

Gravitational entropy and thermodynamics away from the horizon

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Abstract

We define, by an integral of geometric quantities over a spherical shell of arbitrary radius, an invariant gravitational entropy. This definition relies on defining a gravitational energy and pressure, and it reduces at the horizon of both black branes and black holes to Wald's Noether charge entropy. We support the thermodynamic interpretation of the proposed entropy by showing that, for some cases, the field theory duals of the entropy, energy and pressure are the same as the corresponding quantities in the field theory. In this context, the Einstein equations are equivalent to the field theory thermodynamic relation $TdS=dE+PdV$ supplemented by an equation of state.

1 Introduction

Wald's Noether charge entropy (NCE) [1, 2, 3] is a conserved geometric charge that is associated with black hole (BH) horizons. The NCE is related to Bekenstein's area entropy by a specific identification of the dimensionful gravitational coupling [4] and serves as an important conceptual tool for learning about BH thermodynamics and the holographic paradigm (*e.g.*, [5, 6, 7]). The Noether current is defined on the BH horizon, whereas the Noether charge is confined to the horizon bifurcation surface and interpreted thermodynamically as the gravitational entropy of the BH. This thermodynamic interpretation relies on relating the horizon Noether charge to the ADM Noether charge, defined at infinity.

In this paper, the gravitational entropy is liberated from the confines of the horizon and its bifurcation surface. To do so, we introduce local, geometric definitions for an energy density and a pressure, and then apply standard thermodynamics

To be able to define all these quantities, it is necessary to restrict to theories of gravity having a well-defined canonical phase space. We need to consider only UV-complete theories and require that any effective descriptions should reflect their unitarity. (For a detailed discussion on this point, see [8].) Indeed, we have found that it is only sensible to talk about the gravitational entropy away from the horizon for theories having at most two time derivatives in the equations of motions. Any candidate must then either be a theory of Lovelock gravity [9] or, else, made to mimic this class with a suitable choice of boundary conditions. The Wald formalism manages to evade such issues of unitarity because it is valid strictly on the horizon and higher-derivative terms can not directly contribute on this Killing surface [10].

To facilitate the thermodynamic interpretation, we find that it is useful to consider a black brane (BB) with a Poincaré-invariant horizon in an asymptotically anti-de Sitter (AdS) spacetime. In this case, a dual field theory is hosted at the AdS outer boundary [11, 12, 13]. An asymptotically flat spacetime and/or a spherical horizon topology may still be considered but then, in such cases, a clear thermodynamic interpretation is not always so forthcoming. Much of the supporting analysis closely follows that of [14], and so we defer many calculations and caveats to this article.

2 Gravitational entropy away from a horizon

We wish to propose a definition for the gravitational entropy of a region of spacetime. Our concept of a gravitational entropy is that it should be defined in terms of geometric and covariant quantities and should satisfy relations analogous to the first law (and also the second law). Our idea is to define first the energy density ρ and pressure p , and then use the equilibrium relation $sT = \rho + p$ to define the product of the entropy density s and the temperature T . In symmetric enough cases, such as at a stationary Killing horizon or via further inputs and boundary conditions, it is possible to factor the product and define separately s and T . Whereas Wald exploited the former situation, we are considering the second.

For simplicity, we restrict considerations to $d + 1$ -dimensional spacetimes that are either asymptotically flat or asymptotically AdS and can be decomposed into d -dimensional radial slices with either spherical or planar topologies. The line element can then be decomposed as $ds^2 = N^2 dr^2 +$

$h_{\alpha\beta} dx^\alpha dx^\beta$, with Roman/Greek indices denoting spacetime/hypersurface directions. Also, $\sqrt{-g} = N\sqrt{-h}$.

In these cases, we turn to the Brown–York tensor [15, 16] to define the energy density and pressure, $T^{ab} = \lim_{r \rightarrow \infty} \frac{2}{\sqrt{-h}} \left(\delta I / \delta \dot{h}_{ab} \right)$, where $I = \int d^D x \sqrt{-g} \mathcal{L}$ is the action of the gravitational theory with Lagrangian \mathcal{L} and a dot denotes a derivative with respect to r . As this tensor is generically divergent, a renormalization procedure is required. For an asymptotically flat spacetime, one subtracts the reference Minkowski background, whereas standard procedures of holographic renormalization [17, 18] should be applied when the spacetime is asymptotically AdS. Thus, the regularized Brown–York tensor has a well-defined meaning for any spacetime with a timelike or null infinity.

