AN INVESTIGATION INTO THE ROLE OF LESSON STUDY IN DEVELOPING TEACHERS' MATHEMATICS CONTENT AND PEDAGOGICAL CONTENT KNOWLEDGE

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By

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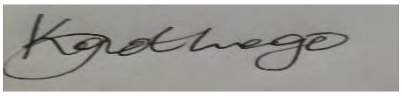
DECLARATION

I, **Naomi Ntsae Kgothego**, hereby declare that the work in this thesis is my own and where ideas from other writers have been used, they are acknowledged in full using referencing according to the Rhodes University Education Guide to References. I further declare that the work in this thesis has not been submitted to any university for degree purposes.



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ABSTRACT

Results of international assessments conducted with South African learners, both in primary and secondary schools, suggest that South African learners underperform in mathematics (Spaull, 2013). While there are numerous explanations for this, one of the key explanations is that teachers are deemed to have inadequate knowledge of both mathematics content and pedagogy. Poor content and pedagogical knowledge are indications that teachers are not adequately trained to teach mathematics (Green, 2011). To improve teachers' content and pedagogical knowledge, well-planned and researched professional development programmes need to be put in place. Current professional development opportunities that centre on workshops are not working as they provide little opportunity for teachers to connect the workshop content to the contexts in which they teach.

Through a collaboration between the Department of Basic Education (DBE) and Japan International Corporation Agency (JICA), the Lesson Study approach is being introduced to support teachers' professional development. This research seeks to research this approach within the context of Foundation Phase mathematics education. The research asks: *How does Lesson Study contribute to the development of teachers' mathematics content and pedagogical content knowledge?* Two sub-questions were developed to support the main question:

- *What mathematics content knowledge do teachers develop as the engage in LS?*
- *What pedagogical content knowledge do teachers develop as they engage in LS?*

Using a qualitative interpretivist case study approach, I worked collaboratively with four Grade 1 teachers from two schools. Data was generated through observations, semi-structured interviews and document analysis as we engaged in the Lesson Study process. The Mathematics Knowledge for Teaching (Ball, Thames & Phelps, 2008) and the Knowledge Quartet (Rowland, Turner, Thwaites & Huckstep, 2009) frameworks were used as analytic and explanatory tools in this research.

This study's findings showed that participation in the interactive cycles of Lesson Study developed the teachers’ confidence, their pedagogical content knowledge and skills and provided them with the opportunity to collaborate and reflect on their knowledge. The study's findings suggest that lesson study can be used as a strategy for improving teacher professional development.

DEDICATION

All thanks and praise to my God and savior for guiding all my ways, and giving me health, strength, wisdom and the opportunity to complete this project. I will forever Praise You, Father.

My late Father, Alfred Moloisane, "LOI" who taught me that "The fear of the LORD is the beginning of wisdom, and the knowledge of the Holy one is insight". You always believed in and encouraged me to pursue my career. May your Soul rest in eternal peace.

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List of Abbreviations

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
CCK	Common Content Knowledge
CDE	Centre for Development and Enterprise
COP	Community of Practice
CPTD	Continuing Professional Teacher Development
DBE	Department of Basic Education
FP	Foundation Phase
HCK	Horizon Content Knowledge
JICA	Japanese International Corporation Agency
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KQ	Knowledge Quartet
LP	Lesson Plan
LOLT	Language of Learning and Teaching
LS	Lesson Study
MKfT	Mathematics Knowledge for Teaching
NEEDU	National Education Evaluation and Development Unit
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
NWDE	North West Department of Education
PCK	Pedagogical Content Knowledge
PGCE	Post Graduate Certificate of Education
SA	South Africa
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1: CONTEXT

1.1 INTRODUCTION

This study considers the role of Lesson Study (LS) in developing Grade 1 teachers' mathematics and pedagogical content knowledge. My motivation relates to: (1) the performance of learners in national and international mathematics assessments; (2) concerns with teachers' content and pedagogical knowledge; and (3) my responsibility as a Senior Education Specialist (Subject Advisor) in the North West Province.

1.2 PERSONAL RESPONSIBILITIES

For several years, I have been a Senior Education Specialist (Subject Advisor) for Foundation Phase (FP) mathematics. I am employed and working in the Rustenburg sub-district office of the North West Department of Education. My responsibilities include: (1) professional guidance and support for teachers in the teaching FP mathematics; (2) practical and relevant in-service training for teachers to promote professional development and growth; (3) support to develop effective learning and teaching; and (4) the implementation and monitoring of educational policies through sustained training of teachers.

1.2.1 Personal Responsibilities - an attempt to solve the 'problem' (through '1+9'),

Despite the provision of several mathematics workshops to support teachers in the Rustenburg sub-district over the past years, classroom practice and learner performance have not shown significant improvement. To change the situation, I introduced a strategy which I called '1+9'. All Grade 3 teachers in the district met for *one day* every two weeks to unpack and sequence the curriculum, plan and prepare lessons for the coming *nine days*, and share good practices. However, this programme did not optimally address the complexity of the subject and pedagogical knowledge needed by teachers to engage with the specific aims of teaching mathematics as required by the National Curriculum Statement (NCS) (South Africa. Department of Basic Education [DBE], 2011).

In 2016 the Department of Basic Education - Japan International Cooperation Agency (DBE-JICA) organized a country-focused workshop for South African primary school mathematics subject advisors in Japan. The workshop's purpose was to learn more about problem-based learning and observe the implementation of LS and how it is used to improve teacher competency in teaching mathematics. As one of the delegates attending the workshop, I learned that the problem-based learning approach introduced by DBE-JICA in our schools and my '1+9 strategy' resemble the Japanese LS. LS is a methodology used to strengthen professional teacher development in teaching (Chokshi & Fernandez, 2004). It is an in-school professional development strategy that involves teachers preparing lessons together, implementing the planned lessons and reflecting on the lessons to improve them. LS is a collective strategy for professional development. It consists of three key components: (1) pre-collaborative work among teachers; (2) lesson observations; and (3) post-collaborative work (Stigler & Hiebert, 1999; Yoshida, 1999).

1.2.2 Personal Responsibilities – an attempt to solve the 'problem' (through Lesson Study)

Since returning from Japan, I was interested in using the LS methodology with teachers in the Rustenburg sub-district. However, to understand why and how the approach works (or does not work), I believed it necessary to engage with its use in context in my research. Hence my decision to register for an MEd qualification and to focus my research on the use of LS to strengthen teachers' mathematics and pedagogical content knowledge.

1.3 LEARNER PERFORMANCE

Results of international assessments done with South African learners, both in primary and secondary schools, suggest that South African learners underperform in mathematics (Spaull, 2013; Reddy, 2006). Results from the 2019 Grade 5 Trends in Mathematics and Science Study (TIMSS) also indicate that South African learners scored on average 374 points; twenty-six points below the international mean of 400 points. This means that there was little change in the learners' performance from 2015 to 2019. Given that three-fifths of South African learners

(63%) did not achieve 400 points or more, Reddy, Isdale, Juan, Visser, Winnaar and Arends (2016) suggest that they lack basic mathematical knowledge.

Regional and national assessments also reveal that primary school learners battle to execute mathematical tasks at the appropriate grade level or the minimum expected competence (Scholar, 2008; Pausigere, 2011; Organisation for Economic Co-operation and Development [OECD], 2012). The Southern and Eastern Africa Consortium for Monitoring Education Quality IV (SACMEQ IV) results show that there has been an increase of 57 points in the learners' mathematics achievement since the SACMEQ III study. The lowest mathematics score was in the North West Province where this research takes place. The SACMEQ IV report suggests the strengthening of in-service and pre-service training of teachers with respect to their pedagogical and subject content knowledge is much needed.

In the 2014 Grade 3 Annual National Assessment (ANA) for mathematics, 56% (national), 49.3% (provincial) and 50% (Rustenburg sub-district) of learners attained a level of performance that reflected partial achievement (i.e., between 49% and 59%) or more. Spaull (2013) maintains that these results are inflated because teachers facilitated and marked the assessments themselves. By contrast, a study conducted by Taylor (2008) in Grade 3 mathematics classes in selected schools indicated that only 10% of the sampled children were on track to complete the year's intended curriculum and reach the desired outcomes. Failure to complete the curriculum contributes to poor learner performance (Reeves, 2005).

According to Pausigere (2011), learners' "poor performance in mathematics tests highlights the need to widen interventions to primary levels" (p. 7). While in the past, national interventions have focused on mathematics learner performance in the Further Education and Training band (Grades 10-12), it is equally necessary to concentrate on mathematics in primary school (Reddy, 2006). A focus on teaching mathematics in primary school is crucial as it contains the foundations for many fundamental concepts of the discipline. These concepts should be built from the early years of schooling (Ma, 2010; Reddy, 2006).

There are numerous explanations for learner underperformance (e.g., insufficient time on task). However, one of the dominant explanations concerns teachers' insufficient content and pedagogical knowledge (e.g., Venkat & Spaull, 2014).

1.4 LIMITED TEACHER SUBJECT AND PEDAGOGICAL KNOWLEDGE

Hugo, Wedekind and Wilson (2010), the National Education Evaluation and Development Unit (NEEDU) (2013), Bolowana (2014), and Venkat and Spaull (2014) have conducted research in an attempt to understand the crisis with learner performance in mathematics in South Africa. These studies agree that one of the key contributing factors to poor learner performance is the teachers' lack of content knowledge.

Venkat and Spaull's (2015) analysis of SACMEQ III teacher data indicated that 79% of Grade 6 mathematics teachers' content knowledge was below Grade 6/7 level. Furthermore, the mathematics knowledge of teachers appeared to be rule-bound and shallow. Teachers were not familiar with the modes of reasoning (conjecture, proof, problem-solving, etc.) required in the curriculum. Research conducted by the Centre for Development and Enterprise [CDE] (2014) confirms that the teaching and learning of mathematics in SA is amongst the worst in the world as teachers struggle to respond to the questions they are teaching from the curriculum and expecting their learners to answer. South African teachers' lack of mathematics subject knowledge, even in the lower grades, is regarded as one contributing factor to low performance in mathematics. Mathematics teachers in South Africa are said to lack the mathematics knowledge required to teach mathematics in ways that enhance conceptual understanding and the effect of this is felt as far back in the education system as Foundation Phase (Chikiwa, 2017).

Taylor (2008) suggests that "in the hands of teachers whose own conceptual frames are not strong, the results are likely to be disastrous where school knowledge is totally submerged in an unorganized confusion of contrived realism" (p.276). This implies that teachers' content knowledge needs to be developed to improve classroom practice. Baumert, Kunter, Blum, Brunner, Voss, Jordan, Kusman, Kraus, Neubrand and Tsai (2010) explain that when teachers do not have a conceptual understanding of mathematics, the representations and explanations they give to learners are likely to be constrained. They stress that inadequate knowledge of mathematical content limits the teachers’ competence to explain the content to learners in a manner that encourages meaning-making.

Adler and Reed (2003) argue that while content knowledge is essential, it is not sufficient. They suggest that teachers also require pedagogical content knowledge, which is the knowledge teachers need to help learners make sense of mathematics. Hill, Rowan and Ball (2005) supported this view. Their study explored whether and how teachers’ knowledge for teaching mathematics contributes to gains in learners’ mathematics achievement. They found that both teachers’ mathematics and pedagogical content knowledge related significantly to learners’ achievement. The knowledge about teaching and learning that teachers bring to the classroom has an impact on whether learners will be able to access the topics that teachers teach. Also, teachers need to know: how learners come to acquire their subject knowledge; reflect on actual classroom practice; utilise a variety of resources in the classroom, including textbooks; and carefully address the issue of the languages of instruction, especially in the primary level (Adler & Reed, 2003; Taylor & Vinjevold, 1999; Kuhne, van den Heuvel-Panhuizen & Ensor, 2005).

Hoadley’s (2012) analysis of various small-scale studies conducted in South Africa confirmed that the majority of primary school teachers generally employ poor pedagogical practices resulting in learner underperformance. She raises several concerns related to teachers’ mathematics teaching. These include: low levels of teacher knowledge; low expectations of learners and low cognitive demand; lack of explicit feedback to learners; a dominance of concrete over abstract representations and meanings; time wastage; collectivized as opposed to individualized learning; poor coherence and slow content pacing of curriculum and activities.

1.5 REASONS UNDERPINNING SOUTH AFRICAN TEACHERS’ LIMITED KNOWLEDGE FOR TEACHING MATHEMATICS

There are various explanations for the limited knowledge of mathematics teachers in all phases of the South African schooling system. The three that form the context for this research are inadequate teacher education, curriculum changes and professional development models.

Carnoy and Chisholm (2008) claim that, despite the overwhelming majority of South African teachers having appropriate teaching qualifications, knowledge of the content and pedagogies appropriate for teaching mathematics is limited. According to Green (2011), the lack of mathematics content and pedagogical knowledge is an indication that FP teachers in South Africa are poorly trained to teach mathematics.

Inadequate teacher preparation is regarded as a critical explanation for teachers' insufficient subject knowledge and poor pedagogical practices. The DBE (2009) maintains that teachers' content knowledge and pedagogical practices are lacking as the universities and teacher-training institutions have not equipped teachers sufficiently to teach mathematics. This inadequate training prevents teachers from achieving the expected education outcomes (SA.DBE, 2009). This does not only threaten the availability of well-qualified mathematics teachers for SA schools, who could improve the development of learners' understanding of the subject mathematics but also the provision of specialists in mathematics and science careers that the country needs so much as well (Adler, 2005).

According to Adler (2005), the legacy of the apartheid system of education is to blame for the challenges faced by the universities and educational institutions in the endeavour to educate teachers. She suggests that there is not enough knowledge about the mathematical preparation needed to prepare teachers and there is limited ongoing support for mathematics teachers after they leave the university. Adler and Davis (2006) argue that most students who enrol for teacher training programs have limited mathematics content knowledge, a result of poor teaching in the schooling system (South Africa. Department of Education [DoE], 2004). In other words, learners leave the schooling system without knowing the content they will ultimately be required to teach. This makes the preparation of teachers who have the required knowledge to teach mathematics complex.

In addition to inadequate teacher education, the CDE (2007) and Pendlebury (2009) maintain that teachers have insufficient understanding of policies (e.g., the curriculum), content knowledge, and the connected pedagogic and assessment requirements. Curriculum reform processes in mathematics in many countries, including South Africa, have resulted in many teachers now having to teach a different curriculum from the one they were trained to teach and/or experienced as learners (Hoadley, 2012). According to Cross, Mungadi, and Rouhana (2002), the Apartheid government used the stringiest measures to control the curriculum and its approach to policy was top-down. Hoadley and Jansen (2012) concur that the curriculum was primarily based on Christian National Education principles, which positioned teachers as passive transmitters of knowledge and was characterised by authoritarian teaching and rote learning. Higher-order thinking, for the majority of learners, was not emphasised and drill and practice were the order of the day. The learners were required to chorus answers, memorise facts and regurgitate taught procedures (Westaway & Graven, 2018)

The South African education system in the post-independence era has been characterised by curriculum flux. There have been three different curriculum iterations since 1994: Curriculum 2005; the Revised National Curriculum; and, the current Curriculum and Assessment Policy. Each iteration of the curriculum articulates a particular orientation and set of practices that directly and indirectly shape teachers' identities (Pausigere, 2011). Teachers' identities are expressed in and through their teaching practices. Westaway and Graven (2018) argue, contrary to Pausigere (2011), that despite all the curriculum changes, teachers' identities have mainly remained the same. As such, there has been little change in teachers' mathematics pedagogical practices.

Stigler and Hiebert (1999) argue that policymakers have underestimated the requirements to improve teaching on a wide scale. They suggest that there are insufficient systems in place to support teachers' professional development. Furthermore, they maintain that teachers have no means of contributing to the gradual improvement of their content knowledge and teaching methods.

The CDE (2014) suggests that to improve the quality of teaching and learning, FP mathematics teachers should share their knowledge and experience in professional learning communities. However, traditional professional development often includes short workshops or seminars that feature departmental officials, university lecturers or outside experts and that occur away from teachers' schools. Rucker (2018) states that lecture-based workshops work neither for learners nor teachers. He maintains that traditional professional development provides little opportunity for participants to connect the content of the workshops to their individual contexts, which is necessary to construct an understanding of teaching and learning. Furthermore, he suggests that workshops provide participants with little opportunity to learn skills or strategies by actively trying them out. Although such professional development strategies can introduce teachers to essential knowledge and skills, they often lack the necessary depth (Chen & McCray, 2012; OECD, 2014). Rucker (2018) suggests that providing time for teacher collaboration, feedback, and reflection enables teachers to develop a deeper understanding of the required content and pedagogical knowledge necessary for classroom implementation.

According to the Lesson Study Group at Mills College (2018), the one-size-fits-all approach of most professional development workshops seldom meets all teachers' needs. LS is a 'bottom-up' professional development approach where teachers work collaboratively to support their learners' needs (Stols & Ono, 2016). Rather than teachers being mere recipients of workshops, LS requires them to take responsibility to improve teaching and learning in their classroom and school (Stigler & Hiebert, 1999). LS "is an ongoing professional development process utilized within Professional Learning Communities (PLCs) to allow teachers the opportunity to create a model for high-quality instructional practices" (Hathcock, 2016, p.4).

1.6 THE RESEARCH AIM AND QUESTIONS

The research aims to ascertain how the provision of opportunities for teachers to be involved in a LS process will strengthen their mathematics content and pedagogical knowledge. My sincere hope is that this study can contribute to the body of knowledge on Lesson Study in countries outside of Japan and, in particular, within a South African context. I intend to examine how Lesson Study can contribute to improving four Grade 1 teachers'mathematics content and pedagogical content knowledge with a view to improving classroom practice.

To achieve this aim, the following research questions were formulated:

How does Lesson Study contribute to the development of teachers' mathematics content and pedagogical content knowledge?

- What mathematics content knowledge do teachers develop as they engage in LS?
- What pedagogical content knowledge do teachers develop as they engage in LS?

1.7 CHAPTER OVERVIEW

My research thesis consists of five chapters, structured as follows:

Chapter 1 provides the context and background for my study. It presents the status of mathematics education in South Africa, as reflected in both national and international research. The key reasons behind this predicament, as evidenced by research, are elaborated as being the teachers' lack of both mathematics content and poor pedagogical practices, poor teacher education and inappropriate professional development models.

Chapter 2 provides an elaborated account of how Lesson Study can be utilised as a professional development approach to develop teachers' content and pedagogical content knowledge. The chapter also reviews three theoretical models related to the development of teachers' mathematics and pedagogical content knowledge. These include: (1) Shulman's Pedagogical Content Knowledge; (2) Ball, Thames & Phelps's (2008) Mathematics Knowledge of Teaching; and (3) the Knowledge Quartet of Rowland, Turner, Thwaites & Huckstep (2009).

Chapter 3 describes the methodology of this study. The research is a qualitative case study underpinned by an interpretive orientation. Four Grade 1 teachers participated in the research. Data collection instruments included observations, interviews and document analysis. The data was analysed using both emic and etic coding.

In **Chapter 4,** I present and analyse the empirical data using both the MKfT and Knowledge Quartet frameworks to illuminate the key findings relating to my research questions.

Chapter 5 concludes the research thesis by discussing the findings, the key contributions, the implications and the limitations of the study, and the opportunities for further research.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORKS

2.1 INTRODUCTION

> *Improving something as complex and culturally embedded as teaching requires the efforts of all the players, including students, parents, and politicians. But teachers must be the primary driving force behind change. They are best positioned to understand the problems that students face and to generate possible solutions (Stigler & Hiebert, 1999, p.13)*

As shown in Chapter 1, South African learners are underperforming in mathematics. One of the reasons put forward in Chapter 1 is teachers' lack of content and pedagogical knowledge. Fennema and Franke (1992) argue that teachers' conceptual understanding of mathematics influences the quality of classroom instruction. Meiliasari (2018) expands on this, suggesting that "to be able to teach a particular subject, a teacher must have a mastery of the subject they teach and understand its structure – the facts, concepts and processes of the subject and the links between them" (p.6). Ball and Bass (2000) and Hill et al. (2005) concur, but add that how teachers teach and what their learners learn is influenced by both their mathematics content and pedagogical knowledge. In this sense, the most significant knowledge domains for teachers are the subject-specific content knowledge and the pedagogical knowledge used in the subject they teach (Pompea & Walker, 2017). This is a concern in South Africa. As explained in Chapter 1, many South African teachers do not have the required content and pedagogical knowledge to teach mathematics, even in the Foundation Phase.

Teachers' seemingly poor content knowledge and pedagogical practices have been attributed to the quaiity of the teacher education system. The DBE (2009) and Adler (2005) observe that both pre-service and in-service Foundation Phase teachers are not adequately prepared for the realities of the classroom. In-service teachers are generally reliant on professional development programmes. These programmes often take the form of one-size-fits-all workshops (Chen & McCray, 2012). Rucker (2018) argues that such workshops do not support the implementation of new knowledge in the classroom. In this chapter, I elaborate on a professional development model that promotes collaborative work and enables teachers to take responsibility for their learning. This model is referred to as Lesson Study. After that, I examine the dominant mathematics knowledge and pedagogy frameworks as proposed by Shulman (1986), Ball, Thames and Phelps (2008) and Rowland (2015).

2.2 AN OVERVIEW OF LESSON STUDY

Lesson Study (LS) was started at the Tokyo Normal School in Japan in the 19th century and is now a common professional development practice utilized by Japanese teachers (Wakabayashi & Shirai, cited in Isoda, 2011). The goal was to foster and improve their learners' mathematical thinking and share and improve teachers' knowledge and pedagogical practices. Recently, teachers in many countries have begun to learn from their Japanese counterparts on how to develop professional learning communities in their schools (Arani, Fukaya, & Lassegard, 2010).

International interest in LS developed in the 1980s and 1990s after some American and Japanese researchers identified a connection between Japanese learners' science and mathematics attainment and the LS practices of their teachers (Isoda, 2007). It is maintained that the TIMSS Video Study and Yoshida's PhD study led to increased attention on LS in the West (Stols & Ono, 2016).

Stigler and Hiebert (1999) and Yoshida (1999) argue that LS has enabled Japanese teachers to develop their learners' conceptual understanding in mathematics by promoting learner-centred approaches. The success of LS in Japan has led to its spread into more than 50 countries around the world. For example, JICA collaborated with various governments in Africa to introduce LS in several countries. South Africa was one such country (Matanluka, Khalid-Joharib, & Matanluk, 2012). Over the past ten years, LS has been the focus of research at universities in Mpumalanga, Free State, Gauteng, Western Cape and Eastern Cape.

2.3 LESSON STUDY

LS is a translation of the two Japanese words: *jugyou* (instruction, lesson(s)); and (2) *kenkyuu* (study/research) (Fernandez 2002, p.394). The term *jugyou kenkyuu* is a strategy that seeks to improve pedagogical practices. Two key features of LS are: (1) the observation of live-action in the classroom based on lessons developed collaboratively by a group of teachers; and (2) the collection of data to analyse and improve teaching practices and learners' learning (Yoshida, cited in Lewis, Perry, & Murata, 2006). Lewis (2002a) claims that the observation of teaching as it happens in the classroom is at "the heart of lesson study" (p. 43). Unlike a video-recorder, the observers can position themselves so that they can see different sections of the classroom and focus on the 'sayings' and 'doings' of different learners. This is important in the collection of evidence of student's learning (Lewis, 2002b). LS is "a systematic investigation of classroom pedagogy conducted collectively by a group of teachers rather than by individuals, intending to improve the quality of teaching and learning" (Tsui & Law 2007, p. 1294). It is a systematic exploration of learning and teaching, which takes place in the classroom during a 'research lesson'.

LS is a 'bottom-up' professional development approach where teachers work collaboratively to support their learners' needs (Stols & Ono, 2016). Rather than teachers being mere recipients of workshops, LS requires them to take responsibility for improving teaching and learning in their classroom and school (Stigler & Hiebert, 1999). It is a "school-based professional development initiative that aims to enhance teaching and learning through the methodology of professional sharing of practice" (Burghes & Robinson, 2010, p.7). According to Ono and Ferreira (2010), LS is characterised as "classroom-situated, context-based, learner-focused, improvement-oriented and teacher-owned" (p.63). As such, LS is regarded as a practical methodology to strengthen professional teacher development in teaching mathematics (Chokshi & Fernandez, 2004; Fernandez & Chokshi, 2002; Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999).

LS is a form of collaborative practice. A group of teachers collaboratively identify an overarching goal (e.g., problem-solving) related to the school's overall vision and mission (e.g., education for all). Having identified the LS goal, a group of teachers plan, observe and analyse their lesson(s). The purpose of the analysis is to generate and refine actual classroom lessons, that is, their research lessons. The teachers then improve the lessons and decide if they want to reteach these ‘research lessons’ (Lewis, 2002b). In their research, Jake and Lee (cited in Matanluk, Khalid-Johari & Matanluk, 2012) indicated that LS provided an opportunity for teachers to collaborate. In the process of planning, observing and reflecting, the teachers learned from each other, reflected on their teaching and developed teaching strategies to improve their teaching. This required teachers to be continually engaged with their learning and to control their professional development.

In the LS model, the thinking and understanding of the learners are most important (Takahashi & Yoshida, 2004). This focus makes teaching more learner-centred as it requires teachers to develop a profound understanding of the curriculum content, their learners' thinking (Murata, 2011) and how to match the content to their learners' prior knowledge and understanding. Burney (2004) regards LS as:

> a process by which practitioners engage as researchers and scholars in their own classrooms by developing and testing lessons and studying their impact on learners. This practice provides a high-fidelity context in which teachers can build their content knowledge and pedagogical skill (p.530).

In other words, he maintains that LS requires teachers to demonstrate support for each other and develop relationships based on trust.

LS supports the development of communities of practice that promote inquiry about learner learning and teaching. Teachers give and receive feedback, examine new ideas and refine their ideas during the post-lesson discussion. In so doing, teachers reflect on their ‘research lessons’, as enacted in the classroom, with a view to improve teaching and learning in the classroom (Ono & Ferreira, 2010).

2.4 THE PROCESS OF LESSON STUDY

LS is an iterative and cyclical process. Researchers tend to differ on the number of stages in the LS process. Stigler and Hiebert (1999) maintain that LS encompasses eight stages, starting from deciding what the learning objective should be to sharing the results of the lesson after the research team reflection. Fernandez and Chokshi (2002) assert that there are six stages starting from the initial group meeting where the general objective is decided to the last stage, where the teachers give final views on the process and write a report. Çelik and Güzel (2018) suggest five stages, starting with research and planning and ending with a reflection on and improvement of the revision lesson. Lewis (cited in Doig & Groves, 2011) suggests that LS is a four-stage process, emerging with identifying a goal and finishing with the re-teaching and improvement of the research lesson. Ono and Stols (2016) suggested LS consists of 3 stages, starting with lesson planning and ending with reflection and post-lesson discussion. A more detailed description of the number of stages per researcher is in Table 2.1.

Authors Name	Stigler and Hiebert (1999)	Fernandez and Chokshi (2002)	Çelik & Güzel (2018)	Lewis in Doig and Groves (2011)	Ono and Stols (2016)
Number of LS Stages	Eight Stages	Six Stages	Five stages	Four Stages	Three Stages
Description of Stages	(1) Define the problem (2) Plan the lesson (3) Teach and observe the lesson (4) Reflect and evaluate the lesson (5) Revise the lesson (6) Teach and observe the revised lesson (7) Reflect and evaluate a second time (8) Share the results	(1) Identifying a lesson study goal and collectively plan the research lesson(s) to achieve the goal(s) (2) Teach and observe the research lesson (3) Post lesson reflection on the observations of the lesson to improve the lesson (4) Revising the lesson plan (5) Re-teaching the revised lesson (6) Debriefing meeting where the refined lesson is reviewed	(1) Lesson study group decides the aims of the lesson, engages in research on the content and plans the lesson (2) Teach and observe the research lesson (3) Reflection on and re-planning the research lesson (4) Teach and observe the revised lesson (5) Reflection on and improvement of the revision	(1) Goal-setting and lesson (2) Teaching and observing the research lesson (3) Post-lesson discussion (4) Consolidations of the lesson and learning	(1) Identification of the main lesson's objective and planning the researched lesson (2) Teach and observe the learner's learning, actions and discussions (3) Reflection and post-lesson discussion

Table 2.1 An explanation of the different stages of the LS as evident in some of the seminal texts

Interestingly, Stigler and Hiebert (1999), Fernandez and Chokshi (2002) and Özaltun Çelik & Bukova Güzel (2018) regards the implementation and reflection on the revised lesson as part of one cycle, whereas Lewis (cited in Doig and Groves, 2011) and Ono and Stols (2016) view the implementation and reflection of the revised lesson as part of the second cycle. Irrespective of these differing opinions, the LS process consists of goal-setting, planning, implementation, reflection, and lesson improvement.

