g. Strauss \& Howe, 2007; Renfro, 2012). The table below gives characteristic similarities of about five generations that the researcher summarized from the articles by, Tapscott (1999), Kelan and Lehnert (2009), Brownlee (2010), Rosen (2011), and Renfro (2012):

## Table B9: Generational characteristic similarities

| Generation's name and the period its individuals were born | Characteristic similarities |
| :---: | :---: |
| Traditional generation (1925-1944) | Grew up through the Great Depression, World War II, and the Cold War. They are characterized by a belief in common goals and respect for authority (Rosen, 2011). |
| Baby Boomer generation $(1945-1964)$ | Born roughly between 1945 and 1964; this generation tends to be optimistic, idealist and communicative (Rosen, 2011). Families for this generation were very large as this generation valued having many siblings, hence the name 'baby boomers'. This generation also tends to value education in general. |
| $\begin{aligned} & \text { Generation } X \\ & \text { (1965-1979) } \end{aligned}$ | The label X signifies that, compared with Traditional generation and Baby Boomers, Generation X cannot be easily categorized. However, this generation also tends to value education and quite a number of its members are very literate. Furthermore, students of Generation X prefer working more individually than in a group, which makes them different from the next Generations Y and Z . |
| $\begin{aligned} & \text { Generation } Y \\ & \text { (1980-1999) } \end{aligned}$ | With the 1980s and the birth of the World Wide Web, the power of cyberspace came to the masses and a new generation of web surfers, very different from their predecessors, was born. This generation is also called the Net Generation to better reflect the impact of the Internet on the lives of its members. Members of this generation tend to favour collaborative work styles in educational institutions, and some of their collective characteristic traits also include being confident, upbeat, open-minded, sociable, technically-literate, adverse to slowness, and highly informed (Kelan and Lehnert, 2009). |
| $\begin{gathered} \text { Generation Z } \\ \text { (2000 - to Date) } \end{gathered}$ | It is believed by many that this generation will hold careers not even yet created and are currently experts with facebook, twitter, cellular phones, IPods, computers and technology as a whole (Brownlee, 2010). This generation prefers media that they can interact with as opposed to passive TV or print texts and comprehend complex graphics better than previous generations (Renfro, 2012). Furthermore, members of this generation are also very collaborative and creative, and so their school tasks and projects need to reflect that. They need to be challenged with active learning and project-based tasks to meet the demands of the future (Renfro, 2012). |

## APPENDIX G: SELECTED DESCRIPTIVE AND INFERENTIAL STATISTICS

## Descriptive statistics

When assessing data for the first time, we start by organizing and summarizing the obtained data. The collection of methods used to do this is called descriptive statistics (Lombaard, van der Merwe, Kele \& Mouton, 2012). Descriptive statistics can be categorized into two groups - measures of central tendency and measure of variability. The first category, measures of central tendency which include the mean, median, lower and upper quartiles, are descriptive statistics that measure the central location or value of sets of scores. They are used widely to summarize and simplify large quantities of data. On the other hand, measures of variability or dispersion include the range, variance and standard deviation, and these are used to show the differences among the scores in a distribution. The statistics for the measures of variability provide an indication of how different or dispersed the scores are from one another within the distribution (McMillan \& Schumacher, 2010). The following are some of the key terms that will be encountered in this section:
a.) The mean is the arithmetic average of a set of scores. It is obtained by adding all the scores in a distribution and dividing by the number of scores.
Mean $\bar{x}=\frac{\sum x_{i}}{n}$, where $x_{1}, x_{2}, x_{3}, \ldots \ldots \ldots . . . . . . . ., x_{n}$ are the $n$ scores in the distribution.
b.) The median $\left(Q_{2}\right)$ is the score in a distribution below which half of the scores fall. In other words, half of the scores are above the median and half are below the median. The median is at the $50^{\text {th }}$ percentile.
c.) The lower quartile $\left(Q_{1}\right)$ is at the $25^{\text {th }}$ percentile while the upper quartile $\left(Q_{3}\right)$ is at the $75^{\text {th }}$ percentile i.e. about $50 \%$ or half of the scores lie between the lower and upper quartiles.
d.) A box and whisker diagram is a graphical method that illustrates the dispersion or spread of data in a distribution. The box represents the central $50 \%$ of the data and the whiskers which extend from the box to the lowest and highest elements give an indication of the overall spread of the data. To be able to draw a box diagram, we need the lowest and highest values or scores as well as the lower quartile $\left(Q_{1}\right)$, median $\left(Q_{2}\right)$ and upper quartile $\left(Q_{3}\right)$, see figure A11 below:


Figure A11: A box and whisker diagram
e.) Range, variance and standard deviation are three measures of dispersion that provide us with information about the spread of a data set. The more spread-out (or more dispersed) the data are, the larger the value of the standard deviation. The more concentrated the data are, the smaller the value of the standard deviation (Lombaard, van der Merwe, Kele \& Mouton, 2012).

## Descriptive statistics for the pre-tests and post-tests data

The following were the test scores obtained by the learners that took part in the research study for both the control and experiment groups:

Table B10: Test scores obtained by both the control and the experiment groups

| CONTROL GROUP (A) |  |  | EXPERIMENT GROUP (B) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| Learner | Pre-test Score | Post-test Score | Learner | Pre-test Score | Post-test Score |
| A1 | 15 | 23 | B1 | 22 | 10 |
| A2 | 42 | 20 | B2 | 39 | 43 |
| A3 | 47 | 31 | B3 | 42 | 57 |
| A4 | 28 | 20 | B4 | 42 | 46 |
| A5 | 17 | 22 | B5 | 37 | 27 |
| A6 | 22 | 21 | B6 | 40 | 39 |
| A7 | 22 | 6 | B7 | 35 | 55 |
| A8 | 25 | 2 | B8 | 50 | 57 |
| A9 | 13 | 4 | B9 | 16 | 12 |
| A10 | 31 | 14 | B10 | 37 | 51 |
| A11 | 29 | 26 | B11 | 38 | 18 |
| A12 | 55 | 76 | B12 | 77 | 86 |
| A13 | 25 | 26 | B13 | 24 | 24 |
| A14 | 33 | 29 | B14 | 52 | 72 |
| A15 | 38 | 40 | B15 | 20 | 48 |
| A16 | 47 | 23 | B16 | 32 | 47 |
| A17 | 43 | 19 | B17 | 49 | 59 |


| A18 | 46 | 68 | B18 | 24 | 29 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A19 | 51 | 40 | B19 | 32 | 47 |
| A20 | 38 | 12 | B20 | 24 | 15 |
| A21 | 16 | 19 | B21 | 29 | 17 |
| A22 | 14 | 25 | B22 | 30 | 19 |
| A23 | 43 | 26 | B23 | 55 | 65 |
| A24 | 28 | 24 | B24 | 11 | 12 |
| A25 | 25 | 28 | B25 | 33 | 18 |
| A26 | 46 | 28 | B26 | 45 | 68 |
| A27 | 48 | 28 | B27 | 22 | 33 |
| A28 | 29 | 25 | B28 | 32 | 53 |
| A29 | 26 | 27 | B29 | 54 | 69 |
| A30 | 7 | 19 | B30 | 21 | 21 |
| $n=30$ | $\sum x_{A i}=949$ | $\sum x_{A i}=771$ | $n=30$ | $\sum x_{B i}=1064$ | $\sum x_{B i}=1217$ |

The following descriptive statistics can be calculated from the data in table B10:

## 1. Pre-test and post-test means for the two groups

$$
\begin{array}{lll}
\bar{x}_{A(\text { Pretest })}=\frac{\sum x_{A i}}{n}=\frac{949}{30}=31.6 \% & \text { and } & \bar{x}_{B(\text { Pretest })}=\frac{\sum x_{B i}}{n}=\frac{1064}{30}=35.5 \% \\
\bar{x}_{A(\text { Posttest })}=\frac{\sum x_{A i}}{n}=\frac{771}{30}=25.7 \% & \text { and } & \bar{x}_{B(\text { Posttest })}=\frac{\sum x_{B i}}{n}=\frac{1217}{30}=40.6 \%
\end{array}
$$

## 2. Pre-test medians, lower and upper quartiles, and ranges

In order to be able to determine the medians, lower and upper quartiles, and range(s) for the pre-tests, the test scores need to be arranged in ascending order as shown in tables B11 and B12:

Table B11

| Control group (A) pre-test scores |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 07 | 13 | 14 | 15 | 16 |
| 17 | 22 | 22 | 25 | 25 |
| 25 | 26 | 28 | 28 | 29 |
| 29 | 31 | 33 | 38 | 38 |
| 42 | 43 | 43 | 46 | 46 |
| 47 | 47 | 48 | 51 | 55 |

Table B12

| Experiment |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 11 | 16 | 20 | 21 | 22 |
| 22 | 24 | 24 | 24 | 29 |
| 30 | 32 | 32 | 32 | 33 |
| 35 | 37 | 37 | 38 | 39 |
| 40 | 42 | 42 | 45 | 49 |
| 50 | 52 | 54 | 55 | 77 |

And from the tables we get:
$\operatorname{Median}(\mathrm{A})=\frac{29+29}{2}=29$,
Lower quartile $(\mathrm{A})=22$,
Upper quartile $(A)=43$,
Minimum score $(A)=7$,
Maximum score $(A)=55$,
Range (A) $=55-7=48$,
$\operatorname{Median}(B)=\frac{33+35}{2}=34$
Lower quartile $(B)=24$
Upper quartile $(B)=42$
Minimum score $(B)=11$
Maximum score $(B)=77$
Range $(B)=77-11=66$

## 3. Post-test medians, lower and upper quartiles, and ranges

Similarly, in order to be able to determine the medians, lower and upper quartiles, and range(s) for the post-tests, we arrange the test scores in ascending order to have tables B13 and B14:

Table B13

| Control group (A) post-test scores |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 02 | 04 | 06 | 12 | 14 |
| 19 | 19 | 19 | 20 | 20 |
| 21 | 22 | 23 | 23 | 24 |
| 25 | 25 | 26 | 26 | 26 |
| 27 | 28 | 28 | 28 | 29 |
| 31 | 40 | 40 | 68 | 76 |

Table B14

| Experiment |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 10 | 12 | 12 | 15 | 17 |
| 18 | 18 | 19 | 21 | 24 |
| 27 | 29 | 33 | 39 | 43 |
| 46 | 47 | 47 | 48 | 51 |
| 53 | 55 | 57 | 57 | 59 |
| 65 | 68 | 69 | 72 | 86 |

And from these tables, we get:
$\operatorname{Median}(A)=\frac{24+25}{2}=24.5$,
Lower quartile $(A)=19$,
Upper quartile $(A)=28$,
Minimum score $(A)=2$,
Maximum score $(A)=76$,
Range (A) $=76-2=74$,
$\operatorname{Median}(B)=\frac{43+46}{2}=44.5$
Lower quartile $(B)=19$
Upper quartile $(B)=57$
Minimum score $(B)=10$
Maximum score $(B)=86$
Range (B) $=86-10=76$

## 4. Variance and standard deviation

The variance $\left(\sigma^{2}\right)$, which is a number that indicates the average dispersion of scores from the mean, is given by the formula

$$
\sigma^{2}=\frac{\sum(x-\bar{x})^{2}}{n}=\frac{\sum x^{2}}{n}-(\bar{x})^{2}
$$

while the standard deviation $(\sigma)$ or square root of the variance is a number that gives a measure of dispersion using the deviation scores expressed in standard units about the mean.

$$
\sigma=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n}}=\sqrt{\frac{\sum x^{2}}{n}-(\bar{x})^{2}}
$$

To calculate the variance and standard deviation of the pre-test from the control group A, we use the control group pre-test scores data from table B10 to have the following table:

Table B15: Control group's pre-test squares to determine the standard deviation

| Learner | Score $(x)$ | $x^{2}$ |
| :---: | :---: | :---: |
| A1 | 15 | 225 |
| A2 | 42 | 1764 |
| A3 | 47 | 2209 |
| A4 | 28 | 784 |
| A5 | 17 | 289 |
| A6 | 22 | 484 |
| A7 | 22 | 484 |
| A8 | 25 | 625 |
| A9 | 13 | 169 |
| A10 | 31 | 961 |
| A11 | 29 | 841 |
| A12 | 55 | 3025 |
| A13 | 25 | 625 |
| A14 | 33 | 1089 |
| A15 | 38 | 1444 |
| A16 | 47 | 2209 |
| A17 | 43 | 1849 |
| A18 | 46 | 2116 |
| A19 | 51 | 2601 |
| A20 | 38 | 1444 |
| A21 | 16 | 256 |
| A22 | 14 | 196 |