Our next step is to move this tensor (once renormalized) from $r \rightarrow \infty$ to a finite radius, R . The resulting tensor $\hat{T}^{ab}(R)$ is, like the Brown–York tensor, precisely defined and finite. As we are dropping terms that diverge at infinity but are otherwise finite, the tensor $\hat{T}^{ab}(R)$ is a projection of the boundary tensor into the bulk and not a reapplication of the Brown–York formula at another radius.

Having identified an energy–momentum stress tensor, we associate a local energy density and pressure with $\hat{T}^{ab}(R)$ and identify these as “gravitational” densities. Then, respectively,

$$\rho(R) = -\sqrt{-g} \frac{\delta(r-R)}{\sqrt{g_{rr}}} \hat{T}_t^t, \quad (1)$$

$$p(R) = \sqrt{-g} \frac{\delta(r-R)}{\sqrt{g_{rr}}} \hat{T}_x^x. \quad (2)$$

It is implicitly assumed that the densities are to be evaluated at fixed time in addition to fixed radius. Hence, we can also obtain a gravitational energy $E(R)$ by integrating $\rho(R)$ over the spatial coordinates, which amounts to integrating over a spherical shell. A straightforward calculation shows that, for a Schwarzschild geometry, $E(R)$ simply gives back the ADM mass.

The standard thermodynamic relation $Ts = \rho + p$ then implies that a gravitational entropy density s and its associated temperature T can also be defined:

$$Ts|_{r=R} = -\sqrt{-g} \left[\hat{T}_t^t - \hat{T}_x^x \right] \frac{\delta(r-R)}{\sqrt{g_{rr}}}. \quad (3)$$

We can, like above, obtain a gravitational entropy $S(R)$ by integrating $s(R)$ over a spherical shell. We interpret s as the gravitational entropy of the region $r \leq R$.

A prerequisite for identifying the entropy is a clean separation between s and T at any given radius. This is not generically possible but rather requires a high degree of symmetry. Thus, it is more meaningful to consider the composite quantity $\Theta \equiv Ts$, as we will often do. In the case of BH and BB geometries, we can identify T with the Hawking temperature. Given that the gravity theory is Einstein's (or any other two-derivative theory that obeys the equivalence principle), the above definition for $S(R)$ then gives back the Bekenstein–Hawking entropy as R tends to the horizon. The validity of this outcome will be made clear by the analysis to follow.

And so a thermodynamic interpretation emerges from the mechanical description. From this point of view, the Einstein equations inform the mechanical interpreter how to change the bulk geometry such that the thermodynamic framework remains valid at arbitrary radial positions.

3 Gravitational entropy and thermodynamics away from the horizon for black branes in AdS

3.1 Background and conventions

We begin here with a $d+1$ -dimensional stationary BB in an (asymptotically) AdS spacetime and the following ansatz for the metric: $ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + g_{xx} dx^i dx_i$, $i = 1, \dots, d-1$, with the horizon of the BB and the AdS outer boundary located at $r = r_h$ and $r \rightarrow \infty$ respectively. We sometimes use “conformal” metrics for which $g_{rr} = -g_{tt}^{-1}$. All the metric components are assumed to depend only on r ; meaning that the (t, x_i) subspace is Poincaré invariant.

At the horizon, it is assumed that g_{tt} has a first-order zero, g_{rr} , a first-order pole and the others are finite. It is also assumed that the gravitational theory asymptotically limits to Einstein gravity at large distance scales. Since the spacetime already asymptotes to pure AdS as $r \rightarrow \infty$, the metric has, with the AdS scale set to unity, the asymptotic form $-g_{tt} \rightarrow r^2$, $g^{rr} \rightarrow r^{-2} \left[1 - \left(\frac{r_h}{r}\right)^d\right]$ and $g_{xx} \rightarrow r^2$. When we use the $d+1$ -decomposed form for

the metric, then $h_{\alpha\beta} = h_{\alpha\beta}(r)$. An index of x or y is used as a representative of the spatial coordinates x_i and, so, should not be summed over.