For my study, each cycle consists of five stages: (1) identification of a problem or concern and goal setting about remedying the identified problem; (2) planning collaborative lessons; (3) presentation and observation of lessons; (4) post-lesson reflection; and (5) planning the improved lesson. This is shown in Figure 2.1

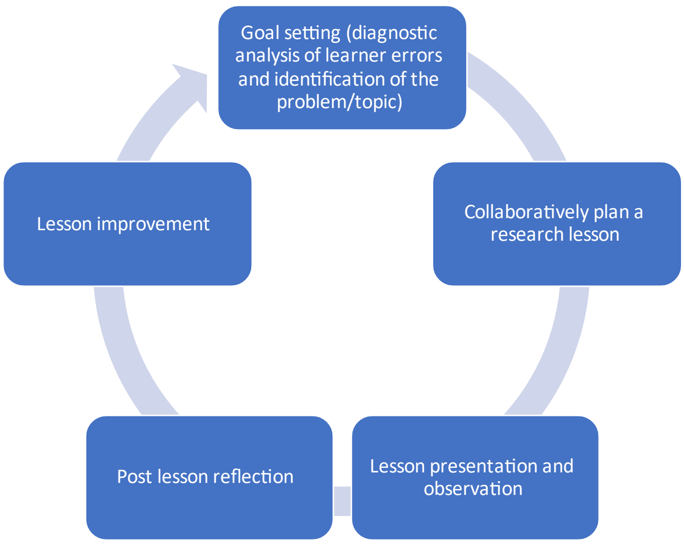

Figure 2.1 The Lesson study process used in this research (adapted from Fernandez, Cannon and Chokshi, 2003 and Sekao, n.d.)

Stage 1: The focus of stage 1 is goal setting. There are different ways in which teachers can identify their LS goals: diagnostic assessment; identifying challenging topics for learners and/or for teachers to teach; and conducting learner error analysis. Diagnostic assessment is "one type of formative assessment, which often takes place at the beginning of a study unit to find a starting point, or baseline, for learning and to develop a suitable learning programme" (OECD, 2013, p.140). Error analysis is a type of diagnostic assessment that can help a teacher determine what types of errors a learner is making and why. An error analysis can help a teacher to:

- identify which steps the learner is able to perform correctly (as opposed to simply marking answers either correct or incorrect);
- ascertain what type(s) of errors a learner is making;
- determine whether an error is a one-time misinterpretation or a constant issue that indicates an important misunderstanding of a mathematic concept or procedure; and
- select an effective instructional approach to address the learners' misconceptions and to teach the correct concept, strategy, or procedure (Brown, Skow & the IRIS Center, 2016).

Lai (2012) attests that identifying learners' consistent errors or misconceptions is the first step to providing remedial or corrective instruction. Teachers identify and analyse learners' errors and misconceptions from their assessment (formal and informal) of the learners' understanding in class. In so doing, teachers can focus on learners' needs. Teachers can also discuss topics they are struggling to teach during the LS meeting and select one of them (topics) as their goal. This can be used as the basis for the LS.

Teachers select a goal(s) by identifying one misconception or topic to focus on during the LS process (Stols & Ono, 2016). The goal(s) serve as a guiding principle throughout the complete LS cycle. For example, suppose one of the long-term goals is to improve learners' problem-solving abilities. In that case, the design of the lessons will be modelled according to typical problem-solving lessons.

Stage 2: Teachers develop 'research lessons' that serve as detailed investigations into the teaching strategies that might enable them to achieve their goal. While planning the lesson, the team needs to be meticulous in considering the following issues:

> investigation/study of curriculum material *(kyouzai kenkyuu in Japanese*), the questions the presenting teacher will ask; the answers the students are expected to give; misconceptions the students might have about the topic that can hinder learning; emphasizing teaching for understanding; the learning approach; and the

rationale for choosing the specific approach, techniques to be applied and plan for chalkboard management *(bansho in Japanese)* (Reynold, 2016, p.15).

During the planning stage, teachers agree on the constructive examples to be used. During this planning phase a More Knowledgeable Other is often consulted or included as part of the group. The Lesson Study Research Group of Teachers College, Columbia University (cited in Takahashi, 2017) identified three major reasons to have More Knowledgeable Others support the development of 'research lessons'. These include: "(1) to provide a different perspective on the work of the group; (2) to provide support the development of teachers' content knowledge, and the implementation of new ideas; and (3) to share the work of other lesson study groups" (Fernandez, Yoshida, Chokshi, & Cannon, 2001, p. 18). Takahashi (2017) mentioned that this support provides teachers with a deeper understanding of the content and the curriculum, support in analysing examples in textbooks, and support in improving their pedagogical practices. According to Ono & Stols (2016) the More Knowledgeable Other should not dominate the LS process, but should guide it by asking questions about possible learner challenges, interpretations and misconceptions.

The reasons for teachers' choices tend to be recorded on the lesson plans to remind the t eachers to observe these choices during the implementation of the lesson in the classroom (Yoshida, 1999). The 'research lesson' should be established in the realities of the school, and it should not differ from the curriculum or everyday experiences of the learners (Fernandez & Chokshi, 2004). The lesson planning session is concluded by a discussion of the different observation procedures to be followed and the observation roles of each teacher during the implementation of the lesson. According to Fernandez and Chokshi (2002), "The key to observing a study lesson in the classroom is to consider this activity as a data-gathering opportunity that can help answer questions of interest for the teachers who planned the lesson and for (the) Lesson Study group as a whole" (p.132).

Stage 3: The core of LS is the teaching of the 'research lesson' as this provides both an opportunity to 'test' the team's lesson plan in the classroom. One teacher teaches the lesson and others observe and record the learners' thinking and learning. The 'teacher-observers' make detailed notes of learners' engagement with the lesson content, thinking, questions and solution strategies. Given that this is a ‘research lesson’, Watanabe (2002, p.38) recommends that observers move around the classroom to observe the learners’ interactions and work. They should hear what the learners are talking about and see what the learners are writing in their exercise books. Teachers observe the lesson to discuss and improve their shared ideas about their understanding of the content, teaching strategies and learners' thinking (Kyle, 2015).

Stage 4: At the end of the lesson, teachers assemble for the post-lesson reflection. The purpose of this meeting is to assess how well the research lesson met the intended objectives. During this reflection session, the emphasis is not on the classroom’s contextual factors, for example, the teacher’s ‘behaviour’ or on delivering a good lesson. It is focused explicitly on the learning of the learners (Stigler, & Hiebert, 1999). The teacher who taught the lesson initiates the discussion and reflects on what s/he thought went well, the difficulties s/he experienced and his/her observations of learners' learning. The team of observers will then share their observation notes from the lesson. The purpose is to understand what the learners learned, the barriers to learning faced, and the necessary support for learning to occur (Reynold, 2016). Evidence of learners’ work in the form of workbooks or worksheets, audio or video material, forms part of the team's data. Key to the discussion is the ‘research lesson’, the learning that occurred and how to improve the lesson. The reflections are never personal, that is, directly about the teacher.

The outcome of a LS cycle is not to create a perfect lesson. As a team collaborates to improve instruction, they deepen their knowledge of content, pedagogy and the learners’ thinking. They also commit themselves to work collaboratively to improve their teaching practices. Through the LS process, teachers evaluate their teaching methods and learners’ learning in a structured and systematic manner. The LS process also prepares the way for a much broader approach to education research by gathering data about learners’ learning directly in the classroom (Lesson Study Group at Mills College, 2018).

Stage 5: At this stage, all ideas discussed during post lessons reflections are consolidated and used to refine and improve the next lesson to be taught in the 2nd cycle. It is likely that the refined lesson that is taught, is on the same topic. The team decides who will re-teach the lesson. It can be repeated in the classroom where it was presented, or a different teacher could teach the lesson to a different class of learners (Sekao, n.d.). Teaching the lesson again, often to a different class, assists in ascertaining its effectiveness.

2.5 THE IMPORTANCE OF LESSON STUDY

Teacher professional development is guided by the need to engage teachers with their teaching practices and beliefs about teaching and learning. The drivers for such needs may include, for example, curriculum change, gaps in teachers' content knowledge, new pedagogies and new classroom technologies. The underlying motivation is to improve learning outcomes, that is, learners' understandings, skills, participation and attitudes (Doig & Groves, 2011). As such, lesson study recognizes the complexity and importance of teaching. It highlights 'best practices' in the classroom. LS provides teachers the opportunity to work collaboratively to "build and refine ideas about 'best practice'" (Lesson Study Group at Mills College, 2018, unpaged). The improvement of Japanese primary school mathematics and science instruction is attributed to the teachers partaking in LS (Burghes & Robinson, 2010).

As discussed in Chapter 1, professional development that offers a one-size-fits-all approach rarely meets the needs of all teachers within a school or district. Drawing on international research, Reynolds (2016) states that LS has the potential to serve as an alternative to the frequently used workshop-based approach to professional development. LS serves as a vehicle to enhance teacher learning and reflection (Cajkler, Wood, Norton & Pedder, 2014; Fernandez & Yoshida, 2007; Ono & Ferreira, 2010, Hart, Alston & Murata, 2011). Among the models that provide professional development, LS has many features of high-quality professional development (Darling-Hammond & McLaughlin, 1996). The authors also suggest that LS could effectively improve mathematics teachers' knowledge of learner thinking.

According to de Vries (2016, p.3), "LS makes different types of knowledge more visible, such as colleagues' ideas about pedagogy and learners' mathematical thinking; thereby enabling teachers to encounter new or different ideas, and to refine their knowledge". The aim of the 'research lessons' is not to develop the perfect lesson, but rather to develop teachers' thinking about their practice, in particular the different teaching strategies that could be used to promote teaching and learning in the classroom (Doig & Groves, 2011). ‘Research lessons’ encourage teachers to engage in specific and general aspects of teaching. Lewis and Tsuchida (cited in Doig and Groves, 2011) regard LS as an essential mechanism for developing teachers’ content and pedagogical knowledge, teaching practices, and assisting when changes are made to the national curriculum.

LS offers the opportunity for more sustained and continuous professional development. It promotes the development of communities of practice that have a commitment to inquiry. It encourages teachers to develop shared goals and fosters a sense of responsibility to the academic project, their fellow teachers and the learners (Lewis et al., 2009). The LS process is often slow to start. Still, with time, teachers begin to include and weave “some of the simpler components of Lesson Study (such as collaborative lesson planning) in with their existing practices, and only later … [grasping] the significance of other ideas such as developing a lesson rationale and documenting their own learning” (Perry & Lewis, 2009, p. 388).

Dudley (2014) affirms that LS opens the ‘black box’ of the classroom. This is a result of it providing:

> teachers with ‘new eyes’ that can observe and see in detail the micro-level, inter-relationships between their learners’ learning and their own teaching – and vice versa, gives teachers new eyes to observe their practice and its effect on learners’ learning allowing them to grow more effectively and more swiftly (Yadeta & Assefa, 2017).

In the process of LS, teaching becomes a collaborative practice. Teachers work collaboratively to share ideas, review their teaching, and work collaboratively to improve teaching within the context of the classroom. During this process, teachers can adjust their underlying beliefs about teaching and learning, and it provides an opportunity for teachers to improve the profession (Kyle, 2015). Teachers learn from each other and gain content and pedagogical knowledge. LS “operates within classrooms, with real learners and in real-time, this allows teachers to form a common vision of what ideas actually look like in practice” (Kyle, 2015, unpaged). It has a strong positive effect on the way they think about teaching and learning; interact with each other and our learners.

In summary, some of the benefits of LS include the:

- improvement and development of teachers' pedagogical skills and subject matter knowledge;
- development of teachers' reflective competence;
- establishment of professional learning communities and breaking down teachers' tendency to work in isolation;
- creation of opportunities for teachers to share 'best' practices and collectively explore problems that impede learners' learning;
- development of a common understanding of what constitutes an effective lesson;
- knowledge of how to sequence and pace of curriculum content; and
- promotion of the learning of the learners (Perry & Lewis, 2009; Kyle, 2015; Stols; & Ono, 2016; Yadeta & Assefa, 2017*).*

2.6 LESSON STUDY IN THE SOUTH AFRICAN CONTEXT

There is a clear body of literature and knowledge relating to the field of LS internationally (Karen, 2010); however, the implementation of LS in SA is not yet common practice. LS was introduced in two provinces (Mpumalanga and Free State) around 2010 but was never sustained nor institutionalized on a grand scale (country-wide). It appeared that LS did not gain the desired traction in South Africa. For example, Japan and Mpumalanga's joint project lasted only nine years, from 1999 to 2008 (Ono & Ferreira, 2010). In 2016, the Department of Education - Japan International Cooperation Agency (DBE-JICA) organized a country-focused workshop for 16 South African primary school mathematics subject advisors (North West and Eastern Cape provinces) in Japan to observe the implementation of LS. Emanating from the training was the inception and institutionalization of an adapted LS model to strengthen the existing Professional Learning Communities (PLCs) in some of the sub-districts of the two provinces. In 2017, this programme was extended to the Kwa-Zulu Natal and Free State Department of Education. The South African team observed Japanese teachers presenting mathematics lessons in the Foundation Phase. Despite these two opportunities to engage with LS, it is still not common practice in SA.

At the end of 2019, I conducted a review of all the DHET accredited South African journals using the keyword ‘lesson study’. The South African journals revealed nine articles and two theses on LS (Appendix 1). The findings from this review are summarized below. While there are limited examples of LS in South Africa, the practice does appear across the teacher development sector, from the more formal pre- and in-service to continuous professional development courses and programmes. The content areas that form the focus of LS appear to be mathematics, physical sciences, biology, and leadership and management.

There was only one thesis that focused on Foundation Phase. In her Masters in Education thesis, Coe (2010) examined the perceived benefit of LS for Foundation and Intermediate Phase teachers. She found that her co-engagement around a shared task had an impact not only on teachers’ practices but also their beliefs about teaching and learning. Using the Knowledge Quartet (Rowland, 2004), she found that the teachers did not develop equally across each of the dimensions of the Knowledge Quartet, that is, Foundation, Transformation, Connection and Contingency Knowledge. An explanation of the Knowledge Quartet is provided later in this chapter.

Hiroaki Ozawa (2010) conducted a study in the province of Mpumalanga. The focus was on the ‘research lessons’ in science and the reflections on the lessons to ascertain how the new curriculum was implemented and discussed by science teachers. The study concentrated on in-service science teachers and teacher trainers. The study affirms that LS can contribute to teachers' professional development as the content knowledge, teaching methodologies, and assessment strategies improve. Similarly, Bayaga’s (2013) research on LS with Post Graduate Certificate of Education (PGCE) students preparing to teach in the Intermediate Phase found that LS supported teacher learning**.** Bayaga (2013) also found that LS led to the development of meta-cognitive skills and improved reflective competence. By contrast, Ono and Ferreira (2010) and Chikamori, Ono, and Rogan (2013) found that teachers struggled to reflect on the lessons during the LS process. They conducted a study in Mpumalanga involving in-service mathematics and science teachers in the Senior and Further Education and Training Phases. These researchers found that the teachers involved in their study did improve their presentation of lessons, but many of them were challenged in learning what to observe during lessons and how to record it.

Coe, Carl and Frick (2010) investigated the role of LS in the continuing professional development of Intermediate Phase teachers. The research found that LS: (1) assisted teachers in transforming new practices into their classroom routines; (2) supported teachers' planning and reflections on lessons; and (3) challenged the apparent tendency of teachers to work in silos. They thus regard LS as a dynamic and effective CPTD programme. Mokhele's (2017) study concentrated on instructional leadership through LS. The data was drawn from a mathematics intervention. The in-service teachers, who participated in the research, recommended LS as a means of improving their teaching and learning of mathematics.

Building on the collaborative element of LS, Van der Walt and De Beer (2016) investigated the affordance of LS approaches in the process of enculturation within a Community of Practice (COP). Their findings suggest that while LS promotes the development of science, technology, engineering and mathematics teachers, it also builds relationships between teachers, and between the theory and practice in the classroom. Ayodele and Gaigher, (2019) also foregrounded the benefit of collaborative planning in their research. They investigated teachers' experiences of teaching electricity and magnetism during a Lesson Study intervention. Findings revealed that collaborative planning was experienced as beneficial by all four participants. However, Ayodele and Gaigher, (2019) did add that Lesson Study may be inefficient in cases where teachers did not have the required content knowledge.

2.7 TEACHERS' MATHEMATICS AND PEDAGOGICAL CONTENT KNOWLEDGE

Shulman (1986, 1987) was one of the first to raise concerns about the focus of teacher education institutions and the development of teachers equipped with the knowledge and skills to teach. Developing teachers' mathematics and pedagogical knowledge is important as it:

> (a) supports teachers in unpacking mathematical concepts, skills, and procedures; (b) allows teachers to connect mathematical ideas within and across mathematical domains; (c) prompts teachers to communicate mathematically in ways that learners can understand and use; and (d) promotes the use pedagogical practices applicable to the discipline of mathematics (Ball & Bass, 2003, pp11-12).

In this chapter, I look at the work of Shulman, Ball and colleagues and Rowland and colleagues, as each build on the former. They all seek to provide teacher educators with a deeper understanding of the knowledge that teachers require to teach with confidence and competence.

2.8 SHULMAN'S PEDAGOGICAL CONTENT KNOWLEDGE

Shulman (1986, 1987), in his research on teacher education programmes, found that subject content knowledge and pedagogical knowledge were treated separately and that teacher education institutions tended to prioritize one over the other. In other words, he maintained that teaching was approached by only focusing on content or pedagogy (Evens, Elen & Depaepe, 2015). For him, teachers are required to know something that is not necessarily understood by other people. They should know the content to be taught, have knowledge of learners and how to represent ideas so that learners can make sense of the content (Shulman, 1987). In an attempt to provide support for pre-service teacher education programmes, he identified seven categories of knowledge that teachers require. These include: content knowledge; general pedagogical knowledge; curriculum knowledge; pedagogical content knowledge; knowledge of the learners; knowledge of educational contexts; and knowledge of the purposes of education (Shulman, 1987). He argued that focusing on one aspect was inadequate for preparing teachers to teach school subjects.

In his view, the key to quality teacher education, and the aspect that he diverted most of his attention to, is the intersection of content and pedagogical knowledge (Shulman, 1986, 1987). He tried to address this dichotomous approach by introducing a specific domain of knowledge, that is, Pedagogical Content Knowledge (PCK). Shulman (1986) regarded PCK as the combination of content and pedagogy unique to teachers' work. He argued that PCK "is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (1987, p.8) as it requires the transformation of the subject matter knowledge into a form that is more accessible to learners (Shulman, 1986, 1987; Grossman, 1990, 1995). PCK is a type of knowledge that is central to the work of teaching, as it is the knowledge that teachers require to interpret and transform the content to facilitate learning.

PCK includes the representations, examples and applications that teachers use in order to make the subject matter comprehensible to learners; an understanding of what makes the learning of specific topics easy or difficult; and the perspectives and preconceptions that learners of different ages and backgrounds bring with them to the learning environment (Shulman, 1986). One might say that at the heart of effective teaching is teachers' PCK.

2.8.1 Critique of Shulman's PCK

Shulman (1987) makes clear that the knowledge base necessary for teaching entails: knowledge of the content of the subject to be taught; knowledge of learners' conceptions and misconceptions related to the content of the subject to be taught; and knowledge of various teaching strategies to address learners' needs. Scheiner (2015) maintains that this view of Shulman places too much emphasis on the content and the act of teaching and seemingly ignores the importance of the the knowledge required to understand how learners learn. This is seemingly a contentious critique. In my view, Shulman does propose in his seven critical categories of knowledge that student teachers require knowledge of learners. One can assume that he meant that student teachers needed an understanding of how learners learn.

Ball, Hill and Bass (2005) argued that Shulman's framework might be difficult to operationalize as the distinction between subject matter knowledge and pedagogical knowledge is not absolute in PCK. In addition, they questioned Shulman's view that disciplinary scholars did not understand the knowledge distinct to teachers' work. Ball and colleague's notion of Specialized Content Knowledge (SCK) emphasizes the kind of knowledge distinctive to mathematics teachers (Scheiner, Montes, Godino, Carrillo, & Pino-Fan, 2017). Interestingly, Jaffer (2020) makes the same comment about Ball and colleagues SCK.

Shulman's framework was developed in the context of pre-service education programmes that prepare students for teaching in secondary schools. In secondary schools, teachers are predominantly subject-specialists. This is different from the Foundation Phase, where teachers are expected to be generalists. In response to this, Ball and her colleagues posited that there might be a difference between secondary and primary school teachers. Also, Shulman's framework is not specific to mathematics but relates to all the disciplines (Ball, 1991; Ball et al., 2008; Grossman, 1990; Fennema & Franke, 1992; Fernández-Balboa & Stiehl, 1995; Marks, 1990). Unlike Shulman's framework, which focused on secondary school teachers, Ball and colleagues developed a framework specific to primary school mathematics education. It is referred to as the Mathematics Knowledge for Teaching (MKfT) framework (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill & Ball, 2004; Hill, Ball, & Schilling, 2008). The MKfT framework focuses specifically on the knowledge teachers require to teach mathematics in primary school.

2.9 BALL'S MATHEMATICS KNOWLEDGE FOR TEACHING

The MKfT framework is a refinement of Shulman's pedagogical content knowledge and content knowledge categories. MKfT deals with what we think and know about mathematics, teaching and, more specifically, teaching mathematics (Reynolds, 2016). Ball et al. (2008) describe MKfT as:

> the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this 'work of teaching' includes explaining terms and concepts to learners, interpreting learners' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing learners with examples of mathematical concepts, algorithms, or proofs (Hill, Rowen & Ball, 2005, p.395).

Like Shulman's PCK, the MKfT framework recognizes the importance of both teachers' Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) in learner performance. However, SMK and PCK are broad, overarching categories of the framework. Each category comprises three domains, as shown in Figure 2.2.

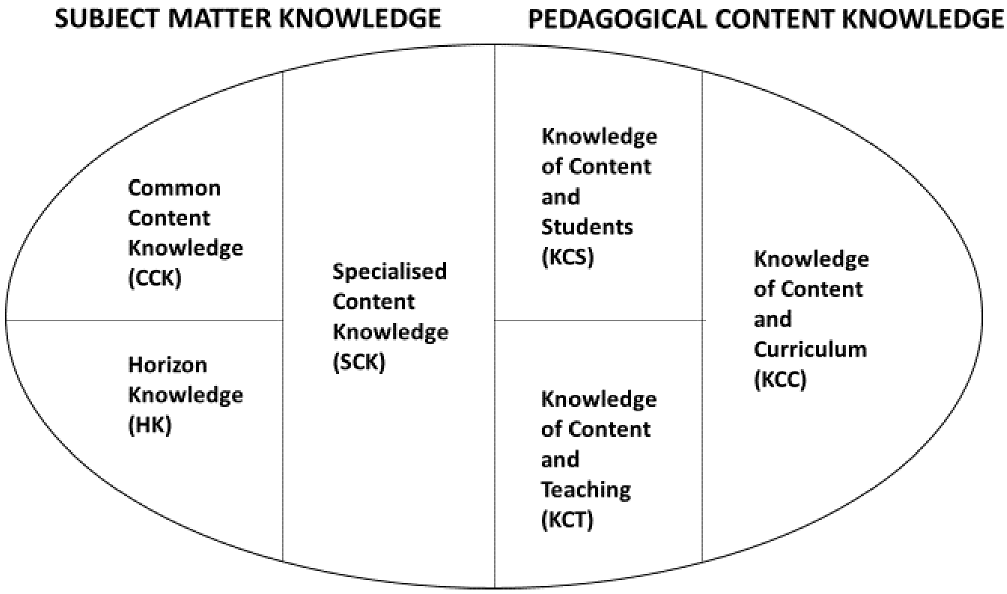

Figure 2.2 MKfT Framework (Ball, Thames & Phelps, 2008)

SMK includes Common Content Knowledge (CCK), Specialised Content Knowledge (SCK) and Horizon Knowledge (HK). CCK refers to the mathematical knowledge that most adults, who have completed their schooling, should have, for example, calculating the cost of groceries or being able to budget. Teachers need to know when learners give incorrect answers, the necessary terminology and notation, and when learning and teaching support materials (e.g., textbooks) contain mathematical errors. When teachers make calculation errors, cannot solve a problem or mispronounce some of the terminologies, learners' ability to make sense of mathematics is seriously curtailed (Ball, et.al., 2008).

SCK is the mathematical knowledge explicitly required of teachers that goes beyond knowledge of the content that most citizens require. For example, teachers do not only have to be able to calculate and identify if a calculation is incorrect; they also need to be able to assess (on the spot) where the learner made the error and how to support them in correcting their errors. In other words, they need to be familiar with different strategies and forms of representation for problem-solving to assist learners' sense-making. Furthermore, teachers need to know how to use mathematical language appropriately and how to justify different mathematical ideas (Ball et.al., 2008).

HK has been conceived of as a domain that involves how mathematics used generally is related to the broader mathematical landscape. It is not directly deployed in instruction in a particular content area, but mathematical knowledge that spans across grades. It engages those aspects of mathematics that may not be included in the curriculum, but are still useful to the learners' learning. HK also includes the knowledge of the connectedness of mathematical topics across the grades and phases (Ball et al., 2008; Ball & Bass, 2009). Teachers' ability to determine the progression of a concept from a lower grade to a higher grade within the phase is an example of HK.

PCK consists of Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS) and Knowledge of Content and Curriculum (KCC). KCT refers to knowledge that combines knowing about teaching and knowing about mathematics content. This includes: identifying and using suitable examples; sequencing and pacing of the examples and activities; knowing which representations are best suited to solving particular problems; knowing when to use and elaborate on a student's comment; and when to instruct or ask questions (Ball, et al., 2008).

KCS focuses on the interrelationship between mathematics knowledge and learners' thinking. In other words, it pertains to the knowledge that combines knowledge of mathematics content and knowledge of learners (Chikiwa, Westaway & Graven, 2019). Teachers can predict what learners may find difficult and confusing and what errors they are likely to make when solving a problem (Herbst & Kosko, 2014). Furthermore, it includes predicting what learners will find exciting and motivating. Teachers with this knowledge are aware of how learners learn, their prior knowledge and experiences, and conceptions and possible misconceptions of specific content (Grossman, cited in Ball et al., 2008). KCT and KCS are complementary domains as they concern knowledge about mathematics, teaching and learners.

KCC incorporates knowledge about how the different curriculum topics connect and how a topic links with the same topic in previous and subsequent years. It includes knowledge of how to plan for learning and how to select material appropriate for learning.

These three domains are separated for methodological reasons only but they are intricately intertwined (Chikiwa, 2017). Table 2.2 provides a summary of the MKfT domains and examples of accompanying indicators.

Ball et al. (2008) Domains	**Definition of MKfT Domains**	**MKfT domains indicators** (adapted from Ball et al., 2008; Herbst & Kosko, 2014; Chikiwa, 2017)
Common Content Knowledge (CCK)	Mathematical knowledge that is also used in contexts or environments other than teaching	The knowledge teachers require to: • Understand the content that they are required to teach. • Recognise when their learners give incorrect responses or when the teacher and learner workbooks have erroneous definitions and calculation errors. • Use terminology and notation correctly.
Horizon Knowledge (HK)	Knowledge of the connectedness of mathematical topics across the grades and phases	Knowledge teachers require to: • Explain how different concepts and topics connect across the grades.
Specialized Content Knowledge (SCK)	Specific knowledge and skills of mathematics for teaching.	Knowledge teachers need to: • Understand different strategies for solving problems that learners may choose to use. • Understand if a strategy learners use is generalizable. • Identify learner errors and the patterns in those errors. • Know the appropriate mathematical language and when to use it. • Understand the benefits of different mathematical representations for different problems. • Share and justify one’s mathematical thinking.