| A 23 | 43 | 1849 |
| :---: | :---: | :---: |
| A 24 | 28 | 784 |
| A 25 | 25 | 625 |
| A26 | 46 | 2116 |
| A27 | 48 | 2304 |
| A28 | 29 | 841 |
| A29 | 26 | 676 |
| A30 | 7 | 49 |
| $n=30$ | $\bar{x}=31.6$ | $\sum x^{2}=34933$ |

And from table B15, the variance and standard deviation for the pre-test of the control group are given by:

$$
\begin{aligned}
& \sigma_{A(\text { Preesst })}^{2}=\frac{\sum x^{2}}{n}-(\bar{x})^{2}=\frac{34933}{30}-(31.6)^{2}=165.87 \\
& \sigma_{A(\text { Pretest })}=\sqrt{165.87}=12.88
\end{aligned}
$$

Similarly for the other test scores of the two groups, we get:

$$
\begin{array}{ll}
\sigma_{B(\text { Pretest })}^{2}=185.48 ; & \sigma_{B(\text { Pretest })}=13.62 \\
\sigma_{A(\text { Postest })}^{2}=227.57 ; & \sigma_{A(\text { Postest })}=15.09 \\
\sigma_{B(\text { Posttest })}^{2}=431.61 ; & \sigma_{B(\text { Posttest })}=20.78
\end{array}
$$

## 5. Histograms for the scores data

The frequency distribution tables for both groups' pre-test and post-test scores data can easily be derived from tables B11, B12, B13 and B14 data that has been arranged in ascending order. Using a class interval of width 10 , the following frequency distribution tables can be drawn:

Table B16: Frequency distribution table for the pre-test and post-test scores

| Mark Range | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ | $70-79$ | $80-89$ | $90-100$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency <br> $f_{A(\text { Pretest })}$ | 1 | 5 | 10 | 4 | 8 | 2 | 0 | 0 | 0 | 0 |
| Frequency <br> $f_{B(\text { Pretest })}$ | 0 | 2 | 8 | 10 | 5 | 4 | 0 | 1 | 0 | 0 |


| Frequency <br> $f_{A(\text { Postest })}$ | 3 | 5 | 17 | 1 | 2 | 0 | 1 | 1 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency <br> $f_{B(\text { Posttest })}$ | 0 | 8 | 4 | 2 | 5 | 6 | 3 | 1 | 1 | 0 |

From the frequency distribution tables, histograms can be derived for the pre-test and post-test scores of both groups to give:


Figure A12: Pre-test histogram for the control group


Figure A13: Pre-test histogram for the experiment group


Figure A14: Post-test histogram for the control group


Figure A15: Post-test histogram for the experiment group

For the analysis and interpretation of the descriptive statistics of the students' test score results, refer to Chapter 6 (Section 6.2) of this research project.

## Inferential statistics

By collecting or observing a representative subset (or fraction of data) from a much greater universe, numerical information (data) is obtained. Conclusions are drawn, or inferences are made about the greater universe based on information obtained from the representative subset. The statistical techniques used to make these inferences about the universe, based on the information from the representative subset, are referred to as inferential statistics (Lombaard, van der Merwe, Kele \& Mouton, 2012). The following are some of the key concepts that are going to be encountered in this inferential statistics section of the Appendix F: null hypothesis, alternative hypothesis, level of significance, $\boldsymbol{t}$-test, degrees of freedom, dependent samples t-test, independent samples $\boldsymbol{t}$-test, and parametric test. And the explanations of the key concepts will be provided in the next section (Inferential statistics for the pre-tests and post-tests data) as it unfolds.