When $\mathcal{L} = \mathcal{L}(g_{ab}, \mathcal{R}_{abcd}, \psi)$ (with \mathcal{R}_{abcd} denoting the Riemann tensor and ψ , additional matter fields), Wald's NCE is often expressed as [1, 2, 3] $S_W = -\frac{2\pi}{\kappa} \oint_{\Sigma} \left(\frac{\partial \mathcal{L}}{\partial \mathcal{R}_{abcd}} \right)^{(0)} \epsilon_{ab} \epsilon_{cd}$, where the integration is over a cross-section of the horizon, the superscript (0) indicates that the evaluation is on-shell and ϵ_{ab} is the horizon bi-normal vector. The surface gravity κ is related in the standard way to the Hawking temperature $T_H = \kappa/2\pi$, which in terms of the brane metric is given by $T_H = -\frac{1}{4\pi} \frac{g_{tt,r}}{\sqrt{-g_{tt}g_{rr}}} \big|_{r=r_h}$. Because the discussion is on a BB for which the “cross-section” is infinite, it is more practical to discuss the entropy density s_W , as defined by the integrand of S_W .

3.2 Gravitational entropy away from the horizon

The Brown–York stress tensor for a Lovelock theory is given by (see Eq. (12)),

$$T^{tt} = 4(d-1) \mathcal{X}^{txtx} K_{xx}, \quad (4)$$

$$T^{xx} = 4 \left[\mathcal{X}^{xtxt} K_{tt} + (d-2) \mathcal{X}^{xyxy} K_{yy} \right], \quad (5)$$

where we have defined the tensor $\mathcal{X}^{abcd} \equiv \frac{\partial \mathcal{L}}{\partial \mathcal{R}_{abcd}}$ and $K_{\alpha\beta}$ denotes the extrinsic curvature,

$$K^t_t = \frac{1}{2} \frac{g_{tt,r}}{g_{tt}} \frac{1}{\sqrt{g_{rr}}}, \quad (6)$$

$$K^x_x = \frac{1}{2} \frac{g_{xx,r}}{g_{xx}} \frac{1}{\sqrt{g_{rr}}}. \quad (7)$$

In terms of the $d+1$ -decomposed brane metric, $K_{\alpha\beta} = \dot{h}_{\alpha\beta}/2N$.

Now suppose that the “couplings” or \mathcal{X} 's do not depend on the polarization; that is, \mathcal{X}^{ab}_{ab} (with $a \neq b$ and no summation of the indices) is independent of the choice of a and b , as would be the case for Einstein and (*e.g.*) $f(\mathcal{R})$ gravity. Then,

$$\Theta(R) = 4\sqrt{-g} \left[K^t_t \mathcal{X}^{rt}_{rt} - K^x_x \mathcal{X}^{rx}_{rx} \right] \frac{\delta(r-R)}{\sqrt{g_{rr}}}. \quad (8)$$

Like before, it is implicitly assumed that time is also fixed to some specific value, so the entropy is obtained by integrating the density on the $d-1$ transverse space — just like is done to obtain the NCE.

The entropy density should, on general grounds, have a structure that is explicitly Poincaré invariant. The second term on the r.h.s of Eq. (8) vanishes on the horizon, as evident from Eq. (7). (At the horizon, the \mathcal{X} 's are always order-unity numbers times $1/16\pi G$.) Then, on a cross-section of the horizon, $\Theta(r_h, t_h) = 4\sqrt{-h}K^t_t\mathcal{X}^{rt}_{rt}$ and Θ reduces to the Wald NCE density,

$$T_H s_W = 4 \left[\sqrt{-g_{tt}} \sigma K^t_t \mathcal{X}^{rt}_{rt} \right]_{r=r_h}, \quad (9)$$

where $\sigma = (g_{xx})^{\frac{d-1}{2}}$ is the area density.