Knowledge of Content and Students (KCS)	A combination of knowledge of mathematics and learners' thinking.	The competence to: • Identify learners' prior knowledge and build on that knowledge. • Foretell what learners might find complicated or what types of errors learners might make when carrying out a given task. • Listen to and clarify learners' thinking and use of language and notation. • Acquaintance with learners' common errors and identifying the ones that learners are most likely to make. • Anticipate learners' conceptions and misconceptions about different mathematical topics. • Select examples that will be of interest and motivating for the learners.
Knowledge of Content and Teaching (KCT)	Integrate knowledge of teaching and knowledge of mathematics and including intensified knowledge needed to determine the best exemplars and depictions to use for given instructional objectives.	The knowledge required to: • Unpack curriculum content and sequence it by building on learners' prior knowledge and experiences. • Identify and use different approaches to teaching • Evaluate the appropriateness of various examples and representations for teaching certain content or ideas. • Select and order examples to ensure coherence in the lesson and to deepen learners' understanding of the concept. • Decide when to instruct, to ask questions, and pay attention to a learner's comment and when to ignore it
Knowledge of Content and Curriculum (KCC)	Incorporates knowledge about how the curriculum topics connect and how a topic links with the same topic	The knowledge to: • Identify the progression of topics within grades and phases. • Sequence skills and concepts of a topic to formulate a unit. • Assess the quality of learning and teaching support materials for teaching a specific concept or topic.

	in the previous and subsequent years.	• Make decisions on which resources will assist learners in developing an understanding of a topic.

Table 2.2 Summary of MKfT Domains adapted from Ball et al. (2008), Herbst & Kosko (2014) and Chikiwa (2017).

Hill and Charalambous (2012) claim that MKfT is vital to the instructional quality and suggest that it is an essential factor for developing teachers' effectiveness in the classroom. They suggest that teachers be informed of this particular framework to use it to reflect on their knowledge and identify the areas where they require development. This should enable them to strengthen any areas in which they may feel they are ill-prepared.

2.9.1 Critique of Ball's MKfT

As seen in Figure 2.2, the famework of Ball et al. (2008) is specific to mathematics and offers a multi-domain perspective. However, there are some concerns with the framework that are based on this categorization. While Ball et al's. (2008) framework assists in operationalizing the mathematics and pedagogical knowledge required to teach primary mathematics, the boundaries between the domains are not clear. In defense of this statement, Coskun and Bostan (2018) claim that being knowledgeable about learners' difficulties and misconceptions could be regarded as both KCS and KCT. Furthermore, they maintain that it may be challenging to decide whether SCK is unique to teaching or common for everyone, for example, being able to tell where an error is made in a calculation. This is an argument also made by Jaffer (2020) in her critique of the MKfT framework. She argues that much of what Ball and colleagues regard as SMK is likely to be CCK for mathematicians and not just pedagogues.

Since Ball and her colleagues examine SMK and PCK separately, it is difficult to discriminate whether the given aspect of the work of teaching belongs to SMK or PCK (Ball et al., 2008). Watson (2008) states that discriminating teachers' knowledge into the categories of SMK or PCK is not helpful. Rowland and Zaskis (2013) agree that these distinctions are artificial and "can veil the essential mathematical activity in which different kinds of knowledge relate and inform each other" (p. 249).

Rowland, Huckstep, and Thwaites (2003) maintain that the different components of teacher knowledge, proposed by Shulman and Ball, interact with each other throughout the teaching and learning process. To emphasise this point, Rowland and his colleagues (2009) use the term "Mathematical Knowledge *in* Teaching" instead of "Mathematical Knowledge *for* Teaching" (Coskun & Bostan, 2018). They aim to understand the knowledge teachers draw on as they teach. They developed the Knowledge Quartet (KQ) framework (Coskun & Bostan, 2018). This framework places less emphasis on the distinction between SMK and PCK. There are four dimensions within the KQ framework. They define the dimensions of the KQ with 'easily remembered labels' that will incorporate important factors in mathematical knowledge in teaching (Rowland & Ruthven, 2011, p. 197).

2.10 KNOWLEDGE QUARTET

The Knowledge Quartet (KQ) is a theoretical framework for examining and expanding the mathematics and pedagogical knowledge required to teach mathematics (Breen, Meheen, O'Shea & Rowland, 2018). It is a 'theory' in the sense that it proposes a way of thinking about mathematics teaching during the day-to-day classroom lessons, with a focus on the disciplinary content (mathematics) of the lesson (Turner & Rowland, 2011). The KQ framework has been used in varied environments, with both pre-service and in-service teachers, and for different reasons. Also, Rowland (2012) used the KQ to scrutinise teachers' mathematical knowledge in both primary and high schools. Like Ball and colleagues (2008), he maintained that teaching mathematics in the primary school was different from that in high school. The KQ provided an opportunity to examine some of those differences.

The KQ was developed for understanding the mathematics and pedagogical knowledge of pre-service teachers, and how their knowledge is enacted in the classroom. It is a structure that was initially utilised for "the discussion of mathematics content knowledge, between teacher educators, trainees and teacher-mentors, in the context of school-based placements" (Rowland et al., 2005, p. 277). However, it has subsequently been used in classrooms with in-service teachers as a means for analyzing their teaching (Rowland, 2005).

The purpose of the research from which the KQ emerged, that is, the Subject Knowledge in Mathematics (SKIMA) inquiry, was to develop an empirically-based conceptual framework for lesson analysis. The focus was to pay particular attention to the mathematics content of the lesson, and the role of the trainees' mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK). The focus of the KQ, as explained in Rowland (2013), is to identify ways that teachers' SMK and PCK can be observed to 'play out' in the practice of teaching. The KQ recognises situations by exploring ways in which teachers' content knowledge and pedagogical content knowledge come into play in the classroom. Abdulhamid and Venkat (2013) assert that the KQ framework is not only aimed at defining what knowledge is needed for mathematics teaching and how such knowledge may be identified, but also provides a way of understanding how such knowledge is developed by teachers.

Ball et al. (2008) refer to MKfT as a "practice-based theory of knowledge for teaching" (Ball & Bass, 2003, p. 5). According to Rowland (2013), the explanation could be adapted to the KQ. While comparisons can be drawn between the methods and some of the outcomes, the two theories are very different. In particular, the theory of Ball and colleagues aims to

> unpick and clarify the formerly somewhat elusive and theoretically-undeveloped notions of SMK and PCK. In the KQ, however, the distinction between different kinds of mathematical knowledge is of lesser significance than the classification of the situations in which mathematical knowledge surfaces in teaching (Rowland, 2013, p.22).

Ruthven (2011) explains that the KQ does not attempt to perfect the model of Shulman. Instead, it serves as a framework to understand "mathematical knowledge-in use" (p. 85) and provides support for the reflection and learning of teachers by observing their lessons in actual teaching. It helps teachers reflect on and develop the mathematical content of their teaching. The KQ creates an opportunity for teacher development by making visible the specifics of a mathematics lesson (Turner & Rowland, 2011). Turner and Rowland (2011) argue that "the reflection in and on practice, with a focus on mathematics content knowledge, will promote change in teachers' content knowledge, as well as their conceptions about mathematics and mathematics teaching" (p 7).

The KQ framework categorises the knowledge and understanding teachers need to teach mathematics into Foundation Knowledge, Transformation Knowledge, Connection Knowledge and Contingency Knowledge (Rowland et al., 2009).

Foundation Knowledge is the first category of the KQ. This knowledge is foundational as it has the likelihood of informing all teaching fundamentally. It consists of the mathematics-related knowledge, pedagogy and beliefs (such as a clear awareness of the purpose of mathematics education) that inform teachers' planning and lesson implementation. This category comprises the knowledge that teachers obtained in their schooling, initial teacher training, in-service teacher education, and during their teaching (Rowland, Huckstep & Thwaites, 2005). The category is considered by Rowland and Turner (2007) to be the starting point for the other three classifications of knowledge in the KQ. The central characteristics of this category are:

- knowledge and understanding of the discipline of mathematics;
- knowledge of the relevance of mathematics and why and how it is learned; and
- knowledge of the research and literature related to the teaching and learning of mathematics (Rowland et al., 2005; Rowland, 2013)

In addition, it includes beliefs about the nature of mathematics, the purposes of mathematics, and how learners learn.

Transformation, Connection and Contingency Knowledge are all related to classroom practice, that is, the teaching and learning of mathematics. These refer to "ways and contexts in which knowledge is brought to bear on the preparation and conduct of teaching" (Turner & Rowland, 2011, p.200). Transformation Knowledge, lies at the heart of the KQ. It focuses on knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself (Rowland et al., 2005). Transformation knowledge is likened to Shulman's PCK in that it refers to the actual teaching of mathematics topics and how to make the content more explicit for their learners (Rowland et al., 2005). Furthermore, Rowland (2013) supported Shulman's utterance that the knowledge base for teaching is determined by "the capacity of a teacher to *transform* the content knowledge s/he possesses into forms that are pedagogically powerful" (pp. 23-24). Rowland et al. (2005) affirm that Foundation Knowledge informs Transformation Knowledge, but that Transformation Knowledge focuses on the learners' learning. This category recognises how the teacher is expected to use what they know when presenting ideas to their learners through examples, procedures, learners' activities, and different forms of representation (Rowland & Turner, 2005; Livy, 2014). Transformation Knowledge is informed by education, research and professional literature, and learning and teaching support materials (Rowland et al., 2005).

The third category is Connection Knowledge. This form of knowledge emerged from the seminal work of Askew, Brown, Rhodes, Wiliam and Johnson (1997) in which they showed the teaching that focused on making 'connections' led to better learner performance in mathematics. It concerns the consistency of the planning or teaching mathematics displayed across an activity, lesson, or series of lessons (Rowland, Huckstep & Thwaites 2003). Connection relates to the teachers' connection of concepts and topics when teaching. This could be the depth and thoroughness of teachers' mathematical knowledge. Teachers are expected to have an awareness of the relative cognitive demands of different mathematics topics and tasks, knowledge of the most practical procedures to sequence instructional topics to make the mathematical concepts more understandable and meaningful for the learners (O'Meara, 2010). The key features of this category include:

- decisions about sequencing;
- making connections between procedures and concepts;
- the anticipation of complexity; and
- recognition of conceptual appropriateness (Abdulhamid & Venkat, 2013).

The final category is Contingency Knowledge. This type of knowledge is the knowledge teachers use to make accurate judgements to deal with unforeseen incidents in their classrooms (O'Meara, 2010). The teacher, in this regard, must be prepared for 'teachable moments' as well as the unexpected in the classroom. They will be required to draw on their Foundation Knowledge in responding to learners' questions and responses (Livy, 2014). Contingency is evident in classroom proceedings that are not anticipated in the teachers' preparations and, thus, requires teachers to be able to 'think on their feet' (Rowland & Turner, 2005; Rowland et al., 2008). Fig. 2.3 provides a summary of the KQ categories.

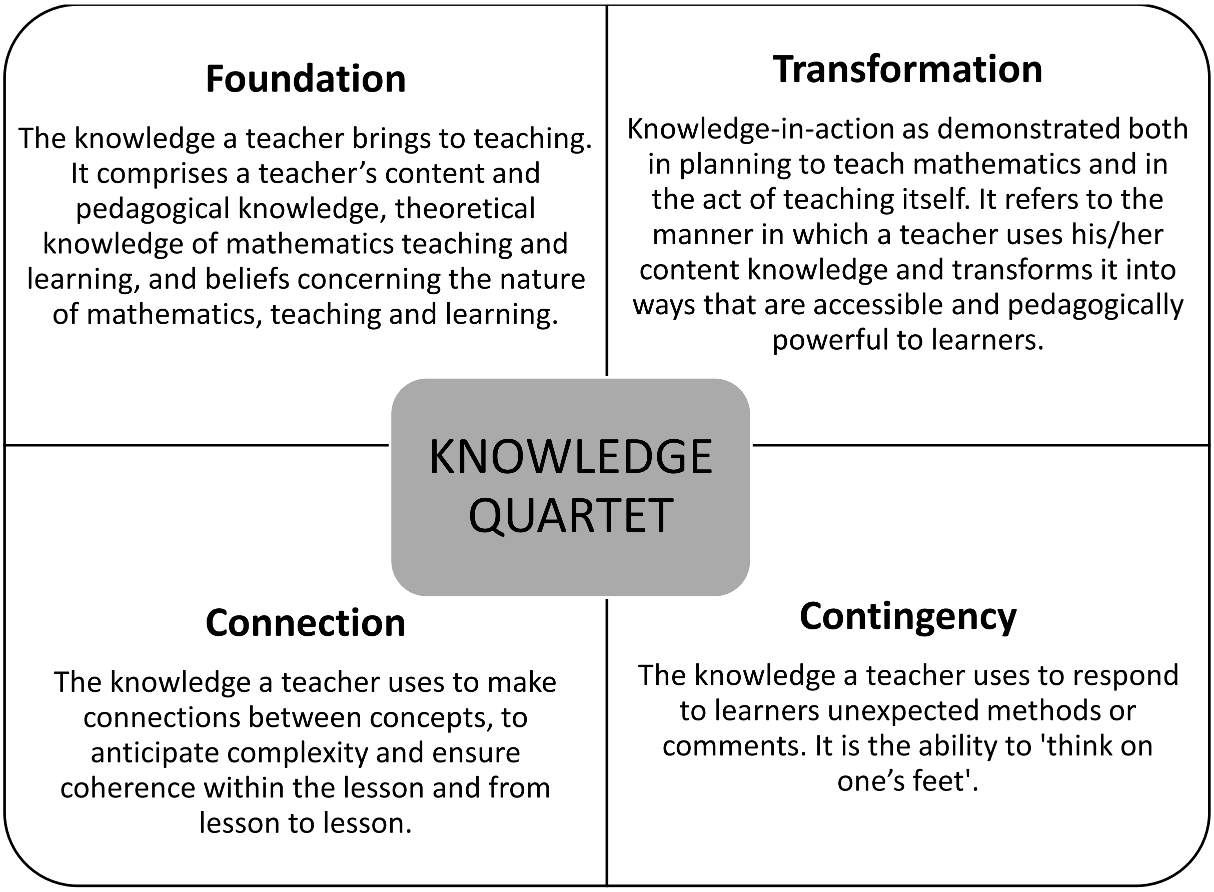

Fig. 2.3 The codes of the 'Knowledge Quartet' and description for each category (Rowland et al., 2009).

2.11 COMBINING THE MKFT AND KQ FRAMEWORKS

The framework of Mathematics Knowledge for Teaching (Ball et al., 2008) and the Knowledge Quartet (Rowland, 2005, 2013) both have relevance for this study. The MKFT framework provides the researcher with the explanatory tools necessary to capture the knowledge that the teachers draw on as they plan and reflect on their lessons. The KQ is not limited to the knowledge required *for* teaching, but also provides the tools to analyse the knowledge that teachers draw on *in* teaching, that is during the actual process of teaching mathematics in the classroom. During the lesson implementation process, the teachers have to transform their knowledge into the classroom, make connections and be able think on their feet. In other words, the KQ recognises situations in which teachers' mathematics content knowledge and pedagogical content knowledge come into play in the classroom.

2.12 CONCLUSION

White and Lim (2008) state that LS assists teachers in designing quality lessons and better their understanding learners' learning. LS is commonly used to support teachers' professional development (Stigler & Hiebert, 1999; Takahashi, 2017). In Chapter 1, I argued that learners' poor performance in mathematics education (TIMSS, SACMEQ, NSC, ANA, etc.) should prompt the South African Department of Basic Education, Education Faculties at universities, District Officials and teachers to consider LS as a professional development approach. Hence, the aim of this research is to ascertain how the provision of opportunities for teachers to be involved in a LS process will strengthen their mathematics and pedagogical content knowledge. To do so, a framework was required to enable me to capture this knowledge in the process of planning, teaching, observing and reflecting. The MKfT and KQ frameworks provide the analytic and explanatory tools to do this. The MKfT framework allows for the methodological distinction between SMK and PCK. The MKfT framework, however, focuses specifically on knowledge *for* teaching. The Knowledge Quartet examines teachers' knowledge *in* teaching. The KQ is a framework for the observation, analysis and development of mathematics teaching, with a focus on the importance of the Foundation Knowledge that teachers bring to the learning and teaching situation (Thwaites, Rowland & Huckstep, 2005; Rowland & Ruthven, 2011, Corcoran & Pepperell, 2011; Rowland, Turner, Thwaites & Huckstep, 2010)

CHAPTER 3 RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

Chapter 2 examined the two key concepts in my research, namely, LS and mathematics and pedagogical content knowledge. The research aimed to ascertain how collaborative engagement in a LS process can strengthen (or not) Grade 1 teachers' mathematics content and pedagogical knowledge. To achieve this aim, the following research questions were formulated:

How does Lesson Study contribute to the development of teachers' mathematics content and pedagogical content knowledge?

- What mathematics content knowledge do teachers develop as they engage in LS?
- What pedagogical content knowledge do teachers develop as they engage in LS?

Four Grade 1 teachers from two Rustenburg Sub-District schools participated in this qualitative case study. The focus was to understand the knowledge developed during the lesson study process. An interpretive orientation underpinned the research as I was interested in the teachers' subjective experiences and perspectives. My ontological and epistemological beliefs, which resonate with interpretivism, guided my selection of a case study and appropriate data collection tools. The tools that I used in my research included observations, interviews and document analysis. While Lesson Study guided the research process, data was analysed using the Mathematics Knowledge for Teaching (Ball et al., 2008) and the Knowledge Quartet (Rowland, 2005, 2013) frameworks. As essential elements to the quality of my research process, I describe at the end of the chapter how I approached issues of validity, positionality and ethics.

3.2 RESEARCH ORIENTATION

My research employed a qualitative research approach using a case study method within an interpretive paradigm (Maree, 2010). Qualitative research, as described by Van Maanen (1979), is "an overall term covering collections of interpretive approaches which seek to outline, interpret, translate, and otherwise come to terms with the meaning, not the frequency, of certain more or less naturally occurring phenomena in the social world" (p.520). I used a qualitative research approach to enable me to understand and explore the richness, depth, context and complexity within which the participants operate (Mason, 2006).

An interpretivist orientation underpins my research. Interpretivism is a philosophy that is often drawn on in qualitative research (Cohen, Manion & Morrison, 2010). According to Alvermann and Mallozzi (2010), interpretivism denotes, "an approach to studying social life with the assumption that the meaning of human action is inherent in that action" (p. 12). Put differently, interpretivism enables one to develop a deeper understanding of human action. Interpretivism enables the generation of rich, detailed descriptions of the phenomenon studied. The view is to support a more comprehensive and detailed explanation of the phenomenon (Cohen, Manion & Morrison, 2010).

Interpretivism is also regarded in the research literature as related to constructivism (Bertram & Christiansen, 2014). Interpretivists believe that 'realism' is established by social practices and people's subjective understanding thereof. They observe that individuals with their own varied culture, beliefs and practices add to the on-going construction of reality in their social contexts through social interaction (Hennink, Hutter & Bailey, 2011). Interpretivists posit that there is no one true reality, but rather multiple interpretations of events in 'the world'. As such, participants and researchers interpret 'the world' based on their own subjective experiences. Meaning is made through interaction with the participants in the study (Bertram & Christiansen 2014). Interpretivist researchers prefer to observe and engage in conversations with the research participants to develop an understanding of their conceptions, meanings and understanding of the 'social world'. As such, interpretivists tend to work with qualitative data that provides rich explanations of social constructs and influences both the methods and data collection tools used in the research. To develop more detailed descriptions during my research, I chose case study as a research method.

3.3 RESEARCH METHOD

This research is a qualitative case study. Yin (2014) describes case study research as "an empirical inquiry that investigates a contemporary phenomenon ('the case') in-depth and within its real-world context when the boundaries between phenomenon and context may not be clear" (p.16). Harling (2002) adds that a case study is an investigation of a phenomenon within a natural setting. The case is dependent on the unit of analysis (Yin, 2014), which in this research was the mathematics and pedagogical content knowledge developed during the LS process. The development of teachers' mathematics and pedagogical content knowledge is bounded by space and time (Creswell, 2002). In other words, the period for the professional development sessions, where the teachers engaged in LS to strengthen their mathematics and pedagogical content knowledge, took five months (May 2019 – September 2019). The naturalistic setting (Creswell, 2002) is the Grade 1 teachers' classrooms.

Case studies allow for the generation of rich, detailed research data from a variety of sources. It encourages an array of data generation tools, which, in turn, can promote the validation of data (Denscombe, 2010). The various sources enable the generation of a broader collection of information needed to provide a detailed picture and set the scene for a more comprehensive analysis (Harling, 2002). In this study, the data generation sources included direct observation, participant observations, interviews and documents.

My study was an instrumental case study, which was administered to develop a broad understanding of the phenomenon, the development of teachers' mathematics and pedagogical content knowledge for teaching through the use of LS. An instrumental case study uses a case to attain insights into a phenomenon. Also, it is used to provide insights into a particular issue (case) and/or refine existing theory (Stake, 1995). In this research, it will be the use of LS to strengthen the mathematics and pedagogical content knowledge of Grade 1 teachers. Figure 3.1 provides and overview of the LS process in my research.

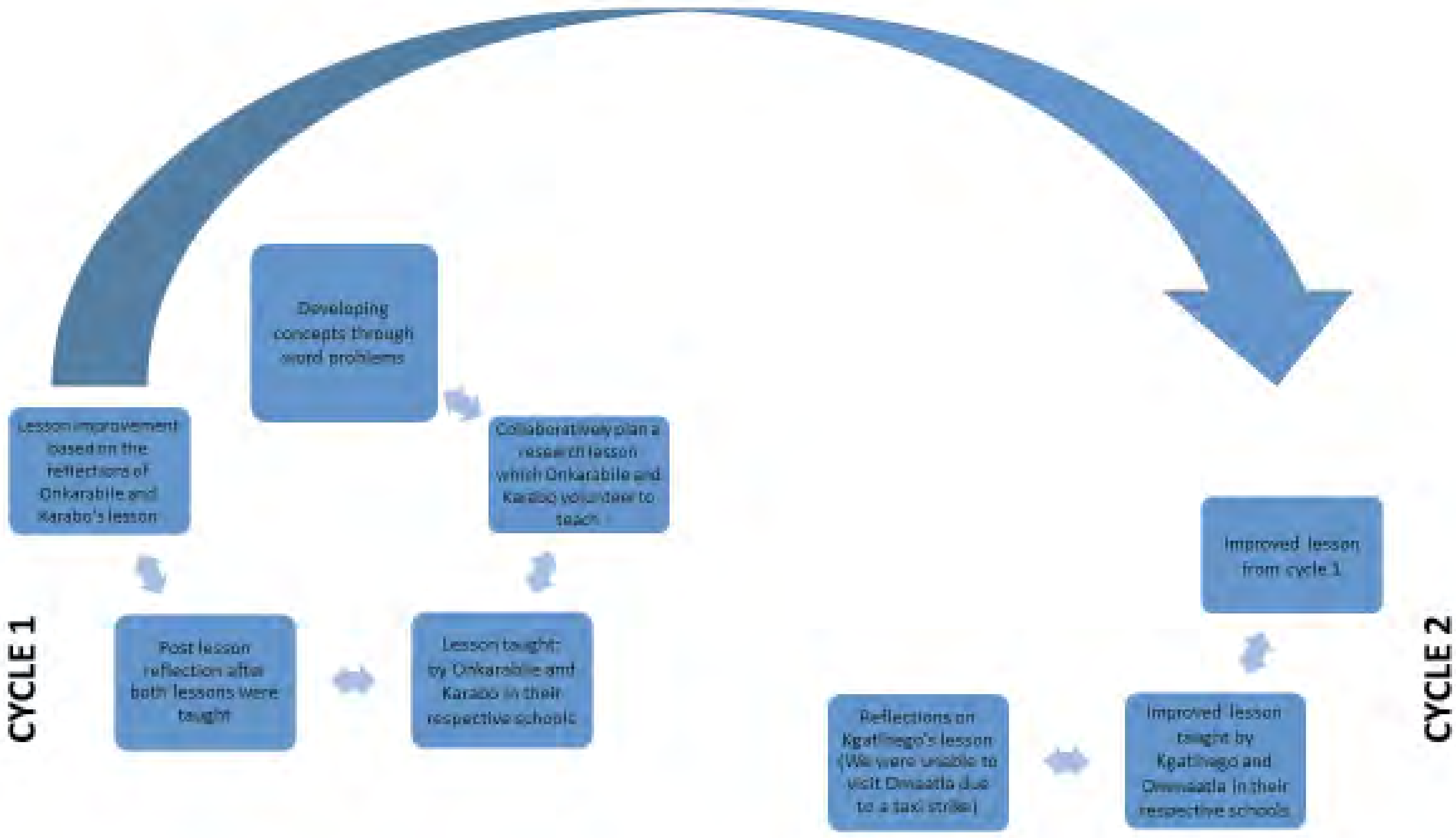

Figure 3.1 A summary of the two Lesson Study cycles

3.4 RESEARCH SITE

This study took place in the North West Department of Education, Rustenburg sub-district, which is located in the Bojanala district. The study focused on Grade 1 teachers at two primary schools (Oaratwa Primary School and Lethabo Primary School - pseudonyms). The Rustenburg sub-district is surrounded by platinum mines where there are a variety of types of schools (e.g., rural schools, farm schools, township schools and former Model C schools). The most common languages used for teaching and learning in the sub-district schools are English, Afrikaans, Setswana and isiXhosa. Oaratwa Primary School uses English as the language of teaching and learning and Lethabo Primary uses Setswana. Oaratwa Primary is a new school with novice teachers, while Lethabo Primary is well-established with very experienced teachers. Both schools are co-educational primary schools from Grade R to Grade 7. Oaratwa Primary is situated in a new suburban area around the mines and Lethabo Primary is situated in the oldest township near Rustenburg town.

The teachers at Oaratwa and Lethabo Primary schools, together with the researcher, met once a week at Lethabo Primary School to plan lessons, map out the schedules for lesson observations, teach and reflect on the taught lessons.

3.5 SELECTION OF PARTICIPANTS

The study used purposive and convenience sampling. Bertram and Christiansen (2014) describe purposive sampling as the selection of specific people, groups and/or objects for the research. Convenience sampling means choosing a sample that is easily accessible to the researcher. This study was conducted with the Grade 1 teachers at two schools. The Grade 1 teachers of the two schools were purposefully chosen as Grade 1 provides the foundations for future mathematics success (Reddy, 2006). The participants in this study are four female Grade 1 teachers (Karabo, Onkarabile, Kgatlhego and Omaatla (pseudonyms)) and the researcher (Naomi). Two teachers were from Oaratwa Primary School and the other two teachers were from Lethabo Primary School. As a Subject Advisor, I work directly with all the teachers at these schools. The schools are in the sub-district where I work and thus, it was convenient for the participants to attend the LS meetings during the research process. The profiles of the participant teachers are provided in Table 3.1.

Participants	**Karabo**	**Onkarabile**	**Kgatlhego**	**Omaatla**
Gender	Female	Female	Female	Female
Age	38 years	48 years	28 years	50 years
Highest Qualification	B. Ed (FP)[1]	UDEP[2] ACE[3]	B. Ed (FP)	UDEP ACE
Home Language	Sepedi	Setswana	Setswana	Setswana
Years of experience as a Foundation Phase teacher	3 years	25 years	3 years	12 years
Years of experience as a Grade 1	3 years	15 years	3 years	5 years

Table 3.1 Profiles of participants

The teachers varied from 28 to 50 years of age and their experience in teaching in the Foundation Phase varied from three years to twenty-five years. Three of the four teachers (Karabo, Kgatlhego and Omaatla) had less than ten years' experience teaching Grade 1. Onkarabile was the only teacher with more than ten years' experience teaching in Grade 1. The two youngest teachers (Karabo and Kgatlhego), who only had three years teaching experience respectively were teaching at Oaratwa Primary School. In contrast, the older teacher (Onkarabile and Omaatla) taught at Lethabo Primary School. All four teachers had the equivalent of a four-year degree.

3.6 DATA COLLECTION METHODS

Data collection "is the systematic approach to gathering information from a variety of sources to develop a complete and accurate picture of an area of interest" (Rouse, 2016, unpaged). The following data generation methods were utilized in my research: interviews; observations

[1] Bachelor of Education (Foundation Phase)
[2] University Diploma in Education (Primary)
[3] Advanced Certificate in Education

of the lesson planning; lesson presentation and lesson reflection sessions; and document analysis. Table 3.2 highlights the data collected and data collection process in each cycle of the research.

CYCLE 1	
Planning meeting (including error analysis and setting the goal for the LS)	
Individual interviews	Transcriptions of interviews
Lesson planning session and development of observation schedules	Video-recording for purposes of 'observation' after the planning session
Lesson implementation and observation	Observation schedules
Reflection session	Video-recording for purposes of 'observation' after the reflection session
Re-plan for 2nd cycle	

CYCLE 2	
Lesson implementation and observation	Observation schedules
Reflection session	Video-recording for purposes of 'observation' after the planning session at Oaratwa Primary School (There was a taxi strike and so the second lesson of cycle 2 could not be video-recorded).
Focus group interview	Transcriptions of interviews

Table 3.2 A summary of the data collected and the data collection process during each of the LS cycles

3.6.1 Observation

Observation is described as the process of collecting live data from the context in which events occur (Cohen et al., 2010). It involves recording systematically that which occurs in a natural setting (Gorman & Clayton, 2005). It is a strategy that involves directly observing behaviour with the purpose of understanding and describing that behaviour.