## Inferential statistics for the pre-tests and post-tests data

The following table shows the test scores obtained by the selected learners in the pre-test exercise:

Table B17: Pre-test scores for calculating the t-statistic value

| Control group pre-test scores |  | Experiment group pre-test scores |  |
| :---: | :---: | :---: | :---: |
| Learner | Score | Learner | Score |
| A1 | 15 | B1 | 22 |
| A2 | 42 | B2 | 39 |
| A3 | 47 | B3 | 42 |
| A4 | 28 | B4 | 42 |
| A5 | 17 | B5 | 37 |
| A6 | 22 | B6 | 40 |
| A7 | 22 | B7 | 35 |
| A8 | 25 | B8 | 50 |
| A9 | 13 | B10 | 16 |
| A10 | 31 | B11 | 37 |
| A11 | 29 | B13 | 38 |
| A12 | 55 | B14 | 77 |
| A13 | 25 | B15 | 24 |
| A14 | 33 | B16 | 52 |
| A15 | 38 | B17 | 20 |
| A16 | 47 | B18 | 32 |
| A17 | 43 | B19 | 49 |
| A18 | 46 | B20 | 24 |
| A19 | 51 | B21 | 32 |
| A20 | 38 | B22 | 24 |
| A21 | 16 | 29 |  |
| A22 | 14 | 30 |  |


| A23 | 43 | B23 | 55 |
| :---: | :---: | :---: | :---: |
| A24 | 28 | B24 | 11 |
| A25 | 25 | B25 | 33 |
| A26 | 46 | B26 | 45 |
| A27 | 48 | B27 | 22 |
| A28 | 29 | B28 | 32 |
| A29 | 26 | B29 | 54 |
| A30 | 7 | B30 | 21 |
| $n=30, \bar{x}=31.6$, | $n=30, \bar{x}=35.5$, |  |  |
| $\sum x=949, \sum x^{2}=34933$ | $\sum x=1064, \sum x^{2}=43372$ |  |  |

Because the test scores data are from two groups or samples that are not related (control group and experiment group) which were selected by means of random sampling, we use the independent samples $t$-test. This t-test (which is a parametric test) is a statistical procedure for determining the probability level of rejecting the null hypothesis which states that there is no significant difference between the two means (or scores averages) from the two groups of learners. We need to calculate the $t$-value for this statistical procedure and the formula for calculating this $t$-test statistic is given by:

$$
t=\frac{\bar{x}_{A}-\bar{x}_{B}}{s_{\bar{x}_{A}-\bar{x}_{B}}}
$$

where $t$ is the $t$-test statistic value, $\bar{x}_{A}$ is mean of one group, $\bar{x}_{B}$ is the mean of the second group and $S_{\bar{x}_{A}-\bar{x}_{B}}$ is the standard error of the difference in means which is calculated using the formula:
$S_{\bar{x}_{A}-\bar{x}_{B}}=s \sqrt{\frac{1}{n_{A}}+\frac{1}{n_{B}}}$ where $s=\sqrt{\frac{\sum x_{A i}^{2}+\sum x_{B i}^{2}}{d f_{A}+d f_{B}}}$ and, $d f_{A}$ and $d f_{B}$ are the degrees of freedom for the two samples (groups) respectively. Pooling the variances of each distribution to result in $s$ we get:

$$
\begin{aligned}
& S=\sqrt{\frac{\sum x_{A i}^{2}+\sum x_{B i}^{2}}{d f_{A}+d f_{B}}}=\sqrt{\frac{34933+43372}{29+29}}=\sqrt{1350.09}=36.74, \text { and } \\
& s_{\bar{x}_{A}-\bar{x}_{B}}=S \sqrt{\frac{1}{n_{A}}+\frac{1}{n_{B}}}=36.74 \sqrt{\frac{1}{30}+\frac{1}{30}}=36.74 \sqrt{\frac{1}{15}}=9.4862
\end{aligned}
$$

$$
\therefore t=\frac{\bar{x}_{A}-\bar{x}_{B}}{S_{\bar{x}_{A}-\bar{x}_{B}}}=\frac{31.6-35.5}{9.4862}=\frac{3.9}{9.4862}=0.41
$$

Similarly for the post-test exercise, we have the following test scores table:

Table B18: Post-test scores for calculating the t-statistic value

| Control group post-test scores |  | Experiment group post-test scores |  |
| :---: | :---: | :---: | :---: |
| Learner | Score | Learner | Score |
| A1 | 23 | B1 | 10 |
| A2 | 20 | B2 | 43 |
| A3 | 31 | B3 | 57 |
| A4 | 20 | B4 | 46 |
| A5 | 22 | B5 | 27 |
| A6 | 21 | B6 | 39 |
| A7 | 6 | B7 | 55 |
| A8 | 2 | B8 | 57 |
| A9 | 4 | B9 | 12 |
| A10 | 14 | B10 | 51 |
| A11 | 26 | B11 | 18 |
| A12 | 76 | B12 | 86 |
| A13 | 26 | B13 | 24 |
| A14 | 29 | B14 | 72 |
| A15 | 40 | B15 | 48 |
| A16 | 23 | B16 | 47 |
| A17 | 19 | B17 | 59 |
| A18 | 68 | B18 | 29 |
| A19 | 40 | B19 | 47 |
| A20 | 12 | B20 | 15 |
| A21 | 19 | B21 | 17 |
| A22 | 25 | B22 | 19 |
| A23 | 26 | B23 | 65 |
| A24 | 24 | B24 | 12 |
| A25 | 28 | B25 | 18 |
| A26 | 28 | B26 | 68 |
| A27 | 28 | B27 | 33 |
| A28 | 25 | B28 | 53 |
| A29 | 27 | B29 | 69 |
| A30 | 19 | B30 | 21 |
| $\begin{gathered} n=30, \bar{x}=25.7 \\ \sum x=771, \sum x^{2}=26488 \end{gathered}$ |  | $\begin{gathered} n=30, \bar{x}=40.6 \\ \sum x=1217, \sum x^{2}=62399 \end{gathered}$ |  |

And for the calculation of the $t$-value, we follow the same procedure to have:

$$
\begin{aligned}
& s=\sqrt{\frac{\sum x_{A i}^{2}+\sum x_{B i}^{2}}{d f_{A}+d f_{B}}}=\sqrt{\frac{26488+62399}{29+29}}=\sqrt{1532.53}=39.15, \text { and } \\
& s_{\bar{x}_{A}-\bar{x}_{B}}=s \sqrt{\frac{1}{n_{A}}+\frac{1}{n_{B}}}=39.15 \sqrt{\frac{1}{30}+\frac{1}{30}}=39.15 \sqrt{\frac{1}{15}}=10.1085 \\
& \therefore t=\frac{\bar{x}_{A}-\bar{x}_{B}}{S_{\bar{x}_{A}-\bar{x}_{B}}}=\frac{25.7-40.6}{10.1085}=\frac{14.9}{10.1085}=1.47
\end{aligned}
$$

Thus, the $t$-values for the pre-test and post-test scores are $t=0.41$ and $t=1.47$ respectively. In Chapter 6 (Section 6.3), the researcher used these calculated $t$-statistic values to determine the probability level of rejecting the null hypothesis (mentioned above) in each case, and this was done using the critical values of student's $t$ distribution table in table B19 which is shown below.

## Critical values of Student's $\boldsymbol{t}$ distribution with $d \boldsymbol{f}$ degrees of freedom

Table B19: Critical values for student's t-distribution

| Probability less | than the critical value $\left(t_{1-\alpha, d f}\right)$ |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| df | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
|  |  |  |  |  |  |  |
| 1. | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.313 |
| 2. | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3. | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 |
| 4. | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5. | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6. | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| 7. | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.782 |
| 8. | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.499 |
| 9. | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.296 |
| 10. | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.143 |
| 11. | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.024 |
| 12. | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.929 |
| 13. | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14. | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15. | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |


| 16. | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 17. | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18. | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19. | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |
| 20. | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21. | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22. | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23. | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24. | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25. | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 |
| 26. | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27. | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28. | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29. | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30. | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |
| 31. | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.375 |
| 32. | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 |
| 33. | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.356 |
| 34. | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 |
| 35. | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.340 |
| 36. | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 |
| 37. | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 3.326 |
| 38. | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 |
| 39. | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 3.313 |
| 40. | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 |
| 41. | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 3.301 |
| 42. | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 |
| 43. | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 3.291 |
| 44. | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 |
| 45. | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.281 |
| 46. | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 |
| 47. | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 3.273 |
| 48. | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 |
| 49. | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 3.265 |
| 50. | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 |
| 51. | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 | 3.258 |
| 52. | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 | 3.255 |
| 53. | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 | 3.251 |
| 54. | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 | 3.248 |
| 55. | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 | 3.245 |
| 56. | 1.297 | 1.673 | 2.003 | 2.395 | 2.667 | 3.242 |
| 57. | 1.297 | 1.672 | 2.002 | 2.394 | 2.665 | 3.239 |
| 58. | 1.296 | 1.672 | 2.002 | 2.392 | 2.663 | 3.237 |
| 59. | 1.296 | 1.671 | 2.001 | 2.391 | 2.662 | 3.234 |
| 60. | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 |
| 61. | 1.296 | 1.670 | 2.000 | 2.389 | 2.659 | 3.229 |
| 62. | 1.295 | 1.670 | 1.999 | 2.388 | 2.657 | 3.227 |
| 63. | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 | 3.225 |
| 64. | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 | 3.223 |
| 65. | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 | 3.220 |
| 66. | 1.295 | 1.668 | 1.997 | 2.384 | 2.652 | 3.218 |
| 67. | 1.294 | 1.668 | 1.996 | 2.383 | 2.651 | 3.216 |
| 68. | 1.294 | 1.668 | 1.995 | 2.382 | 2.650 | 3.214 |
| 69. | 1.294 | 1.667 | 1.995 | 2.382 | 2.649 | 3.213 |
| 70. | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 |
| 71. | 1.294 | 1.667 | 1.994 | 2.380 | 2.647 | 3.209 |
| 72. | 1.293 | 1.666 | 1.993 | 2.379 | 2.646 | 3.207 |


| 73. | 1.293 | 1.666 | 1.993 | 2.379 | 2.645 | 3.206 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 74 | 1.293 | 1.666 | 1.993 | 2.378 | 2.644 | 3.204 |
| 75. | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 | 3.202 |
| 76 | 1.293 | 1.665 | 1.992 | 2.376 | 2.642 | 3.201 |
| 77. | 1.293 | 1.665 | 1.991 | 2.376 | 2.641 | 3.199 |
| 78 | 1.292 | 1.665 | 1.991 | 2.375 | 2.640 | 3.198 |
| 79. | 1.292 | 1.664 | 1.990 | 2.374 | 2.640 | 3.197 |
| 80. | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 |
| 81 | 1.292 | 1.664 | 1.990 | 2.373 | 2.638 | 3.194 |
| 82. | 1.292 | 1.664 | 1.989 | 2.373 | 2.637 | 3.193 |
| 83 | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 | 3.191 |
| 84 | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 | 3.190 |
| 85. | 1.292 | 1.663 | 1.988 | 2.371 | 2.635 | 3.189 |
| 86. | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 | 3.188 |
| 87. | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 | 3.187 |
| 88 | 1.291 | 1.662 | 1.987 | 2.369 | 2.633 | 3.185 |
| 89. | 1.291 | 1.662 | 1.987 | 2.369 | 2.632 | 3.184 |
| 90. | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 |
| 91. | 1.291 | 1.662 | 1.986 | 2.368 | 2.631 | 3.182 |
| 92. | 1.291 | 1.662 | 1.986 | 2.368 | 2.630 | 3.181 |
| 93. | 1.291 | 1.661 | 1.986 | 2.367 | 2.630 | 3.180 |
| 94. | 1.291 | 1.661 | 1.986 | 2.367 | 2.629 | 3.179 |
| 95. | 1.291 | 1.661 | 1.985 | 2.366 | 2.629 | 3.178 |
| 96. | 1.290 | 1.661 | 1.985 | 2.366 | 2.628 | 3.177 |
| 97. | 1.290 | 1.661 | 1.985 | 2.365 | 2.627 | 3.176 |
| 98. | 1.290 | 1.661 | 1.984 | 2.365 | 2.627 | 3.175 |
| 99. | 1.290 | 1.660 | 1.984 | 2.365 | 2.626 | 3.175 |
| 100. | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |

Source: Engineering Statistics Handbook (2012). e-Handbook of Statistical Methods. Online available at http://www.itl.nist.gov/div898/handbook/. Retrieved November 25, 2016 from http://www.itl.nist.gov/div898/handbook/eda/section3/eda3672.htm
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