Things are not so simple for theories with polarization-dependent couplings like (*e.g.*) Gauss–Bonnet gravity. A theory whose couplings depend on the polarization can also be viewed as Einstein supplemented by additional vector or higher-form fields. So, it is expected that additional thermodynamic parameters (chemical potentials) would be needed to fully describe the thermodynamics. However, these cases are not conceptually different from the Einstein case, as long as they have a well-defined phase space and allow for the separation of the entropy and the temperature; for example, by supporting a radially dependent but Poincaré-invariant solution.

4 Entropy and thermodynamics of the dual field theory

4.1 Canonical variables and phase space

Before identifying the field-theory dual of Θ , we recall some basics. According to the standard holographic dictionary, the dual of a bulk operator Φ is defined by $\langle \tilde{\Phi}(x^i) \rangle = \lim_{r \rightarrow \infty} \frac{1}{\sqrt{-h}} \Pi_\Phi(r, x^i)_{ren}$, where the conjugate momentum Π_Φ is, as usual, $\Pi_\Phi(r, x^i) \equiv \frac{\partial \sqrt{-g} \mathcal{L}}{\partial \Phi(r, x^i)}$. The subscript *ren*, above, is meant as a reminder that appropriate procedures of subtraction and renormalization are required at the boundary. (See, *e.g.*, [17, 18] for further details.)

The identification of the canonically conjugate variables or, equivalently, the Hamiltonian decomposition of the action requires a well-defined variational principle and, so, well-formulated boundary conditions, a Cauchy surface and a set of boundary terms. This restriction eliminates all higher-derivative gravitational theories except for those of the Lovelock class [19].

Given that the canonically conjugate variables are definable, these can be deduced by inspecting the decomposed form of the action I , which is given

in Eq. (34) of [14] (and see [20] for the original work):

$$I = \int dr \int dt \int d^{d-1}x \sqrt{-h} \left(4 U^{r\beta\gamma r} \dot{K}_{\beta\gamma} + 2NU^{\alpha\beta\gamma\delta} K_{\alpha\gamma} K_{\beta\delta} + \dots \right), \quad (10)$$

where U^{abcd} is an auxiliary field which is equal to \mathcal{X}^{abcd} once the field equations have been imposed. The second term is the Hamiltonian.

The conclusion of [14] is that \mathcal{X}^{rt}_{rt} and K^t_t are canonical conjugates and, likewise, for the pair \mathcal{X}^{rx}_{rx}, K^x_x .

4.2 The field theory dual of the gravitational entropy

Recalling the standard relationship between the on-shell action and canonically conjugate variables, we observe that the dual of an operator of the form $\Phi \Pi_\Phi$ is simply $\Pi_\Phi \Phi$, as the roles of the source and operator get interchanged but the original term in the on-shell action remains the same.

Therefore, the field theory dual to Θ goes as

$$\tilde{\Theta} = \lim_{r \rightarrow \infty} 4\sqrt{-h} \left[\mathcal{X}^{rt}_{rt} K^t_t - \mathcal{X}^{rx}_{rx} K^x_x \right] \Big|_{ren}, \quad (11)$$

subject to the usual renormalization procedures at the boundary.

The gravitational entropy and its field-theory dual both originate from the action term $U^{r\beta\gamma r} \dot{K}_{\beta\gamma}$ which is of the usual adiabatic-invariant form $J = \int^R \Pi_\Phi(r) d\Phi/dr dr$. Here, the radial coordinate r should be thought of as replacing time, the integration over t and the x_i 's is implied, and we have used that $K_{\alpha\beta}$ commutes with the Hamiltonian (the second term in the action (10)) to replace the partial derivative by an exact one. As neither the Hamiltonian nor J depend explicitly on r , we can then follow the standard discussion on adiabatic invariants to convert J into an integral on a phase-space surface of constant energy, $J = \oint \Pi_\Phi d\Phi$.

The adiabatic invariance of Θ and $\tilde{\Theta}$ now follows from imposing the on-shell condition, leaving only a boundary contribution of the form $\Pi_\Phi \Phi$ or $\Phi \Pi_\Phi$. That the entropy operator and its dual are adiabatic invariants is not too surprising, as the temperature and entropy already turn up as a canonical pair in most any version of the thermodynamic first law.