McLoed (2015) maintains that observation is a data collection method that can be structured or unstructured. Structured observation is usually carried out using a structured observation schedule. That is, a schedule where the observation criteria are decided in advance. Typically, when researchers use a structured observation schedule, they do not deviate from the criteria for observation on the observation schedule. By contrast, an unstructured observation is carried out in an open and unrestricted manner where no explicit criteria are set in advance (McLeod, 2015). Cohen et al. (2010) include semi-structured observations as a data collection method. Here the researcher has guidelines in what to observe but allows for flexibility during the observation process.

This study conducted semi-structured observations. The researcher and participants met to develop the observation schedules prior to the lessons. These schedules had specific criteria, but also space to allow the teachers to note anything else that may have been worth noting. The observation schedules were used during the teaching of the lessons. An example of an observation schedule can be found in Appendix 2.

The researcher was a participant observer in this study. Becker and Geer (1970) define participant observation as either a covert or overt activity "in which the observer participates in the daily life of the people under study . . . observing things that happen, listening to what is said, and questioning people, over some length of time" (p.133). As the researcher, I employed overt observations as the participants were aware of my interests and research, they had signed consent letters in which the research agenda was explained, and they participated in all aspects of the research. I was part of the group that I studied which enabled me to gain deeper insight into their knowledge of teaching and learning of mathematics and the development thereof through participation in the LS process.

While I was a participant observer who made the observation process explicit to the research participants, Adler and Adler (1994) suggest that it is incumbent on the researcher to remain "strongly research oriented" at all times and "not cross into the friendship domain" (p. 380).

This is not an easy suggestion as these teachers are in my sub-district and we work closely together. While we are always professional with each other, there is a bond that has developed between us.

Observation is a complicated research method because it required me to play a number of roles and to use a number of methods to collect data. I had two roles: (1) to participate in the LS process; and (2) to make use of video-recorded data to observe the LS process so that I could extricate myself and examine the data from a researcher stance. I participated in the lesson study process as I engaged with the teacher participants in the lesson planning, observation and reflection stages of the LS. This made it practically impossible to observe properly the LS process while events were occurring. Despite my level of involvement with the research process, I had to ensure that I also remained appropriately detached to analyze the collected data (Baker, 2006). I thus decided to video-record the lesson planning sessions and also the reflection sessions. I did not record the process of lesson planning in cycle 2 because we watched the two videos of the presented lessons in cycle 1 and reviewed some of the activities for the lessons in the 2nd cycle at the same time. I was also not able to video-record the reflections on Omaatla as there was a taxi strike at that time. This meant that Onkarabile and Omaatla could not travel to Oaratwa Primary School were our lesson planning and reflection stages took place.

The video-recordings enabled me to observe the interactions and decisions taken during the planning and reflection sessions at a later stage. As I reviewed the video-recorded data, I identified the sections to transcribe. I began to analyse the mathematics and pedagogical content knowledge drawn on and developed in each of the stages of the LS process.

3.6.2 Document Analysis

In addition to the lesson observations, I analysed the lesson plans that the teachers and I developed during the planning session of the LS process and the observation schedules that were completed during the lesson observations. The lesson plans developed at the start of the process and the reworked versions, were useful for tracking any changes that occurred in the teachers' mathematics and pedagogical content knowledge as they engaged in the LS process.

The researcher and teachers developed the observation schedules collaboratively during the planning sessions. The purpose of the observation schedules was to focus our observations while watching each lesson enacted in the classroom. These observation schedules were also used in the reflection sessions. I collected the completed lesson observation schedules at the end of the reflection session in order to analyse the mathematics and pedagogical content knowledge domains evident in the teachers' reflections of their observations of the taught lessons.

3.6.3 Interviews

Mathers, Fox and Hunn (1998) describe interviews as "an important data gathering technique involving verbal communication between the researcher and the subject" (p. 1). Interviewing, like observations and document analysis, is one of the dominant procedures qualitative researchers develop and utilize to collect data for their research studies (Gubrium & Holstein, 2003; Kvale & Brinkmann, 2008; Rubin & Rubin, 2006). Many of these researchers prefer conducting naturalistic research that focuses on discovery-oriented inquiries rather than utilizing pre-established questionnaires (Gubrium & Holstein, 2003).

Bertram & Christiansen (2014) describe an interview as "a conversation between the researcher and the respondent … it is a structured conversation where the researcher has in mind particular information that he or she wants from the respondent and has designed a particular set of questions to be answered" (p.80). In this research, the researcher made use of interviews to collect information about the participants' experiences, perceptions, opinions and interpretations.

The individual interviews in my research were semi-structured. In other words, I did not follow a strict formalized list of questions so that I could ask follow-up questions for further explanation. Cohen, Manion and Morrison (2011) suggest that open questions enable participants to use their own understanding when answering questions and the researcher may ask follow-up questions as the participant talks. Open-ended questions facilitated the free expression of participants' views on the LS process and their learning during the process. The first semi-structured interview took place prior to the start of the LS process. In this interview, general information was gleaned from the teachers (e.g., last professional qualification, number of years of teaching in the Foundation Phase, knowledge of error analysis, knowledge of LS) (Appendix 3).

Once the teachers completed the feedback and reflection sessions and redesigned the lessons, I conducted a focus group interview with all four participants involved in the research (Appendix 4). According to Maree (2010) focus group interviews involve discussion among a group of people with the purpose of collecting in-depth qualitative data. All interviews were video-recorded. These recordings were transcribed and member-checked for accuracy.

3.7 DATA ANALYSIS

Qualitative data analysis refers to the processes and procedures used to form some interpretation, understanding and explanation of the phenomenon being researched from the data that has been generated (Bhattacharjee, 2014). Merriam (2009) describes data analysis as "the process of making sense out of the data, which involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read – it is the process of making meaning" (p. 178). The data analysis process includes data collection all the way through interpretation and presentation in the thesis.

The data collection and analysis processes were iterative in my research. Creswell (2012) and Miles and Huberman (1994) state that data collection, recording and analysis ought to be done concurrently as interrelated simultaneous procedures rather than individual processes done in a linear form. In this process, I made use of the following steps developed by Creswell (2012). The process included data collection, preparing the data for analysis, engaging in an initial exploration of the data, coding the data, representing the findings, and validating the accuracy of the findings. While I present the steps in a linear manner here, the process itself was not linear.

1. *Collecting the data*. The data collected in this study included: (1) observations of the planning and reflections stages of the LS; (2) lesson plans; (3) lesson observation schedules from the participating teachers; (4) individual interviews with the teachers; and a (5) focus-group interview.

2. *Preparing and organizing the data for analysis*. All the video-recorded data from the planning and reflection sessions were viewed a number of times and relevant sections transcribed. It was not possible for me to transcribe all the video data as that would have generated too much data. It was through the iterative process of data collection and analysis that I was able to identify the segments in the video data that should be transcribed. All the interviews were transcribed in full.

3. *Engaging in an initial exploration of the data.* I read through the data to get a general sense of the data collected. I (re)read the interview transcripts, lesson plans and video transcripts. I developed preliminary notes in order to organize data.

4. *Coding the data*. This process required coding and re-coding the data. Initial coding was done inductively to 'see' what emerged from the data. These codes were then categorized into themes. After that, I recoded the data using the six MKfT domains. My reason for coding the data without applying the MKfT pre-determined codes first, was to ascertain whether there were possible categories that may not be included in the MKfT framework. One of the purposes of research is theory-building, so it was important for me to code the data without applying the pre-determined MKfT codes. After coding the data using the MKfT framework, I then applied the Knowledge Quartet codes. My reason for doing this is that I found it difficult to code using the MKfT framework as the domains overlap. For example, when a teacher was using a particular strategy to develop the learners' understanding of the concept, there were multiple domains that s/he could draw on (e.g., KCS, KCT, SCK). I thus had to make decisions and generate my own indicators for each of the domains (Chapter 2). The KQ also provided me with a framework to examine the teachers' knowledge during the teaching process.

5. *Represent the findings* and reflect on the impact of the findings and examine the literature that might inform the findings. This took place once I had coded and categorized the data in

accordance with the MKfT and KQ frameworks. In representing the data and findings, I used the words of the teachers exactly as they appeared in the transcripts. In other words, I did not change or correct the grammar and structure of the participants' comments.

6. *Validate the accuracy of the finding* to guarantee that there is a good correlation between the researcher's discovery and the views and understanding of the research participants. To do this, I used inter-rater reliability by asking my supervisor and a critical reader to code sections of my data. I also involved the participants in checking both the transcribed data and the findings that emerged from the data. The latter took place during one of the professional development sessions after the LS process.

3.8 POSITIONALITY

As a subject advisor, I am well aware of the power relations between the participants and me. Over the past 10 years, I have established good working relations with the teachers in the district. We have worked together for many years. As noted in Chapter 1, many of these teachers have participated in various teacher development programmes with me. During a 'pilot LS process' with the Grade 3 teachers, teachers in the other grades expressed interest in participating in LS. That being said, I realised my positionality as their 'senior' may still influence their decision of whether to participate or not in this research. I thus made it clear (repeatedly) that they could withdraw from the research process at any time and that their withdrawal would have no negative consequences. I was explicit about the aims and focus of the study. I also explained that while I am familiar with the LS approach, they have recent classroom experience, which is central to the research. Throughout the research process, I reflected on my role as a subject advisor and researcher.

3.9 VALIDITY AND TRUSTWORTHINESS

Validity is conceptualized as trustworthiness, rigor and quality in qualitative research. It is also through these connections that the researcher seeks to eliminate bias and increase the trustworthiness of the data and propositions about some social phenomenon (Denzin, 1978).

As mentioned above, I collected data from different sources. These included observations (video transcriptions), interviews (interview transcriptions), and data analysis (lesson plans and observation schedules). In this way, the findings were not gleaned from single events. Instead, they are triangulated by the multiple data sources. Triangulation is defined to be "a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study" (Creswell & Mille, 2000, p.126).

I further sought to ensure trustworthiness by applying the strategy referred to as member checking. A number of researchers (Barbour, 2001; Doyle, 2007; Harper & Cole, 2012; Lincoln & Guba, 1985) defined member checking as a quality control process whereby the researcher intends to improve the accuracy, credibility and validity of what was captured during research interviews and reflections. "Member checking continues to be an important quality control process in qualitative research as during the course of conducting a study, participants receive the opportunity to review their statements for accuracy and, in so doing, they may acquire a remedial benefit" (Hamper & Cole, 2012. P 1). This is regarded as a "very essential approach for establishing credibility" (Lincoln & Guba, 1985, p.314) and it allowed me to verify my transcriptions of the video-recorded planning and reflection sessions and the interviews. Realising that we (the participants and myself) bring out own experiences, understandings and perceptions to the research site, I needed to attempt to develop shared meanings through our interactions and create an opportunity for the participants to 'member-check' my transcriptions. Lincoln and Guba (1985) believe in another kind of member checking that occurs near the end of the research project. They propose that once the data is analyzed it should be reported to the participants to review the authenticity of the work. Member checking is a process whereby "the final report or specific description or themes" are taken back to the participants (Creswell, 2009, p. 191). I shared the findings with the participants during one of the professional development sessions after the LS process was completed and before concluding the thesis writing process. The goal was to obtain verification that my findings (as researcher) were congruous with the perspectives of participants and to seek out areas in which there is a lack of coherence and the reasons for it.

In addition, my supervisor and a critical friend also coded sections of my data to ensure that the codes I identified were reliable. This form of inter-rater reliability is used to ensure the reliability of the identified codes.

3.10 ETHICAL CONSIDERATIONS

Ethical concerns are essential in research in which human and non-human primates are involved in order to minimize damage or wrongdoing to the participants (Abdulhamid, 2016). The research adheres to the standards of ethical conduct required by the Rhodes University Ethics Standards Committee (Appendix 5), as well as the schools' policies, and takes all necessary steps to eliminate the risk by using pseudonyms.

Informed consent, respect for the rights of research participants, confidentiality and anonymity are some of the ethical considerations that were attended to prior to conducting the investigation. I wrote letters to the North West Department of Education and Sports Development (Appendix 6) and the school principals of the two schools (Appendix 7) to request permission to conduct my research. The research purpose and objectives were discussed with the participants before they were given the written consent forms. I sought the teachers' written consent to participate in the research after a discussion about my research focus and my data collection plan. I requested them to be part of my research by allowing me to: (1) observe their lessons; (2) interview them individually and/or in a group; (3) conduct planning and feedback sessions with them; and (4) have access to documents, such as, lesson plans and observation schedules (Appendix 8). I also sought consent to record the class presentations and interviews and ensured them that my presence in their classrooms would not interfere with their daily teaching routines. I explained to participants that they have the right to refuse participation and to withdraw their data and participation at any time during the research process. Participants were guaranteed of confidentiality and pseudonymous were given to protect their identities. They were informed timeously that there would be no remuneration to participate in this research.

Given that I would be observing the teachers' teaching in their classrooms, I also requested consent form the parents/caregivers in the classes of the four teachers. The participating teachers sent the consent letters home with the learners for their parents/caregivers to sign (Appendix 9).

Interview transcripts and all observation notes are kept in a locked, safe place to ensure that no one other than the researcher has access to the information. During the first meeting with the teachers, we developed a timetable for the research so as not to interfere with schools' daily programmes. This is reflected in Table 3.3.

Activity	**Responsibility**	**Venue**	**Date**	**Time**
Meeting and individual interviews	Researcher and the four participants	Lethabo Primary school	7 August 2019	14h00 – 16h00
Planning research lesson	All participants	Lethabo Primary school	13 August 2019	14h00 – 16h30
Presentation, observation, feedback and reflection of the 1st cycle lesson	1st lesson presented by Onkarabile	Lethabo Primary	27 August 2019	12h30 - 13h30
Presentation, observation, feedback and reflection of the 1st cycle lesson	2nd lesson of cycle 1 presented by Karabo	Oaratwa primary	28 August 2019	12h30 – 13h30
Review 1st lesson plan and improvement of the 2nd cycle lesson	All participants	Oaratwa primary	04 September 2019	14h00 – 17h00
Presentation, observation, feedback and reflection of the 2nd cycle lesson	1st improved lesson presented by Kgatlhego	Oaratwa primary	11 September 2019	13h00 – 14h00

Presentation, observation, feedback and reflection of the 2nd cycle lesson	2nd improved lesson presented by Omaatla	Lethabo Primary	12.09.2019	08h00 – 9h00

Table 3.3 The collaborative schedule plan for the LS process.

3.11 CONCLUSION

In this chapter, I have provided insight into the research process by focusing on the orientation, research methods, the research site, the selection of participants, the data generation methods, and analytic process. Throughout the process, I tried to maintain the trustworthiness of the study. The ethical considerations for qualitative research and my position as researcher has also been considered. Table 3.4 provides a summary of the research process.

Research Design	• Qualitative Research • Underpinned by an interpretivist orientation • Case Study method
Data Collection Tools	• Individual semi-structured interviews with each of the four participants • Observations of planning, implementation and reflection stages of the LS • Document analysis of lesson plans and observation schedules • Focus group interview with the four participants
Data Analysis	• Transcription of interviews (individual and focus group) and sections of video-recorded data of planning and reflections stages of the LS • Data management and reduction • Inductive coding • Coding using MKfT and KQ frameworks
Validity	• Triangulation • Member checking • Inter-rater reliability

Ethics	• Informed consent from the NW DoE, principals, teachers and parents of the learners • Confidentiality and anonymity • The autonomy of the participants

Table 3.4 Research design and methodology overview

CHAPTER 4: DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

This chapter addresses the research question: *How does Lesson Study contribute to the development of teachers' mathematics content and pedagogical content knowledge?* In so doing, I first focus on the two sub-questions:

- What mathematics content knowledge do teachers develop as they engage in LS?
- What pedagogical content knowledge do teachers develop as they engage in LS?

In order to answer these questions, I present the data from the planning, teaching and observation, and reflection sessions of the LS cycle. Onkarabile (the oldest teacher) and Karabo (one of the younger teachers) agree to teach the research lessons in the first cycle, at their respective schools. All of the teachers and the researcher participated in the planning sessions, observation of the lessons and reflection sessions after the lessons. The lesson reflections occurred after watching the videos of each of the presented lessons. The reflection session led to changes being made to the lessons that were taught in the second cycle by Kgatlhego and Omaatla.

This chapter presents the data obtained through interviews, observations of planning and reflection sessions, lesson observations and document analysis as the teachers and I participated in the LS process. I begin the chapter with the planning, implementation and reflection of the first LS cycle before progressing to the second LS cycle. The data is analysed using the MKfT framework of Ball et al. (2008) and later, the KQ framework of Rowland (2005). This analysis enabled me to answer the two sub-questions. At the end of the chapter, I draw on the focus group interview with the participating teachers to examine how LS contributed to the development of their mathematics and pedagogical knowledge.

4.2 LESSON STUDY CYCLE 1

Prior to the planning of the first lesson, I met with the teachers to identify the concerns they had with the teaching and learning of mathematics in their classrooms. All of the teachers identified problem-solving as a concern. We decided that our goal for the LS would be problem-solving, and in particular, word problems. The lesson below is collaboratively planned. The topic and objectives taught in these lessons are drawn from the Annual Teaching Plan as per the Curriculum and Assessment Policy Statement (CAPS). The teachers also referred to the Department of Education - Japan International Cooperation Agency (DBE-JICA) Mathematics Project Manual (Term 3) for possible teaching activities. The teachers follow the CAPS aligned structure for a lesson, that is, counting and mental maths (10 minutes), concept development (30 minutes), individual or group work activities (10 minutes) and consolidation activities (10 minutes). The topic to be developed is grouping and sharing leading to division. According to CAPS, at the end of the lessons, the learners should be able to "solve word problems in context and explain own solution to problems involving equal sharing with whole numbers up to 15 and with answer that may include remainders" (DBE, 2011, p. 42).

4.2.1 Planning the lesson

Having looked at the lesson objectives in the Annual Teaching Plan, the teachers start their discussion by focusing on counting. This is the first part of the mental mathematics session of the lesson. Kgatlhego reminds her colleagues, "**We said [that we] will count forward in 2s and 5s using 10-frames and let learners count in 2s by moving out counters from the 10-frames to 12, 14 and 16" (LP1[4]).** They continue to discuss the steps to follow to ensure the learners understand how to count in 2s and 5s using the 10-frames. Onkarabile was aware of her children's prior knowledge and explained that she did not want to start counting up to 12 or 14. "**Will it be wrong if I start by asking learners to say how many 2s are there in 4, 6, and 8"?** The team indicates that there should be opportunities to differentiate learning. Karabo asks a clarifying question, **"How are the learners going to count"?** She then emphasizes her question by demonstrating what she thinks should happen when learners count with the 10-frame. She counts in 2s by taking two counters out of the frame and says, the **"learners will put them 2, 2, 2 until they count 12".** This is shown in Figure 4.1 below.

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[4] In this section of my data presentation and analysis (4.2.1), all quotes are from Lesson Planning session 1. For ease of reading I have not referenced 'LP1' after each quote, rather I have decided to write in bold the teachers' voices. Where a statement is not from 'LP1' in section 4.2.1, I will reference it.

Figure 4.1 Karabo showing how the learners will count

Onkarabile adds, "**the best way is to put them in groups of 2s then count the groups**". The teachers continue discussing the mental maths section of the lesson, but the focus shifts to the rapid recall of addition facts. Kgatlhego suggests that they should use flashcards and ask the learners to give answers based on bonds of 5. She writes some sums on the cards to demonstrate what she means (e.g., 3 + 2 = □)**.** Karabo asks if they will only concentrate on addition facts because CAPS states that the learners need to engage with rapid recall of both addition and subtraction facts. Kgatlhego wants to **"do addition only. Sometimes when you give them addition and subtraction, they get confused".**

Moving to the concept development, Kgatlhego mentions that the topic to be developed is grouping and sharing. Karabo adds to Kgatlhego's statement by reading the actual requirement in full from CAPS **"According to the curriculum, the topic should be grouping and sharing leading to division"**. She adds, "**however, for this lesson, we will focus on equal sharing"**. Karabo reads the knowledge and skills to be acquired in this topic from the CAPS **"Solve word problems in context and explain own solutions to problems involving equal sharing and grouping with whole numbers up to 15 and with answers that may include remainders".**

The teachers unpack the objectives, explain and demonstrate all the steps to be done when solving a word problem. Karabo says to the group, **"don't forget to write the word problem neatly on the board in short sentences"**. The problem reads as follows **"My friend and I share 8 sweets equally. How many sweets will I get"?** Kgatlhego explains the steps that the teachers should follow for teaching word problems as per the DBE-JICA manual:

- Read the word problem aloud several times.

- Read the word problem with the learners, sentence by sentence;
- Ask the learners questions about the word problem, e.g., “How many people are in the story"? "What must they do"? “What must they share”? “How must they share"? What do we mean by sharing equally”?
- Ask the learners to identify the numbers and the main question in the word problem, e.g., **“**How many sweets will I get”? Learners must underline the question.
- Emphasise the key words in the word problem (e.g., share equally). The teacher can write the key words on the flashcards.
- Let the learners solve the problem on a worksheet or in their classwork books.
- Allow the learners to explain their solutions and do corrections after they explain.

Onkarabile asks for clarification, **“How are the learners going to sit when solving the problem? In groups or in pairs”?** Kgatlhego explains, **“They should work in pairs”.** Karabo adds, **“they should work in pairs while they [are] seated in their tables”.** Naomi (researcher) also comments, **“let them first work in pairs [to] share sweets amongst themselves”. “Learners can sit on the carpet while the teacher is doing the demonstration activity and they will go back to their tables to do activity 2”** (Omaatla). Kgatlhego emphasises that learners should share sweets one by one between themselves.

Naomi asks, **“What are you going to emphasise for them to remember what to do when sharing”** to check if they still remember the steps of teaching word problems mentioned earlier by Kgatlhego. Onkarabile answers, **“share equally”.** Naomi explains that emphasizing the words **“sharing equally”** will strengthen learners’ mathematics vocabulary. **“We can even write the words on the flashcard and put it on the board”** (Kgatlhego). Onkarabile asks if **“it’s compulsory to say, sweets, because I have changed to say counters,”** to which Karabo replies, **“there is no problem. That will assist to avoid confusion”**.

Onkarabile explained that she is still confused with the process of introducing word problems. She asks the following questions in trying to get clarity **“Do I have to read word problems to learners? Learners reading after me. Ask them questions about the word problem? Let them solve problems in pairs? I was thinking of working the first problem with them”.** The group agrees that she can demonstrate solving the first problem together with learners. Onkarabile needs further elaboration**, "How am I going to introduce the word problem? After reading the word problem, can I demonstrate on the board to show learners how they must share equally while they are sitting on the carpet?"** After a lengthy debate about whether to demonstrate the first activity to the learners or to leave them with the problem to solve, we agreed that we should demonstrate the activity to motivate and stimulate learners' interest in the lesson. The teachers agreed that this whole-class activity would provide learners with opportunities to develop their conceptual and procedural understanding (Takahashi, 2006). Brown (1994) contends that when learners are passively listening to teachers, their opportunities to understand mathematical concepts and procedures are not maximized. Rather than just listening to teachers talk, learners need to be actively involved in mathematics and do various mathematical activities.

Karabo suggests, **"Let the teacher put 8 counters (representing sweets) on the board, put two big faces represent 2 friends and demonstrate equal sharing".** Naomi adds that learners can also have counters and share with their friends while the teacher is demonstrating on the board. **"You can also let learners in pairs to have 8 counters, and they role-play sharing when you demonstrate"** (Naomi). Omaatla proposes that instead of the learners working with counters on their tables, they should sit on the carpet for all the whole class activities. **"I think it will work better if they do counting, mental maths and concept development while sitting on the carpet".** Onkarabile concurs with Omaatla **"especially for concept development, I want them to have full understanding of the concept before working in pairs or individually".** Karabo agrees, **"then after the demonstration activity learners can then work in their tables working in pairs solve another word problem".** The teachers agree that everyone will do the demonstration lesson to suit their classroom context, that is, either as a whole class sitting on the carpet or whole class sitting in their tables.

Kgatlhego reminds the teachers to let the learners explain their solutions strategies after solving the problem. **"Let the pairs explain how they share the sweet/counters".** She further explains that they should conclude the activity by explaining the key words to strengthen the vocabulary that relates to equal sharing. We need to **"confirm with the whole class at the end that we halve something into equal parts when sharing"** Karabo explains that they should emphasis equal sharing by showing learners that you give one to yourself, and one to your friend by **"demonstrating"** until all the items are shared and everyone has the same amount.

In order to differentiate learning, the teachers organize the booklets according to colour. While checking the activities in the booklets, Karabo reads the word problems and asks questions for clarity in the activity 1. **"How many sweets are they going to share? How many friends? Where are they going write answer?"** Onkarabile adds that **"10 sweets must be shared equally between two friends".** Kgatlhego asks if it okay for them to **"share [the] sweets and not write the answer?"** As they talk, Onkarabile and Kgatlhego demonstrate how to share the sweets (Figure 4.2).

Figure 4.2 Onkarabile and Kgatlhego demonstrating how learners will share sweets in the boxes.

The teachers discuss the activity further and Karabo suggests that they should create a space in the activity book where learners can write their answers. She suggests they write **"each friend will get ______sweets"**. In this way, the learners will realise that they **"must write answer to understand that problems have solutions and must be written in the space provided all the times"** (Figure 4.3).

Figure 4.3 Karabo indicating space where the answer should be written.

Naomi also agrees that after the learners share sweets equally in the boxes (on the worksheet), they should count and see how many sweets each friend gets, and they should write the answer in the space provided (Figure 4.4).

Figure 4.4 Naomi is explaining and demonstrating.

The teachers agree that they should read the word problems to the learners. Karabo demonstrates this and says, "**we will read the word problem several times, learners individually use the sweets on the other page to share in the boxes, then write the answer"** (Figure 4.5).

Figure 4.5 Karabo is demonstrating

Omaatla, suggests that the **"learners must put counters on top of the pictures of sweets in the booklet, so that when sharing, physically they will remove sweets and share equally to the friend's boxes"** (Figure 4.6). This will assist them in solving the problem and seeing how many sweets each friend will get and how many sweets will be leftover.

Figure 4.6 Omaatla is demonstrating

The teachers continue checking and discussing other activities in the booklet and agree that they should keep the number of sweets consistent, but change the number of friends that the sweets should be shared between. Karabo commented, "**the next activity is the same number of sweets, but number of learners changed to 5",** then we have the "**same number of sweets but number of learners changed to 4 and that means this one will gives us a remainder".** She reads the question, **"How many sweets will each friend get and how many sweets left?"**

The teachers agree to include a word problem that will give a remainder (i.e. 'Share 10 sweets equally amongst 4 friends. How many sweets will each friend get?) "**Learners will recognise that in some activities there will be some sweets left that cannot be equally shared"** (Kgatlhego).

The teachers agree to include a formative assessment task that learners will complete individually in their classwork books. Kgatlhego suggests that they should photocopy the worksheet activity in the Platinum series (a mathematics textbook used in the two schools). Onkarabile suggests that they develop their own worksheets. She shows them an example of a worksheet she has made. The teachers compare and discuss the two worksheets, that is, the one that Onkarabile developed and the one in the Platinum textbook. Kgatlhego asks if they have to give a formative activity, **"are we having one activity?"** To which Onkarabile responds, **"I prefer 1, they have completed many activities practically we just want to see if individually can apply what they have learned".** The teachers agreed to use either of the two worksheets for the independent classwork. The worksheets are shown in Figure 4.7.

Figure 4.7 Suggested worksheets by Kgatlhego and Onkarabile

Naomi advises the teachers to move around the classroom as the learners are working independently. We should **“move around the groups marking their work to establish learners who are still struggling and need individual assistance while moving around select few learners who will come to the board to justify their solutions and explain how they have shared”.** Naomi suggests that when learners are done with the activity, teachers should choose learners who use different strategies for solving problems and allow them to explain their solutions on the chalkboard**.** Naomi proposes that the teachers **“carefully select learners’ solutions and place them on the board for whole class discussion”**. She wants to encourage the teachers to give all learners enough chances to explain and constructively criticize one another. **“We want to see learners explaining and justifying**” (Naomi).

