An analysis of the influence of question design on learners' approaches to number pattern generalisation tasks

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This paper reports on a study of the extent to which question design affects the solution strategies adopted by children when solving linear number pattern generalisation tasks presented in pictorial and numeric contexts. The research tool comprised a series of 22 pencil-and-paper exercises based on linear generalisation tasks set in both numeric and two-dimensional pictorial contexts. The responses to these linear generalisation questions were classified by means of stage descriptors as well as stage numbers. The method or strategy adopted was analysed and classified into one of seven categories. In addition, a meta-analysis focused on the formula derived for the n^{th} term in conjunction with its justification. The results of this study strongly support the notion that question design can play a critical role in influencing learners' choice of strategy and level of attainment when solving pattern generalisation tasks. An understanding of the importance of appropriate question design has direct pedagogical application within the context of the mathematics classroom.

Introduction and background

The connection between mathematics and the notion of pattern is prevalent at all levels of mathematical endeavour. Goldin (2002: 197) describes mathematics as "the systematic description and study of pattern" while Sandefur and Camp (2004: 211) suggest that patterns are "the very essence of mathematics, the language in which it is expressed." Perhaps more generalised and all-encompassing, Steen (1988: 616) broadly defines mathematics as "the science of patterns." Pattern, in a broad sense of the word, is by no means restricted to numeric or pictorial patterns, although this is the usual context in the word for most school Mathematics syllabi.

Working with number patterns or number sequences in the classroom offers valuable opportunities for recognising, describing, extending and creating patterns (Hargreaves, Threlfall, Frobisher & Shorrocks-Taylor, 1999: 67). It has been suggested that these processes have considerable value as a precursor to formal algebra (Lannin & Warren, 1998). Searching for patterns is also an important strategy for mathematical problem-solving (Stacey, 1989: 147). Furthermore, in their seminal paper on an organising principle for Mathematics curricula, Cuoco, Goldenberg and Mark (1996) identify the search for pattern as a critical habit of mind.

The study of pattern has become an integral component across all grades of the South African school Mathematics curriculum (Department of Education, 2002; Department of Education, 2003b). In the Intermediate Phase (grades 4-6) the importance of number pattern activities is in "laying the foundation for the study of formal algebra in the Senior Phase while at the same time developing important mathematical thinking skills" (Department of Education, 2003a: 37). Number pattern activities in the Senior Phase (grades 7-9) are essentially an extension of the Intermediate Phase. However, in grades 8 and 9 there is an expectation that learners "use algebra and algebraic processes in their description of these patterns" (Department of Education, 2003a: 39). Within the Further Education and Training (FET) band (grades 10-12) learners will "solve problems related to arithmetic, geometric and other sequences and series" as well as "explore real-life and purely mathematical number patterns and problems which develop the ability to generalise, justify and prove" (Department of Education, 2003b: 12).

There are a variety of different number patterns which fall under the above framework, including: linear or arithmetic sequences, quadratic sequences, power sequences, geometric sequences, and Fibonacci-type sequences. While number patterns can be explored purely numerically – namely, in terms of patterns presented as a sequence of numerical symbols – implicit in the requirement that learners be able to "provide explanations and justifications and attempt to prove conjectures" (Department of Education, 2003b: 18) is the condition that at least some of the