4.3 Thermodynamics at the boundary

Let us recall the AdS analogue of the Brown–York (boundary) stress tensor [15, 16]. Prior to renormalization procedures, this can be determined from

the defining relation $T^{ab} = \frac{2}{\sqrt{-h}} \frac{\partial K_{\alpha\beta}}{\partial h_{ab}} \frac{\delta I}{\delta K_{\alpha\beta}}$. Since the only dependence on $K_{\alpha\beta}$ comes through the Riemann tensor, $T^{ab} = \frac{1}{N\sqrt{-h}} \delta^a_\alpha \delta^b_\beta \frac{\delta(\int \sqrt{-g}\mathcal{L})}{\delta \mathcal{R}_{pquv}} \frac{\partial \mathcal{R}_{pquv}}{\partial K_{\alpha\beta}}$, yielding the final result $T^{ab} = \delta^a_\alpha \delta^b_\beta \mathcal{X}^{pquv} \frac{\partial \mathcal{R}_{pquv}}{\partial K_{\alpha\beta}}$. In a general theory of gravity, calculating T^{ab} would be complicated; however, we have already restricted our attention to Lovelock theories, and for these the task becomes simpler,

$$T^{\alpha\beta} = 4\mathcal{X}^{\alpha\gamma\beta\delta} K_{\gamma\delta}. \quad (12)$$

This outcome can be verified by direct calculation but is best understood by way of the well-known Einstein result, $T_E^{\alpha\beta} = -\frac{1}{8\pi G_{d+1}} (K^{\alpha\beta} - Kh^{\alpha\beta})$, which can also be recast as Eq. (12) by using $\mathcal{X}_E^{abcd} = \frac{1}{32\pi G_{d+1}} [g^{ac}g^{bd} - g^{ad}g^{bc}]$. The relative simplicity of the Einstein stress tensor comes about because terms in the action with explicit (covariant) derivatives do not contribute. A term with (*e.g.*) $X_E^{r\alpha\beta a} \nabla_a K_{\alpha\beta}$ can be eliminated by integration by parts. Hence, all contributions to T^{ab} must come from the $\Gamma\Gamma$ part of the Riemann tensor $\mathcal{R} \sim \nabla\Gamma + \Gamma\Gamma$. But the same must be true of any other Lovelock theory. In essence, these theories satisfy the defining identity (see Eq. (3.6) in [9]) $\nabla_a \mathcal{X}^{abcd} = 0$.

Now, to obtain the desired expression for the entropy, we again invoke the standard relations $Ts = \rho + P = -\sqrt{-h} [T_t^t - T_x^x]$. Denoting this quantity by Θ_{FT} and then plugging in the Brown–York tensor (12), we obtain

$$\Theta_{FT} = \lim_{r \rightarrow \infty} 4\sqrt{-h} \left(\mathcal{X}^{xt}_{xt} K_t^t - [(d-1)\mathcal{X}^{xt}_{xt} - (d-2)\mathcal{X}^{xy}_{xy}] K_x^x \right)_{ren}, \quad (13)$$

again subject to renormalization procedures.

The operators $\tilde{\Theta}$ and Θ_{FT} agree at radial infinity. This is so because the tensors \mathcal{X}^{ab}_{ab} (no summation and $a \neq b$) become insensitive to the polarization as the metric asymptotically approaches pure AdS space at the boundary. Adopting the notation $\mathcal{X}_\infty = \lim_{r \rightarrow \infty} \mathcal{X}^{ab}_{ab}$, we then find that

$$\tilde{\Theta}(r \rightarrow \infty) = \Theta_{FT}(r \rightarrow \infty) = 4\sqrt{-h} [\mathcal{X}_\infty K_t^t - \mathcal{X}_\infty K_x^x]. \quad (14)$$

Eq. (14) should be understood as an equivalence between the two field-theory operators, $\tilde{\Theta}$ and Θ_{FT} . Normally, this would be subject to the matching of conformal factors but, here, the subtraction procedures are built-in. This is because, for pure AdS space, $\rho + p = 0$ or equivalently $K_t^t - K_x^x = 0$; and so these operators are, unlike the Brown–York tensor, already finite at the AdS boundary.