Towards the end of a lesson, a teacher should pull all the different strategies and ideas together to assist the learners to see the connection (Takahashi, 2006). Karabo reminds the teachers that at the end of the lesson, they should consolidate or wrap up the lesson by quickly going through key points on the chalkboard. Naomi explains the importance of using the chalkboard properly in order to follow all stages during consolidation **“your chalkboard must be divide in to 3 parts. The first part is for the teacher to introduce the lesson, middle part to display learners’ workings and the third part it is used by the teacher for consolidation”.** She draws on the work of Yoshida (cited in Takahashi, 2006) to explain how teachers should use the chalkboard in their lessons

> 1) to keep a record of the lesson (2) to help students remember what they need to do and to think about (3) to help students see the connection between different parts of the lesson and the progression of the lesson (4) to compare, contrast, and discuss ideas that students present (5) to help to organize student thinking and discovery of

new ideas (6) to foster organized student note-taking skills by modelling good organization (p. 42).

Teachers should use the chalkboard to summarize, organize, and link a sequence of lesson events to facilitate collective thinking, more than merely displaying information or solutions (Emerling, 2015).

4.2.2 Lesson presentation

Karabo and Onkarabile taught the lesson in cycle 1. They taught the first lesson that we planned in their respective schools. Karabo's school uses English as Language of Learning and Teaching (LOLT) and Onkarabile uses Setswana.

Onkarabile adapted the planned lesson to suit the context in which she is teaching. Her school is a full-service school, which means she has learners with special needs in her class. In the lesson, she made the following changes:

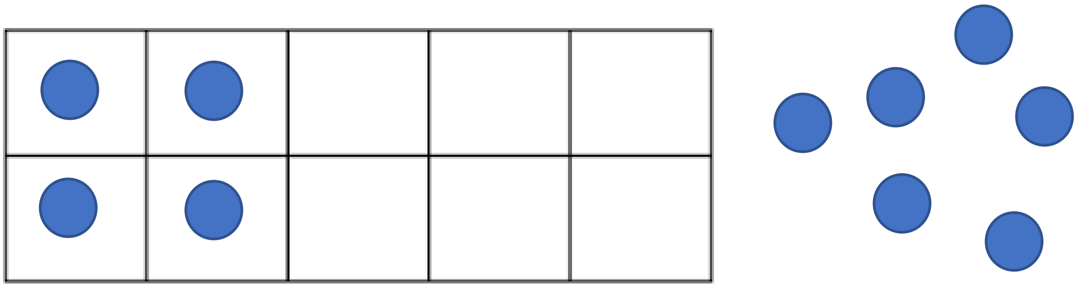
- used as counting rhyme;
- got the learners to count in 2s up to 10 using the 10-frames; and
- skipped the number bonds activity.

The lessons are written as vignettes. The actual lesson plans have been included in Appendix 10.

Lesson 1 (Onkarabile)

Counting and Mental Maths The learners are seated at their desks. Onkarabile starts the lesson with the daily routine (asking questions, such as, "what date/day it is today"). She introduces the lesson by counting with the learners in 2s to 20. The learners clap their hands as they count. This is followed by a counting rhyme: 2, 4, 6 clap your hands 8, 10, 12 stretch your arms 2, 4, 6 clap, clap, clap 8, 10, 12 stretch, stretch, stretch

She introduces the 10-frame by counting the blocks with the learners and explaining to them that they will use two 10-frames.



Each learner gets two 10-frames with counters. She asks the learners to both 10-frames with their counters, emphasizing that they should fill all the blocks on the 10-frame. She moves around the class to assist individuals who are struggling. Afterwards, she asks them to remove 2 counters from the 10-frame. The learners do this up to 4, 6, and then 10. Onkarabile asks them how many groups of 2 there are in 4 (6, 8 & 10). She focuses on a smaller number range (e.g., up to 10) to accommodate learners with learning barriers. The next instruction is for learners to count up to 12, 14 and 16 and say how many groups of 2 there are in each number. The learners repeat the steps above, but this time they count in groups of 5 up to 15 still using the 10-frames. She asks a few learners to write number sentences for counting in 2s up to 16 on the chalkboard.

Onkarabile skips the mental maths activities because she spent more time with the 10-frame activities than we had initially planned. The learners also take time to understand and grasp what she intends to develop with the counting.

Concept development

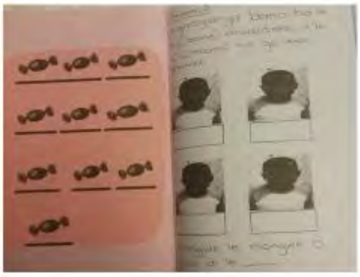
Onkarabile reads the word problem *"My friend and I share 8 sweets equally. How many sweets will I get?*" She follows all the activities as we had planned them. The learners try to solve the word problem on their own. She asks one learner to solve the problem on the board while the others compare their solutions. The majority of learners use equal sharing, that is, giving each learner one sweet at a time. However, one boy gives his 'friend' 4 counters. He counts the remaining counters and takes them for himself.

The learners get an activity to complete individually using the worksheet booklet that we developed. Onkarabile explains the activities in the booklet thoroughly. She reads each word problem from the worksheet:

- Share 10 sweets equally between 2 children.
- Share 10 sweets equally amongst 5 children
- Share 10 sweets equally amongst 4 children.

She reads with them and lets them solve the word problems individually. She moves around to check and gives assistance to learners who are struggling. She identifies a boy who is still struggling with identifying the activity to be completed. She demonstrates to him how to page through the booklet to find the correct activity.





She instructs them to page to another activity and reads the word problem with them: "Share 10 sweets equally between 4 children". She lets a few children come in front to explain how they worked out the problem and explain their solution.

Lesson Consolidation and Conclusion

Onkarabile did not get a chance to consolidate the lesson due to time constraints.

A detailed copy of the lesson plan is in Appendix 10. Karabo taught the same lesson in her school. She did not make any changes to the planned lesson. In other words, unlike Onkarabile, she did not skip the mental mathematics section of the lesson.

4.2.3 Reflections

All the feedback and reflections of the two lessons took place immediately after the lesson presentations in the respective classrooms. This assisted the team to reflect using evidence on the chalkboard and learners' activity books.

4.2.3.1 Refection on the lesson taught by Onkarabile

In keeping with the LS process, during the feedback sessions, the presenters of the lessons first had a chance to reflect on the lesson. Onkarabile indicated that her learners were not participating in the lesson. **"My learners were so quiet and not flexible, every time I have to push and push to get response from them, I was scared that I might end up telling them answers instead of letting them to think and give answers"** and **"they also frustrated me because they were not actively participating" (RL 1[5]).** She explained that she did not reach the lesson's objectives and could not consolidate or wrap up the lesson. **"I couldn't make to the end of the lesson and my objective was partially reached because I can't say my learners have grasp the skill as expected".** Time management was a challenge for her; that is why she couldn't finish and achieve her objectives. **"Starting from the beginning of the lesson, I skipped mental maths because I realised that I spent lot of time for counting in a 10-frame"**

Kgatlhego agrees with Onkarabile **"we did not do mental maths we dwell too much in counting".** Onkarabile commented that in her lesson, she rushed learners because she wanted to cover all the activities planned. She **"forgot some aspects of teaching word problems, e.g., identifying numbers from the word problem"** (Onkarabile). Despite counting taking up too much time in the lesson, all of the teachers, who were observing the lesson, commented that the counting was well conducted and that the learners were motivated because they were clapping their hands as they counted. The "**learners' interest was stimulated by doing counting with clapping hands in a rhyme"** (Karabo). Omaatla added that the **"learners enjoyed the counting that was performed in the form of rhyme clapping"**. Naomi suggests that to assist the counting further in the improved lesson, "**presenters should**

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[5] In this section of my data presentation and analysis (4.2.3.1), all quotes are from Reflection Session 1. For ease of reading I have not referenced 'RL 1' after each quote, rather I have decided to write in bold the teachers' voices. Where a statement is not from 'RL 1' in section 4.2.3.1, I will reference it.

have their own 10-frames on the board and demonstrate to learners how to count filling up the 10-frame". She demonstrated this to the teachers.

The teachers commented that there were too many practical activities and that forced Onkarabile to rush. Onkarabile added, **"We had many activities in the booklet".** Kgatlhego suggested that in the future, teachers should give learners a chance to focus on one activity and spend time on letting the learners justify their solutions. The teacher can then consolidate after each activity. She continued explaining that she had **"observed that some learners did not understand the first activity and it was very difficult for them to execute the next activities". "In future, we need to reduce the practical activities so that we don't rush learners to complete without understanding"** (Karabo). Naomi mentioned that the teachers should "**allow learners enough time to think of the activity. Let them use [their] own strategy to share but be able to justify, and always help the learners having challenges".**

The teachers agreed that during the individual activities, especially when the learners explain their solutions strategies, the teachers must provide big charts to work on so that all learners can be engaged. **"My suggestion is that we should give them enlarged pictures so that when it is paste on the board it is visible to everybody"** (Omaatla).

Karabo commended Onkarabile as **"individual support was given to struggling learners from the counting activity to the last activity".** She added that despite Onkarabile's concerns that she did not meet the objective, most learners showed an understanding during practical activities and were able to complete the individual assessment activity.

4.2.3.2 Reflection on the lesson that Karabo taught

Karabo mentioned that she forgot some aspects that the group agreed upon during planning when presenting the lesson. **"Before counting on a 10-frame I forgot to demonstrate first on my big 10-frame how to pack counters while counting"** (Karabo, RL 2[6]).

Karabo was satisfied with the lesson. She explained, **"I have achieve my objectives because when I walk around during pairs activities I realised that most of the learners can share sweets equally in the given numbers"**. She suggested that the practical activities should be reduced in the re-planned lesson so that learners have more time to constructively work on the problem and get a chance to justify their solutions.

The amount of reflection on the lesson that Karabo taught was limited as she was the second teacher to present. She followed the lesson as was planned by the team. She infused and effected the recommendations mentioned by team members from Onkarabile's lesson. Kgatlhego applauded Karabo for emphasing key words when reading the problem with the learners. However, she mentioned that the presenter forgot to show the learners the key words on the flash cards (e.g "sharing equally"). Karabo wanted to know if it is wrong to give learners a problem to solve before demonstrating it to them. Naomi indicated that there is no problem because that will allow learners to show their understanding of the word problem and provide them with the opportunity to use different strategies. She continued that in using different strategies they have chance to explain their thinking and justify their solution strategies. Onkarabile asked about the strategy used by one learner for sharing **"Was is it correct for the boy to give friends sweets in twos or was he supposed give one to each until the sweets finished"**? This facilitated a lengthy discussion. Onkarbile mentioned that the presenter should have corrected the boy as the focus was on equal sharing not grouping**.** Kgatlhego disagreed and argued that the learners should be encouraged to use different methods to share the sweets. Given that the child who was being refered to had used a grouping method as opposed to an equal sharing method to solve the word problem, Kgatlhego voiced an opinion the **" maybe that boy is advanced in terms of counting and his**

[6] In this section of my data presentation and analysis (4.2.3.2), all quotes are from Reflection Session 2. For ease of reading I have not referenced 'RL 2' after each quote, rather I have decided to write in bold the teachers' voices. Where a statement is not from 'RL 2' in section 4.2.3.2, I will reference it.

number sense is well developed". Naomi consolidated the session by explaining, clarifying and demonstrating some of the activities in the lesson.

The analysis above describes the LS process, which is lesson planning, lesson presentations, and feedback and discussion sessions. During cycle 1 of the LS process, different MKfT domains emerged and showed the potential for teachers to increase their knowledge for teaching mathematics, as highlighted below.

4.3 MATHEMATICS KNOWLEDGE FOR TEACHING: CYCLE 1

Each of the MKfT domains evident in the Onkarabile and Korabo's lessons are highlighted below. The analysis focuses on the pedagogical content knowledge in the MKfT framework. It appeared from the planning, implementation and reflection stages that the teachers were familiar with the mathematics content required to teach Grade 1 mathematics.

Knowledge of Content and Curriculum refers to the "grasp of the materials and programs that serve as "tools of the trade" for teachers" (Shulman, 1987, p.8). This includes knowledge of the mathematics content in curriculum policies and the choice of resources that the teachers use to support learners' learning, that is, the Learning and Teaching Support Materials (LTSM) (e.g., textbooks, manipulatives).

The teachers showed that they were familiar with the curriculum expectations, that is, that learners had to solve contextual problems that involved 'equal sharing with remainders'. They were also familiar with the lesson format, as highlighted in CAPS. Furthermore, the teachers were able to identify suitable LTSM to assist the learners in making sense of division with equal sharing. For example, they knew that counters would assist learners with 'equal sharing' as they can physically share the counters 'between friends' (e.g., "My friend and I share 8 sweets equally. How many sweets will I get?"). They drew on the DBE-JICA manual for ideas of activities that they could use in their lesson. They developed an activity 'booklet' for the learners to lead them from 'equal sharing leading to division' to 'equal sharing leading to division with remainders'. Table 4.1 provides a summary of the KCC related to 'equal sharing with a remainder' evident during the planning phase.

KCC	• Know the topics in the curriculum (e.g., equal sharing with a remainder) relevant to Grade 1
	• Know the CAPS prescribed structure of a mathematics lesson
	• Know that the mental mathematics section should include both counting and rapid recall of number facts
	• Identify, adapt and develop suitable LTSM (e.g., counters, worksheets)
	    • Drew on activities from the DBE-JICA manual
	    • Adapt lesson activities from the Platinum textbook
	    • Generate worksheets suited to the topic of the lesson
	    • Use counters for solving the word-problems

Table 4.1 KCC for LS cycle 1

Knowledge of Content and Teaching "combines knowing about teaching and knowing about mathematics" (Ball, Thames and Phelps, 2008, p. 401). It includes the knowledge needed to decide on appropriate examples, representations and strategies to use to illuminate a concept for a learner (Ball et al., 2008). KCT also includes the kinds of questions teachers ask and the manner in which they explain concepts to promote learning (Shuilleabhain, 2015). For example, the teachers decided that they should first introduce 'equal sharing' with a word problem that did not have a remainder. Before the learners were introduced to remainders in the worksheet, they completed two word problems that did not include remainders. The teachers sequenced the examples working from what the learners were able to do, that is, 'equal sharing' to the new knowledge, 'equal sharing with reminders'. The teachers decided that the learners should be given the opportunity to represent the word problems by using concrete objects (counters). The main teaching strategies included facilitation, demonstration, explication and questioning in unpacking the word problem with the learners. Onkarabile and Karabo asked several productive questions (e.g., **"What is the story about? What numbers do you see in the story? What do my friend and I do?"***)*. In so doing, she supported her learners in understanding the word problems. Both teachers moved around the class to assess the learners' understanding and to assist learners with challenges. A summary of the KCT expressed in the first LS cycle is in Table 4.2 below.

KCT	• Know how to support learners in developing an understanding of how to interpret word problems • Sequence the mathematics activities in the lesson to start with 'equal sharing' and progress to 'equal sharing with remainders' • Use different teaching strategies to assist in the development of the concept (e.g., demonstration, explication, questioning, facilitation) • Ask productive mathematical questions to ensure teaching for understanding • Select examples of different representations to develop the learners' mathematical understanding (e.g., counters, drawings, ten-frames) • Adjust teaching based on the prior knowledge of the learners and assessment of learners as they engage in the lesson and complete the various tasks

Table 4.2 A summary of the KCT expressed in the first LS cycle

Focusing on learners' mathematical thinking provides teachers with an opportunity to enhance their knowledge of how content can be best developed in order to build learners' understanding (Carpenter et al., 1989; Jacobs, Lamb & Philipp, 2010; van Es & Sherin, 2008). Knowledge of Content and Students relates to the knowledge required to connect content with learners. According to Baumert et al. (2010), KCS links mathematical content with how learners' think about that content. KCS refers to teachers' understanding of how learners learn particular content and includes teachers' knowledge of common misconceptions learners might have (Shuilleabhain, 2015). KCS is focused on teachers' understanding of how learners learn particular content and encompasses teachers' knowledge of common student errors and conceptions or misconceptions about mathematical topics (Ball et al., 2008; Hill et al., 2008). According to Ball et al. (2008), KCS "implies an understanding of students' thinking and what makes the learning of particular concepts easy or difficult but does not include knowledge of teaching moves (p. 378)".

The teachers were aware of the learners' prior knowledge as they anticipated that counting using 10-frames to 12, 14 and 16 might be a challenge for some learners. They decided that they would build on the learners' prior knowledge and first use the 10-frames to count to 4, 6 and 8. They realised that doing addition and subtraction in the mental mathematics section of the lesson might be confusing for the learners. In this way, they anticipated what the learners may find challenging.

The teachers scaffolded the learners thinking by developing activities which progressed from easy to more complex (e.g., division with equal sharing without a reminder may be more straightforward than with a remainder). They developed relevant examples to gain the learners' interest. For example, using pictures of two faces and sweets on the chalkboard as a means of contextualising division. They emphasised the importance of learners learning the relevant mathematics terminology and engaging in practical work. In addition, the teachers deliberated on how best to arrange the learners so that they could learn from each other.

The teachers identified which learners were not understanding the lesson content and considered how to support them. Noticing learners' responses is an explicit objective and becomes integral to teaching through participation in lesson study (Dooley, Dunphy and Shiel with Butler, Corcoran, Farrell, NicMhuirí, O'Connor, &Travers, 2014)

While the teachers were cognisant of the importance of learners explaining and justifying their solution strategies, they did not consider possible strategies that the learners may use other than equal sharing in the planning session. For example, the learner who used grouping as a strategy rather than equal sharing. Table 4.3 provides a summary of the teachers' KCS in the first LS cycle.

KCS	• Identify learners' prior knowledge • Predict what learners will find exciting and motivating when choosing an example • Highlight challenges learners may have in the lesson • Anticipating what learners are likely to think or find confusing (with regards to the possible confusion between addition and subtraction

	during the mental mathematics, but not in relation to the strategies the learners use for solving division word problems) • Explain that learners learn when they are allowed to share and justify their solution strategies • Know that learners should develop the mathematics vocabulary and pay particular attention to this • Know that learners learn by engaging in practical work • Consider how to encourage learners to work together and learn from each other • Differentiate the activities to suit the learners' needs • Realise that some learners need additional support

Table 4.3 A summary of the KCS expressed in the first LS cycle

In the first LS cycle, the teachers drew on their KCC, KCT and KCS in planning and reflecting on the lessons.

4.4 LESSON STUDY CYCLE 2: IMPROVED LESSON PLAN

Kgatlhego and Omaatla taught the improved lessons in cycle 2. They taught the lesson we improved after the first LS cycle in their respective schools. Kgatlhego's school uses English as Language of Learning and Teaching and Omaatla'a uses Setswana.

4.4.1 Planning the lesson

After the first lesson reflection, we listed the aspects that needed to be changed for the improved version of the lesson plan. This brief conversation was not recorded, as explained in Chapter 3. We did not record the process of lesson planning in cycle 2 because we watched the two videos of the presented lessons and reviewed some of the activities for the lessons in the 2nd cycle at the same time.

The changes suggested were:

- We will have a big 10-frame on the board to demonstrate how to pack counters while counting.
- We will allow struggling learners to use manipulatives during the mental maths part of the lesson.
- We will demonstrate the first 'equal sharing' activity with the whole class and allow learners to role-play (act out) the word problem.
- The number of activities to be executed by learners in pairs or individually will be reduced to give learners the opportunity to use their own strategies for calculating and to ensure that there is time for the learners to explain their solution strategies to the class.
- The independent assessment activities will be reduced to one.
- The activities booklet will have different colour pages to guide learners.

Lesson 2 (Kgatlhego)

Counting and Mental Maths The learners are seated at their desks. Kgatlhego starts the lesson with daily routines (day, date, and checking learners' birthdays). She places a big 10-frame on the board and introduces the 10-frame to the learners. She calls it 'base ten' instead of 10-frame. She asks the learners to fill the frame while counting. She moves around the classroom and checks if all the learners have filled their 10-frames with counters. She asks the learners to count backwards in 2s by taking two counters from the frame. Kgatlhego asks them how many groups of 2 are there in 12. The next instruction was for the learners to count up to 14 and 16 and tell her how many groups of 2 there are in each number. Learners repeat the steps above, but instead of counting and grouping numbers in 2s, they moved to grouping numbers in 5s. For mental maths, Kgatlhego uses flashcards to focus on rapid recall of addition facts. Concept development Kgatlhego reads a word problem to the class: "My friend and I share 8 sweets equally. How many sweets will I get?" After reading the word problem, she clarifies the problem by

emphasising the key words, identifying the numbers in the story and highlighting the question in the word problem. Kgatlhego and the learners read the word problem together. She then asks questions to clarify the problem, e.g.,

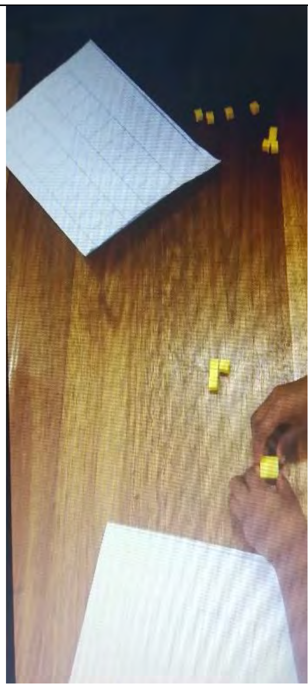
- "How many people are here?"
- "What must they share?"
- "How must they share?"

She has key words emanating from the word problem written on the flashcards (e.g., 'share'). She shows these to learners, reads them, and then gives the learners a chance to read them after her. She puts all the key words on the chalkboard

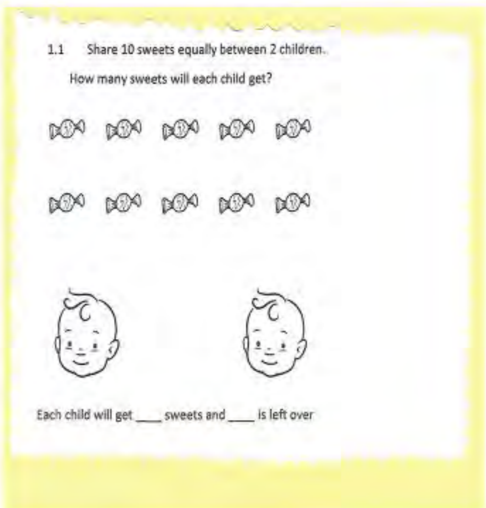
She demonstrates how the 2 friends will share 8 sweets using pictures that she has drawn on the board. She draws 2 faces on the board to represent her friend and herself. She counts 8 sweets, from a plastic tub, with the whole class. She explains that she will share the 8 sweets equally by giving each friend one sweet at a time. She takes the sweets from the plastic tub and gives one to each of the 'faces' (representations of her and her friend) on the board. She keeps asking the learners if she should continue sharing until all sweets are finished in the tub.

She requests the learners to work in pairs. She asks each learner to take 8 counters from the containers at their desks. She asks each learner to share his/her 8 counters with a friend. She moves around the tables to check their solutions while they are working and assists those who are struggling.





Kgatlhego gives the learners an activity to complete individually using a booklet with different colours (yellow and blue). The coloured pages guide the learners to the relevant activity. She reads the word problems on the yellow page. She asks the learners to cover the picture of the sweets with counters so that when they share, they manipulate 'real' objects. She moves between the tables to assist those who are struggling. She invites a few learners to come to the front of the class and explain their solutions.

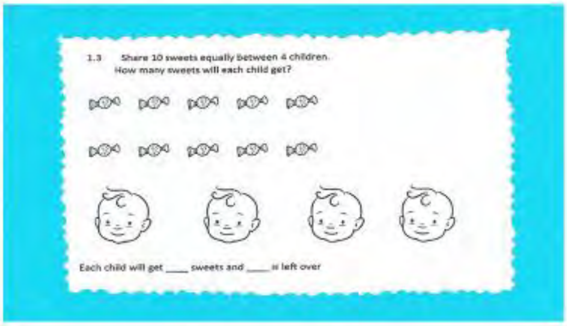

1.1 Share 10 sweets equally between 2 children.

How many sweets will each child get?

Each child will get ____ sweets and ____ is left over

She instructs the learners to turn to the blue page of the booklet. She reads the word problem with them: "Share 10 sweets equally between 4 children. How many will each get?"

She asks them to follow the same steps they used to solve the problem on the yellow page[7].



Lesson Consolidation and Conclusion

She gives the learners a worksheet to complete as a consolidation exercise. She is not able to conclude the lesson because she runs out of time.

Lesson 2 (Omaatla)

Omaatla taught the lesson as it was improved by all teachers. She implemented the recommendations agreed upon during the review of lessons 1 and 2 in cycle 1. For example, she had big 10-frames for the teacher to demonstrate packing out counters and counting in multiples of 2 and 5 (Figure 4.8) and allowed learners to role-play equal sharing when she introduced the word problem.

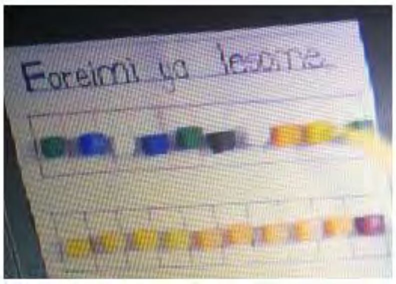


Figure 4.8 Teachers' ten-frames

Figure 4.9 shows the activity she gave to learners for individual work. She did not manage to consolidate and wrap up the lesson due to time management.

[7] While the LS process tries to promote progressive teaching methods, and while we tried to engage with that in our planning, shifting teachers' approaches to teaching and learning takes time.

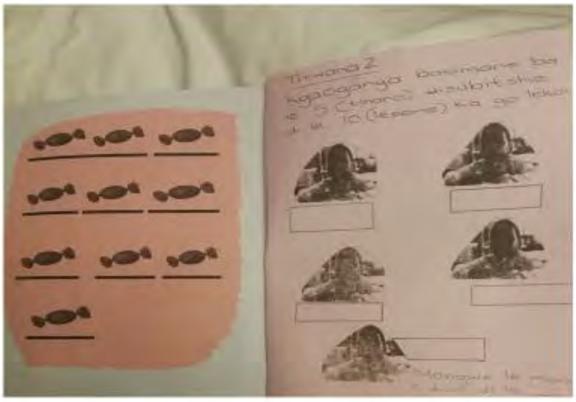


Figure 4.9 Learner activities

4.4.2 Reflections on the improved lesson

Reflections on the improved lesson only occurred after Kgatlhego's lesson (RL 3[8]) as a taxi strike made it impossible to reflect on Omaatla's lesson (Chapter 3).

4.4.2.1 Reflections on the improved lesson that Kgathlego taught

As in the LS process, the person who taught the lesson begins the reflection process. Kgatlhego mentioned that she forgot to explain the words on the flashcards while she was unpacking the word problem and so did it afterward. Onkarabile echoed this comment later in the session, but added that **"in terms of the word problem the presenter did not follow all steps as planned",** that is, read the word problem alone, read with learners, explain and emphasise the key words in the problem, let learners underline the numbers and the question. The observers indicated that some of the presenters' instructions to learners were confusing. Kgatlhego explained that she did not follow the improved lesson exactly as it was developed "**I didn't follow the lesson as planned. I have jumbled activities, e.g., I was supposed to explain and demonstrate the counting on the 10-frame, I read and flash key words later in the lesson not where we planned them"**

[8] In this section of my data presentation and analysis (4.4.2.1), all quotes are from reflection session 3. For ease of reading I have not referenced 'RL 3)' after each quote, rather I have decided to write in bold the teachers' voices. Where a statement is not from 'RL 3' in section 4.4.2.1, I will reference it.

Onkarabile noted that she **“was also confused by the presenter's instruction**” as we had decided that each pair should take out 8 counters from the containers on their desks. However, the presenter asked each learner to take out 8 counters and so each of “**the pairs end up having 16 counters and when sharing they end up each having 8 again”.** Karabo confirms that “**the instructions in most activities was confusing as the colleague has alluded”.**

Karabo mentions that when the presenter gave the learners the word problem where they had to share 10 sweets amongst 4 learners, she **“did not explain or emphasize the "left over" or remainder after sharing equally”.** When explaining and justifying her solution, one of the learners indicated that all friends would get 2 sweets, then there will be 2 left, which she will eat. “**I think the presenter should have explain about the remainder at that stage”** (Karabo)

Karabo commented that **“individual support was given to struggling learners from the counting activity to the last activity”.** Onkarabile added that she did **“not see the proper use of chalkboard as that part of lesson presentation”.** Naomi emphasised that **“the proper use of the chalkboard assists to keep a record of the lesson, help learners remember what they need to do and to think about, help learners to see the connection between different parts of the lesson and the progression of the lesson”.** All teachers agreed that they did not get a chance to wrap-up or consolidate the lessons in the way they would have liked.

4.5 MATHEMATICS KNOWLEDGE FOR TEACHING: CYCLE 2

Utilising the same framework of Mathematical Knowledge for Teaching (Ball et al., 2008), the following indicators of KCC, KCS and KCT emerged in LS cycle 2 that is from the improved lesson. Rather than repeating the KCC, KCS and KCT from cycle 1, I focus on the changes that emerged as a result of the reflections.

4.5.1 Knowledge of Content and Curriculum

Knowledge of Content and Curriculum refers to the knowledge of the curriculum content requirements and the materials that can be used to teach the relevant content (Chikiwa, Westaway & Graven, 2019). Teachers showed their knowledge of KCC when they: located the topic in CAPS; followed the structure of a mathematics lesson; identified, adapted and designed suitable LTSM; explain that the mental mathematics section of the lesson should include counting and rapid recall. Table 4.4 indicates the knowledge that the teachers drew on as they reflected on and planned the lesson for cycle 2 of the LS.

KCC	• Realise the limitations of the time allocation in the recommended structure of the mathematics lesson as they struggled to include both counting with the 10-frames and rapid recall of addition facts in the allocated 10 minutes. • Ensure that the selected materials are large enough for the class to observe the demonstration

Table 4.4 A summary of the KCC expressed in the second LS cycle

4.5.2 Knowledge of Content and Student

Knowledge of Content and Students refers to teachers' understanding of how learners learn particular content and encompasses teachers' knowledge of common student errors and conceptions or misconceptions about mathematical topics (Ball et al., 2008; Hill et al., 2008).

The improved lesson taught in cycle two was based on the same KCS that the teachers drew on in the first cycle of the LS. This is reflected in Table 4.5 below.

KCS	• Learners require clear instructions in order to understand the content they are expected to learn • Time needs to be created for learners to share their thinking and justify their solutions

Table 4.5 A summary of the KCS expressed in the second LS cycle

4.5.3 Knowledge of content and teaching

The teachers' improved lesson drew on the same KCT that formed the focus of the lesson presented in the first cycle. For instance, they used different teaching strategies, asked productive questions, selected appropriate forms of representation, sequenced the mathematical activities to move from the known to unknown, and adjusted their teaching to suit the learners in the class. Table 4.6 provides a summary of the KCT that came to the fore in the second cycle.

KCT	• Decide when to explain a strategy and when to allow learners to come up with the answers themselves • Emphasise and explain the key words • Demonstrate the activity to the learners before requiring them to do the activity • Use the board appropriately to record the lesson

Table 4.6 A summary of the KCT expressed in the second LS cycle

4.6. THE MKFT THAT THE TEACHERS DREW ON DURING THE LESSON STUDY PROCESS

The LS process created a space for teachers to work collaboratively on developing lessons to achieve a particular goal. During the course of their interaction, the teachers drew on their KCC, KCS, KCT. Table 4.7 provides a summary of the PCK that the teachers drew on during the lesson planning and reflection stages of the LS.

KCC	• Know the topics in the curriculum (e.g., equal sharing with a remainder) relevant to Grade 1 • Know the CAPS prescribed structure of a mathematics lesson • Know that the mental mathematics section should include both counting and rapid recall of number facts • Realised the limitations of the time allocation in the recommended structure of the mathematics lesson as they struggled to include both counting with the 10-frames and rapid recall of addition facts in the allocated 10 minutes.

<table>
<tr><td></td><td>Identify, adapt and develop suitable LTSM (e.g., counters, worksheets)
Drew on activities from the DBE-JICA manual
Adapt lesson activities from a teachers’ resource book
Generate worksheets suited to the topic of the lesson
Use counters for solving the word-problems
Ensure that the selected materials are large enough for the class to observe the demonstration</td></tr>
<tr><td>KCT</td><td>Know how to support learners in developing an understanding of how to interpret word problems
Emphasise and explain the key words
Sequence the mathematics activities in the lesson to start with ‘equal sharing’ and progress to ‘equal sharing with remainders’
Use different teaching strategies to assist in the development of the concept (e.g., demonstration, explication, facilitation)
Deciding when to explain and when to enable learners to come up with the answers themselves
Demonstrate the activity to the learners before requiring them to do the activity
Use the board appropriately to record the lesson
Ask productive mathematical questions to ensure teaching for understanding
Select examples of different representations to take learners more in-depth into mathematical content (e.g., counters, drawings, ten-frames)
Adjust teaching based on the prior knowledge of the learners and assessment of learners as they engage in the lesson and complete the various tasks</td></tr>
<tr><td>KCS</td><td>Identify learners’ prior knowledge
Highlight challenges learners may have in the lesson
Predict what learners will find exciting and motivating when choosing an example</td></tr>
</table>

	• Anticipating what learners are likely to think or find confusing
	    • Recognise that learners require clear instructions in order to understand the content they are expected to learn
	• Consider how to encourage learners to work together and learn from each other
	• Know that learners should develop the mathematics vocabulary and pay particular attention to this
	• Know that learners learn by engaging in practical work
	• Explain that learners' learn when they are allowed to share and justify their solution strategies
	    • Suggest that time needs to be created for learners to share their thinking and justify their solutions
	• Differentiate the activities to suit the learners' needs
	• Realise that some learners need additional support

Table 4.7 A summary of the PCK expressed in the two LS cycles

I assumed during the planning sessions of the LS process, that the teachers had the required CCK, SCK and HK. These aspects of the MKfT framework were not highlighted in the lesson planning and reflection sessions. Put differently, they appeared to be familiar with the content required to teach Grade 1 learners. However, I cannot guarantee that they did not consult each other outside of the LS on this matter.

I used Ball et al's. (2008) framework to assist me in identifying the MKfT that the teacher drew on as they engaged in the LS process. This framework provided an opportunity for me to operationalize the mathematics and pedagogical knowledge required to teach primary mathematics. I soon realised that the dominant MKfT category that emerged during the lesson planning and reflection sessions was PCK. However, in an attempt to identify the PCK domains, it became evident that the boundaries between the domains are not rigid. For example, choosing and adapting LTSM appropriate to the lesson and the development of the particular concepts to be taught is part of teachers' KCC. Adapting the LTSM to build on learners' prior knowledge, provoke their interests, and cater to different learning needs is part of KCS. Sequencing the examples in a worksheet and ensuring that there are different modes of representation is an example of KCT. I found it difficult to operationalize the PCK category of the MKfT framework and had to develop clear criteria relating to each of the domains (Chapter 2). I thus turned my attention to the KQ.

4.7. THE KNOWLEDGE QUARTET

Drawing on the Knowledge Quartet framework (Rowland, 2004), I analyse the data in terms of four knowledge categories, that is, Foundation Knowledge, Transformation Knowledge, Connection Knowledge and Contingency Knowledge.

As explained in Chapter 2, Foundation Knowledge consists of the foundational knowledge that is necessary to teach. This knowledge includes content knowledge, pedagogical knowledge and beliefs. Foundational Knowledge is developed in the course of schooling, teacher education and teaching. In my research, all the teachers showed evidence that they understood the Subject Matter Knowledge required to teach mathematics in Grade 1. By that I mean, they had Common Content Knowledge, Specialised Content Knowledge and Horizon Knowledge. With regards to their pedagogical content knowledge, they were able to link the content to curriculum, students and teaching. In other words, they showed evidence of KCC, KCS, KCT as they engaged in the LS (as highlighted in Table 4.7).

4.7.1.Foundation Knowledge

The Foundation dimension includes teachers' mathematics and pedagogical content knowledge, and beliefs about mathematics, and teaching and learning mathematics (Rowland et al., 2009). The four teachers displayed knowledge of the curriculum they were expected to teach in the Grade 1 class, they were able to locate the topic in the policy document, unpack the curriculum, identify the topic and verify its relevance for the grade. They adapted teaching and learning activities from the DBE-JICA manual, developed activities and created assessment tasks for learners

They were able to differentiate learning and teaching by adapting learning opportunities to meet individual learner needs. This was observed during planning when they agreed to start counting with numbers at a lower range to accommodate learners who were struggling. However, it should be noted that the entire class worked with the lower number range rather than the learners who were battling. In the planning sessions, teachers anticipated learners' responses in the lessons, for example, they were concerned that focusing on the recall of addition and subtraction facts in one lesson might cause confusion for the learners.

In the planning session, teachers selected examples that were of interest and motivating for the learners, they identified and discussed counting and mental mathematics activities, and selected appropriate word problems. During the collaborative planning sessions the teachers emphasized the importance of learners being active in the lesson (e.g., through role-play, explaining how they got to the solution). In this way their beliefs about how learners' learn mathematics shifted. The teachers also recognised the value of using multiple representations, that is concrete, pictorial and abstract (symbolic).

Table 4.8 provides a summary of the teachers' Foundation Knowledge that they drew on and developed as they participated in the Lesson Study.

KQ dimension: Foundation Knowledge Knowledge and understanding of mathematics per se and of mathematics specific pedagogy, beliefs concerning the nature of mathematics, the purposes of mathematics education, and the conditions under which learners will best learn mathematics
The teachers: ▪ Demonstrate an accurate understanding of mathematical ideas or concepts: they could unpack the curriculum (CAPS), identify the topic, verify if it is relevant for the grade and discuss the objectives to be achieved. ▪ Demonstrate an accurate understanding of division with and without remainders. ▪ Awareness of the purpose of the lesson: teachers read and discuss the objective of the lesson from the CAPS document. ▪ Demonstrate an understanding of the learners' prior knowledge. ▪ Select examples that will be of interest and motivating for the learners: identify and discuss counting and mental mathematics activities, and select and discuss word problems activities that will motivate learners to participate. ▪ Use appropriate teaching strategies: teachers use rhymes, role play,

demonstrations, group and individual activities. ▪ Teachers agreed on the steps to follow when teaching word problem and how they will encourage learners to complete individual practice activities. ▪ Develop learners' understanding rather than focus on procedures: allowing learners to explain and justify their solutions to check understanding. ▪ Demonstrate knowledge of the importance of mental mathematics: agreed on activities that encouraged the recall of addition facts. ▪ Adapt LTSM for teaching: teachers adapted teaching and learning activities from the DBE-JICA manual, developed booklet activities and create assessment worksheets for learners)

Table 4.8 Teachers' Foundation Knowledge

4.7.2.Transformation Knowledge

As explained in Chapter 2, Transformation Knowledge focuses on knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself (Rowland et al, 2005). It is likened to Shulman's PCK as it focuses on the actual teaching of mathematics and how to make the concepts accessible to the learners (Rowland et al., 2005). This category focuses on instructional strategies, the examples used, the different forms of representation, and the resources necessary to make the content accessible to the learners (Rowland & Turner, 2005; Livy, 2014). The indicators of this category highlighted in Table 4.9 were observed during the lesson presentations when the teachers used different teaching strategies for demonstration and ensuring that learners understood the concept of equal sharing with and without remainders.

The teachers sequenced their teaching examples for both counting and equal sharing carefully to ensure that the learning occured incrementally. The use of different resources, carefully sequenced and structured examples, and instructional techniques that focus on questioning, demonstration, individual and pair work, all worked together to make the content more accessible to the learners and support learners' learning and understanding.

The teachers used teaching and learning materials which were of interest to the learners, encouraged participation and a teaching approach that began to foreground the learner. The learners' interest was stimulated by concrete, pictorial and abstract (symbolic) representations that they used to solve and explain solutions of division problems.

The teachers anticipated the complexity of equal sharing and decided to demonstrate, role-play and let learners engage in pair and individual activities to assist them in understanding both equal sharing and equal sharing with remainders.

In the two lessons in the 2nd cycle, the changes made by the teachers were to use a large 10-frame on the board to assist the learners with counting, promote role-playing to assist the learners in solving the word problems and to reduce the number of activities in the booklet.

KQ dimension: Transformation Knowledge The presentation of ideas to learners in the form of analogies, illustrations, examples, explanations and demonstrations.
The teachers: ▪ Sequence their examples for both counting and equal sharing carefully to ensure that learning occurs incrementally. ▪ Use a variety of resources (ten-frames, counters, worksheets in the activity books) were used to support learning. ▪ Use different forms of visual representations to strengthen the understanding of equal sharing with a remainder. ▪ Use large 10-frames during the 2nd LS cycle to demonstrate how to count in 2s and 5s. ▪ Reduce the number of activities for the learners to complete on their own. ▪ Make use of demonstrations (i.e., showing the learners how to share), ask pertinent questions about the word problems to ensure that they learners understand, and provide opportunities for whole class, pair and individual work. ▪ Require learners to role-play the word problem as a means to assist them in understanding the word problem in the 2nd LS cycle

Table 4.9 The teachers' Transformation Knowledge

4.7.3. Connection Knowledge

Connection Knowledge concerns the consistency of the planning or teaching displayed across an episode, lesson or series of lessons (Rowland, Huckstep & Thwaites 2003). Connection relates to the teachers' connection of concepts and topics when teaching. The ability to make connections could be an indicator of the depth and breadth of teachers' mathematical knowledge.

In the first cycle during the lesson planning process, the teachers considered the learners' previous knowledge when planning the counting and mental maths activities. They decided to change the number range for the counting activity.

The teachers realised that doing both addition and subtraction in the mental mathematics section of the lesson might be confusing for the learners. In this way, they anticipated what the learners may find challenging. One could argue that they didn't connect addition and subtraction because they did not explore the inverse relationship.

The teachers ensured that their teaching activities during the concept development stage of the lesson connected with one another. They agreed to sequence them from simple to abstract. For example, they agreed to start with word problems with no remainders and end with the ones having remainders. According to Turner (2012) understanding the connection between mathematical concepts and making a logical sequence of teaching relates to the teachers' knowledge of curriculum and subject matter (Meliarasi, 2018, p.20)

KQ dimension: Connection Knowledge The sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks.
The teachers: ▪ Make appropriate conceptual connections within the subject matter – Decide to start with word problems with no remainders and end with the ones having remainders to overcome learners' difficulties. ▪ Decide that they would build on the learners' prior knowledge and first use the 10-frames to count from 4 to 10.

▪ Realize that doing addition and subtraction in the mental mathematics section of the lesson might be confusing for the learners. In this way, they anticipated the learners' mathematical responses and what they may find challenging. One could argue that they didn't connect addition and subtraction because they did not explore the inverse relationship. ▪ Start by teaching sharing leading to division using word problems (context) with the support of concrete objects and pictures and then connecting that to the symbols.

Table 4.10 Teachers' Connection Knowledge

4.7.4. Contingency Knowledge

The Contingency dimension concerns teachers' ability to act or answer in the contingent moments (Rowland et al., 2009). Put differently, it is about the teachers' responses to unanticipated moments, such as, responding to unpredicted learner solutions and questions in the lesson. This includes, how teachers handle incorrect answers, deviate from the lesson plan, and respond to the lack of resources, give alternative explanations and 'think on their feet' (Breen et al., 2018, p 4). The teachers generally followed the lesson plan as it was developed during the planning stages of the lesson study. Onkarabile was the only teacher to deviate from the lesson plan. She chose to skip the rapid recall aspect of the mental mathematics due to her concerns with time.

In the post discussion session of cycle 1, the teachers were concerned that Karabo ignored the learner who used the grouping strategy rather than the equal sharing strategy. In cycle 2, teachers identified moments where the presenter of the lesson should have deviated from the lesson and used incidental opportunities to strengthen the learners' understanding. The teachers agreed that the presenter could have responded to the teachable moment when a learner explained her solution, but did not know what to do with the remainder.

KQ dimension: Contingency Knowledge The ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned event.
The teachers: ▪ Use opportunities: teachers reflected on the events that happened during lessons where presenters could have used that opportunities to strengthen learning and conceptual understanding.

Table 4.11 Teachers' Contingency Knowledge

4.8. CHANGES IN THE TEACHERS' MATHEMATICAL AND PEDAGOGICAL CONTENT KNOWLEDGE FROM CYCLE 1 TO CYCLE 2

As mentioned ealier in this chapter, the teachers were all familiar with the mathematics content they were required to teach. Put differently, they knew how to divide using equal sharing and grouping. Where their knowledge improved during the LS process was in how to teach the learners to divide. In other words, the LS process led to some new insights with regards to their pedagogical content knowledge.

The teachers appeared to develop both their individual and collective pedagogical content knowledge during the planning and reflection session. For example, in the planning session of LS cycle 1, Onkarabile asked numerous questions that required further elaboration by the group. Onkarabile wanted to understand whether:

- It would be more beneficial to give the learners the word problems to solve individually or in groups;
- She should first demonstrate how to solve the word problem before allowing the learners to solve the word problems on their own.

In asking such questions, she developed both her KCS and KCT as the focus is how to connect the content to the students as she teachers them.

During the reflection sessions in LS cycle 1, the teachers learned the importance of obtaining the learners interest in the mathematics classroom (encouraging them to clap while counting), providing an opportunity for them to justify their thinking, encouraging different strategies (accepting the use of a grouping strategy to solve an equal sharing problem), and reducing the number of activities so that the learners are not rushed and their understanding of the content compromised. Each of these reflections concern knowledge of content and teaching, and knowledge of content and students.

In their planning for the second LS cycle, the teachers improved their lesson by:

- Demonstrating what they wanted the learners to do (e.g. packing out the counters using a large 10-frame on the board, and demonstrating how to solve a word problem that requires equal sharing).

- Reducing the number of mental mathematics activities and independent activities so that more time could be spent on division using equal sharing and the development of a deeper understanding thereof.

The teacher used this new knowledge to inform the improvement of their lesson in the second cycle.

4.9. THE LESSON STUDY PROCESS

At the end of the LS process, I conducted a focus group interview with the teachers to ascertain their perceptions of the LS process (Appendix 4).

In analysing the transcript, five key themes emerged: team-work; collaboration; learning from the LS; learner focused teaching; and confidence.

4.8.1. Teamwork

“LS improves teamwork” (Omaatla). All teachers noted that collaborative planning promotes teamwork. They mentioned that the LS encouraged sharing of good practices and that they learn from each other through brainstorming ideas. This they suggest leads to a better understanding of how to teach the learners. Teachers believe that teamwork supports individual learning, increases participants’ engagement and strengthens friendly interpersonal relationships.

4.8.2. Collaboration

The teachers indicated that collaboration was a significant benefit for them because it breaks working in silos and promotes teamwork. Teachers felt a sense of empowerment after a collaborative lesson planning sessions. Onkarabile and Kgatlhego agreed that LS gives teachers an opportunity to collaborate, plan, prepare and present a lesson while others are observing. Onkarabile adds **“Collaborative planning promotes better understanding of curriculum and concepts”. “**Regular collaboration with peers about curriculum objectives, teacher instruction, and information learned from field experts helped the participants learn new approaches to instructing students” (Rock & Wilson, 2005, p 86). All teachers agreed that regular collaboration with colleagues about curriculum objectives, teacher instructions, and information learned from reflection or feedback has completely changed their approach to mathematics instruction. Kgatlhego indicated that she gained more teaching strategies through collaboration with other participants. **“My knowledge of maths teaching has increased”**

4.8.3. Learning from LS

All the teachers indicated that participation in LS had improved their effectiveness as teachers as it had assisted them to reflect upon their teaching and helped them grow as teachers. They mentioned that they have learned about clarifying learners’ conceptions, misconceptions, and errors before planning a lesson. Kgatlhego mentioned that understanding learners’ strengths and misconceptions **“will assist teachers to identify gaps learners have in terms of the concept, direct teachers to focus during planning and would help teachers to choose relevant teaching materials”.** Karabo adds **“it will assist me to focus and know where to put more efforts”.** All the teachers highlighted that they have developed and improved in teaching strategies, have moved from traditional approaches (teacher centred) to modern approaches concerned with learners learning in the lessons. All the teachers indicated that their colleagues would benefit from participating in a LS**,** Kgatlhego elaborated **“they will improve in mathematics teaching, learn from their colleagues.”** Teachers emphasized that the involvement of other teachers in LS will developing the quality of school mathematics. Kgatlhego supports team reflection and add that **“our education system will improve”** if we reflect on our teaching together. She added that LS deepened her strategies of teaching mathematics and she will no longer be scared when officials visit her class.

Teachers mentioned that they found the feedback process beneficial and useful because they received more clarity on the ‘what’, ‘why’ and ‘how’ of teaching mathematics. According to Takahashi and McDougal (2016) “the purpose of lesson study is to gain new knowledge for teaching and learning, not to perfect a lesson plan” (p.515).

4.8.4. Learner focused teaching

All the teachers highlighted that getting feedback and reports from observers expanded their knowledge, teaching strategies and improved their classroom instructions. Karabo elaborate **“feedback from observers about my teaching has given me opportunity to focus on learners**

learning and how to teach them mathematics" According to Stepanek, Appel, Leong, Mangan and Mitchell (2007) feedback and reflection, contribute significantly to the improvement of the lesson as well as to the professional development of teachers.

4.8.5. Confidence

Teachers also indicated that they experienced increased confidence in approaching instruction as a result of engaging in the LS cycles. They highlighted that they were not comfortable having observers in the classroom when they were presenting the lessons, Katlhego highlighted **"I was scared, overwhelmed by observers, I end up mixing the plan activities and confused learners with unclear instruction in some activities"**. Teachers indicated that it was not easy to be observed when teaching. Karabo said that "**It was scary at the beginning but at the end I was relaxed and confident".** Kgatlhego exclaimed **"Iyhoo!! (laughing) I was scared and nervous but now I'm confident and proud of myself."** The teachers indicated that feedback from others has influenced their confidence level of teaching math. "**The viewpoint of others, especially those experienced in teaching, is important if you want to grow and become a better teacher"** (Kgatlhego). All the teachers agreed that feedback and observers recommendations added to their knowledge of teaching and classroom expertise and that this will enable them to apply new knowledge in the classroom and to teach in ways that will develop learners' problem-solving, reasoning, and communication skills as called for by the curriculum requirements.

4.9. CONCLUSION

In this chapter the data collected from the four participants who took part in two LS cycles were analysed, interpreted and presented. The data were collected from the planning, teaching and observation, and reflection sessions in each of the LS cycles. The data was analysed using the MKfT framework of Ball et al. (2008) and later the KQ framework of Rowland (2005).

In analysing the data from the LS process and interviews, it was noted that teachers' participation in LS can improve their pedagogical content knowledge. The analysis showed that the participants in this study acquired new knowledge about teaching mathematics and learners' learning in Grade 1. In short, a teacher with strong pedagogical content knowledge knows how to transform his/her subject matter knowledge to make it comprehensible to learners (Hoadley, 2012).

The findings presented by this study both support and are supported by the work of other researchers in the field. They attest that teachers' participation in LS improves their pedagogical content knowledge. The data generated through interviews shows that teachers, as a collective, found much value from participating in LS. The teachers in this study demonstrated improvement in their PCK and showed a shift in the reflections of their classroom practices.

In general, this study confirms the view of Rowland et al. (2009) in that the Foundation Knowledge is demonstrated in the other three dimensions. One important finding of this research is that LS enabled the development of teachers' Foundation Knowledge, in particular their PCK. The Foundation Knowledge strengthened in this LS was predominantly KCS, KCC and KCT as the teachers had the required mathematics content knowledge to teach Grade 1.

CHAPTER 5: CONCLUDING REMARKS

5.1. INTRODUCTION

The purpose of this research was to ascertain how the provision of opportunities for Grade 1 teachers to be involved in a Lesson Study process would strengthen their mathematics content and pedagogical content knowledge. To achieve this, the study was informed by the following question: How does Lesson Study contribute to the development of teachers' mathematics content and pedagogical content knowledge? Two sub-questions were developed to support the main question:

- What mathematics content knowledge do teachers develop as the engage in LS?
- What pedagogical content knowledge do teachers develop as they engage in LS?

In Chapter 1 it was attested that both international and national assessments of educational achievement have consistently shown that South African learners are underperforming. There are numerous explanations for learner underperformance (e.g. insufficient time on task). One of the dominant justifications however concerns teachers' insufficient content and pedagogical knowledge (e.g. Venkat & Spaull, 2014). While there are many explanations for this, one of the key explanations is that teachers, including Foundation Phase teachers, are deemed to have inadequate knowledge of both content and pedagogy (NEEDU, 2013; Taylor & Taylor, 2013; Venkat & Spaull, 2015). This problem has been attributed to the quality of the teacher education system.

The Department of Basic Education has underestimated the problem of how to improve teachers' content and pedagogical content knowledge on a wide scale (Spaull, 2013). Both the DBE (2009) and Adler (2005) observed that pre-service and in-service Foundation Phase teachers are not adequately prepared for the realities of the classroom. In order to improve the quality of teaching and learning, FP mathematics teachers should share their knowledge and experience in professional learning communities. However, traditional professional development often includes short workshops or seminars that feature departmental officials, university lecturers or outside experts, and that occur away from the teaching context that is schools (CDE, 2014). The chapter argues that LS may be a professional development process that enables teachers to take responsibility for improving their mathematics content and pedagogical knowledge.

In Chapter 2, I present the case for the use of LS in my research. LS is a professional development process that enables teachers to take responsibility for their own learning. In so doing, they plan, implement and reflect on what are known as 'research lessons'. LS *"*operates within classrooms, with real learners and in real-time, this allows teachers to form a common vision of what ideas actually look like in practice" (Kyle, 2015, unpaged). In this chapter, I draw on the research on LS to show that it is regarded as a successful professional development approach as it promotes inquiry about learner learning and teaching (Stigler & Hiebert, 1999). Despite the successes of LS in improving teachers' content and pedagogical knowledge in different subject areas, most of the research has been conducted with teachers who teach in the Intermediate and Senior Phases and Further Education and Training Band. There is a dearth of research with Foundation Phase teachers; hence, this study aims to explore and ascertain the extent to which LS develops Grade 1 teachers' mathematics and pedagogical content knowledge.

In addition to arguing for LS in Chapter 2, I also examine the theoretical frameworks that assist in analyzing and explaining teachers' mathematics and pedagogical content knowledge. I draw on three theoretical models: Shulman's Pedagogical Content Knowledge; Ball and colleagues Mathematics Knowledge for Teaching (MKfT); and the Knowledge Quartet (KQ) of Rowland, Huckstep, and Thwaites. I specifically focus on the MKfT and KQ frameworks to investigate the mathematics and pedagogical content knowledge of the teachers in my research. The MKfT framework focuses specifically on knowledge *for* teaching, while the KQ examines teachers' knowledge *in* teaching.

Chapter 3 presents the research methodology. Drawing on a case study methodology, this research was situated within an interpretive paradigm. Four Grade 1 teachers from two Rustenburg Sub-District schools participated in this qualitative research. The focus was to understand the knowledge developed during the lesson study process. The data was generated through the use of observations, interviews and document analysis. While LS guided the research process, data was analysed using the Mathematics Knowledge for Teaching (Ball et al., 2008) and the Knowledge Quartet (Rowland et al 2009) frameworks.

Chapter 4 presented, analysed and interpreted data collected from two cycles of LS. The data was collected from during the lesson planning, lesson observation and reflection stages of the LS. The analysis highlighted the development of teacher's Foundation Knowledge in particular the pedagogical content knowledge as they participated in lesson study.

5.2. FINDINGS

I specifically focus on the MKfT and KQ frameworks to investigate the mathematics and pedagogical content knowledge of the teachers in my research. The findings of this study showed that through the participation in LS, the teachers developed their Knowledge of Content and Curriculum, Knowledge of Content and Teaching and Knowledge of Content and Students. While the questions guiding this research focus on both the mathematics knowledge and pedagogical content knowledge, it became evident during the LS that the teachers have knowledge of the content required to teach Grade 1.

5.2.1. Foundation Knowledge

During the LS cycles, the teachers drew on and developed their Foundation Knowledge. This was particularly evident during the planning and reflection stages of the LS process. The specific knowledge that the teachers used during these phases and that informed their teaching can be linked to the PCK category. This includes Knowledge of Content and Curriculum, Knowledge of Content and Students, and Knowledge of Content and Teaching.

5.2.2. Knowledge of Content and Curriculum

While planning the lessons the teachers provided evidence of their Foundation Knowledge in the manner in which they:

- unpacked the curriculum, located it in grade 1 curriculum overview and annual teaching plan;

- articulated the content requirements of the curriculum they had to present;
- identified suitable resources for the lessons and adapted activities from the various teachers' manuals;
- and recognised the important key elements of the lesson plan.

5.2.3. Knowledge of Content and Students

During the planning sessions, the teachers:

- based their lessons on the learners' prior knowledge;
- anticipated what learners are likely to think or find confusing in the lesson;
- anticipated learners' responses to the instructions or questions developed for the research lesson; and

When reflecting in debriefing session teachers thought of:

- adapting the activities and assessment to enable greater learner engagement and deeper conceptual development;
- planning differentiated activities based on learners needs.