4.4 Thermodynamics in the bulk revisited

Comparison between the bulk and the boundary pictures at finite r requires more information. The simplest case is for polarization- and radially independent couplings. From the viewpoint of the boundary theory, this corresponds to a truly conformal field theory whose couplings do not depend on scale. This translates in the bulk to Einstein gravity with its conformal metric, and the thermodynamics can then be described by a temperature, an energy density and a pressure such that the trace of the associated energy-momentum tensor vanishes on any radial slice. That this can be imposed at any radial position is guaranteed by the Einstein equations, which translates to [21, 14] $\frac{d}{dr} [\sqrt{-h} (K_t^t - K_x^x)] = 0$ or $\dot{\Theta}_E = 0$.

The simplest example of radially dependent but polarization-independent couplings is an $f(\mathcal{R})$ theory of gravity. This is essentially Einstein gravity coupled to a scalar field $\psi(r) = f'(\mathcal{R})$ (a prime denotes a derivative with respect to the argument). It is still possible to describe the thermodynamics at different radial positions, but we then need one additional parameter, a chemical potential for the dual of the scalar field.

One finds for an $f(\mathcal{R})$ theory that [14]

$$\Theta_{f(\mathcal{R})} = f'(\mathcal{R}) \left\{ \frac{\sqrt{-h}}{8\pi G} [K_t^t - K_x^x] \right\}. \quad (15)$$

The term in the curly brackets is, as already mentioned, a radial invariant and what would normally be identified as the Einstein energy density and pressure, $\rho_E + p_E$. We can retain these definitions by adding a new term to the thermodynamic potential

$$\Theta_{f(\mathcal{R})} = \rho_E + p_E + \mu n, \quad (16)$$

where n is a “scalar-charge” density [22],

$$n = \sqrt{-h} [f'(\mathcal{R}) - 1] \quad (17)$$

and the associated chemical potential μ takes the form

$$\mu = \frac{1}{8\pi G} [K_t^t - K_x^x]. \quad (18)$$

Now, since μn is the product of a radial invariant times a quantity $f'(\mathcal{R}) - 1$ which typically is not, it follows that

$$\frac{d\Theta_{f(\mathcal{R})}}{dr} = \sqrt{-h} \mu \frac{d \left[\frac{1}{\sqrt{-h}} n \right]}{dr}, \quad (19)$$

which is generally non-vanishing. Again, the Einstein equations inform the mechanical interpreter how to change the bulk geometry such that the thermodynamic framework remains valid at arbitrary radial positions. The difference here is that the consistency requires that Θ changes as a function of r .

Finally, it is amusing to consider what happens in the limit of weak gravity or, equivalently, for $r \gg r_h$. In this case, we can use the asymptotic Einstein solution and re-express the metric in terms of the Newtonian potential $\phi(r)$, $g_{tt} = -r^2 [1 + 2\phi(r) + \dots]$. A straightforward calculation then reveals that, to leading order in ϕ , $\Theta = 4r^{d+1} \mathcal{X}_\infty \phi$. This is the leading order result for any theory of gravity that asymptotes to Einstein for $r \gg r_h$. The association of sT with the Newtonian potential is reminiscent of the “emergent gravity” proposal [23]. Following [23] and identifying an “Unruh temperature” $T = r^2 \dot{\phi} / 2\pi$, we then obtain $s = 8\pi \mathcal{X}_\infty r^{d-1} = r^{d-1} / 4G_{d+1}$ or an off-horizon form of the Bekenstein–Hawking area law.

5 Summary

Let us briefly summarize: Given a UV-complete theory of gravity, we defined a gravitational energy density and pressure, and then used these to define a gravitational entropy. Next, working in the context of an AdS black brane spacetime, we identified these geometric constructs with the energy density, pressure and entropy of the field theory at the boundary.

From our results, the following picture emerges: The field theory can be viewed as a means for defining the thermodynamic quantity sT and the equation of state $\rho(p)$. The boundary theory also tells us how to connect these via its standard thermodynamic interpretation. Meanwhile, the bulk geometry endows us with the Einstein equations, but these are fiducial. They simply instruct a geometrically inclined interpreter on how to change the bulk geometry so as to maintain the validity of the thermodynamic interpretation at an arbitrary radial position.

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