5.2.4. Knowledge of Content and Teaching

When planning teachers decided:

- on questions that best provide learning opportunities for particular learners;
- on the sequence the mathematical activities in the lesson;
- which teaching strategies to use to assist in the development of the concept;
- on productive mathematical questions to ensure teaching for understanding; and
- on which representations would deepen learners' mathematical thinking;

During feedback and reflection session teachers:

- review and adjusted teaching based on the assessment of learners; and
- engaged with variety of strategies to enhance the learners understanding.

In addition to the knowledge, domains relating to Ball and colleagues PCK, the beliefs of the teachers also had an impact on how they talked about the learning and teaching of mathematics and how they enacted their lesson plans in the classroom.

5.2.5. The role of Lesson Study in developing teachers' Foundation Knowledge

Key changes in the teachers' Foundation Knowledge that occurred between cycle 1 and 2 of the LS process included:

- realising the importance of obtaining the learners' interest in the mathematics classroom (encouraging them to clap while counting);
- providing an opportunity for the learners to reason mathematically about division with and without remainders and justify their thinking;
- encouraging the learners to use different strategies for calculating; and
- reducing the number of activities in an effort to ensure that learners develop a conceptual understanding.

5.2.6. The role of Lesson Study in Teacher Development

LS provides the opportunity for teachers to make teaching a shared practice by working collaboratively to improve teaching. Through teachers' participation in the interactive cycles of LS they explained that the LS process built their confidence; assisted in developing their teaching skills; made them aware how to orientate their practices towards learners' learning; developed their professional knowledge; and provided them the opportunity to reflect on their practice knowledge.

5.2.7. The role of of a 'more knowledgeable other'

Vygotsky (1978) refers to a 'more knowledgeable other' as the person(s) who assists an individual or group by creating opportunities to support their learning. As noted in Chapter 3, I was aware that the participants considered me their senior and looked to me for support. It is my contention that the LS process, as a professional development approach, may need a 'more knowledgeable other' to assist the teachers in developing their PCK.

5.3. LIMITATIONS OF THE RESEARCH

There were a number of limitations in relation to my research. These relate to the generalizability of the research, my positionality as a subject advisor, time and data collection constraints.

This research was a small-scale case study with four Grade 1 teachers from two different schools in the Rustenburg sub-district. The results are therefore not generalizable and no claims can be made that this represents the experiences of LS with Grade 1 teachers.

In Chapter 3, I elaborated on my position as a subject advisor working with the participants to support, guide and monitor their curriculum implementation. I explained at the beginning of the research process that I was new to the implementation of LS and that, as a result, we would be learning together. However, throughout the research process, I was aware that the teachers regarded me as their senior. During the planning and reflection phases, they would look to me to contribute, advise and affirm. Given my role as a subject advisor, it became very difficult for me to restrain myself during the planning and reflection sessions. In an effort to elaborate on their responses, push their thinking or share a new idea, I found myself conducting a workshop with the teachers. In so doing, I encouraged the teachers to be reliant on me during the planning and reflection phases.

Time was a challenge for both the researcher and the participants. Numerous competing activities (e.g. workshops, meetings) at national, provincial and district levels hindered our plans. Finding time for the five of us to meet on a regular basis also proved difficult as the teachers had a relatively full and demanding school load. Due to a taxi strike, all participants did not observe the second improved lesson as the teachers were not able to travel between the two schools. Time also impinged on the success of our planning. Our lengthy discussion during the reflection stage or the two taught lessons in the first cycle, meant that we were not able to plan the improved lessons in the level of detail we would have liked. The issue with time thus had implications on the data collected for my research.

5.4. RECOMMENDATIONS

In Chapter 2, I explained that there is limited research on the use Lesson Study in strengthening Foundation Phase teachers' mathematics and pedagogical content knowledge in South Africa. Given that the results of this research are not generalizable, it may be worthwhile conducting a more extensive LS programme with teachers in the Foundation

Phase. This would be important to ascertain if LS is suited to the Department of Basic Education's Professional Teacher Development plans at national and provincial levels

It would be useful to ascertain whether LS would work in a context where teachers were responsible for their own learning, without the support of a subject advisor or more knowledgeable other. I found myself (having to) workshop the teachers during the planning and reflections stages as they seemingly were recycling what they already knew rather than building new knowledge.

5.5. MY PERSONAL GROWTH

This study has made an important contribution to my growth as a subject advisor in teacher education and as a researcher. Improving and strengthening the Foundation Phase teachers' mathematics and pedagogical content knowledge, classroom practice and learner performance has been the focus of my work as a subject advisor. Having reflected on numerous programmes developed by the DBE and me to support teachers learning, I found it useful to formally research my work

My interaction with teachers benefited me as subject advisor. It provided me with the opportunity to engage with the practicalities of the classroom *in* the classroom. I was able to identify the pedagogical content knowledge gaps that the teachers had through the discussions in the planning and reflections sessions of the lessons. The research illuminated areas that I need to redress when facilitating teacher development workshops. The research also taught me how to work with teachers in a more collaborative manner.

REFERENCES

Abdulhamid, L. (2016). *Primary mathematics in-service teaching development: elaborating 'in-the-moment'*. Unpublished Doctorate of Philosophy, University of the Witwatersrand, Johannesburg

Abdulhamid, L. & Venkat, H. (2013). Using the 'knowledge quartet' to analyse primary mathematics teaching in South Africa: the case of Sibongile. In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, University of Cape Town, 1, 47 – 58.

Adler, J. (2005). Holding the past, living the present and creating a future: trends and challenges in research on mathematics teacher education. In R. Vithal, J. Adler, and C. Keitel (Eds). *Researching mathematics education in South Africa: perspectives, practices and possibilities.* Cape Town: HSRC Press

Adler, J., & Davis, Z. (2006). Opening another black box: research mathematics for teaching in mathematics teacher education. *Journal for research in mathematics education,* 37(4), 270–296.

Adler, P. A., & Adler, P. (1994). Observational techniques. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 377–392). Thousand Oaks, CA: sage publications

Adler, J. & Reed, Y. (2003). *Challenges of teacher development: an investigation of take-up in South Africa*. Pretoria: van Schaik publishers

Al-Jaro, M. S., Asmawi, A., Hasim, Z. (2017). Content analysis of the pedagogical content knowledge in the curriculum of Yemeni EFL teacher education programme. *Arab World English Journa*l, 8(1).DOI: https://dx.doi.org/10.24093/awej/vol8no1.19.

Alvermann, D. E., & Mallozzi, C. A. (2010). Interpretive research. In A. McGill-Franzen & R. L. Allington (Eds.), *Handbook of reading disability research* (pp. 488-498). New York: Routledge

Arani, M. R. S., Fukaya, K. & Lassegard, J. P. (2010). Lesson study as professional culture in Japanese schools: a historical perspective on elementary classroom practices. *Japan Review* 22, 171–200.

Askew, M., Brown, M., Rhodes, V., William, D., & Johnson, D. (1997). *Effective teachers of numeracy: report of a study carried out for the Teacher Training Agency*. London: King's College

Ayodele, O. & Gaigher, E. (2019). *Developing physics teachers' pedagogical content knowledge: reflection on a lesson study intervention.* A paper presented by at the 27th annual conference of the Southern African association for research in Mathematics, Science, and Technology Education in university of KwaZulu-Natal, Durban, South Africa

Baker, L. (2006). Observation: a complex research method. *Library Trends*, 55(1), 171-189. doi: 10.1353/lib.2006.0045

Ball, D. L. (1991). Research on teaching mathematics: making subject-matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching* (Vol. 2) (pp. 1-48). Greenwich: JAI Press

Ball, D.L. and Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: knowing and using mathematics. In J. Boaler, (Ed.). *Multiple perspectives on the teaching and learning of mathematics.* Westport, CT: Ablex Publishing

Ball, D. L., & Bass, H. (2009). Towards a picture-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* (pp.3-14) Edmonton: CMESS

Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: know mathematics for teaching to learners' mathematical futures.* Paper presented at the 43rd Jahrestagung der Gesellschaft für Didaktik der Mathematik, Oldenburg, Germany

Ball, D. L., Hill, H. C. & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator,* 29(1), 14-22.

Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed.) (pp. 433- 456). New York, NY: Macmillan

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes it Special? *Journal of Teacher Education,* 59(5), 389-407.

Barbour, R. S. (2001). Checklist for improving rigour in qualitative research: a case of the tail wagging the dog? *British Medical Journal*, 233, 1115-1117. doi: http://dx.doi.org/10.1136/bmj.322.7294.1115

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. (2010). Teachers' Mathematical Knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180.

Bayaga, A. (2013). Mathematics teaching via the lesson study model. *International Journal of Education Science,* 5(1): 11-18.

Becker, H. S., & Geer, B. (1970*).* Participant observation and interviewing: A comparison. In W. J. Filstead (Ed.), *Qualitative methodology: first hand involvement with the social world* (pp.133–142). Chicago: Markham

Bertram, C., & Christiansen, I. (2014). *Understanding research: an introduction to regarding* *research.* Pretoria: Van Schaik

Bhattacharjee, A. (2014). *Qualitative data analysis-what is it about*. Retrieved on 20 February 2019 from https://blog.udemy.com/qualitative-data-analysis

Bolowana, A. (2014, December Thursday 4). *Ineffective teaching behind poor performance in schools:* Motshekga. SABC News

Breen, S., Meehan, M., O'Shea, A. &Tim Rowland, T. (2018). An analysis of university mathematics teaching using the Knowledge Quartet. *INDRUM 2018*, INDRUM Network, University of Agder, Kristiansand, Norway

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences, 2*(2), 141–178. https://doi.org/10.1207/s15327809jls0202_2

Brown, A.L. (1994). The advancement of learning. *Educational researcher, 23* (8), 4-12.

Burghes, D. N., & Robinson, D. (2010). *Lesson study: enhancing mathematics teaching and learning*. Reading: CfBT Education Trust

Burney, D. (2004). Craft knowledge: the road to transforming schools. *Phi Delta Kappan, 85*(7), 526-531.

Cajkler, W., Wood, P., Norton, J., & Peddar, D. (2013). Lesson study: towards a collaborative approach to learning in Initial Teacher Education? *Cambridge Journal of Education*, 43(4), 537-554.

Carnoy, M., Chisholm, L., 2008. *Towards understanding student performance in South Africa: a pilot study of grade 6 mathematics lessons in Gauteng province*. Cape Town: HSRC Press

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: an experimental study. *American Educational Research Journal,* 26(4), 499-531.

Çelik, A. Ö. and Güzel E.B. (2018). Describing lesson study designed for improvement of mathematics teachers' knowledge of student thinking, I*nternational Journal for Mathematics Teaching and Learning,* 19(2), 176-204.

Centre for Development and Enterprise (CDE). (2007). *Doubling for growth: addressing the math and science challenge in South Africa's schools*. Johannesburg: CDE

Centre for Development and Enterprise (CDE). (2014). *What does research tell us about teachers, teaching and learner performance in mathematics.* Johannesburg: CDE.

Chikiwa, S. (2017). *Investigating mathematical knowledge for teaching required to develop grade 2 learners' number sense through counting*. Unpublished master's thesis, Rhodes

University, Grahamstown

Chikiwa, S., Westaway, L. & Graven, M., (2019). What mathematics knowledge for teaching is used by a Grade 2 teacher when teaching counting? *South African Journal of Childhood Education*, 9(1), a567. https://doi.org/10.4102/ sajce.v9i1.56

Chikamori, K., Ono,Y., & Rogan, J. (2013). A lesson study approach to improving a Biology lesson. *African journal of research in Mathematics, Science and Technology education*, 17(1-2), 14-22.

Chokshi, S. & Fernandez, C. (2004). Challenges to importing Japanese lesson study: concerns, misconceptions, and nuances. *Phi Delta Kappan*, 85(7), 520-525.

Cohen, L., Manion, L., & Morrison, K. (2010). *Research Methods in Education* (6th ed.). New York: Routledge

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education* (7th ed*.).* London: Routledge

Chen, J. Q., & McCray, J. (2012). A conceptual framework for teacher professional development: the whole teacher approach. [NHSA Dialog]. *A Research-to Practice Journal for the Early Childhood Field,* 15(1). doi:10.1080/15240751.2011.636491.

Coe, K. L. (2010). *The process of lesson study as a strategy for development of teaching in primary school: a case study in Western Cape Province, South Africa*. Unpublished PhD dissertation, Stellenbosch University, Stellenbosch

Coe, K.L., Carl, A.E., & Frick, B. L. (2010). Lesson study in continuing professional teacher development: a South African case study. *Acta Academica*, 42(4), 206–230.

Corcoran, D., & Pepperell, S. (2011). Learning to teach mathematics using lesson study. In T. Rowland & K. Ruthven (Eds.), *Mathematical Knowledge in Teaching* (pp. 213–230). Dordrecht: Springer.

Coskun, S.D., & Bostan, M.I. (2018). The analysis of a novice primary teachers’ mathematical knowledge in teaching: area measurement. *International Journal for Mathematics Teaching and Learning*, 19(1), 1-21.

Creswell, J.W. (2002). *Research design: qualitative, quantitative and mixed methods approaches* (3rd ed.). London: Sage

Creswell, J.W. (2012). *Educational research: planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Boston: Pearson

Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into Practice*, 39, 124–134.

Cross, M., Mungadi, R. & Rouhani, S. (2002). 'From policy to practice: curriculum reform in South African education'. *Comparative education,* 38 (2), 171-187.

Darling-Hammond, L. & McLaughlin, M., (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan*, 77 (7), 597-603.

Denscombe, M. (2010). *The good research guide for small-scale social research projects* (5th ed.). Maidenhead: OUP/McGraw-Hill Education

Denzin, N. K. (1978). *The research act*. New York: McGraw-Hill

De Vries, S. (2016). Lesson study in professional learning communities 2014-2016: experiences from the Netherlands. Paper presented at *WALS,* Exeter, United Kingdom.

Doig, B. & Groves, S. (2011). Japanese lesson study: teacher professional development through communities of inquiry. *Mathematics Teacher Education and Development*, 13(1), 77–93.

Dooley, T., Dunphy, E., Shiel, G., Butler, D., Corcoran, D., Farrell, T. …. Travers, J. (2014). Mathematics in early childhood and primary education (3-8 years), *National Council for Curriculum and Assessment,* Educational Research Centre, Charles Stuart University, Australia

Doyle, S. (2007). Member checking with older women: a framework for negotiating meaning. *Health Care for Women International*, 28(10), 888-908.

Dudley, P. (2014). *Lesson study: professional learning for our time*. New York: Routledge

Evens, M., Elen, J., & Depaepe, F. (2015). Developing pedagogical content knowledge: lessons learned from intervention studies. *Education Research International* Journal. doi:10.1155/2015/790417

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan

Fernandez, C. (2002). Learning from Japanese approaches to professional development: the case of lesson study. *Journal of Teacher Education*, 53(5*),* 393-405.

Fernandez, C., & Chokshi, S. (2002). A practical guide to translating lesson study for a U.S. setting. *Phi Delta Kappan*, 84(2), 128-134.

Fernandez, C., Cannon, J., & Chokshi, S (2003). AUS- Japan Lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education*, 19, 171-185.

Fernandez, C., & Yoshida, M. (2004). *Lesson study: a case of a Japanese approach to improving instruction through school-based teacher development*. Mahwah, NJ: Lawrence Erlbaum

Fernández-Balboa, J. M. & Stiehl, J. (1995). The generic nature of pedagogical content knowledge among college professors. *Teaching and Teacher Education*, 11(3), 293-306.

Gess-Newsome, J. (1999). Secondary teachers' knowledge and beliefs about subject matter and their impact on instruction. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp. 51–94). Dordrecht, Netherlands: Kluwer Academic Publishers

Gorman, G. E., & Clayton, P. (2005). *Qualitative research for the information professional* (2nd ed.). London: Facet

Green, W. (2011) *Approved projects: foundation phase teacher education programme*. Pretoria: Government printers

Grossman, P. L. (1990). *The making of a teacher: teacher knowledge and teacher education*. New York: Teachers College Press

Grossman, P. L. (1995). Teachers' knowledge. In L. W. Anderson (Ed.), *International Encyclopaedia of teaching and teacher education* (2nd ed.) (pp. 20-24). UK: Elsevier Science Ltd

Gubrium, J. F., & Holstein, J. A. (2003). From the individual interview to the interview society. *Postmodern interviewing* (pp. 21–49). Thousand Oaks: Sage

Harling, K. (2002). *An overview of case study*. Retrieved on 22 March 2019 from http://www.farmfoundation.biz/news/ article files/1028-1 harling.pdf

Harper, M., & Cole, P. (2012). Member checking: can benefits be gained similar to group therapy. *The Qualitative Report,* 17(2), 510-517.

Hart, L. C., Alston, A. S., & Murata, A. (2011). *Lesson study research and practice in mathematics education: learning together*. New York: Springer

Hennink, M. M., Hutter, I., & Bailey, A. (2011). *Qualitative research methods*. London: Sage

Herbst, P., & Kosko, K. (2014). Mathematical knowledge for teaching and its specificity to high school geometry instruction. In J. Lo, K. R. Leatham, & L. R. Van Zoest (Eds.), *Research trends in mathematics teacher education* (pp. 23–45). New York, NY: Springer

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education,* 35(5), 330-351.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal* *for Research in Mathematics Education*, 39(4), 372-400.

Hill, H.C., & Charalambous, C.Y. (2012). Teacher knowledge, curriculum materials, and quality of Instruction: lessons learned and open issues. *Journal of Curriculum Studies,* 44(4), 559-576.

Hill, H.C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371–406.

Hoadley, U. K. (2012). What do we know about teaching and learning in South African primary schools? *Education as Change,* 16(2), 187-202.

Hoadley, U. & Jansen, J. (2012). *Curriculum: organizing knowledge for the classroom*. Cape Town: Oxford University Press

Hugo, W., Jack, M., Wedekind, V., Wilson, D., (2010). The state of education in Kwa Zulu Natal: *a report to the provincial treasury*. Pietermaritzburg: KZN Provincial Treasury

Isoda, M. (2007). *Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement*. Hackensack, NJ: World Scientific

Isoda, M. (2011). Problem solving approaches in mathematics education as a product of Japanese lesson study. *Journal of Science and Mathematics Education in Southeast Asia,* 34(1), 2–25.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169-202.

Jaffer, S. (2020). Assessment of number in Foundation Phase Initial Teacher Education programmes in four South African universities. In P. Vale, L. Westaway, Z. Nhase, & I. Schudel (Eds.), *Proceedings of the 28th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 76-89). Eastern Cape, South Africa: SAARMSTE.

James, E. A., Milenkiewicz, M. T., & Bucknam, A. (2008). *Participatory action research for educational leadership.* Thousand Oaks, CA: Sage

Kühne, C., Van den Heuvel-Panhuizen, M. & Ensor, P. (2005). Learning and teaching early number: teachers' perceptions. In H. L. Chik & J. L.Vincent (Eds.), *Proceedings of the International Group for the Psychology of Mathematics Education (PME)* (pp.217-224), University of Melbourne, Australia

Kvale, S., & Brinkmann, S. (2008). *Interviews: Learning the craft of qualitative research interviewing* (2nd ed.). Thousand Oaks, CA: Sage

Kyle P. (2015). *Four Ways Lesson Study Improves Teaching, Middle Tennessee Grand Division Teacher of the Year.* Retreived on 20 February 2019 from http://tnclassroomchronicles.org

Lai, C.F. (2012) *Error analysis in mathematics*, Technical Report 1012. Behavioral Research and Teaching, University of Oregon. Retrieved on 16 April 2019 from http://brt.uoregon.edu

Lewis, C. (2002a). Does lesson study have a future in the U.S.? *Nagoya Journal of Education and Human Development,* 1, 1-23.

Lewis, C. (2002b). *Lesson study: a handbook for teacher-led improvement of instruction.* Philadelphia: Research for Better Schools

Lewis, C. C., Perry, R. R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education,* 12, 285–304.

Lewis, C. and Tsuchida, I. (1998), "A lesson is like a swiftly flowing river: research lessons and the page 19 of 20 improvement of Japanese education," *American Educator*, 22(4), 14-17, 50-52.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage

Livy, S. L. (2014). *Development and contributing factors in primary pre-service teachers' mathematical content knowledge.* Unpublished doctoral dissertation, Deakin University, Melbourne, Australia. Retrieved on 02 April 2018 from http://dro.deakin.edu.au/view/DU:30067361

Ma, L. (2010). *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J: Lawrence Erlbaum Associates

Maree, K. (Ed). 2010. *First steps in research.* Pretoria: Van Schaik Publishers

Mason, J. (2006). Mixing methods in a qualitatively driven way. *Qualitative Research,* 6 (1), pp. 9-25.

Matanluka, K., Khalid-Joharib, K. and Matanluk, O. (2012). The perception of teachers and students toward lesson study implementation at rural school of Sabah: a pilot study. *6th International Conference on University Learning and Teaching,* University of Malaysia Sabah, Sabah, Malaysia

Marks, R. (1990). *Pedagogical content knowledge in elementary mathematics.* Unpublished doctoral dissertation, Stanford University, Palo Alto, CA

Mathers, N., Fox, N., Hunn, A. (1998). *Trent focus for research and development in primary health care: using interview in a research Project*. Sheffield: Trent Focus Group

McLeod, S. A. (2015). *Observation methods*. Retrieved on 6 June 2019 from https://www.simplypsychology.org/observation.html

Meiliasari (2018) Developing *Pre-service teachers' pedagogical content knowledge through lesson study***.** Unpublished Doctor of Philosophy, Deakin University, Australia

Merriam, S.B (2009). *Qualitative research: A guide to design and implementation* (2nd ed.). San Francisco: Jossey Bass

Mills College Lesson Study Group. (2005). *How Many Seats? Excerpts of a lesson study cycle* [DVD]. Oakland, CA: Mills College Lesson Study Group

Miles, M., & Huberman, A. M. (1994). *Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage

Mokhele, M.L. (2017). Lesson study as a professional development model for improving teachers' mathematics instruction. *E-Bangi Journal of Social Sciences and Humanities*, 2(17) 49-56.

Murata, A. (2011). Conceptual overview of lesson study: Introduction. In L. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education: Learning together* (pp. 1–12). New York, NY: Springer

National Education Evaluation and Development Unit (NEEDU). (2013). *National report 2012: The State of Literacy Teaching and Learning in the Foundation Phase, April 2013*. Pretoria: National Education Evaluation and Development Unit

O'Meara, N. (2010) 'Knowledge (s) for effective mathematics teaching'. *National centre for excellence in mathematics and science teaching and learning: Resource and Research Guide* Limerick: University of Limerick

Ono, Y. & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa*. South African Journal of Education*, 30(1), 59-74.

Pausigere, P. (2011). *A longitudinal study of numeracy teacher learning and identity within a Community of Practice- Inquiry setting.* Unpublished PhD research proposal, Rhodes University Grahamstown

Organization for Economic Cooperation and Development (OECD). (2012). *Programme for international student assessment (PISA): Results from 2012*. Retrieved on 11 April 2014 from htpp://www.oecd.org/pisa/keyfindings/PISA-2012-results-japan

Organization for Economic Cooperation and Development OECD (2013) *PISA 2012* Results: *What Students Know and Can Do: Student Performance in Mathematics, Reading and*

Science (Vol 1). Retrieved on April 15, 2014, from http://www.oecd.org/pisa/keyfindings/pisa-2012-results-volume-I.pdf

Organization for Economic Cooperation and Development OECD. (2014). *PISA 2012 result in focus what 15-year-olds know and what they can do with what they know.* Retrieved on 12 January 2018 from https://www.oecd.org/pisa/keyfindings/pisa-2012- results-overview.pdf

Pendlebury, S. (2009). Meaningful access to basic education. *South African Child Gauge,* 24–29.

Perry, R. R., & Lewis, C. C. (2009). What is successful adaptation of lesson study in the US? *Journal of Education Change,* 10, 365–391.

Pompea, S.M. & Walker, C.E. (2017*). The importance of pedagogical content knowledge in curriculum development. Proceedings from the 14th conference on educational training in optics and photonics (ETO) (pp.1-15),* Hangzou, China

Reddy, V. (2006). *Mathematics and science achievement at South African Schools in TIMSS 2003.* Cape Town: Human Sciences Research Council

Reddy, V., Isdale, K., Juan, A., Visser, M., Winnaar, L., and Arends, F. (2016). *TIMSS 2015: highlights of mathematics achievement of Grade 5 South African learners*. Cape Town: Human Sciences Research Council

Reeves, C. (2005). *The effect of opportunity-to-learn and classroom pedagogy on mathematics achievement in schools serving low socio-economic status communities in the Cape Peninsula*. Unpublished PhD thesis, University of Cape Town, Cape Town

Reynolds, M.S.J. (2016). *Mathematics teachers' reflection in the context of Lesson Study and the development of their knowledge for teaching*. Unpublished MEd thesis, University of Pretoria, Pretoria

Rock, T. C., & Wilson, C. (2005). Improving teaching through lesson study. *Teacher Education Quarterly,* 32(1), 77–92.

Rouse, M. (2016). *How has your organization data collection strategy evolved the past 10 years?* Retrieved on 17 May 2016 from https://searchcio.techtarget.com/definition/data-collection

Rowland, T. (2004). The psychology of maths education in the early primary years: a response to Munn. *Psychology of Education Review,* 28(1), 17-19.

Rowland, T. (2005). The knowledge quartet: A tool for developing mathematics teaching. *Paper presented at the 4th Mediterranean conference on mathematics education* (pp. 69-81), Nicosia, Greece.

Rowland, T. (2008). The purpose, design and use of examples in the teaching of elementary mathematics. *Educational Studies in Mathematics,* 69(2), 149-163.

Rowland, T. (2012) Contrasting knowledge for elementary and secondary mathematics teaching. *For the Learning of Mathematics* 32(1) 16-21.

Rowland, T. (2013). The Knowledge Quartet: the genesis and application of a framework for analysing mathematics teaching and deepening teachers' mathematics knowledge. *Sisyphus Journal of Education,* 1(3), 15-43.

Rowland, T. (2015) Teacher learning provoked by teaching: equal-area triangles. *Tangenten,* 2, 43-48.

Rowland, T., Huckstep, P., & Thwaites, A. (2003). The knowledge quartet. *Proceedings of the British Society for Research into Learning Mathematics*, *23*(3), 97–102.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education,* 3(8), 255-281.

Rowland, T. & Ruthven, K. (Eds). (2011). *Mathematical knowledge in teaching.* Dordrecht: Springer

Rowland, T., & Turner, F. (2007). Developing and using the 'Knowledge Quartet': a framework for the observation of mathematics teaching. *The Mathematics Educator,* 10(1), 107-124.

Rowland, T., Turner, F., Thwaites, A. and Huckstep, P. (2009). *Developing primary mathematics teaching: reflecting on practice with the Knowledge Quartet*. London: Sage

Rowland, T. and Zazkis, R. (2013) Contingency in the mathematics classroom: opportunities taken and opportunities missed. *Canadian Journal of Science, Mathematics and Technology Education,* 13(2), 137–153.

Rubin, H. J., & Rubin, I. S. (2005). *Qualitative interviewing: the art of hearing data* (2nd ed.). Thousand Oaks, CA: Sage

Rucker, K. (2018). The six flaws of "traditional" professional development. Retrieved on 08 December 2019 from http://www.gettingsmart.com/2018/02/the-six-flaws-of-traditional-professionaldevelopment

Ruthven, K. (2011). Conceptualising mathematical knowledge in teaching. In T. Rowland and K. Ruthven (Eds.), *Mathematical knowledge in teaching,* (pp. 83 – 98). Dordrecht: Springer

Scheiner, T. (2015). Shifting the emphasis toward a structural description of (mathematics) teachers' knowledge. In K. Bewick, T. Muir, & J. Wells (Eds.). *Proceedings of the 39th* *conference of the International Group for the Psychology of Mathematics Education (PME),* (pp. 129–136), Hobart, Australia

Scheiner, T., Montes, M. A., Godino, J. D., Carrillo, J. & Pino-Fan, L. (2017). What Makes Mathematics Teacher Knowledge Specialized? Offering Alternative Views. *International Journal of Science and Mathematics Education.* Retrieved on 20 June 2019 from https://link.springer.com/article/10.1007/s10763-017-9

Sekao R.D. (n.d.). *Lesson study cycle*. Retrieved on 19 July 2019 from https://www.up.ac.za/lesson-study/article/2725785/the-lesson-study-cycle

Schollar, E. (2008). *The primary mathematics research Project 2004-2007.* Johannesburg: Zenex Foundation

Shuilleabhain, A.N. (2015). *Developing mathematics teachers'pedagogical content knowledge through lesson study: A multiple case study at a time of curriculum change.* Unpublished doctoral dissertation, Trinity College Dublin Library

Shulman, L.S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher,* 15, 4-14.

Shulman, L. S. (1987). Knowledge and teaching: foundations of the new reform*. Harvard Educational Review,* 57(1), 1-22.

South Africa. Department of Education (SA.DBE). (2004). *Qualitative overview of the further education and training sector: a sector in transition*, Department of Education, Pretoria

South Africa. Department of Basic Education (SA.DBE). (2009). *National examinations and assessment: report on the national senior certificate examination Results (Part 2).* Pretoria: Department of Basic Education

South Africa. Department of Basic Education (SA.DBE). (2011). *Curriculum and assessment policy statement*. Pretoria: Department of Basic Education

Spaull, N. (2013). *South Africa's education crisis: the quality of education in South Africa 1994-2011*. Retrieved on 21 May 2013 from http://nicspaull.com/2013/10/18/southafricas-education-crisis-1994-2011-my-new-cde-report

Stake, R. E. (1995). *The art of case study*. Thousand Oaks, CA: Sage

Stepanek, J., Appel, G., Leong, M., Mangan, M. T., & Mitchell, M. (2007). *Leading lesson study: a practical guide for teachers and facilitators*. Thousand Oaks, CA: Corwin Press

Stigler, J. and Hiebert, J. (1999). *The teaching gap: best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press

Stols, G. & Ono, Y. (2016). *Lesson Study: an implementation manual*. University of Pretoria, South Africa & Naruto University of Education, Japan

Takahashi, A. (2006). Characteristics of Japanese mathematics lessons. Retrieved on 10 September 2008 from http://www.criced.tsukuba.ac.jp/math/sympo_2006/takahashi.pdf

Takahashi A. (2017). Lesson Study: the fundamental driver for mathematics teacher development in Japan. In: Kaur B., Kwon O., Leong Y. (Eds.) *Professional development of mathematics teachers' mathematics education: an Asian perspective* (p. 47-61). Springer, Singapore

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: maximizing the impact of lesson study. *ZDM*, 48(4), 513-526.

Takahashi, A., & Yoshida, M. (2004). Ideas for establishing lesson-study communities. *Teaching Children Mathematics*, 10(9), 436-437.

Taylor, N. (2008). *What's wrong with South African schools?* Johannesburg: Joint Education Trust

Taylor, N., & Taylor, S. (2013). Teacher knowledge and professional habitus. In N. Taylor, S. Van der Berg, & T. Mabogoane (Eds.), *Creating Effective Schools.* Cape Town: Pearson

Taylor, N., & Vinjevold, P. (1999). Teaching and learning in South African schools. In N. Taylor & P. Vinjevold (Eds), *Getting learning right: report of the president's education initiative research Project.* Braamfontein: Joint Education Trust

Taylor, S. (2012). *A note on matric result trends.* Retrieved on 16 April 2020 from https://nicspaull.files.wordpress.com/2013/01/taylor-2012-matric-results-note-stephen-taylor.pdf

Tsui, B.M., and D.Y.K. Law. (2007). Learning as boundary-crossing in school-university partnership. *Teaching and Teacher Education*, 23, 1289-1301.

Turner F., & Rowland T. (2011) The Knowledge Quartet as an organising framework for developing and deepening teachers' mathematics knowledge. In Rowland T., & Ruthven K. (Eds), *Mathematical knowledge in teaching* (pp. 195-212). Dordrecht: Springer

Van der Walt, M. & De Beer, J. (2016, October). *The affordances of adapted lesson study in South Africa,* two cases. Paper presented at ISTE Conference on Mathematics, Science and Technology Education. UNISA, Kruger Park, South Africa

Van Maanen, J. (1979). Reclaiming qualitative methods for organizational research: a preface. *Administrative Science Quarterly*, 24(4): 520–526.

Venkat, H., & Spaull, N. (2014). *What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007 (Working Paper 13).* Stellenbosch: Stellenbosch University

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development*, 41, 121–130. doi: http://dx.doi.org/10.1066/j.ijedudev.15.02.002

Vygotsky, L. S. (1978). *Mind in society.* Harvard University Press

Watanabe, T. (2002). Learning from Japanese lesson study. *Education Leadership.* 59(6), 36 – 39.

Watson, A. (2008). Developing and deepening mathematical knowledge in teaching: being and knowing. In *MKiT 6, Nuffield Seminar Series*. Retrieved on 22 December 2019 from http://www.mkit.maths-ed.org.uk/MKiT5_Watson_distribution-version.pdf

Westaway, L., & Graven, M. (2018). Exploring grade 3 teachers' resistance to "take up" progressive mathematics teaching roles. *Mathematics Education Research Journal,* 31(1) 1–20. http://doi.org/10.1007/s13394-018-0237-7

White, A. L., & Lim, C. S. (2008). Lesson study in Asia Pacific classrooms: local responses to a global movement. *ZDM—The International Journal on Mathematics Education,* 40, 915–925.

Yadeta, T. & Assefa, T. (2017). The Practices and Challenges of Teacher Educators' Professional Development through Lesson Study: *Focus on Oromia Colleges of Teachers Education,* College of Education and Behavioral Sciences, Jimma University, Jimma

Yin, R. K. (2014). *Case study research: design and methods* (5th ed). Thousand Oaks, CA: Sage

Yoshida, M. (1999, April). Lesson study in elementary school mathematics in Japan: a case study. *Paper presented at the American Educational Research Association Annual Meeting*, Montreal, Canada

APPENDICES

Appendix 1

Summary of selected South African research literature on Lesson Study

Researcher/s	Article	Purpose of Research	Research Question	Methodology	Findings /Implications
Ono and Ferreira (2010)	In-service mathematics and science teachers attending the Mpumalanga Secondary Science Institute (MSSI).	The project was aimed at improving mathematics and science learning of secondary school learners through the use of lesson study for teacher development.		Case study	These writers found that the teachers involved in their study did improve their presentation of lessons, but many of them were challenged in learning what to observe during lessons and how to record it.
Hiroaki Ozawa (2010)	Lesson Study in Mpumalanga Province, South Africa	This study described how the lesson study approach was implemented ,and how the teachers and teacher trainers accepted lesson study for teacher professional development. The study also focused on the content of science lessons and reflections of the lessons to ascertain how new curriculum was implemented and discussed by science teachers.	How the concept of lesson study was accepted and implemented by teachers. How the science lessons were planned and conducted based on the new curriculum. How lesson study was implemented and how it could be improved,	Case study	Science teachers of Mpumalanga who participated in lesson study were very positive with the lesson study approach. They thought that lesson study can contribute to their professional development The researchers maintain that teachers could learn from each other through the planning process. They could learn about content knowledge, teaching methodology and assessment.

Karen Lee Coe (2010) MED Thesis	The process of Lesson Study as a strategy for the development of teaching in primary schools, South Africa	The purpose of this qualitative research study was to determine the value that a group of teachers in South Africa would place on the process of lesson study as a model for their own learning and instructional improvement.	What value will a group of South African teachers place on the process of lesson study as a model for their own learning and instructional improvement?	Case Study	Participants in this study found that their feelings of isolation diminished as they progressed from one cycle to the next. The participants commented that the biggest difference was that the collaboration they experienced during this study was focused specifically on the context of the classroom with particular emphasis on teaching strategies and the learners themselves. The second area identified by the participants as a result of effective collaboration was the introduction of several specific instructional techniques or strategies Foundation phase teachers did not continue with the processes.
Posthuma (2011)	Nature of mathematics teachers' reflection was explored in the context of a Lesson Study.				Posthuma found that teachers had a limited understanding of the concept of reflection and they reflected mostly at the level of recall and rationalisation.

Bayaga (2013)	Mathematics Teaching via the Lesson Study Model	These teachers were preparing to teach in the Intermediate Phase (grades 4-6)			It was found that by employing Lesson Study as a professional development model, several improvements resulted. These included, in particular, the development of meta-cognitive skills, reflection and teacher learning.
Chikamori, Ono, & Rogan (2013)	A Lesson Study Approach to Improving a Biology Lesson.	Introducing the Japanese concept of Lesson Study to a group of science and mathematics educators from Mpumalanga province, South Africa, during a workshop held in Naruto University of Education (NUE), Japan.	This research sought to answer two specific questions regarding change in the content and level of post-lesson discussion over time, as well as the extent of its contribution to the improvement to the lesson.	Case Study 4 weeks workshop at Naruto university	These writers examined the changes that took place in the focus of the teachers' discussion and the quality of the reflection in terms of insight and substance. They concluded that "reflection is a skill which can be learned, but needs to be developed as one of the components of a Lesson Study workshop programme" (p.22).
Maria Susanna Johanna Reynolds (neé Vlok (2016) MED Thesis	Mathematics teachers' reflection in the context of Lesson Study and the development of their knowledge for teaching.	The purpose of this study is to establish the content of the teachers' reflection in the context of a Lesson Study and to analyse this content against the framework of the Knowledge Quartet.	Research question 1: What is the nature of mathematics teachers' reflection in the Lesson Study cycle? Research question 2: In what ways does reflection in Lesson Study contribute to the development of	Action research	As far as research question 1 is concerned, the results of this study suggest that participation in Lesson Study provides an opportunity for teachers to engage collaboratively in a common task, which although experienced

		(Grades 7-9 teachers, Senior Phase)	mathematics teachers' knowledge?		differently, can change profoundly their beliefs about education and classroom practice. The results may also indicate that teachers develop according to their readiness to change in a specific area of teaching. The second aim of this study was to determine whether the reflection by the teachers, during the Lesson Study post-lesson discussions, contributed to the development of their knowledge in respect of teaching mathematics. The researcher used the four dimensions of the Knowledge Quartet to categorise the development of knowledge for teaching mathematics. It was clear from the analysis of the data that teachers did not develop equally in the different dimensions.
Mokhele (2017)	Instructional Leadership through a Lesson Study Project	The purpose of this article is to explore teachers' experiences on one of the mathematics interventions that they participated in		Case study	In this study teachers found the lesson study informative and recommended it as a tool to improving their teaching and learning of

					mathematics. Based on the data collected, lesson study is one of the professional development models that teachers would be happy to participate in.
Coe, Arend & Frick (2010)	Lesson study in continuing professional teacher development: a South African case study	Sought to determine the value that a group of teachers would place on the process of lesson study as a model for their own learning and instructional improvement.	This article explores the value of lesson study as an approach to facilitate CPTD in South Africa, thereby making it more effective as a catalyst for instructional improvement.	Case Study	The findings highlight several areas where lesson study can be considered an effective CPTD programme within the South African context.
Van der Walt & De Beer (2016)	The affordances of adapted lesson study in South Africa	To investigate the affordances of lesson study approaches in the process of enculturation within a community of practice.	How can an adapted lesson study facilitate student teachers' professional development, pedagogical content knowledge and reflection?"	Case Study	Findings suggest that lesson study has potential to facilitate the building of relationships between theory and practice as well as the professional development of STEM teachers within communities of practice.
Ayodele & Gaigher (2019)	Benefits and challenges of lesson study: A case of teaching Physical Sciences in South Africa	The study seeks to explore teachers' experiences regarding the teaching of electricity and magnetism during a Lesson Study intervention.	The study addressed three questions: 1. How does Lesson Study influence teachers' professional knowledge? 2. How does Lesson Study influence teachers' attitudes and beliefs? 3. What are the contextual factors affecting teachers'	Case Study	Findings revealed that the collaborative planning was experienced as beneficial by all four participants. However, it was also found that Lesson Study may be inefficient in cases where there are gaps in teachers' content knowledge. It is therefore

			participation in the Lesson Study process?		essential that a subject specialist participate in Lesson Study meetings to provide support where inadequate content knowledge obstructs meaningful cooperation.
Adler and Alshwaikh (2019)	A Case of Lesson Study in South Africa	Wits Maths Connect Secondary project (WMCS) is a research-linked professional development project aimed at improving the learning and teaching of mathematics in previously disadvantaged secondary schools in one province in South Africa.	What changes occur in the example set across the lesson plans over a cycle? How do these changes evolve?	Case Study	The study confirms that exemplification is a key element of the mathematics teaching framework developed in the project to support planning and reflection. The teachers' reflection suggested that working on examples had been enabling and empowering.

Appendix 2

MATHEMATICS LESSON OBSERVATION SHEET
FOR
THE LESSON STUDY

Observation Schedule

The observation schedule was developed in collaboration with the teachers participating in my research. During the planning of lessons, we discussed what the focus of our observations should be.

We have developed this schedule in line with the focus on a lesson that seeks to develop the learners' competence in solving grouping and sharing word problems using counters and pictures as a method of representation.

OBSERVER'S NAME…………………………………..

OBSERVER'S INSTITUTION…………………………….

DATE OF LESSON OBSERVATION…………………….

Name of lesson presenter:	
Name of school:	
Grade:	
Topic:	
Lesson objective(s):	
Duration of the lesson:	
Questions:	**Observation notes**
How did the learners respond to the introduction of the lesson?	
How did the learners respond to the mental activities that the teachers gave to them? (What were some of the strategies they used for calculating?)	

How did the learners respond to the teachers' explanation of the *word problem*?	
Sufficient time was given to learners to think, discuss and respond either individually, in pairs or in groups (Explain your answer)	
What challenges did the learners experience in sharing equally?	
How did the teacher respond to these challenges?	
How did the teacher adapt the lesson to support the development of learners' experiencing learning barriers?	
Were the objectives of the lesson met? (Explain your answer)	
Was the lesson consolidated (Motivate your answer)	

Appendix 3

Individual interview questions

Interview Schedule

The interviews will follow this procedure. Below are examples of possible questions. The list is not definitive.

Phase 1 (Individual) (Before classroom observation and lesson presentation)

1. How long have you been a teacher in Foundation Phase?
2. Have you been trained to conduct error analysis?
3. What is your understanding of error analysis?
4. What is the impact of error analysis on identifying the focus concept of the lessons?
5. How do you formulate lessons objectives? / Where do you draw you lessons objectives?

Appendix 4

Focus group interview questions

Phase 2 (Focus Group) (After the classroom observations, lesson presentations and reflections)

1. Explain your understanding about Lesson Study?
2. What benefits or challenges, have you experienced in engaging in the Lesson Study?
3. What did you learn about identifying learners’ conception, misconception and errors before planning a lesson?
4. Based on your experience, how do you think Lesson Study can be used to develop teachers’ mathematics and pedagogical knowledge?
5. Will you encourage other teachers to participate in Lesson Study? Why? / Why Not?
6. What are your thoughts about planning lessons with other teachers?
7. How did you feel after being observed by other teachers?
8. What has been the impact of lesson reflection to your teaching?

Appendix 5

ETHICS PERMISSION FROM RHODES UNIVERSITY

RHODES UNIVERSITY
Where leaders learn

Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

20 May 2019

Dr Lise Westaway

Email: l.westaway@ru.ac.za

Dear Dr Lise Westaway

Re: The use of Lesson Study to develop teachers' Mathematics Knowledge for Teaching ,The use of Lesson Study to develop teachers' Mathematics Knowledge for Teaching , 0321 , May ; 2019

Principal Investigator: Dr Lise Westaway

Collaborators: naomi kgothego , Mrs Naomi Kgothego , ,

This letter confirms that the above research proposal has been reviewed by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee and **PROVISIONALLY APPROVED PENDING GATEKEEPER PERMISSION**.

Gatekeeper permission is required from:

North-Wets Department of Education

School Principal

Once the Gatekeeper permission letter/s have been received please forward to the Ethics Coordinator, (ethics-committee@ru.ac.za) in order to finalise your ethics approval.

Sincerely

Prof Joanna Dames

Chair: Human Ethics sub-committee, RUESC- HE

Appendix 6

Department of Education and Sport Development
Departement van Onderwys en Sportontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko
NORTH WEST PROVINCE

1st Floor, East Wing,
Private Bag X2044,
Mmabatho 2735
Tel.: (018) 388-3433
Fax.: 086-514-0126
e-mail: motlhabanej@nwpg.gov.za

OFFICE OF THE SUPERINTENDENT-GENERAL

Enq. : Dr T Phorabatho
Tel. : 018 388 3071/3433

To: Dr Lise Westaway
Rhodes University
Faculty of Education

From: Mrs S M Semaswe
Superintendent-General

Date : 17 July 2019

PERMISSION TO CONDUCT RESEARCH: MS Ntsae Naomi Kgothego

Permission is hereby granted to your student Ms NN Kgothego to conduct research in the department as requested, subject to the following conditions:

- She contacts the relevant School Principals for her target schools about her request with this letter of permission.
- Considering that your research will involve both Educators and Learners, the general functionality of the school should not be compromised by the research process.
- The participation in your project will be voluntary.
- The principles of informed consent and confidentiality will be observed in strictest terms, and
- The findings of your research should be made available to the North West Department of Education and Sport Development upon request.

Best wishes

Mrs S M Semaswe
Superintendent-General

17/07/2019
Date

"Towards Excellence in Education and Sport Development"

Appendix 7

No 3 Palmboom treet
Geelhoutpark Ext 4
Rustenburg
0300
Date: ___________________

Mr/s. _________________ (Principal)
____________________School
____________________ Street

Dear Mr/s __________________

Re: Permission to conduct research

My name is Ntsae Naomi Kgothego, a Masters of Education student at Rhodes University. I am conducting research in the field of Mathematics Education under the supervision of Dr. Lise Westaway. The title of my thesis is: *The use of Lesson study to strengthen foundation phase teachers' Mathematics Knowledge for Teaching* in Rustenburg sub-district. The provincial Department of Education has given me permission to approach your school for my research. This research has also met the requirements of the Rhodes University Education Faculty Ethics Committee. Copies of both documents are attached to this letter.

The research aims to develop teachers' mathematics knowledge and pedagogical practices to improve learner performance in ways that are sustainable. I intend to use the Lesson Study approach where the Grade 1 teachers take responsibility for their professional development and work collaboratively to improve their mathematics and pedagogical knowledge.

I will collect data through observations, interviews and document analysis. I will ask for permission from the Grade One teachers and the parents of the learners in the Grade One classes. While this research does not focus on the learners, they are indirectly involved in the research. The research will involve collaborative lesson planning, observation and reflection sessions with participating teachers. I will videotape teachers' lessons for the purposes of collaborative reflection. Data collected will be treated with confidentiality and pseudonyms will be used for the participants and school. Participation will be voluntarily and the teachers may withdraw from the research at any time. The role of the school is also voluntary and the principal may withdraw the school from participating at any time.

Once I have received the permission to approach the Grade One teachers to participate in the study, I will clearly inform them about the research. I will make arrangements with the school for data collection phases that is, the lesson observations to take place. After the research, a copy of the research report will be made available to the school. Once the teachers have consented, I will approach the parents and request their consent.

Should you require further information, please can contact me on 0729583218 or nkgothego@gamail.com.

If you grant me permission to conduct research, please complete and return the attached form.

Thank you for taking time to read this information.

Yours Sincerely
Ntsae Naomi Kgothego

____

(Rhodes University)

INFORMED CONSENT FORM

School Principal

I give permission for you to conduct your research with Grade 1 teachers at my school. Grade 1 teachers will be research participants in a Lesson study with the view to strengthen their Mathematics Knowledge for Teaching and to improve their classroom practice.

I have read the research information explaining the purpose of the research and understand that:

- The role of the school is voluntary
- I may decide to withdraw the school's participation at any time
- All information obtained will be treated with confidentiality
- Participants and school names will not be written in the study (Pseudonyms will be used)
- A report of the findings will be made available to the school
- I may contact Ntsae Naomi Kgothego on 0729583218 or nkgothego@gmail.com

____________________________________ _________________________

Principal Signature

____

Date

Appendix 8

RHODES UNIVERSITY
Where leaders learn

No 3 Palmboom street

Geelhoutpark Ext 4

Rustenburg

0300

Dear ________________________

Re: Invitation to participate in a research study

My name is Naomi Kgothego and I am the subject advisor for Foundation Phase mathematics in the Rustenburg sub- district. I have registered for Master's in Education at Rhodes University in Grahamstown, South Africa. I am interested in e*xploring 'The use of Lesson Study to strengthen Foundation Phase teachers' Mathematics Knowledge for Teaching in Rustenburg sub-district'.* As a Grade 1 teacher you are invited to be one of my participants.

The research aims to develop teachers' mathematics knowledge and pedagogical practices in ways that are sustainable in order to improve learner performance. I propose the use of Lesson Study which is a collaborative learning approach. Together we will plan mathematics lessons, observe each other teaching the jointly planned lessons, and reflect on the lessons and make suggestions of how to improve them. During the lesson presentations, the focus of the observations will be on the learning of the learners rather than the teaching.

I will collect data through observations, interviews and document analysis. I might request to videotape some of the lessons for collaborative reflection purposes. Data collected will be treated with confidentiality and pseudonyms will be used for the names of participants and school.

If you agree to participate, I will explain in more detail what would be expected of you and provide you with more information to understand the details of the research. These guidelines would include potential risks, benefits, and your rights as a participant. Once this study has been approved by the Ethics Committee of the Faculty of Education you will be sent the letter of ethical approval.

Participation in this research is voluntary and you have the right to withdraw at any given time during the research. To participate, you will be asked to sign a consent form to confirm that you understand and agree to the conditions, prior to collection of data.

Thank you for your time and I hope that you will respond favourably to my request.

Yours sincerely,

Student name: Ntsae Naomi Kgothego Signature: ___________________

INFORMED CONSENT FORM

Research Project Title:	The use of Lesson Study to strengthen Foundation Phase teachers' Mathematics Knowledge for Teaching
Principal Investigator:	Ntsae Naomi Kgothego
Participation Information	
• I understand the purpose of the research study and my involvement in it. • I understand the risks and benefits of participating in this research study. • I understand that I may withdraw from the research study at any stage without any penalty. • I understand that participation in this research study is done on a voluntary basis. • I understand that while information gained during the study may be published, but that I will remain anonymous and no reference will be made to me by name. • I understand that audio and video recording may be used. • I understand that I will be given the opportunity to read and comment on the transcribed interview notes. • I confirm that I am not participating in this study for financial gain.	
Information Explanation	
The above information was explained to me by: Ntsae Naomi Kgothego	
The above information was explained to me in English and I am in command of this language.	
Voluntary Consent	
I, hereby voluntarily consent to participate in the above-mentioned research.	
	Date: / /
Investigator Declaration	
I, Ntsae Naomi Kgothego, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: / /

Appendix 9

No 3 Palmboom street

Geelhoutpark Ext 4

Rustenburg

0300

Dear Parent/Guardian

Request for parental/ guardian consent

My name is Naomi Kgothego and I am the subject advisor for Foundation Phase mathematics in the Rustenburg sub- district. I have registered for Master's in Education at Rhodes University in Grahamstown, South Africa. I am interested in e*xploring 'The use of Lesson Study to strengthen Foundation Phase teachers' Mathematics Knowledge for Teaching in Rustenburg sub-district'*. .

For the successful completion of my Master's degree with Rhodes University, I need to do research in your child's school. S/he will experience a lesson taught by a teacher while other teachers and I will observe the lesson. The teachers will discuss the observations made from the lesson after the lesson was taught in the reflection session. The focus is not on the learner as a person, but how do learners learn mathematics.

Any information that is obtained in connection with this study and that can be identified with your child will remain confidential. Confidentiality will be maintained by means of using pseudonyms in the coding procedures; the name of the school will also not be disclosed.

Data will be locked up and kept for a period of five years after the thesis is submitted. No persons other than the teachers, my supervisor and I will have access to the data.

Should you require further information, please can contact me on 0729583218 or nkgothego@gamail.com.

Yours sincerely,

Student name: Ntsae Naomi Kgothego Signature: ___________________

INFORMED CONSENT FORM

Parent/Guardian

I the undersigned, hereby give Mrs. N.N Kgothego permission to conduct her research for her Master's degree, in this school, while my child is present in the class.

Learner's Name: ____________________________ Grade: _____________

Parent/Guardian: ________________________________ Date: ______________

Appendix 10

MATHEMATICS DAILY PLAN	
GRADE: 1	**DAY AND DATE:** Wednesday, 11 September 2019
WHOLE CLASS ACTIVITIES (20 MINUTES)	
COUNT OBJECTS & COUNTING FORWARD & BACKWARDS Count forward 1-20 in 2's and 5's. • Let learners fill in two 10 frames with 20 counters in total. • Let the learners count forwards in 2s from 1 by moving 2 counters out of the 10 frames. • Stop learners at 12. • ask the following. - How many 2s in 12? six 2s - how many 2s in 10? five 2s - how many 2s in 16? eight 2s - who can write a number sentence for counting in 2s up to 16 on the chalkboard? - 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = ……………………………………………………………………. • let the learners fill in two 10 frames with 20 counters in total. • Let the learners count forwards in 5s from 1 by moving 5 counters out of the 10 frames. • stop the learners at 15. • Ask the following. - how many 5s in 15? three 5s - how many 5s in 10? two 5s - how many 5s in 20? four 5s - who can write a number sentence for counting in 5s up to 20 on the chalkboard? - 5 + 5 + 5 +5 = 20	**MENTAL MATHS** Recall addition fact up to 5. Calculation strategy. Put the larger number first in order to count on or count back 3 + 2 = 4 + 1= 2 + 2 + 2 =

<table>
<tr><th colspan="2">CONCEPT DEVELOPMENT: WHOLE CLASS (30 MINUTES)</th></tr>
<tr>
<td>
TOPIC: Grouping and sharing leading to division

SKILLS/OBJECTIVES:

Solve word problems in context and explain own solution to problems involving equal sharing with whole numbers up to 15 and with answer that may include remainders

TEACHING ACTIVITIES:

Write a word problem with one sentence per line.
My friend and I share 8 sweets equally. How many sweets can I get?
Read the word problem aloud several times.
Ask the following:
when learners understand the story, let them read, following after you sentence by sentence.
form pairs and let each pair have counters.
let them share 8 counters equally.
Let a pair present how they shared the 8 counters.
confirm with the whole class that we halve something when we share something equally into 2 parts by circling each group of 4 counters.
when the pairs try to share 8 counters equally, they have to take one counter at a time and move it to one of 2 parts equally. Eventually, each person gets 4 sweets.
Learners will refer to the booklet

Let learners open booklet page 1
Read the word problem aloud several times.

</td>
<td>
LEARNING ACTIVITIES:

Platinum Learner book page 62 no. 1 (a-b)
worksheet attached

1. There are five children and ten crayons.
how many crayons will each child get?

2. There are 15 crayons and six children.

How many crayons will each child get?

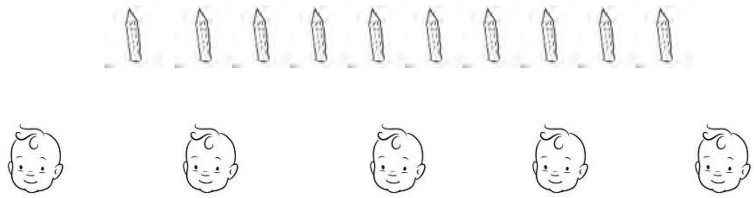
How many crayons left?
</td>
</tr>
</table>

• when learners understand the story, let them read, following after you sentence by sentence. • underline the numbers and question. • Learner work in pair of 2 • Ensure that each pair has 10 counters • Learners share bottle tops to faces • Let a pair present how they shared the 8 counters. • when the pairs try to share 10 counters/sweets equally, they have to take one counter at a time and move it to one of 2 parts equally.	
ASSESSMENT: Informal Assess learners that can do sharing up to 15 and with answers that may include remainders.	

SHARING

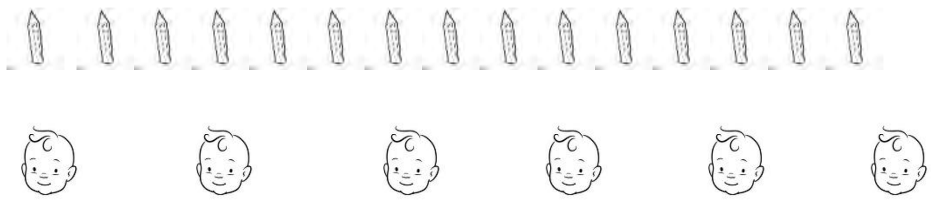
1. Solve the following:

   1. share the crayons equally between five children.
      How many crayons will each child get?



Each child will get ______ crayons.

   2. Share the crayons equally between six children.
      How many crayons will each child get?
      How many crayons are left?



Each child will get ______ crayons and _____ are left over.